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On Refined Neutrosophic Finite p-Group

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Abstract

The neutrosophic automorphisms of a neutrosophic groups $G(I)$, denoted by Aut(G (I)) is a neu-trosophic group under the usual mapping composition. It is a permutation of $G(I)$ which is also a neutrosophic homomorphism. Moreover, suppose that $X_1 = X(G(I))$ is the neutrosophic group of inner neutrosophic auto-morphisms of a neutrosophic group G (I) and X_n the neutrosophic group of inner neutrosophic automorphisms of X_{n-1}. In this paper, we show that if any neutrosophic group of the sequence $G(I), X_1, X_2, \dots$ is the identity, then G (I) is nilpotent.

Keywords: Neutrosophic automorphism, Commutator subgroup, Neutrosophic subgroup, Minimal condition, Mapping composition, Nilpotency.

1 | Introduction

The concepts of refined neutrosophic algebraic structures and studies of refined neutrosophic groups in particular were introduced by Agboola [1]. After the successful feat, many other neutrosophic researchers have as well tried to establish more further studies on the refined neutrosophic algebraic structures [2]. Further studies on refined neutrosophic rings and refined neutrosophic subrings, their presentations and fundamentals were also worked upon.

Also, Agboola [3] has examined and as well studied the refined neutrosophic quotient groups, where more properties of re ned neutrosophic groups were presented and it was shown that the classical isomorphism theorems of groups do not hold in the refined neu-trosophic groups. The existence of classical morphisms between refined neutrosophic groups $G(I_1; I_2)$ and neutrosophic groups $G(I)$ were established. The readers can as well consult [4–7] in order to have detailed knowledge concerning the refined neutrosophic logic, neutrosophic groups, refined neutrosophic groups and neutrosophy, in general. Please note the following: throughout this paper, our binary operation is strictly the usual ordinary addition (as the operation of multiplication may not be de ned due to the fact that I_1 does not exist).

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Definition 1 ([3]). Suppose that (X (I_1; I_2); +; .) is any re ned neutro-sophic algebraic structure. Here, + and . are ordinary addition and multiplication respectively. Then I_1 and I_2 are the split components of the indeterminacy factor I that is $I = \alpha_1 I_1 + \alpha_2 I_2$ with α_i in C (the set of complex numbers); i = 1; 2.

Definition 2 ([3]). Suppose that (G; *) is any group. Then, the couple (G (I_1; I_2); *) can be referred to as the refined neutrosophic group. Furthermore, this group can be said to be generated by G, I_1 and I_2 and (G (I_1; I_2); *) is said to be commutative if for all x; y in for all G (I_1; I_2); we have x*y = y*x: otherwise, (G (I_1; I_2); *) can be referred to as a non-commutative refined neutrosophic group.

Here, I has been refined as I_1 and I_2: note that it is possible to refine T and F as well as T_1, T_2 and F as F_1, F_2 (see [8] for some details on this. We hope to make substantial contributions and relevant considerations in this regards as future possible studies.)

Theorem 1 ([3]). 1) every refined neutrosophic group is a semigroup but not a group, and 2) every refined neutrosophic group contains a group.

Corollary 1 ([3]). Every refined neutrosophic group (G (I_1; I_2); +) is a group.

Definition 3 ([3]). Let (G (I_1; I_2);*) be a refined neutrosophic group and let A (I_1; I_2) be a nonempty subset of G (I_1; I_2): A (I_1; I_2) is called a refined neutrosophic sub-group of G (I_1; I_2) if (A (I_1; I_2);*) is a refined neutrosophic group. It is essential that A (I_1; I_2) contains a proper subset which is a group. Otherwise, A (I_1; I_2) will be called a pseudo refined neutrosophic subgroup of G (I_1; I_2).

Definition 4 ([3]). Let H (I_1; I_2) be a refined neutrosophic subgroup of a refined neutrosophic group (G (I_1; I_2); .): define x = (a; bI_1; cI_2) in G (I_1; I_2).

Theorem 2 ([3]). Let (G (I_1; I_2); +) be a refined neutrosophic group and let (G (I); +) be a neutrosophic group such that where $I = xI_1 + yI_2$ with x; y in C. Let ϕ: G (I_1; I_2)⟶G (I)be a mapping defined by ((a; xI_1; yI_2))=(a; (x + y)I) for all (a; xI_1; yI_2) in (G (I_1; I_2) with a; x; y in G: then ϕ is a group homomorphism.

An interesting type of neutrosophic isomorphism of a neutrosophic groups G (I) would occur when the image neutrosophic group G (I) coincides with G (I). The classical group concepts as regards to this has been discussed by [9]. A neutrosophic isomorphism α: G (I)⟶G (I) of G (I) onto itself can be called a neutrosophic automorphism of G (I). In particular, permutes the elements of G(I). The collection of all neutrosophic automorphisms of G (I) forms a neutrosophic group under composition of maps.

If β: G (I)⟶G (I) is another neutrosophic automorphism, we denote the product of α and β by $\alpha\beta$. The group of all neutrosophic automorphisms of G (I) denoted Aut (G(I)) can be called the neutrosophic automorphism group of G(I). The unit element of G(I) is the neutrosophic identity automorphism i. This which leaves every element of G (I) fixed i.e.,

$$ix = x, \left(\left(a;\ bI_1;\ cI_2\right) = x \in G(I)\right).$$

Definition 5. A neutrosophic group G (I) can be said to be nilpotent if it has a normal series of a finite length n. That is,

$$G(I) = G0(I) \geq G1(I) \geq G2(I) \geq \ldots \geq Gn(I) = \{e\},$$

where

$$G_i(I) / G_{i+1}(I) \leq Z(G(I) / G_{i+1}(I)).$$

By this notion, every finite neutrosophic p-group G(I) is nilpotent. The nilpotence property is an hereditary one. Thus

I. Any finite product of nilpotent neutrosophic group is nilpotent.
II. If G (I) is nilpotent of a class c, then, every neutrosophic subgroup as well as the neutrosophic quotient group of G (I) is nilpotent and of class $\leq$ c.

Definition 6. Suppose that (W (I); #) and (V (I); $\oplus$) are two neutrosophic groups. Define a neutrosophic homomorphism from α: W (I) to V (I) to be a mapping: W (I) $\oplus$ V (I) such that α(x#y) = α (x) α (y) where x = (a_1; b_1I_1; c_1I_2), and y = (a_2; b_2I_1; c_2I_2). A neutrosophic homomorphism α which maps a neutrosophic group W (I) on itself is called a neutrosophic endomorphism. A bijective neutrosophic endomorphism is known as a neutrosophic automorphism.

Now, let t = (a; bI_1; cI_2) be a fixed element of a group W (I). The mapping β_t: W (I)$\longrightarrow$W (I) which could be defined by β_t(x) = txt-1 for all (x_1; x_2I_1; x_3I_2) = x in W (I) is known as an inner neutrosophic automorphism of the group W (I).

Every other neutrosophic automorphism of W (I) is called outer neutrosophic automor-phism. (The classical group concepts on this was also discussed in [10] and [11].)

Theorem 3. A neutrosophic abelian group G (I) of order $p_1^{\alpha 1} p_2^{\alpha 2} \ldots p_n^{\alpha n}$, where $p_1\, p_2 \ldots p_n$ are distinct primes, is the direct product of groups G_{p1}(I), G_{p2}(I), G_{p3}(I), … G_{pn}(I) of respective orders $p_1^{\alpha_1}$, $p_2^{\alpha 2}$, … $p_n^{\alpha_n}$.

The subgroup G_p(I) is formed of all the operations of G (I) whose orders are powers of p with the identical operation (see also [12] for the classical group concepts.)

2 | Statement of Proof of the Main Results

We are now about to prove the main results. Already, an inner neutrosophic automorphism of a neutrosophic group has been de ned. Now, given that X_1 = X(G(I)) is the neutrosophic group of inner neutrosophic automorphisms of a group W (I). Also X_n is the neutrosophic group of the inner neutrosophic automorphisms of X_{n-1}, n, an integer.

Definition 7. Suppose there exists the lower central series of a group G (I) given by:

G(I)=$G_{(0)}$(I) $\supseteq$ $G_{(1)}$(I) $\supseteq$ $G_{(2)}$(I) $\supseteq$ …. Here, $G_{(i)}$(I)=[$G_{(i-1)}$(I), G(I)], i>0. i.e., $G_{(1)}$(I)=[$G_{(0)}$(I), G (I)]=[G (I), G (I)]=G''(I), the commutator subgroup of G (I) such that the lower central series terminates at {ε} after a finite number of steps (i.e. $G_{(n)}$(I) = {ε}, for some integer n). Then G (I) is said to be nilpotent.

Define $u^{-1}v^{-1}uv$=[u; v], the commutator of u and v, in a group G (I).

And $u^v = v^{-1}uv$. Here, u = (u_1; u_2I_1; u_3I_2), and $v = \left(v_1;\ v_2 I_1;\ v_3 I_2\right) u^{-1} v^{-1} uv$

$$
\begin{aligned}
&= \left(u_1^{-1};\ u_2^{-1} I_1;\ u_3^{-1} I_2\right)\left(v_1^{-1};\ v_2^{-1} I_1;\ v_3^{-1} I_2\right)\left(u_1;\ u_2 I_1;\ u_3 I_2\right)\left(v_1;\ v_2 I_1;\ v_3 I_2\right) \\
&= \left(u_1^{-1} v_1^{-1};\ u_2^{-1} v_2^{-1} I_1;\ u_3^{-1} v_3^{-1} I_2\right)\left(u_1 v_1;\ u_2 v_2 I_1;\ u_3 v_3 I_2\right) \\
&= \left(u_1^{-1} v_1^{-1} u_1 v_1;\ u_2^{-1} v_2^{-1} u_2 v_2 I_1;\ u_3^{-1} v_3^{-1} u_3 v_3 I_2\right) = \left[u;\ v\right].
\end{aligned}
$$

By the definition of inner neutrosophic automorphism,using induction on G (I),

$$
\begin{aligned}
X(G(I)) = X_1 &= \left\{x_1^{-1} g x_1 j(a_1;\ a_2 I_1;\ a_3 I_2) = g \text{ in } G(I)\right\} \\
&= \{g^{x1} \mid\ g \in G(I)\} = \{\ g g^{-1} x_1^{-1} g x_1 j(a_1;\ a_2 I_1;\ a_3 I_2) = g \in G \text{ and } x_i;\ a_1;\ a_2;\ a_3; \in G\} \\
&= \{\ g[g;\ x_1] j(a_1;\ a_2 I_1;\ a_3 I_2) = g \in G(I)\}.
\end{aligned}
$$

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If n=2. Then X_2 is the neutrosophic group of the inner neutrosophic automorphisms of $X_1 = \{g[g;\ x_1] / (a_1;\ a_2I_1;\ a_3I_2) = gG(I)\}$.

Hence, there exists $x_2 \in G(I)$ such that

$$X_2 = \{x_2 -^1 (g^{x1})x_2 \mid g^{x1} \in X_1\} = \{(g^{x1})^{x2} \mid g^{x1} \in X_1\}.$$

Following similar trends, there exists $x_3 \in G(I)$ such that

$$X_3 = \{x_3 -^1 (g^{x_1x_2})x_3 | g^{x_1x_2} \in X_2\} = \{g^{x_1x_2x_3} \mid g^{x_1x_2} \in x_2\}.$$

Also, there exists $x_4 \in G(I)$, such that

$$X_4 = \{x_4^{-1}(g^{x_1x_2x_3})x_4 \mid g^{x_1x_2x_3}\ X_3\} = fg^{x_1x_2x_3x_4} \mid g^{x_1x_2x_3} \in X_3\}.$$

And for $n = k$, there exists $x_k \in G(I)$, $k \in N$, such that

$$X_k = \{X_k^{-1}(g^{x_1x_2x_{k-1}})X_{k+1} \mid g^{x_1x_2x_k} \in X_{k+1}.$$

If the truth of the last statement is assumed, there exists $x_{k+1} \in G(I)$, $k \in N$ such that

$$X_{k+1} = \{x_{k+1}^{-1}(g^{x1x2\ xk})x_{k+1} \mid g^{x1x2\ xk}\ X_{k+1}\}.$$

We have that

$$G(I) = X_0 \supseteq X_1 \supseteq X_2 \supseteq X_k \supseteq X_{k+1} \supseteq \ldots \supseteq X_n \supseteq \ldots.$$

Definition 8. A neutrosophic group A (I) is said to satisfy the Descending Chain Condition (DCC) for any neutrosophic subgroups if every descending chain, $A_1(I) \supseteq A_2(I) \supseteq$ i.e, of neutrosophic subgroups terminates, i.e., there exists t in N (the set of natural numbers) such that for all $n \geq t$, $A_n(I) = A_t(I)$. Hence, every non-empty subset of the neutrosophic subgroups of A (I) has a minimal element. By the original hypothesis, let X_{n+1} be the identity $\{\varepsilon\}$ of the sequence G (I), X_1, X_2, … .

Then, the minimal condition implies that

$$G(I) = X_0 \supseteq X_1 \supseteq X_2 \supseteq \ldots \supseteq X_n \supseteq X_{n+1} = \{\varepsilon\} \text{ and } X_i = X_{i-1}(G(I)).$$

This actually shows the nilpotence of G (I) (for more and extensive discussion regarding to the classical group concepts, please see [10] and [13].)

3 | Applications

This findings can be fully applicable to every other nite group in general, most especially those nite groups that are nilpotent.

4 | Conclusion

Finally, the nilpotent characteristics of every nite p-group has been observed to be highly hereditary and so, any other neutrosophic product groups formed which have origin from nite p-group would de nitely display neutrosopiy.

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Conflicts of Interest

The authors declare that there is no competing of interests.

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