Situating Frege's Look into Language

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To Ernst Tugendhat,
light in a dark world.

Contents

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§ 1. Introduction

§ 1.1 The Concept-Script (*Begriffsschrift*), a Rationalized, Aphonetic Language

Gottlob Frege, who lived from 1848 to 1925, was professor of mathematics at the University of Jena, Germany, from 1874 to 1918. He taught geometry, analysis or function theory, algebra, number theory, the foundations of arithmetic, and logic, as well as Newtonian physics and analytical mechanics (the mechanics founded by Joseph Louis de Lagrange in which all problems of mechanics are reduced to solving equations). His writings reveal a man who had undivided admiration for Kant and Leibniz and whose thought was shaped and oriented by the achievements of modern mathematics, science, and technology (see, e.g., *BLHP*).[1] As far as scientific reason is concerned, he was a man of the Enlightenment. I tempered the last assertion, for his late political and anti-Semitic views[2] hardly qualify as instances of enlightened value rationality.[3]

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1. For the abbreviations of the titles of Frege's works, as well as of selected works by others, see the Bibliography, which follows this essay.

2. The German manuscript propounding these views was published 15 years ago. See Gottlob Frege, "[Tagebuch]," ed. Gottfried Gabriel and Wolfgang Kienzler, *Deutsche Zeitschrift für Philosophie* 42 (1994), 1067–98; in addition to providing annotations and very helpful historical elucidations to Frege's text, the editors also wrote an introduction: "Gottlob Freges politisches Tagebuch," ibid., 1057–66. An English translation, by Richard L. Mendelsohn, is available: "Diary: Written by Professor Dr Gottlob Frege in the Time from 10 March to 9 April [*sic*], 1924," *Inquiry* 39 (1996), 303–42 (there is an error in the title of this translation: it should say '9 May', not '9 April'). Also relevant is Eckart Menzler-Trott's presentation and discussion of Frege's views and their historical context in "Ich wünsche die Wahrheit und nichts als die Wahrheit . . . ," *Forum* 36 (1989), 68–79.

Aside from Frege's decidedly right-wing political positions, as manifested by his opposition to the liberal democratic state and many, albeit not all (e.g. he does countenance the separation of church and state in the entries of March 18 to 22), of the acquisitions of the French Revolution or other strands of the Enlightenment, his blood-and-soil nationalism, his anti-Semitism, etc., the diary entries reveal a man conflicted between enlightenment reason and counter-enlightenment sentiment ("patriotic sentiment," entry of April 24; "German sentiment," entry of April 27; primacy of the heart (*Gemüt*) over the understanding (*Verstand*) in the relationship to one's country, entry of May 2).

As Gabriel and Kienzler point out in their introduction (1065) in somewhat different language, the conflict becomes particularly acute when, as in the entry of April 22, Frege entertains the notion of framing legislation that would curtail the rights of Jews and is thereby led to consider the concept of being Jewish: on that occasion, he states that the concept is fuzzy, which is to say that it is not a criterion that enables one to decide unerringly who does and who does not fall under it (entries of April 22 and esp. April 30), yet, although he backs down somewhat in the entry of April 30, he does not do the logically—not to mention morally—correct thing of dismissing the concept as unreliable in truth-claiming and thus legal contexts (as he does, e.g., with the concept of heap in *BS*, § 27), thus betraying "the rigorous discipline of thought" (*die strenge Zucht des Denkens*) hailed and

Frege's theoretical life's project was to clarify the arithmetical component of mathematics. As he sought to realize that ambition in natural language—"the language of life," as Frege calls it—and its script (its syntax and its phonetic written signs, the alphabet), he saw himself forced by the difficulties he encountered to devise a new language. He did for logic what mathematicians had done for their science centuries before him: he set aside the phonetic signs of natural language in favor of an ideography (also described as a symbolic language or notation), a language of ideograms or aphonetic signs—signs, that is, made solely for the eye and the hand.[4] Frege first made public this ideography in a little book entitled *Begriffsschrift*, which was published in 1879. The exact English translation of that word is

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upheld in the opening entry of the diary (March 10), his appeal to reason and science on March 30, and his call for "truth, nothing but the truth" on May 9. The truth is that Frege should have discarded the concept in question, as demanded by the following passage from his *Posthumous Writings*: "we have to throw aside concept words that do not have a meaning [or significance] [*Bedeutung*]. These are not such as, say, contain a contradiction—for a concept may indeed be empty—but such as have vague boundaries. It must be determinate for every object whether it falls under a concept or not; a concept word that does not meet this requirement on its meaning [*Bedeutung*] does not have meaning [*bedeutungslos*]" (*PW*, 122/*NS*, 133; see also 195/212, where the requirement of sharp boundaries is again stated and discussed).

Frege, a Protestant who yearned for a renewal of religion (entry of May 8) and called for a more truthful and historically informed account of the life of Jesus (May 9), is so taken with producing an unexceptionable means to identify Jews that it does not occur to him that the project to deny equal rights to Jews would be immoral and irrational, even if the concept of being Jewish had sharp edges.

3. Here I am using Max Weber's distinction between *Wertrationalität* (value rationality) and *Zweckrationalität* (purposive rationality).

4. Semiotically, the language of mathematics is ideographic, at least in intent. In fact, it is hybrid, being a composite of ideographic and phonetic signs: e.g. in the equation 'y = 3x + 11', 'y' and 'x' are phonetic signs borrowed from the Roman alphabet, whereas the other signs in the expression are ideographic. It is also true, however, that the phonomimetic nature itself of the phonetic signs is pretty much neutralized in mathematics. The ideographic signs greatly facilitate the beholding of the sentences of mathematics and thereby ease their (practical and theoretical) manipulation, which stands under the guidance of the eyes. The ideographic signs achieve this by making the mathematical relations more perspicuous than they would be if they were expressed in a phonetic language such as English: consider, e.g., the difference between 'y = 3x + 11' and its phonetic equivalent 'y is equal to three times x plus eleven'. The latter is still a very simple example; one can see, however, that longer and more complex mathematical sentences become unintelligible when expressed phonetically. Perspicuity is achieved by replacing the complex phonetic signs by simple ideographic signs: e.g. by substituting '=' for '. . . is equal to . . . '. In that respect, the language of modern logic very much resembles that of mathematics. It is because the signs of mathematics are primarily ideographic that the language of mathematics does not need to be translated. No doubt, we may read the formulae of mathematics aloud, but when we do so, we are translating without even noticing that we are.

'concept (*Begriff*) script (*Schrift*)'—the additional 's' between the two German words being a genitival ending—and the full title of the book is *Concept-Script, A Formalized Language of Pure Thought Modeled upon the Formalized Language of Arithmetic*.

§ 1.2 The Function-Argument Distinction

Frege's fashioning of the new language was not restricted to devising new signs: it also endowed the ideography with a syntax that, unlike the grammar of natural language, is semantically transparent and thus makes visible the semantic behavior of its signs. This second innovation was guided by two mathematical acquisitions, the modern distinction between function and argument and the more general distinction between constants and placeholders. Placeholders are predominantly called 'variables' in mathematics and the sciences, as the more descriptively correct Fregean word 'placeholder' did not gain currency. Frege ushered both distinctions into logic in *Concept-Script*. The modeling of the new logical language upon these structures yielded a rationalized language in which the project of clarifying arithmetic was to be carried out. Before arithmetic could be properly elucidated, the medium in which this clarification was to take place had to be itself clarified. Very generally, to say that the new ideography was a clarified language is to say that it was stripped of all exceptions to grammatical rules (irregular verbs being a case in point) and of all meaning ambiguities that are the lot of natural languages. More specifically, Frege sought to eliminate the ambiguities and confusions that arise as a result of the multiple uses of the subject-predicate structure in natural language. He did so by replacing it, as he states in the preface to *Concept-Script*, by the function-argument pattern. Before turning my attention, in the remainder of the paper, to a more detailed account of what this work of linguistic clarification amounts to, it will prove instructive to rehearse in somewhat greater detail how Frege described his importation of the function-argument distinction into logic and what in the history of mathematics prepared that move.

Frege was able to introduce the mathematical structure of function and argument into logic, for he recognized that it could be generalized further than it had already been in the function theory of his day by the German mathematician Peter Gustav Lejeune Dirichlet. The latter had noticed that the function-argument pattern is not an exclusively mathematical structure but a broader mereological structure and that mathematical concepts (such as being a rational number) other than the arithmetical operations or the operations of the calculus qualify as functions.[5]

5. One must distinguish between operation and computation. An operation—say, an arithmetical operation such as that of addition—is a function chosen for a possible calculation: the function rules the activity of computing in that it sets the particular modality the computing is to take. A computation or calculation is a fixed and set procedure (an algo-

Roughly, a function is a rule of correspondence between the elements of two sets. More precisely, we say that a function **f** is a rule of correspondence that assigns a unique element **y** from a set **B** (the range of the function) to each element **x** of a set **A** (the domain of the function). In the expression '45 + 55 = 100', the function '+' assigns 100 to the pair {45, 55}. In the expression 'y = 3x + 11', '3x + 11' is the function that assigns a unique element y from the range (say, the set of natural numbers) to each element x from the domain (again, the set of natural numbers). If x = 2, then 3 · 2 + 11 = 17. The rule of correspondence '3x + 11' assigns 17 to 2.

In 'y = 3x + 11', 'x' is the placeholder or independent variable of the function, whereas 'y' is the value or dependent variable of the function. Within the function, 'x' holds the place that is to receive the arguments from the domain of the function. To make this ideographically manifest, we may write '3x + 11' in this way: 3(. . .) + 11. This shows that 'x' is nothing other than a convenient way of writing '(. . .)', of holding a place for an argument.[6] The value is determined by the function; we say that the value is a function of, or depends upon, the argument, or, more loosely, that the dependent variable is a function of the independent variable. The rule of correspondence is thus marked by dependency: that is to say, the elements that stand in a functional relation, or are set into correspondence by a function, are such that some of the elements are dependent on other elements.

The examples I have adduced above show that the expression of a function is an incomplete expression (of a rule of correspondence between the elements of two sets) that needs filling or completing (or saturation, to use Frege's metaphor)

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rithm). There are many operations in arithmetic and higher mathematics. The first step to be taken prior to computing is to settle on an operation or string of operations. The point is that calculation presupposes a choice of an operation (or a string of operations), whether the operation be chosen by the calculator or some other party. To perform an arithmetical operation is to apply it to numbers and to write down the result of the operation: e.g. to perform the operation of addition upon 45 and 55 is to constitute their sum; the sum is the result of the performance of the operation and is written down as '45 + 55'. All results of arithmetical operations are forms.

Now, to perform a computation of 45 + 55 is to subject this form to a transformation that aims at giving it as reduced or simple a form as possible: it is to give another form to the sum of the two numbers, that of a third number. The fact that the third number is another form of the sum of the two numbers is expressed by placing the sign of identity between the sum and the third number, as follows: 45 + 55 = 100. Calculation in arithmetic is thus the process of giving another form to a sum, a product, a difference, or a quotient. The calculation replaces the result of an operation performed on two numbers, by a third number and expresses that replacement by means of a statement of identity.

In these remarks, I am indebted to Stella Baruk's exceptional work, *Dictionnaire de mathématiques élémentaires* (Paris: Seuil, 1995). Baruk has raised the notion of a dictionary of mathematics to an unprecedented degree of pedagogical thoroughness and conceptual clarity. Reading this book is a truly aesthetic pleasure.

6. See *PW*, 121/*NS*, 131–32, where Frege uses this device to mark places in predicates.

by an argument (or by more than one argument, if it is a more than one-place function) to yield a value. Again, the incompleteness is made manifest by the placeholders.

Dirichlet formulated his insight into the concept of function as follows:

> The letters *a* and *b* are to be thought of as two constant values, and *x* as a variable magnitude, which is to assume all the values lying between *a* and *b*. Now, if to each *x* there corresponds a unique, finite *y*, and if while *x* continuously runs through the interval from *a* to *b*, *y* = *f(x)* also gradually changes, then *y* is a continuous function of *x* in this interval. It is not necessary that in this interval *y* be dependent upon *x* in accordance with the same law—in fact, one does not even need to think of a dependence expressible through mathematical operations.[7]

The last sentence of the citation lifts the function structure out of the domain of mathematical operations. The correspondence between *y* and *x* need no longer be expressible by means of mathematical operations. A function is now thought of merely as a dependence of one variable upon another, and *this dependence may be specified by conceptual determinations other than those offered by standard mathematical operations*. Here is a famous example given by Dirichlet of such an emancipated function:

f(x) = 1, if x is a rational number

f(x) = 0, if x is an irrational number.

As one would expect of any professional mathematician of that time, Frege was perfectly familiar with it: he mentions the example in "Function and Concept" (1891).[8] Using the device of the parentheses introduced earlier, we may rewrite these two lines as follows:

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7. Peter Gustav Lejeune Dirichlet, *Über die Darstellung ganz willkürlicher Functionen durch Sinus- und Cosinusreihen* [*On the Representation of Entirely Arbitrary Functions by Sine and Cosine Series*] (1837), ed. Heinrich Liebmann (Leipzig: Engelmann, 1900), 3–4.

8. *FB*, 25: "Man ist weiter gegangen und sogar genötigt worden, zu der Wortsprache seine Zuflucht zu nehmen, da die Zeichensprache der Analysis versagte, wenn z.B. von einer Funktion die Rede war, deren Wert für rational Argumente 1 für irrationale 0 ist." TRANSLATION: "People went further still, and were even obliged to seek refuge in ordinary language, since the ideography of analysis failed; this happened, for example, when there was talk of a function, whose value is 1 for rational and 0 for irrational arguments." Notice, in this passage, by the way, that Frege speaks of a failure of the ideography of function theory (which he calls *Analysis*); his ideography sought, among other things, to correct that shortcoming.

Regarding Frege's knowledge of Dirichlet's contribution to function theory, note also that in "What Is a Function?" (*WF*, 90) Frege refers to Hermann Hankel's definition of function in *Untersuchungen über die unendlich oft oszillierenden und unstetigen Funktionen. Ein Beitrag zur Feststellung des Funktionsbegriffes überhaupt* (1870), which is so indebted to Dirichlet's own characterization that Hankel calls it the "Dirichlet concept."

f(. . .) = 1, if (. . .) is a rational number

f(. . .) = 0, if (. . .) is an irrational number.

The two sentences thus say that if (. . .) is a rational number, then the value of the function will be 1; and if (. . .) is an irrational number, the value of the function will be 0. The *concept* of being a rational and that of being an irrational number *are now thought of as functions*.

One will note that the concept expressions used ('is a rational number' and 'is an irrational number') include what for centuries had been considered to be the copula (in this case, 'is') in the Western logical tradition. The finite forms of 'to be' are not treated as separate from the adjectives or descriptive words ('rational' and 'irrational'). The verbal component of the expressions is not understood as linking or tying (the Latin word *copula* means 'leash') a subject term to a predicate term (or a concept term), as if predication were an operation of putting things together, a synthesizing of sorts. Rather, predication consists in completing an incomplete expression. The completing yields a sentence that is susceptible of being true or false. Predicates are thus understood as basically in need of completion, as parts of sentences, and not as independent entities, in the way that objects are. This clearly foreshadows the sharp distinction that Frege will make between concepts and objects.

Although Dirichlet's definition extends the function concept[9] (itself a higher-order concept) beyond mathematical operations, the concepts and relations that fall under it are still such as characterize mathematicals: in other words, it does not lift the tacit stipulation that the arguments that are to fill the places held by *x* and *y* (or by parentheses) within the function be numbers or magnitudes. Frege's act of generalization will consist in doing precisely that. Proceeding from the most recent acquisitions of the function theory of his day, Frege will take a further step by lifting all restrictions off the function-argument structure: he will extend what may count as a function to *any concept and to any n-adic relation* (where $n \geq 2$ and ranges over the natural numbers), and extend what may count as an argument to *any individual entity or object* (and not merely mathematical entities). Frege introduces this completely generalized notion of function in § 9 of *Concept-Script* as follows:

> Let us entertain the notion that the fact that hydrogen is lighter than carbon dioxide is expressed in our formal language. We can then replace the sign for hydrogen with the sign for oxygen or that for nitrogen. This so changes the meaning that 'oxygen' or 'nitrogen' now stands in the relations in which 'hydrogen' once stood. As we think of an expression as variable in this manner, it splits up into a stable component, representing the totality of relations, and into the sign that is thought of as replaceable

9. See the occurrence of the word *Functionsbegriff* at *BS*, § 10, p. 19.

> by other signs and that denotes the object occurring in these relations. The former component I call a function, the latter its argument. (*BS*, 15)

That Frege found this mereological structure in mathematics and that he himself understands his introduction of it into logic as one of unfolding (*entwickeln*), of disclosing its relevance to non-mathematical concepts, relations, and objects, is stated at the end of § 10 of *Concept-Script*: "One thereby sees quite clearly that the function concept of analysis, to which I generally kept, is far more restricted than the one developed here."[10]

The text from § 9 yields the following proportion:

$$\frac{\text{'... is lighter than carbon dioxide'}}{\text{'hydrogen' or 'oxygen' or 'nitrogen'}} :: \frac{\text{stable component}}{\text{replaceable sign}} :: \frac{\text{totality of relations}}{\text{object}} :: \frac{\text{function}}{\text{argument.}}$$

In this example, the expression for the function is '... is lighter than carbon dioxide'. As expected, it is an incomplete expression, and the function is a one-place (monadic) function. Borrowing Dirichlet's suggestion that any concept under which numbers may fall is a function, Frege thinks of any concept under which any object may fall, and of any relation in which objects may stand, as functions. In the article "Function and Concept," we read: "We thus see how closely what is called a concept in logic is connected with what we call a function. Indeed, we may say at once: a concept is a function."[11] And:

> Assertoric propositions in general, just as equations or analytical expressions, can be thought of as split up into two parts: one complete in itself, and

10. *BS*, § 10, p. 19: "Man sieht hieran besonders klar, dass der Functionsbegriff der Analysis, dem ich mich im Allgemeinen angeschlossen habe, weit beschränkter ist als der hier entwickelte." Frege is more expansive on these two points in "Function and Concept," where we read: "My starting-point is what is called a function in mathematics. The original meaning of this word was not as wide as the one it has since acquired; it will be well to begin by dealing with this first usage, and only then consider the later extensions. [...] The first place where a scientific expression appears with a clear-cut meaning is where it is required for the statement of a law. This case arose as regards functions upon the discovery of higher analysis. Here for the first time it was a matter of setting forth laws holding for functions in general. So we must go back to the time when higher analysis was discovered if we want to know what the word 'function' was originally taken to mean" (*FB*, 18). See also *PW*, 119/*NS*, 129: "Here I am borrowing the term 'function' from analysis and, while retaining what is essential to it, using it in a somewhat extended meaning, a procedure for which the history of analysis itself offers a precedent." It seems reasonable to suppose that the precedent in question is the Dirichlet concept.

11. *FB*, 28: "Wir sehen daraus, wie eng das, was in der Logik Begriff gennant wird, zusammenhängt mit dem, was wir Funktion nennen."

> the other in need of completion, or unsaturated. Thus, for example, we split up the sentence
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> 'Caesar conquered Gaul'
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> into 'Caesar' and 'conquered Gaul'. The second part is unsaturated. It contains an empty place; only when this place is filled with a proper name, or with an expression that stands in for a proper name, does a complete sense appear. Here, too, I give the name 'function' to what is signified by this unsaturated part. In this case, the argument is Caesar.
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> We see that here an extension has been made in this other direction, namely as regards what can occur as an argument. Not merely numbers but objects in general are now admissible. . . . We must go further and admit objects without restrictions as values of functions.[12]

This text allows us to add the following ratio to our proportion:

$$:: \frac{\text{expression in need of completion, or unsaturated expression}}{\text{expression complete in itself.}}$$

Two things are to be noted about concepts, relations, and predicates (i.e. function words). Concepts and relations, as we have just seen, are functions, and functions are "essentially predicative" (*UBG*, 75), which is to say that they are essentially in need of completion. Predicates are also said to be incomplete: they are incomplete expressions. As stated in the last citation, functions are the significance (*Bedeutung*) of predicates. Frege thus uses the same language to describe both predicates

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12. *FB*, 29: "Behauptungssätze im allgemeinen kann man ebenso wie Gleichungen oder analytische Ausdrücke zerlegt denken in zwei Teile, von denen der eine in sich abgeschlossen, der andere ergänzungsbedürftig, ungesättigt ist. So kann man z.B. den Satz

'Caesar eroberte Gallien'

zerlegen in 'Caesar' und 'eroberte Gallien'. Der zweite Teil ist ungesättigt, führt eine leere Stelle mit sich, und erst dadurch, daß diese Stelle von einem Eigennamen ausgefüllt wird oder von einem Ausdrucke, der einen Eigennamen vertritt, kommt ein abgeschlossener Sinn zum Vorschein. Ich nenne auch hier die Bedeutung dieses ungesättigten Teiles Funktion. In diesem Falle ist das Argument Caesar.

Wir sehen, dass hier zugleich eine Erweiterung in der anderen Richtung vorgenommen ist, nämlich hinsichtlich dessen, was als Argument auftreten kann. Es sind nicht mehr bloß Zahlen zuzulassen, sondern Gegenstände überhaupt. . . . Wir müssen weiter gehen und Gegenstände ohne Beschränkung als Funktionswerte zulassen."

I have translated *Bedeutung* by 'what is signified', not by the customary 'reference', for reasons that will be given momentarily, in § 1.3, the subsection entitled "Semantics." Note that in Gottlob Frege, "Funktion und Begriff," *Kleine Schriften*, ed. Ignacio Angelelli (Hildesheim: Olms, 2d ed., 1990), 125–42, here 134, *Gleichungen* in the first sentence is followed by *oder Ungleichungen*, which is not the case in Patzig's edition, which I have used here.

and their significance. Secondly, what pre-Fregean logicians called the copula is now no longer treated separately, but is instead considered to be a piece of the predicate, albeit a formal one. For Frege, the copula no longer has an independent semantic status: this is to say that, as we shall see in detail in § 3 below, the finite forms of 'to be' always have a predicative aspect, which is not always explicit, however.

Having shown how Frege completed the generalization of the function-argument structure, and how deliberately he imported the mereological pattern into the wider context of ordinary language (in contrast, that is, to the specialized language of mathematics), thereby making explicit its linguistic character, let us briefly return to Dirichlet's emancipated function. If we proceed from the formulations of the function just given, we can get even closer to the ideographic formulation of present-day predicate logic, as follows:

GLOSSARY

. . . is a rational number: R(. . .)

. . . is an irrational number: I(. . .)

IDEOGRAPHIC TRANSLATION

R(. . .) = 1

I(. . .) = 0

What we have here is the mere correlation of the number 1 with every number that falls under the concept '(. . .) is a rational number' and the correlation of the number 0 with every number that falls under the concept '(. . .) is an irrational number'. Neither of the two concepts involves a mathematical operation; as such, neither invites the kind of transformation of a mathematical expression that we ordinarily call 'computation'. What yields the value of the function (i.e. 1 or 0) for a given number is not a transformation of a form (all results of the performance of an operation are forms) the result of which would be a new number that could replace the initial form. Rather, what yields the value of the function is whether it is true that such and such a number is rational or irrational: if it is true that the argument is rational, then the value of the function is 1, whereas if it is true that the argument is irrational, then the value of the function is 0.

Is the fact that a certain concept word applies to an object (or, as Frege often says, that an object falls under a certain concept) comparable to the transformation that a computation is? The outcome of a computation—or the value assigned by the realized operation—is another number, whereas the outcome of the application of the predicate '. . . is rational' to a number (i.e. the value it takes as that number is plugged into the predicate) is either truth or falsity. On the basis of that outcome, the value '1' is then correlated (or not) with the initial number. The concept becomes saturated by its argument and yields a truth-value, whereas an arithmetical operation is saturated by two arguments and yields a number (its

value). It is only when the value is equated to the initial form that the question of the truth or falsity of the equality may arise.

In other words, whereas the matter of truth arises after the numerical value of the mathematical function has been determined, it arises before the determination of the numerical value in the emancipated function. As such, while demoting mathematical operations in the constitution of a function, the Dirichlet function promotes truth to the fore, for the dependency characteristic of the correspondence established by a function is no longer set by the specificity of an operation and by the result of computation but rather by the relationship between a concept and an object, a relationship that, in a truth-claiming (or aletheic) discursive context, is immediately susceptible of being true or false. It is this precedence of truth in the wider function-argument structure that Frege receives and unfolds into a new logical theory that will give rise to a logic of truth-functional sentences and a logic of singular and general sentences (sentential and predicate logic, respectively).

§ 1.3 Semantics

As the province of meaning, semantics seeks to understand how the meaning of a sentence depends upon, or is a function of, the meaning of its constituent parts. It aims at understanding the ways in which the meanings of the parts of a sentence contribute to the meaning of the whole they form. This contribution is variously described as 'semantic contribution', 'semantic behavior', or even simply 'semantics'. To formulate sentences, we have two sorts of words at our disposal: form words and content words. A sentence may be composed either of form words and content words, or merely of content words. The simplest and the most basic sentences consist of content words only: these are sentences such as 'Dieter loves Elisabeth', 'Ernst is a great philosopher', or 'She wants an orange', namely singular predicative sentences. The complexity of a sentence increases with the number of its form words. Logic is made possible by the forms exhibited by sentences, which is to say that there is no logical theory without at least some rudiments of semantics. To understand the meaning of a sentence is thus to understand how that meaning is a function of the meaning of its form and content words, or simply of its content words.

For the sake of clarity and comprehensiveness, one must note that the sentences of concern are assertoric, or truth-claiming, sentences: as Ludwig Wittgenstein will make explicit in his *Tractatus logico-philosophicus* (see proposition 4.024), the meanings of their components set the conditions under which the sentences are true, the conditions, in other words, that determine the fit or agreement between sentences and world. This is certainly so for Frege and has been so for logical theory since Aristotle. However, it need not be so, and in the second part of the conclusion, entitled "On the Classification of Speech Acts," we shall see that the unifying concept of illocutionary forces put forth by both Searle and Tugendhat permits an extension of semantics to non-assertoric utterances.

The parts of sentences may be either further sentences or subsentential components. Sentences composed of other sentences are said to be compound, complex, or molecular sentences: for example, the sentence 'John is singing and Mary is talking to Jane' is composed of the two sentences 'John is singing' and 'Mary is talking to Jane', the sentences being connected by the form word 'and'. Not surprisingly, if the parts of a sentence are themselves sentences, those sentences will be connected by means of certain signs (phonetic or ideographic signs, depending on whether the language is phonetic or ideographic). These connecting signs are form signs. In English, the form signs used to connect sentences are expressions such as 'and', 'or', 'if-then', or 'if and only if'. Used truth-functionally, the latter signs are signs for those specific functions known as truth-functions (the signs in question, being natural-language signs, are not always used truth-functionally; e.g. see n. 43 below). Their ideographic counterparts, which are used exclusively truth-functionally, are called truth-functional connectives or operators; they are the standard connectives of sentential or truth-functional logic. Although the form word 'not' does not connect sentences, it is usually treated as a connective in modern logical theory and its associated semantics; for when it bears upon a sentence (namely when it can be replaced by 'it is not the case that'), it modifies the truth-value of that sentence and is thus a truth-function. As we shall see, the English form words 'all' and 'some', too, are function signs: in logic, they are called universal and existential quantifiers, respectively.

In the terminology that gained currency after Frege, sentences made of only subsentential parts are often said to be atomic: both 'John is singing' and 'Mary is talking to Jane' are examples of such sentences. Subsentential components are singular terms and function terms (or general terms). Singular terms are words that designate individuals, such as 'Mary', 'this lion', 'the highest mountain on Earth'. More will be said about them, particularly in § 4. Function terms, or general terms—what we will also call predicates—are expressions such as 'is singing', 'is green', 'is talking to', 'loves', 'lies between . . . and', about which more will also be said. The adjective 'atomic' thus does not say that atomic sentences are devoid of parts, but only that their parts are not themselves sentences.

One more set of remarks is in order regarding the concept of form as it was used in the foregoing remarks. The main form word of a sentence endows it with its characteristic physiognomy, as it were. For instance, when 'and' connects sentences, as it does in 'John is singing and Mary is talking to Jane', and when it is not used to mean 'and then' (thus introducing a temporal relation between the sentences it joins), it makes its sentence of occurrence a truth-functional one, namely one the truth of which is a function of the truth-value (false or true) of the sentences it connects. Used as just specified, the form word 'and' sets the conditions or circumstances under which its sentence of occurrence is true: the sentence is true just in one case, namely when both of the sentences connected by

'and' are true. Every truth-functional form word sets specific truth conditions for the sentence of which it is the main form word. (In any contemporary manual of elementary sentential logic, one will find tabular definitions of the basic truth-functional connectives.)

But when are the singular predicative sentences 'John is singing' and 'Mary is talking to Jane' true? A singular predicative sentence is true when its predicate does indeed apply to the object picked out by its subject term or, in the case of a relational predicate, when the term of relation applies to the objects picked out by its subject, direct-object, and indirect-object terms: when 'is singing' applies to John, 'John is singing' is true, and when 'is talking to' applies to the ordered pair consisting of Mary ('Mary' being the subject term) and Jane ('Jane' being the indirect-object term), the sentence 'Mary is talking to Jane' is true. One may also describe the relation in question as one of subsumption: in the sentence 'John is singing', if the object John falls under the concept of singing, the sentence is true, and in the sentence 'Mary is talking to Jane', if the ordered objects Mary and Jane fall under the relation of talking-to or stand in the relation of talking-to, the sentence is true. What is the relation between the one description and the other? The predicate applies to the object (or objects in the case of a relation term) because the object falls under the concept, and is thus classified by the concept. Furthermore, as we have seen above, concepts or functions are the significance of predicates (see *FB*, 29 and *PMC*, 63).

This conception of the relation between object and concept is fundamental to the semantics and to the account of general or quantified sentences that Frege discovered (the latter being often referred to as Frege's quantification theory). This *falling of object under concept* or, more linguistically said, this application of concept word to object is the very form of singular predicative sentences, and since those sentences are the most basic, it *is the most basic semantic form*,[13] although it is not explicitly treated as such in standard predicate logic.

Please note that I may occasionally speak of functions when I really should be speaking of signs for functions; however, the context usually makes clear what is meant. Let me also state right now that the following expressions will be used synonymously (the reader may in fact already have noticed that from what was said in the first paragraphs of this subsection): 'predicate', 'function word', and 'general term', 'common noun' (but see Frege's reservation about the latter at *PW*, 124/*NS*, 135). At times, I may use the term 'concept word' when, strictly, I should be saying 'function word' or 'general term'. Indeed, we have seen that functions include both concepts (monadic, or one-place, functions) and relations (polyadic, or more-than-one-place, functions). Frege often uses 'concept word' in the wide sense of 'n-place predicate' (where $n \geq 1$ and ranges over the natural numbers). The word *Begriff* in the title *Begriffsschrift* is obviously meant in such a wide sense.

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13. See *PW*, 118/*NS*, 128: "The fundamental logical relation is that of an object's falling under a concept: all relations between concepts can be reduced to this."

As I translate Frege's technical term *Bedeutung* in a way that departs from custom (see n. 2 above), it will be helpful to make some general remarks about the translation of that key word, and to render explicit at the outset certain important characteristics of Frege's semantics.

The rendition of *Bedeutung* as 'significance' was suggested to me by Ernst Tugendhat's 1970 paper on the term *Bedeutung* in Frege (see *BAB*). Tugendhat rightly thinks that there is a semantic theory in Frege and that the notion of *Bedeutung* lies at its center. According to Tugendhat, the *Bedeutung* of an expression (a sentence or a part of such) is its truth-value potential. Here is how he introduces this concept:

> Frege says that the significance [*Bedeutung*] of a part of a sentence is of interest to us only insofar as we are concerned with the truth-value of the sentence. Does this not mean that the significance of the parts of a sentence, in particular that of names, consists in their contribution to the truth-value of their sentence of occurrence? If this is so, then we should have to consider the significance of sentences as primary. Instead of transferring the features of the significance of names to the significance of sentences, we should invert the order and seek to define the significance of names by means of the concept which defines the significance of sentences. To this end, I propose to use the technical phrase 'truth-value potential'.[14]

He then provides a general definition of the concept: "*two expressions φ and ψ have exactly the same truth-value potential if, when each is completed into a sentence by one and the same expression, the two resulting sentences have the same truth-value*."[15] The *Bedeutung* of an expression is its truth-value *potential*: this is to say that the *Bedeutung* of an expression is *the specific contribution it is suited to make to the determination of the truth-value of its sentence of occurrence*. In this paper, I have primarily kept to using 'significance' when rendering *Bedeutung*; in some cases, for reasons of style, 'what is meant', 'what is signified', or 'the meaning' has been used, and when speaking of the *Bedeutung* of singular terms, I have often used 'reference' ('reference' is the proper translation of *Bedeutung* when the *Bedeutung* of singular terms is under discussion, and, strictly speaking, it should be reserved for that purpose).

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14. *BAB*, 234: "Frege sagt, daß uns die Bedeutung eines Satzteils nur insofern interessiert, als es uns auf den Wahrheitswert des Satzes ankommt. Heißt dies aber nicht, daß die Bedeutung der Satzteile, insbesondere der Namen, in ihrem Beitrag zum Wahrheitswert der Sätze, in die sie eingehen, besteht? Dann müßten wir die Bedeutung der Sätze als primär ansehen. Anstelle einer Übertragung der Bedeutungsmerkmale von Namen auf die Bedeutung von Sätzen sollten wir die Reihenfolge umkehren und die Bedeutung von Namen mittels jenes Begriff zu definieren versuchen, durch den die Bedeutung von Sätzen definiert ist. Zu diesem Zweck schlage ich den *terminus technicus* 'Wahrheitswertpotential' vor."

15. *BAB*, 234–35: "*zwei Ausdrücke φ und ψ haben genau dann dasselbe Wahrheitswertpotential, wenn, sofern jeder durch ein und denselben Ausdruck zu einem Satz ergänzt wird, die beiden Sätze denselben Wahrheitswert haben*."

Instead of the significance of an expression, one may speak of its semantic relevance, import, or contribution to its sentence of occurrence, as I have in fact already done above. The translators of *Posthumous Writings* opted for the term 'meaning'; for the most part, I chose not to do so owing to the great proximity of the words 'sense' and 'meaning' in English, 'sense' being the preferred translation of the technical word *Sinn*.

The concept of *Bedeutung* may easily mislead one into believing that it is simple; in other words, that there is only one way in which expressions shape the truth-value of their sentences of occurrence, namely by referring to, or standing for, an object. However, as the reader will see more concretely in § 3 in the discussion of the ideographic expression of functions and their semantic role, such is not the case. *There are in fact as many different ways of contributing to the determination of the truth-value of a sentence as there are different types of logical expressions*. The types of logical expression are thus semantic forms. We shall become acquainted with several sorts of logical expression in § 3 while exemplifying and discussing the semantic substructures that are operative within natural language. For instance,

- A singular term's contribution to the truth-value of its sentence of occurrence consists in referring to an object, to which a predicate is applied (see the more extensive discussion of singular terms in § 4 below). The singular term's reference to an object is its truth-value potential (*BAB*, 236).
- A general term's (concept word or relation word) contribution to the truth-value of its sentence of occurrence consists in characterizing, classifying, and thus differentiating the object(s) picked out by the singular term(s) that saturate(s) the general term; that is to say, a concept word provides a criterion by which objects are distinguished and separated off into groups or classes:[16] it draws, at it were, a boundary for the differentiation of objects, and objects fall either within or without the boundary. The concept word's supplying such a criterion is its truth-value potential (*BAB*, 241).
- A term of quantity contributes to the truth-value of a sentence by saying something about one or more concepts: in the case of the universal quantifier, by saying that the concept or concepts under its scope are instantiated by all the objects being spoken about; in the case of the existential quantifier, by

16. That Frege indeed understands a concept to be a criterion by which objects are separated into those that fall under it and those that do not fall under it is evidenced by the following passage from *PW*, 122/*NS*, 133: "we have to throw aside concept words that do not have a significance [*Bedeutung*]. These are not such as, say, contain a contradiction—for a concept may indeed be empty—but such as have vague boundaries. It must be determinate for every object whether it falls under a concept or not; a concept word that does not meet this requirement on its significance [*Bedeutung*] does not have significance [*bedeutungslos*]."

saying that the concept or concepts under its scope are instantiated by at least one of the objects being spoken about.

- Used truth-functionally, sentence connectors such as 'and', 'or', 'if . . . then', or the adverb 'not' contribute to the determination of the truth-value of their sentence of occurrence by setting the patterns of truth-values of the connected sentences that make the resulting sentence true or false.

The foregoing are different ways of meaning; one may also speak of different modes of use of words or signs (*Gebrauchsweisen*; *UBG*, 68). This multiplicity of semantic behaviors refutes the thesis of objectual semantics that every case of meaning is a case of referring (to an object), or, differently stated, that the "name to bearer-of-the-name" relation is the paradigmatic model of the meaning relation.[17]

17. The discovery that there are multiple ways of meaning marks a major divergence between Frege and Husserl, and, more generally, between analysis—or, more precisely, the analytical philosophy of language—and phenomenology. Indeed, Husserl is firmly committed to an objectual semantics, as one may see, e.g., from the following passages from his 1906–1907 introductory lectures to logic and epistemology, *Einleitung in die Logik und Erkenntnistheorie Vorlesungen 1906/07*, ed. Ullrich Melle, Husserliana XXIV (Dordrecht: Nijhoff, 1985):

- 38: "Jedes Wort darin und jede zusammenhängende Wortkomplexion hat ihre Bedeutung, und diese Bedeutung bezieht sich auf ein Gegenständliches. Jedes Satzsubjekt nennt einen Gegenstand und nennt ihn vermittels der Bedeutung."
- 51–52: "Was wir aber nicht in nähere Erwägung gezogen haben, das war die Korrelation zwischen Bedeutung und Gegenstand, den Umstand, daß es unabtrennbar zu der Bedeutung gehört, sich auf eine Gegenständlichkeit zu beziehen."
- 52–53: "Bedeutung und Gegenstand stehen *apriori*, d.i. evident notwendig ihrem Sinn nach, in Korrelation. Der Gegenstand ist für das Denken nur gegeben eben als gedachter Gegenstand, und dann bezieht sich das Denken auf ihn durch seinen Bedeutungsinhalt, durch Begriff und Satz. Und umgekehrt, zum Wesen der Bedeutung gehört es, dass sie einen Gegenstand sei es als nominale Vorstellung vorstellt oder als Satz setzt."

We find the same commitment in *Logical Investigations*. See, in particular, *LU* II/1, §§ 12–14 of the First Investigation, from which I have culled the following citations:

- § 12, 52: "Jeder Ausdruck besagt nicht nur etwas, sondern er sagt auch über Etwas; er hat nicht nur seine Bedeutung, sondern er bezieht sich auch auf irgendwelche Gegenstände."
- § 14, 56: "Die beziehenden Reden von *Kundgabe*, *Bedeutung* und *Gegenstand* gehören wesentlich zu jedem Ausdruck. Mit einem jeden ist etwas kundgegeben, in jedem etwas bedeutet und etwas genannt oder sonstwie bezeichnet."

As is to be expected from the generalizations just quoted, Husserl understands the semantic office of predicates no less objectually, as is evidenced by this passage from *LU* II/1, § 12, 53: "Wenn wir einmal sagen *Bucephalus ist ein Pferd*, und das andere Mal *dieser Karrengaul ist ein Pferd*, so ist im Übergang von der einen zur anderen Aussage mit der sinngebenden Vorstellung offenbar eine Änderung vorgegangen. Ihr 'Inhalt', die Bedeutung des Aus-

If these characterizations are not yet entirely clear to the reader, they will become so in § 2 below, where examples will be discussed.

The concept of truth-value potential is in essentials the same as the concept of semantic value with which Michael Dummett elucidates Frege's notion of *Bedeutung*.[18]

Sections 2–4 (with the exception of § 2.1) will discuss and illustrate Frege's language, the difficulties it is meant to avoid, and the transparency it affords. The concluding remarks will not only address its limitations but also, as I have said above, indicate how the assertoric semantics can be extended .

§ 2. The Concept-Script and the "Language of Life": Breaking Out of the Mesocosmos

When, in the preface to *Concept-Script*, Frege turns to the subject of the script's relation to natural language, he specifies that relation by means of the following comparison:

> I believe that the relation of my concept-script to the language of life [*Sprache des Lebens*] can be most clearly brought out if I compare it to the microscope's relation to the eye. Because of the range of its uses and the versatility with which it can adapt to the most diverse circumstances, the eye is far superior to the microscope. It is true that when considered as an optical instrument, it shows many imperfections, which ordinarily go unnoticed only as a result of its intimate connection with our mental life. However, as soon as scientific purposes require greater sharpness of discrimination, the eye proves to be insufficient. The microscope, on the other hand, is perfectly suited to precisely such purposes, but that is just why it is useless for all others. (*BS*, xi)

The comparison comes in the form of a proportion stating that the ideography is

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druckes *ein Pferd* ist zwar ungeändert geblieben, aber die gegenständliche Beziehung hat sich geändert. Mittels derselben Bedeutung stellt der Ausdruck *ein Pferd* das eine Mal den Bucephalus, das andere Mal den Karrengaul vor."

For a more detailed critical discussion of Husserl's objectual semantics, see Tugendhat, *VEP*, chap. 9–11.

18. In this respect, see also Michael Dummett's well-taken critical review of Gordon P. Baker and P. M. S. Hacker's *Frege: Logical Excavations* (New York: Oxford University, 1984), "An Unsuccessful Dig," in Crispin Wright, ed., *Frege: Tradition and Influence* (Oxford: Blackwell, 1984), 194–226; for the concept of semantic value, see 203–6. Dummett discusses Tugendhat's paper in *FPL*, 199–203 and 405–7. Tugendhat's reply to Dummett now appears as a postcript to *BAB*, at *PA*, 246–50.

Incidentally, it seems to me that Ernst Tugendhat's 1970 paper anticipatorily refuted by 14 years a central thesis of Baker and Hacker's hyperbolically contentious book—the thesis, namely, that there is no semantics in Frege and that, as a result, Frege had no influence on the development of semantics in the twentieth century.

to the language of everyday life as the microscope stands to the naked eye.[19] The microscope expands the reach of the gaze; magnifying its object, it enables the eye to see the fine, subsensory patterns of matter, microbes, and minute organic processes that would otherwise remain inaccessible—and for the most part unsuspected or left to mere speculation and verbosity—to the natural, unaided sightings that occur within the average world of daily life, that is, within the mesocosmos. Whereas the telescope had allowed the astronomers of the seventeenth century to observe for the first time the forms of macroscopic matter that lie at vertiginous distances from our average world, the microscope broke through the mesoscopic world of ordinary experience at its opposite end, so to speak. Similarly, the script of pure thought was to reveal certain substructures that remain largely inaccessible to, or at the very least unsuspected in their complexity by, speakers in their ordinary intercourse and activities and that heretofore had largely been only imperfectly, if at all, noticed by philosophers and logicians.[20] To gain a better appreciation of the deep transformations eventually brought about by these forays, the initial steps of this migration out of the mesocosmos and its far-reaching consequences deserve to be briefly rehearsed.

Before turning to those transformations of the mesocosmic *forma mentis*, I would like to point out that, on Frege's understanding, the concept-script reveals the universal structure that lies within the language of life. The first step to this thought is to be found in the following passage from *Posthumous Writings*: "If our language were more logically perfect, we would perhaps have no further need of logic, or we might pick it off [*ablesen*] the language [of life]" (*PW*, 252/*NS*, 272). The logical imperfection of the language of life stands in the way of our picking logic off our language: it has the effect of a screen, blocking our access to the substructures that form our language. If the patterns that constitute logic form our language, what conceals them from us? It is the fact that they are not the only constituents of our language, namely that natural language is a hybrid: "grammar, which has a significance for language analogous to that which logic has for judgment, is a mixture of the logical and the psychological" (142/154; see also 6/6). Natural language is a composite of natural (psychological) and non-natural (logical) factors (see 269/288: "Certainly the logical disposition of man was at work in the formation of language but equally alongside this many other dispositions—such as the poetic disposition. As such, language is not constructed in accordance with a

19. In "Towards a Universal Characteristic," Leibniz had also compared the characteristic (another term for ideography) to a microscope; see Gottfried Wilhelm Leibniz, *Selections*, ed. Philip P. Wiener (New York: Scribner's, 1951), 17–25, here 23.

20. See, e.g., *PW*, 195/*NS*, 212: "We are very dependent on external aids in our thinking, and there is no doubt that the language of everyday life—so far, at least, as a certain area of discourse is concerned—had first to be replaced by a more sophisticated instrument, before certain distinctions could be noticed."

logical yardstick."). The psychological features occlude the logical forms operative within language: "Such a yardstick [i.e. logical form] is operative even within language, obstructed though it may be by the many illogical features that are also at work in language" (266/285). Frege says that "if this were not so, all languages would necessarily have the same grammar" (142/154; see also 6/6). In other words, remove the "psychological trappings" (142/154) from natural language and the grammar common to all languages (the yardstick) will become manifest. The key thought here is 'if logical form were not obstructed, then all languages would have the same grammar'. Clearly, the consequent of this counterfactual conditional sentence is *universally* quantified. A universal structure dwells within language. The script will make it visible, for, like a microscope, it will enable us to see past the obstructions.

§ 2.1 Peering Out of the Mesocosmos into the Macrocosm: The Rise of Appearances

At the dawn of modernity, in the sixteenth century, Copernicus's heliocentric model of the universe initiated the systematic undoing of Aristotle's teachings that is inseparable from the constitution of modern science and philosophy, and it did so in the form of a rejection of his cosmology, dislodging the Earth from the center of the cosmos and placing the Sun in its stead. Aristotle's cosmology and Ptolemy's attendant astronomy were formulated within the limits of the mesocosmos (the world of natural, unaided perception and its concerns). Copernicus replaced them by a conception of the world that stands at a great remove from the immediate perceptual givens of which it had to give an account and which continue to be the lot of average everyday life, even in our techno-scientific culture.[21] Ptolemy's astrono-

21. In this regard, one may observe that modern science encounters all manner of resistance within the very societies that have created it and still remain its primary producers, the refusal of the theory of evolution by certain religious constituencies being only one of the noisier examples of this unease. A striking instance is to be found in the widespread difficulties people have in accepting the principle of inertia, a phenomenon documented by Michael McCloskey in his fascinating papers on naive or intuitive physics, which show that not only in its *untutored* state, but even after being exposed to a first course in physics, the common sense of the greater part of the inhabitants of industrialized countries tends to be closer to the medieval account of motion (impetus theory) than to the modern, Newtonian one. See Michael McCloskey, "Intuitive Physics," *Scientific American* 248 (1983), 122–30, and "Naive Theories of Motion," in Dedre Gentner and Albert L. Stevens, eds., *Mental Models* (Hillsdale, N.J.: Erlbaum, 1983), 299–324; and McCloskey and M. K. Kaiser, "The Impetus Impulse: A Medieval Theory of Motion Lives On in the Minds of Children," *The Sciences* 24 (1986), 40–45, as well as McCloskey, Alfonso Caramazza, and Bert Green, "Curvilinear Motion in the Absence of External Forces: Naive Beliefs About the Motion of Objects," *Science* 210 (1980), 1139–41. See also Pierre Costabel, "Galilée, hier et aujourd'hui," in Paul Poupard, ed., *Galileo Galilei 350 ans d'histoire 1633–1983* (Tournai: Desclée Inter-

my offered a system of geometrical transcriptions—in the form of combinations of circular motions—and of localization of the objects of (unaided) visual observation, of the phenomena, that is, and as such was a geometry of the visible world. Whereas Ptolemy's aim was "to save the phenomena," to account for the things manifest to the senses, by means of hypotheses concerning the visible features of the heavenly bodies (their forms, motions, and relations) and framed in such a way as to be faithful to these features, by maintaining their integrity and not reducing them (by turning them into epiphenomena, for instance),[22] Copernicus's astronomy will instead attempt to give a mathematical account of the physical causes of the motions of celestial bodies. These causes will turn out to differ considerably from what the world of unaided perception has to offer. As Gérard Simon so aptly writes, "Copernicus is thus led to *interpret* the observational givens that the ancients *transcribed* as so many *realities*: he interprets them as so many indices of an order that robs them of their reality."[23] Gradually, some of the natural world's most

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national, 1983), 197–208. At the end of the article, Costabel reports the findings of a survey conducted in France at the time. To the question whether the Sun revolves around the Earth, 37 percent of the people polled answered 'yes'.

22. On the alliance between the imperative to be faithful to the givens and the recourse to hypotheses, see Pierre Duhem, *To Save the Phenomena: An Essay on the Idea of Physical Theory from Plato to Galileo*, trans. Edmund Doland and Chaninah Maschler (Chicago: University of Chicago, 1969). Regarding this motif, Duhem cites (p. 5) the following from Simplicius: "Plato lays down the principle that the heavenly bodies' motion is circular, uniform, and constantly regular [i.e. always in the same direction]. Thereupon he sets the mathematicians the following problem: what circular motions, uniform and perfectly regular, are to be admitted as hypotheses so that it might be possible to save the appearances [or 'to preserve the phenomena', either of which renders *diasozein ta phainomena*] presented by the planets?" The principle attributed to Plato in this citation is the central methodological device of ancient astronomy, namely that of geometrical transcription: astronomy is to save the phenomena available to the careful sky-gazer by means of combinations of uniform, circular motions. The linguistic description and geometrical modeling of the combinations constitute the hypotheses. In other words, the aim of ancient astronomy was to devise combinations of uniform, circular motions such that the geometrical paths resulting from these combinations would model exactly the observed trajectories of the heavenly bodies. When this goal was achieved, the hypotheses were deemed to have saved the phenomena. The first model of the phenomenal behavior of celestial objects, i.e., the first answer to Plato's question, was supplied by his pupil Eudoxus (ca. 408–ca. 355 BC). Eudoxus's solution consisted in placing each planet on the inner sphere of a set of two or more interconnected, concentric spheres whose simultaneous rotation about different axes resulted in the observed motion of the body. Eudoxus's model was short-lived since it failed to account for the planets' variation in brightness during retrogression. It was replaced by the theory of epicycles and deferents. Ptolemy inherited and refined that theory. Thomas Kuhn has given an account of both theories in chapter 2 of *The Copernican Revolution* (Cambridge, Mass.: Harvard University, 1957).

23. Gérard Simon, *Kepler astronome astrologue* (Paris: Gallimard, 1979), 258. In the above remarks, I am drawing on Simon's beautiful book and on Izydora Dambska, "L'épisté-

conspicuous and reliable phenomena, such as the sun's daily route through the sky, were coming under the *skepsis*, the prying and suspicious gaze of modern inquiry, and were thereby being turned into mere appearances (eventually, to be called 'representations').[24] Although in its early stages modern inquiry did not yet draw a sharp line between science and philosophy, its polemics against a certain type of philosophizing would become quite widespread in the seventeenth century, and the strife—which had already begun in the previous century after the publication in 1543 of Copernicus's *On the Revolutions of Celestial Bodies*—between modern inquiry and mesocosmic beliefs, primarily embodied in the Christian ethos and the then powerful Catholic Church (which, through thinkers such as Thomas Aquinas, had assimilated many of Aristotle's teachings), would become vehement, threatening, and, in some cases, led to repression (in the form of censorship, house arrest, and incarceration or worse).

In 1610, having appropriated Copernicus's novel theory of the heavens and his greatly simplified and more accurate account of the motions of the planets and of the (now merely apparent) trajectory of the Sun through the Earth's skies, the physicist, mathematician, and astronomer Galileo Galilei published his freshly collected astronomical observations in a 56-page booklet entitled *Sidereus Nuncius*. In translation, the nearly full title of the report reads as follows:

> *Message of the Stars, Disclosing Large* [magna] *and by far Admirable Views* [spectacula]*, to which it urges all to raise their eyes—especially, however, the philosophers and the astronomers. Performed by means of a spyglass* [perspicillum] *recently discovered by the author, the observations concern the face of the moon, innumerable fixed stars, the Milky Way, the nebulae, and above*

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mologie de Ptolémée," in Centre International de Synthèse, *Avant, Avec, Après Copernic* (Paris: Librairie Blanchard, 1975), 31–37. This entire section is generally indebted to Simon's work.

24. Note the following, which Frege cites as an example in *USB*, 52: "'Kopernikus glaubte, daß der Schein der Sonnenbewegung durch die wirkliche Bewegung der Erde hervorgebracht werde.'" TRANSLATION: "'Copernicus believed that the apparent motion of the sun is produced by the real motion of the Earth.'" The text of "Über Sinn und Bedeutung" contains the topic of the rent between reality and appearance—on the one hand, as it manifests itself in modern science in the theory of its first great initiator, Copernicus, and, on the other, as it appears in Frege's reflections on the semantic workings of natural language. See my discussion of this text in § 3.1.

For the appearance-reality distinction in Copernicus, see the following passage from Copernicus cited in Kuhn, *Copernican Revolution*, 152: "Why then hesitate to grant Earth that power of motion natural to its [spherical] shape, rather than suppose a gliding round of the whole universe, whose limits are unknown and unknowable? And why not grant that the diurnal rotation is only apparent in the Heavens but real in the Earth? It is but as the saying of Aeneas in Virgil—'We sail forth from the harbor, and lands and cities retire.' As the ship floats along in the calm, all external things seem to have the motion that is really that of the ship, while those within the ship feel that they and all its contents are at rest."

> *all, the four planets that go around the star Jupiter, at irregular intervals and periods, and at wonderful velocity; these planets unknown to all until now were newly discovered by the author for the first time. . . .*[25]

What news have the stars (in the then current sense of 'celestial bodies') sent to us in our average abode? They have beamed big (*magna*)[26] views or aspects (*spectacula*) to us, and admirable ones by far (*longe admirabilia*). They have revealed the uneven looks of the moon, a sheer profusion of stars (in our sense of 'star') unseen by the naked eye, the true nature of the Milky Way (it is nothing but an aggregate of clusters of countless stars) and of the nebulae (also, nothing but "herds" of stars), and the hitherto unknown satellites of Jupiter. The conveyance of their message had to be enabled, however, by the *perspicillum*, or spyglass (a year later, the word 'telescope' was to be coined), an instrument (*organum*)[27] the author himself constructed in November 1609. The instrument had a magnification of twenty: thanks to it, the astronomer's eyes could see far (*tele-skopein*), much farther than in their unaided state. His gaze channeled and enhanced by the *novum organum*, he "leav[es] the terrestrial things behind [to] turn to the observation of the Heavens. . . ."[28]

Galileo performed his observations during the first weeks of 1610, and it is probably fair to say that they yielded the most important single sequence of discoveries ever made by a human being in such a short period of time. They brought to completion the process of undoing Aristotelian cosmology set into motion by Copernicus in 1543 and lent considerable support to the new, Copernican cosmology, which Galileo explicitly embraced already in the prefatory dedication of *Sidereus Nuncius* to the Grand Duke of Tuscany, Como the Second of Medici. However, the full defense and justification of heliocentricity, which is separate

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25. All translations from the Latin text of *Sidereus Nuncius* are mine. In English, this work's title is often rendered as *The Starry Messenger*. At issue is the translation of the Latin word *nuncius* (spelled *nuntius* in classical Latin). In Latin, the term may mean 'message', 'news', or 'messenger'. Galileo himself states that he understood the word in the first sense. Furthermore, the title on the first page of the text begins with the words *Astronomicus Nuncius*, not with *Sidereus Nuncius*, and in that phrase the term means 'news' or 'message'. I have opted for that meaning, too, in my translation of the title: it very fittingly expresses the arrival of news from the macrocosm to the mesocosmos. See Edward Rosen's discussion in his "The Title of Galileo's *Sidereus Nuncius*," *Isis* 41 (1950), 287–89.

26. The adjective *magnus* is the first word to occur after the phrase *sidereus nuncius* on the title page. It is also the first word of the two opening paragraphs of the text.

27. I count five occurrences of the word *organum* (and always with its first letter capitalized) in the first three pages of the text: see Galileo Galilei, *Le Opere* (Florence: G. Barbera, 1899–1909; reprint 1968), vol. 3, part I, 59–61.

28. Ibid., 61: "Sed, missis terrenis, ad Caelestium speculationes me contuli . . ." See also the phrase *in caelum migrans* (migrating to the sky) in the dedication to the Grand Duke of Tuscany.

from the critique of geocentricity, would require the account of motion that Galileo set forth in his *Discourses and Mathematical Demonstrations Regarding Two New Sciences* of 1638 and that would culminate in Newton's inertial physics.

The empirical dismantling of the old cosmology began in fact in the sixteenth century, with two observations made by the astronomer Tycho Brahe (1546–1601). He is the first scientist to have empirically confuted central beliefs of Aristotelian cosmology. In 1572, Brahe witnessed the appearance of a new body in the sky. Its brightness made it visible in broad daylight. The object was what is now known as a supernova, namely a collapsing star. Brahe recorded it as a star. In 1577, he established that the trajectory of a comet he had been observing led it through the spherical shell that, according to Aristotle, was carrying Venus. These two observations refuted three Aristotelian cosmological teachings: 1) the thesis of the incorruptibility of the superlunary region; 2) that of the sublunary localization of comets; and 3) that of the division of the heavens into separate regions by crystalline spherical shells that allegedly bore the planets. Brahe also proposed that the shape of the comet's orbit was not circular but oval, thereby questioning for the first time ever, it would seem, the Greek and medieval belief that celestial bodies move in circular orbits. The latter conviction was common to both the geocentrist Ptolemy and the heliocentrist Aristarchus of Samos (ca. 310–ca. 230 BC) and lasted into the seventeenth century, until Kepler showed that the orbits of the planets were elliptical in shape. Copernicus himself rejected neither the belief in crystalline spheres nor the dogma of circularity. Moreover, Tycho Brahe is the first astronomer to have challenged Ptolemy's observational records, whereas Copernicus did not question their accuracy and, for the most part, relied on ancient empirical data. Brahe freed astronomers from their dependence on former observational records by introducing an unprecedented demand for precision and verification protocols into the observational situation. However, Brahe still retained the mesocosmic craftsman's loyalty to the integrity of natural perception. It is true that he enhanced the accuracy of the reports of natural perception by refining and giving greater stability to the conditions under which it supplied its data, but he accepted that giving and its limitations and did not seek to go beyond those limits by modifying the structure (its elements and their relations to each other) of perception itself. The latter step will be taken by Galileo, who can be said to have inaugurated the instrumentally mediated phase of scientific inquiry, that is, the instrumental regimentation of perception itself, of its giving, and of scientific protocol in general.[29]

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29. In these remarks on the rise of instrumentally dependent scientific inquiry, I owe much to Fernand Hallyn's very informative and insightful introduction to his translation of *Sidereus Nuncius*: see Galileo Galilei, *Le Messager des étoiles* (Paris: Seuil, 1992), 89–91; hereinafter cited as 'Hallyn' with page reference. See also Alexandre Koyré, *From the Closed World to the Infinite Universe* (Baltimore, Md.: Johns Hopkins University, 1970), 90: "the

There is now nearly no department of modern inquiry that does not depend on some instrumentation for its further elaboration and explorations, and, as Koyré has rightly seen, theory articulation and instrumentation are now mutually founded upon one another: scientific research zigzags between the two. This ever-increasing reliance on instrumentation that, *inter alia*, has characterized the growth and unfolding of modern science would seem to afford sufficient ground to justify the claim that in its modern form science is necessarily, intimately, and not contingently, connected to technology, which increasingly will require a certain form of economic organization of scientific societies (namely societies productive of and vitally dependent upon science and its technological implementation).[30]

In 1604, on the occasion of the appearance of another supernova in the sky, Galileo himself was able to question the mesocosmic belief in the incorruptibility of the superlunary realm on empirical grounds. The *Message of the Stars* is clearly at odds with Aristotelian teachings in the following two respects:

1) Since the careful observations of the Moon and Galileo's analogical reflections on them reveal it to have an irregular topography, made of crevasses, mountains, and valleys, similar to that of the Earth,[31] and not at all a smooth and polished surface as Aristotle and the medievals thought, they confute the

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Message of the Stars . . . played a decisive part in the whole subsequent development of astronomical science, which from now on became so closely linked together with that of its instruments that every progress of the one implied and involved a progress of the other. One could even say that not only astronomy, but science as such, began, with Galileo's invention, a new phase of its development, the phase that we might call the instrumental one." And see Maurice Clavelin, *La philosophie naturelle de Galilée* (Paris: Albin Michel, 1996), 403–4.

30. One need only evoke here an instrument such as a subatomic particle accelerator, the sheer size of such an instrument, the large and complex industrial infrastructure required to construct it and to maintain it in operation (consider the electrical power needed to operate the machine and the computers necessary to record the data and organize it), and the crew of workers and scientists (physicists, engineers, and mathematicians) necessary to run and maintain the machine and collect, process, and interpret the data.

31. The thought that the Moon has an uneven surface had been entertained since antiquity, e.g., by Plutarch in his *De facie in orbe lunae*. Other anticipations of modern conceptions on the part of ancient thinkers include heliocentricity, which was put forth by Aristarchus of Samos (for other ancient heliocentrists, of whom Copernicus was aware, see Kuhn, *Copernican Revolution*, 142) and the conception of the Milky Way as a heap of countless stars, which we owe to Democritus. However, these thoughts, albeit brilliant and ingenious, were premature, for they failed to have the conceptual analyses and empirical observations on their side that would have been required to counter the prevailing mesocosmic beliefs and to win over at least some of the inhabitants of the mesocosmos to a different picture of heavenly phenomena. I say 'some' because, as I pointed out in n. 20 above, the counter-intuitive, non-natural character of modern science and philosophy, make them alien to the average person and require tutoring and labor to be acquired. Frege, a man who

thesis of the incorruptibility of the Moon and thereby of the immutability of the superlunary region; as such, they strengthen Tycho Brahe's and Galileo's previous challenges to the belief in such immutability.

2) The news that Jupiter has satellites showed that at least one other body in the universe acts as a center of gravity and therefore refutes the geocentric thesis that the cosmos contains only one center of gravity, the Earth. Moreover, if there are two centers of gravity in the cosmos, might there not be more? The discovery of the satellites of Jupiter has far-reaching consequences.

Furthermore, one may argue, as Fernand Hallyn has done,[32] that Galileo's interpretation of his discovery of an abundance of stars that no earthling had ever seen reveals a heliocentric a priori, but we need not pursue that here.

§ 3. Microstructures of Thought, or How Natural Language Became an Appearance

Having recalled these at times exhilarating, at times painful modifications of the mesocosmic *forma mentis*, we return to the topic of the instrument, to the spyglass, as I have called Galileo's telescope. That, in turn, will allow me to go back to the elucidation of Frege's characterization of the ideography's relation to natural language begun in § 2 above.

'Spyglass' renders the neologism *perspicillum* and best captures the literal sense of the Latin term, which is 'little through-seer', and the literal sense is obviously the appropriate one here. With the through-seeing glass, the eye sees more of the cosmos and sees it better; more, because it sees through the mesocosmos into the macrocosm; and better, because the glass strips the object of what does not belong to it insofar as it cuts down the glare that forms around stars when they are seen by the naked eye. The quantitative enhancement was already canvassed above. As for the qualitative improvement, we find it in this passage:

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always tried to replace intuition by labor (namely, by arguments, analyses, definitions, discussions, examples or models) knew that, of course; see, e.g., *UBG*, 67: "Das Logischeinfache ist nun ebensowenig wie die meisten chemischen Elemente von vornherein gegeben, sondern wird erst durch wissenschaftliche Arbeit gewonnen." TRANSLATION: "That which is logically simple is no more given from the start than are most chemical elements, but is gained only through scientific labor." Frege's entire theoretical life exemplifies this and bears it out over and over again; the one exception to this generalization is to be found in the reflections he set forth in his 1924 diary entries (see n. 2 above), where vague intuitions, ineffable feeling (see the praise of the virtues of *Gemüt* in the entry of May 2), love of fatherland, prejudice, and limping analogies have taken the place of the "rigorous discipline of thought," and ideology stoked by rage, frustration, and humiliation has supplanted "the truth, nothing but the truth."

32. Hallyn, 64–65.

> worthy of remark is the fact that the stars, fixed as well as errant, when they are seen by means of the spyglass, do not at all increase in size in the same proportion as other objects, including the Moon, increase in size. Indeed, in the case of the stars, this increase appears much slighter. . . . The reason for this is that when the stars are seen by natural sight, they do not present themselves to us in their real and, so to speak, bare size, but are surrounded by a halo and fringed with sparkling rays, particularly when the night is advanced: as such, they appear much larger than if they were stripped of these adventitious fringes. . . . The spyglass . . . removes the accidental and adventitious splendors and then enlarges their true globes . . . .[33]

Both enhancements make for the critical power of the spyglass, for "thanks to the spyglass, we can see so well that all the altercations that have tortured philosophers for so many centuries are laid to rest by the certitude of the eyes; and that we are freed of verbose disputations."[34] The through-seeing glass enables us to sort out those beliefs worth retaining from those that are to be discarded, and thereby frees us of disputes that used to turn prolix and otiose, for there was no way of adjudicating the claims made in them.

Like the telescope, the microscope lets us see more and better. In his comparison of the concept-script to a microscope,[35] Frege emphasizes the qualitative enhancement, since he speaks of "sharpness of discrimination." Extending Galileo's terminology, one might call the script a *perspicillum microideologicum*,[36] or a micro-

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33. Galilei, *Le Opere*, vol. 3, part 1, 75–76.

34. Ibid., 78.

35. In "Über Sinn und Bedeutung," Frege uses the telescope as an analogy in order to usher in his tripartite distinction of significance (*Bedeutung*), sense (*Sinn*), and representation (*Vorstellung*), and to elucidate the relationship among those three factors: "Die Bedeutung eines Eigennamens ist der Gegenstand selbst, den wir damit bezeichnen; die Vorstellung, welche wir dabei haben, ist ganz subjektiv; dazwischen liegt der Sinn, der zwar nicht mehr subjektiv wie die Vorstellung, aber doch auch nicht der Gegenstand selbst ist. Folgendes Gleichnis ist vielleicht geeignet, diese Verhältnisse zu verdeutlichen. Jemand betrachtet den Mond durch ein Fernrohr. Ich vergleiche den Mond selbst mit der Bedeutung; er ist der Gegenstand der Beobachtung, die vermittelt wird durch das reelle Bild, welches vom Ojektivglase im Innern des Fernrohrs entworfen wird, und durch das Netzhautbild des Betrachtenden. Jenes vergleiche ich mit dem Sinne, dieses mit der Vorstellung oder Anschauung" (*USB*, 44–45). TRANSLATION: "The significance of a proper noun is the object itself to which we refer by means of the noun; the representation that we have on that occasion is entirely subjective; in between lies the sense, which is no longer subjective as the representation is, yet is also not the object itself. The following comparison may be suited to clarify these relationships. Someone observes the Moon through a telescope. I compare the Moon itself to the significance; it is the object of observation, which is mediated by the real image—which is formed by the lens within the telescope—and by the retinal image of the observer. The real image I compare to the sense, and the retinal image, to the representation or intuition." For further discussion of *USB*, the analogy, and the appearance-reality divide, see § 3.1 below.

36. The term *perspicillum* was also used in connection with the microscopic world.

ideoscope, in that it enables one to bring into view the fine structures and substructures of thoughts (in Frege's sense of 'thought', of course). Examples of such structures are: a) the relations of dependency of complex thoughts upon their constituent thoughts (some of these relations being now called 'truth-functions'); b) the subsumption of an object under a concept and the standing of *n* objects in an *n*-place relation (where *n* is ≥ 2 and is a natural number), each of which brings with it the distinction between singular and general terms; c) the subordination of functions; d) the falling of a function within another one or the attribution of properties to concepts or relations, the latter bringing with them a typology of functions or a hierarchy of orders of generality; and e) the relation of identity. These microstructures, or semantic forms, occur in the following illustrative sentences of natural language.[37]

1) The **subsumption** or **falling** of an object **under** a concept or of *n* objects under an *n*-place relation:

   Galileo is Tuscan

   The highest mountain on Earth is on the border of Nepal and Tibet

   This man robbed the liquor store on Astor Place

   Julia loves Robert

   The Earth has one satellite

   The horse is in the backyard! (said to my daughter in response to her wondering where her horse is)

2) The **subordination** of concepts:

   The horse is an herbivorous animal

   Man is a political animal

   A square is a four-sided figure

   All men are mortal

3) The **attribution of a property to a concept or a relation**, or the **falling** of a concept or a relation **within** another concept:

   Being a father is an intransitive relation

   In the set of real numbers, addition is commutative

   There are strange people

See, e.g., the essay by William Coles (1626–1662) entitled "Perspicillum microcosmologicum," in his work of medical botany, *The Art of Simpling: An Introduction to the Knowledge and Gathering of Plants* (London: Nath Brook, 1656).

37. Although some truth-functions will appear in the examples and will, of course, be present in their ideographic counterparts, I shall not be treating them under a separate heading, for such is not necessary for the point to be made.

4) The relation of **identity**:

Pope John XXI is Peter of Spain

The morning star is the evening star

2457 + 186 = 2643

Of the above sentences, several present the same grammatical or syntactic pattern, the same appearance. Consider 'Galileo is a Tuscan', 'The horse is in the backyard!', 'Pope John XXI is Peter of Spain', 'The horse is an herbivorous animal', 'A square is a four-sided figure', 'Man is a political animal', 'Being a father is an intransitive relation', and 'Addition is commutative'. According to the syntax we all learned in grammar school and which classifies the parts of speech and describes their roles in sentences, all of the sentences just cited have a subject, a copula, and a predicate. The subjects are, in order, 'Galileo', 'the horse', 'Pope John XXI', 'the horse', 'a square', 'man', 'being a father', and 'addition'. In each sentence, the copula is in the third person singular of the indicative present. The predicates are 'Tuscan', 'in the backyard', 'Peter of Spain', 'an herbivorous animal', 'a four-sided figure', 'a political animal', 'an intransitive relation', and 'commutative'. Since expressions such as 'Galileo' and 'Pope John XXI' clearly refer to persons who once lived, and since, for various reasons, the relation of the name to what it names has traditionally been considered to be the model of the relation of meaning, the prevalent temptation has been to think that the other subject terms likewise have references.[38] In the case of the second sentence, one would be right in so thinking, since the subject term refers to my daughter's horse. In the last five sentences, however, it is less obvious to what their subjects might be referring. The difficulty in establishing the reference seems to be greatest in the case of the word 'addition'—after all, what does 'addition' allegedly stand for? The sign '+' or a synthetic act of the human mind, as some have theorized? To suggest that the word 'addition' refers to the sign '+' merely shifts the locus of the question, for of the sign '+' one may again ask what it is supposed to stand for. It might not seem entirely unreasonable to think that 'addition' refers to an act of synthesizing or putting together as long as one confines one's attention to cases of adding natural numbers and considers the latter as heaps of units that would seem amenable to a kind of aggregative transformation. However, what does one combine when one adds zero and any other natural number? Furthermore, what is being synthesized when negative integers are added, not to mention fractional, irrational, and imaginary numbers?

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38. *PW*, 124/*NS*, 135: "The word 'common name' leads to the mistaken assumption that a common name is related to objects in essentially the same way as is a proper name, the difference being only that the latter names just one thing whilst the former is usually applicable to more than one. But this is false, and that is why I prefer 'concept-word' to 'common name'."

Similarly, what does 'the horse' in 'The horse is an herbivorous animal' refer to? It does not seem to stand for any particular individual. Indeed, if told that the horse is not an herbivorous animal, no competent speaker of English would reply: "Which horse do you mean?" Or, upon hearing the same assertion, no one would respond: "Well, the horse you are referring to may not be, but that horse over there certainly is." Rather, a more likely response from, say, a horse rancher would be something like this: "No, I can assure you that horses are herbivorous animals." Or perhaps: "No, you're wrong; all horses are herbivorous animals." The utterance 'The horse is an herbivorous animal' is not about an individual entity, as are all the sentences under the first and fourth rubrics (be it a human being, a mountain, a planet, or an animal, or numbers), but it has a general point: it states that all the things that are horses are also herbivorous, or that, for all things, if they are horses, then they are herbivorous. In other words, it says that there is a certain relation between horses and the group of herbivorous animals such that all things that are horses are included in the larger group of herbivorous animals. Using the language of falling under a concept, we may rephrase the sentence 'For all things, if they are horses, then they are herbivorous' as follows: For all things, if they fall under the concept horse, then they fall under the concept of being herbivorous. The utterance 'The horse is an herbivorous animal', despite its grammatical kinship to 'The horse is in the backyard'—or despite appearances—is thus not about an individual's being such and such or having such and such a property, but is rather, as Frege was the first to point out clearly and distinctly, about a *relation* between two concepts, the relation, that is, of the concept horse's being subordinated to the concept of being herbivorous.[39]

As was just shown, the grammar of natural language assimilates the two sentences 'The horse is an herbivorous animal' and 'The horse is in the backyard' in that it construes them to have the same syntactic structure, and, in so doing, it is not wrong. *What is unwarranted and misleading is to assume that the syntactic similarity of two sentences of natural language is necessarily a reliable indication that the semantics of the sentences in question are also the same; and, more generally, that the syntax of a sentence of natural language is a trustworthy index of the semantics of its constituents*, namely of their mode of use, or the way in which they contribute to the meaning of their sentence of occurrence. It is this assumption that leads one to think that the subject term of 'The horse is an herbivorous animal' refers just as the subject term of 'The horse is in the backyard' does, reference being precisely a *semantic*, *not* a syntactic relation. Frege's microscope shows the assumption of a syn-

39. *PW*, 213/*NS*, 230–31: "We must not think that I mean to assert something about an African chieftain from darkest Africa who is wholly unknown to me, when I say 'All men are mortal'. I am not saying anything about either this man or that man, but I am subordinating the concept man to the concept of what is mortal. [. . .] By the sentence 'All men are mortal' I say 'If anything is a man, it is mortal'."

tactic-semantic parallelism in natural language to be unjustified. The ideographic expression of the two sentences will capture and exhibit the microstructures operative within them that were revealed by the foregoing reflections.

Before continuing with the analysis of the last two sentences under consideration, some general and anticipatory remarks will help to orient the reader. Using not Frege's but current ideograms, I shall gradually make explicit the four microstructures or semantic forms listed above (i.e. subsumption, subordination, falling within, and identity) in the sentences offered as their illustrations. The semantic forms will be laid bare by discussing and paraphrasing the English sentences and then translating them into the ideography of logic. Save for the last sentence ('The horse is in the backyard') under the rubric of subsumption, the present section will treat exclusively of the structures of subordination, falling within, and identity. Section 4 will complete the discussion by dealing with the other sentences that exemplify subsumption.

I shall not be using Frege's ideography for two reasons of unequal weight: first, its two-dimensional character requires large amounts of space and is difficult to typeset; secondly and more importantly, the fact that it uses only four form signs (now called operators or logical constants in logical theory) makes it more difficult to grasp and discern the semantic forms than in a ideography that uses more form signs.

Let us now return to the discussion of the sentences 'The horse is an herbivorous animal' and 'The horse is in the backyard' and make explicit their respective forms in the following translations.

- The horse is in the backyard.

GLOSSARY

'the horse' (i.e. my daughter Chloé's horse, Bailey) will be rendered by the singular term (also called individual constant in logical theory) 'b'

'…is in the backyard' is to be translated by the one-place predicate 'Y(…)'[40]

IDEOGRAPHIC TRANSLATION

Yb

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40. The expression '. . . is in the backyard' is treated here as a one-place predicate, although, if the context required it, it could be further analyzed. That is to say, it could be split up into the dyadic predicate '. . . is in . . .' and the singular term 'the backyard'. What is meant by 'if the context required it' is something like this: the sentence in question may be a piece of an argument that requires that the polyadicity of the predicate be made explicit in the ideographic rendition of the argument, for the validity of the argument depends, among other things, upon that dyadic structure.

- The horse is an herbivorous animal.

The ideographic rendition of this sentence will be based on the paraphrastic analysis already given above, namely on this formulation:

- For all things, if they are horses, then they are herbivorous animals.

In fact, this sentence will undergo one more step of analysis before we make the transition to the ideographic rendition:

- For every x, if x is a horse, then x is an herbivorous animal.

This unnatural and inelegant (to be sure, from the standpoint of natural language, not from that of logic[41]) version of 'The horse is an herbivorous animal' separates very clearly the term of quantity 'every' from the two predicates of the original sentence, '. . . is a horse' and '. . . is an herbivorous animal'. The placeholder 'x' throughout the paraphrase serves to make explicit the fact that the word of quantity 'every' applies to the two predicates: the sign of quantity that forms the prefix of a given placeholder *binds* all occurrences of that placeholder in the expression that follows it. It is said to bind the placeholder insofar as it indicates the (quantitative) extent to which the predicates wherein the placeholder occurs can be instantiated:[42] the predicates may be instantiated by all things or universally, or by at least one thing or existentially, depending on whether the word of quantity under which they stand is 'every' or 'some' (or 'at least one'), respectively. In our example, the word 'every' applies to the subordinated predicates, and it indicates how many of the things under discussion (the universe of discourse) the predicates hold true of: it tells us that the *subordinated* concepts apply to, and thus characterize and classify, *all* things being talked about. The unnatural formulation also makes explicit the relation of conditionality (or material conditional) expressed by 'if . . . , then . . . '.

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41. *PW*, 7/*NS*, 7: "There is no reproach the logician need fear less than the reproach that his way of formulating things is unnatural . . ." The unnaturalness of many formulations in logical theory and, more broadly, in philosophical logic is in fact quite apt, insofar as it disrupts our familiarity with, our closeness to natural language, and reminds us that no matter *how much language is always already a part of us, it never thinks for us*, and that if we let it, the risk is great that it will mislead us. It is a testimony of our ability to reflect on that which is natural, customary, or implicit.

42. To instantiate a general sentence such as 'For every x, if x is a horse, then x is an herbivorous animal' consists in turning it into a singular sentence by substituting all occurrences of 'x' with an appropriate singular term. For example, since 'Bailey' is the name of my daughter's horse, I may substitute 'Bailey' in the general sentence and obtain 'If Bailey is a horse, then Bailey is an herbivorous animal'.

GLOSSARY

For all x, or For every x: (∀x)

if . . . , then . . . : . . . ⊃ . . .[43]

. . . is a horse: Hx

. . . is an herbivorous animal: Vx

IDEOGRAPHIC TRANSLATION

(∀x) (Hx ⊃ Vx)

Although this is not Frege's terminology, the ideogram '(x)' is now commonly called 'universal quantifier', and sentences such as this one are said to be universally quantified. Under the third rubric, we shall translate a sentence whose predicates stand under the quantity word 'some', also called 'existential quantifier'.

The next sentence from the second category is 'Man is a political animal'. According to the syntax of natural language, it is an indefinite sentence in the singular, and "[n]umber is that form of a word which indicates whether we are speaking of *one* or *more than one*."[44] The indefinite plural of the same sentence would be 'Men are political animals'. If we take our bearings by the syntax of the two sentences, and if we assume that 'man' and 'men', being subjects of sentences, must, like 'Galileo' in the above sentence, be referring to some object, we might venture the proposal that 'man' refers to the universal—say, the essence of man or, alternatively, the class of men—whereas 'men', one might surmise, refers to an indefinite plurality of human beings, or refers indefinitely. On the first hypothesis, 'Man is a political animal' would thus be stating that the essence of human being is a political animal. That surely must be wrong, for essences are not animals. Similarly, if we entertain the notion that 'man' refers to the class of human beings, we are left with the absurd consequence that a class is an animal. The suggestion that 'man' refers to an essence, to a feature that is common to all humans, goes in the right direction insofar as it captures, albeit very clumsily, the fact that the sentence says something

43. The English, or natural language, conjunction 'if . . . , then . . .' (in contrast to the ideographic sign '⊃') has a number of uses, and it rarely behaves merely like the logical relation of conditionality. The logical relation of conditionality, or material conditional, is a truth-functional relation, which is to say that a sentence such as 'A ⊃ B', where 'A' and 'B' are two arbitrary truth-claiming sentences, is such that its truth or falsity exclusively depends on (is a function of) the truth and falsity of its component sentences, 'A' and 'B'. This is not so for most conditional sentences of natural language, as there are additional constraints on their truth conditions. For an excellent discussion of conditional sentences, see David H. Sanford, *If P, then Q Conditionals and the Foundations of Reasoning* (London: Routledge, 1989). The first part of the book is devoted to a history of the philosophical treatment of conditionals.

44. George O. Curme, *A Grammar of the English Language*, 2 vols. (Essex, Conn.: Verbatim, 1986), I: 112.

general. The sentence makes a statement about human beings in general: of human beings it says that they are political animals. More precisely, it says something about the concept of human being, namely that it is subordinated to that of political animal. In other words, semantically speaking, we are dealing here with a universally quantified sentence, that is, a sentence the form of which is the same as that of 'The horse is an herbivorous animal'.

- Man is a political animal.
- For every thing, if it is a man, then it is a political animal.
- For every x, if x is a man, then x is a political animal.

GLOSSARY

For every x: $(\forall x)$

if . . . , then . . . : . . . $\supset$ . . .

. . . is a man: Mx

. . . is a political animal: Px

IDEOGRAPHIC TRANSLATION

$(\forall x)\ (Mx \supset Px)$

The subject of 'A square is a four-sided figure' is the common noun 'square', modified by the singular indefinite article. Obviously, all squares are four-sided figures, and our sentence, despite the fact that it is in the singular, says nothing less than that—and nothing more. It says exactly the same as 'All squares are four-sided-figures'.

- A square is a four-sided figure.
- For every x, if x is a square, then x is a four-sided figure.

GLOSSARY

For every x: $(\forall x)$

if . . . , then . . . : . . . $\supset$ . . .

. . . is a square: Sx

. . . is a four-sided figure: Fx

IDEOGRAPHIC TRANSLATION

$(\forall x)\ (Sx \supset Fx)$

Of all the sentences in the second category, the last one, 'All men are mortal', is the most straightforward, for its syntax is more consonant with its semantics than that of the others: it is universally quantified and its syntax *shows* it to be so,

although even it can use a pinch of analysis to make explicit the relation of the quantifier to the remainder of the sentence and to bring out the conditional relation between its two predicates:[45]

- For all things, if they are men, then they are mortal.
- For every x, if x is a man, then x is mortal.

GLOSSARY

For every x: $(\forall x)$

if . . . , then . . . : . . . $\supset$ . . .

. . . is a man: Hx

. . . is mortal: Mx

IDEOGRAPHIC TRANSLATION

$(\forall x)\,(Hx \supset Mx)$

The ideographic rendition of the thoughts expressed by the two sentences 'The horse is an herbivorous animal' and 'The horse is in the backyard' is syntactically faithful to their semantic (sub)structures, whereas natural language is at best only erratically so, which is to say that its appearance is unreliable. The ideography's syntax allows the greater complexity of the former sentence to come into view. The structural sameness exhibited by the natural-language formulations of the two thoughts is thus a merely outward aspect, a semblance likely to mislead. *The discrepancy between syntax and semantics* (in natural language) brought to light by this pair of sentences shows very well what Frege meant when he excluded the subject-predicate distinction from logic on the ground that it conceals distinctions that are at work in the sentences of natural language: "We shall have no truck with the expressions 'subject' and 'predicate', of which logicians are so fond, especially since they not only make it more difficult for us to recognize the same as the same, but also conceal distinctions that are there" (*PW*, 143/*NS*, 155). In fact, Frege's text identifies two types of divergence between syntax and semantics that may occur in natural language and two sorts of error to which, as a result, one is exposed when one relies without further ado on the subject-predicate analysis of sentences:

1) One mistake consists in assuming that **if** two or more sentences have the **same syntax**, **then** they must have the **same semantics**, whereas in reality a similarity in syntax may occlude a difference in semantics (what Frege calls 'concealment of distinctions that are there').

This case of sentences' having same syntax but different semantics has just been illustrated by the sentences 'The horse is an herbivorous animal' and 'The horse is in

45. For Frege's analysis of this sentence, see *PW*, 213/*NS*, 231.

the backyard'. We shall encounter further instances of this divergence when the semantics of the following sentences is made manifest in the remainder of the present section and in § 4: 'A square is a four-sided figure', 'Man is a political animal', 'Being a father is an intransitive relation', 'Addition is commutative', 'Pope John XXI is Peter of Spain', and 'Galileo is a Tuscan'.

2) Another error consists in assuming that **if** two or more sentences have **different syntax**, **then** they must have **different semantics**, whereas in reality a difference in syntax may hide a similarity in semantics (what Frege calls 'the difficulty in recognizing the same as the same').

The case of sentences' having different syntax but same semantics was documented in the preceding analysis of the sentences 'Man is a political animal' and 'All men are mortal'. This type of discrepancy between syntax and semantics is particularly pronounced in the case of the singular terms of natural language, to which § 4 will be devoted.

These two phenomena thus militate against the logical reliability of the subject-predicate distinction and its nuances (e.g. the distinction between the plural and the singular, or the diverse faces of natural language's singular terms).

It should also be clear from the foregoing that although the script gives a truthful depiction of the forms of thoughts, the forms first have to be teased out of their natural-language garb by reflection upon linguistic use and behavior, which, incidentally, encompasses the giving of arguments or the construction of proofs (paradigmatically in mathematics). This laying bare of the forms can prove delicate and requires practice;[46] it may also lead to controversy. Frege recognized this fact when he encouraged those interested in acquiring a logical education to be conversant in foreign languages. This is what he says on the subject:

> From this we can see the value of learning foreign languages for one's logical education. [. . .] This is how the difference between languages can facilitate our grasp of what is logical. But still the difficulties are not wholly removed in this way and our logicians still keep dragging in a number of things which are really of no logical concern, though they belong to the grammar of languages akin to our own, if not to others. For this reason it is

46. This is not to say, however, that it is merely a matter of practice and that the making explicit of the forms is left to a kind of logical intuition. Such is not the case. The discernment of the substructures of sentences, or the translation of natural language into the ideography of modern logic, can be discussed and theorized about at length. See, e.g., Graeme Forbes, *Modern Logic* (New York: Oxford University, 1994), 12–26 (devoted to truth-functions); 149–64 (universal and existential quantifiers); 231–40 (identity—which will be introduced through the examples illustrating the fourth rubric). A systematic—perhaps the most systematic—treatment of translation from natural language into ideography is to be found in Ernest Lepore, *Meaning and Argument: An Introduction to Logic through Language* (Oxford: Blackwell, 2000).

> useful to be acquainted also with a means of expression of a quite different kind, such as we have, for instance, in the formula-language of algebra. (*PW*, 6/*NS*, 6; see 142/154)

Competency in logic will be greatly enhanced if one practices at least one foreign language and is versed in an ideography, such as that of algebra. For example, if one masters two languages that share common roots, the phenomenon of false cognates (the French *déception*, which means 'disappointment', and the English 'deception' offer an instance of such false cognates) acquaints one with the discrepancy between the morphological similarity of certain lexical items—owing to their common etymology—and their divergent meanings.

We now turn our attention to the sentences under the third rubric. The sentence 'Being a father is an intransitive relation' may also be formulated as follows: the phrase '. . . is the father of . . .' describes an intransitive relation. Unlike the sentences under the first and the fourth rubrics, which are about individual things, this sentence is about a certain dyadic relation, and of that relation it tells us that if, *for instance*, Robert is the father of Paul, and Paul is the father of Amandine, then Robert cannot be the father of Amandine, although he is her grandfather.[47] The latter is *an illustration*, *as well as an explication*, of what it means to say that the relation of being someone's father is intransitive. In other words, the sentence under consideration ascribes a property to the relation of being a father: the relation is said to have the property of being intransitive. Note that the sentence ascribing the property is utterly general: the property of intransitivity holds of the relation of being a father, regardless of the individuals who stand in the relation of father and child. That is, all individuals who are related as father and child will be such that their relation has the property of being intransitive, which can be paraphrased and *explicated* as follows: for all people, if one person is the father of a second person, and if the second person is the father of a third person, then the first person is not the father of the third person. The analysis can be sharpened, that is, the form can be made more perspicuous, if we mark the places in the relation term held by the ordinal adjectives by means of the last letters of the alphabet:[48] for all people, if **x** is the father of **y**, and if **y** is the father of **z**, then **x** is not the father of **z**. Finally, the fact that the negation operative in the then-clause applies to the predicate can also be made sharper: for all people, if **x** is the father of **y**, and if **y** is the father of **z**, then **it is not the case** that **x** is the father of **z**. As indicated by 'for all people', the universe of discourse for this sentence happens to be that of human beings, although it obviously need not be.

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47. Generally stated, a relation is intransitive if and only if, if one thing bears that relation to a second, and the second to a third, then the first cannot bear it to the third.

48. If, that is, we substitute individual placeholders for the ordinal phrases. Conventionally, the last letters of the alphabet are employed in logical theory as individual placeholders. An individual placeholder holds a place for a singular term.

GLOSSARY

For all x: (∀x)

For all y: (∀y)

For all z: (∀z)

. . . is the father of . . . : F(. . .)(. . .)

. . . and . . . : . . . & . . .

if . . . , then . . . : . . . ⊃ . . .

it is not the case that . . . : ~ . . .

IDEOGRAPHIC TRANSLATION

(∀x) (∀y) (∀z) [(Fxy & Fyz) ⊃ ~ Fxz]

A few more remarks about the structure at work within this sentence are in order. The relation of being a father is a first-order relation, which is to say that the predicate '. . . is the father of . . .' takes singular terms (referring to individuals, human or otherwise) as its subject and as its complement (the complement being the word that completes the phrase 'the father of . . .'). On the other hand, the property of being intransitive that characterizes the first-order relation is a second-order concept. Consequently, the sentence under consideration is about the relation between a first-order relation and a second-order concept: Frege describes this relation by saying that the first-order relation *falls within* the second-order concept (or, more generally, that the n^{th}-order relation *falls within* the n^{th}+1-order concept).[49] It is helpful to contrast this relation of a concept's or relation's falling within another concept with the subordination of two concepts. Whereas the relation of falling-within obtains between a relation (or a concept, as the case may be) and a concept of *different orders*, subordination relates two concepts of *the same order*: we saw that one first-order concept is subordinated to another first-order concept if and only if everything that falls under the first also falls under the second. However, the ideographic rendition of the sentence given above does not merely say that a certain first-order relation has a certain property. Rather, the rendition shows what the property consists in.

The next sentence under the third rubric, namely 'In the set of real numbers, addition is commutative', ascribes a property to an arithmetical operation: it says that, for any pair of real numbers, the operation of adding them is commutative. To say that the operation of addition is commutative in the set of real numbers is to say that:

- For every x and every y, if x and y are real numbers, then x + y = y + x.

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49. See *PW*, 110/*NS*, 120–21 and 254–55/274–75; and *GA*, § 53, p. 65.

This sentence can be further analyzed:

- For every x and every y, if x is a real number and y is a real number, then x + y = y + x.
- For every x and every y, if x is a member of the set of real numbers and y is a member of the set of real numbers, then x + y = y + x.

Again, as in the preceding example, the property of commutativity is displayed ideographically and not merely named.

GLOSSARY

For all x, or For every x: $(\forall x)$

For all y, or For every y: $(\forall y)$

x is a member of . . . : $x \in \ldots$

the set of real numbers: $\mathbf{R}$

. . . and . . . : . . . & . . .

if . . . , then . . . : . . . $\supset$ . . .

IDEOGRAPHIC TRANSLATION

$(\forall x)\,(\forall y)\,\{[(x \in \mathbf{R})\,\&\,(y \in \mathbf{R})] \supset (x + y = y + x)\}$[50]

As for the third sentence, 'There are strange people', it too ascribes a property to a concept. In fact, as we are about to see, it ascribes a property to the conjunction of two concepts. Let me explain. To say that there are strange people is to say that of all people, some are strange. The latter is the same as saying that some people are strange. To say that some people are strange is, in turn, equivalent to saying that:

- Some person is such that he or she is strange.

This sentence can again be paraphrased as follows:

- Some thing (or at least one thing) is such that it is a person and it is strange.

In the last sentence, the two concept terms '. . . is a person' and '. . . is strange' are now separated from the quantity term 'some', which stands at the beginning of the sentence. To lend this separation greater sharpness the term of quantity is further set off, and to indicate that the two predicates stand under the scope of the quantity term, the sentence is rewritten with the help of the individual placeholder 'x':

- For some x, x is a person and x is strange.

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50. The parentheses, square brackets, and braces used in the script serve as punctuation, namely as means of disambiguating complex sentences.

It should now be plain that the sentence says something about the two concepts, namely that neither concept is empty,[51] or that their conjunction is such that it holds true of at least one entity. In other words, it ascribes to them the property of being instantiated by at least one thing. Using the metaphor of falling within, one may say that the two concepts in question fall within the concept 'concept that applies to at least one thing', which is a second-order concept.

GLOSSARY

For some x, or For at least one x: (∃x)

. . . is a person: Px

. . . is strange: Sx

. . . and . . . : . . . & . . .

IDEOGRAPHIC TRANSLATION

(∃x) (Px & Sx)

Although this is not Frege's terminology, the ideogram '(x)' is now typically called 'existential quantifier', and sentences such as this one are said to be existentially quantified.

There remain the sentences that illustrate the fourth rubric. According to the syntax of natural language and traditional logic, just as 'Tuscan' is the predicate of the sentence 'Galileo is Tuscan', so too 'Peter of Spain' is the predicate of the sentence 'Pope John XXI is Peter of Spain'. However, whereas 'Tuscan' is an adjective that qualifies the proper name 'Galileo', that is, whereas it is a concept word that characterizes and classifies the individual Galileo (he belongs to the class of Tuscans), 'Peter of Spain' is not a concept word but rather a proper name, just as 'Galileo' and 'John XXI' are proper names. If someone were to disagree with the assertion that Pope John XXI is Peter of Spain, that person would not reply: "No, Pope John XXI is tall." Such a response would be utterly inappropriate, for 'being tall' is not a suitable alternative to 'being Peter of Spain'. Rather, the person might say: "I thought Pope John XXI was Peter Abelard" (a falsehood, of course). In other words, disagreement about the statement that Pope John XXI is Peter of Spain is one about who Pope John XXI is, what his identity is, not about his characteristics, physical or psychological. To make this explicit, one could paraphrase the initial sentence as follows:

- Pope John XXI is identical with Peter of Spain.

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51. *PW*,107/*NS*,116–17: "I have called existence a property of a concept. How I mean this is best made clear by an example. In the sentence 'there is at least one square root of 4', we have an assertion, not about (say) the definite number 2, nor about –2, but about a concept, *square root of 4*; viz. that it is not empty."

On the other hand, such a paraphrase could not be supplied for 'Galileo is Tuscan', for 'Galileo is identical with Tuscan' is plainly ill-formed; it is not even correct English. The sentence is thus about a relation, the two-place relation '. . . is identical with . . . '. What may fill the gaps of such a relational phrase are singular terms, in this case, proper names or definite descriptions. Consider the statement of identity 'Theodore Kaczinsky is the Unabomber'. The sentence contains a proper name and a definite description: 'Unabomber' is an ambivalent (abbreviated) definite description (the unabbreviated expression being 'university-airline bomber'), as it is nearly always spelled with an initial capital but used with the definite article.

GLOSSARY

Pope John XXI: j

Peter of Spain: p

. . . is identical with . . . : . . . = . . .

IDEOGRAPHIC TRANSLATION

j = p

To distinguish the use of 'is' in 'Galileo is Tuscan' from its use in 'Pope John XXI is Peter of Spain', one could also have pointed out that whereas in the latter sentence 'is' expresses a symmetrical relation (as shown by the fact that the order of its proper names is reversible as follows: Peter of Spain is Pope John XXI), such is not the case with the former ('Tuscan is Galileo' is patent nonsense).

The second sentence under the last rubric is Frege's famous 'The morning star is the evening star'. It consists of two definite descriptions (namely, 'the morning star' and 'the evening star') that flank the finite form 'is' of the verb 'to be', as that form is used to express the dyadic and symmetrical relation of identity (as was the case in the preceding example).[52] Both 'the morning star' and 'the evening star' re-

52. See the contrast drawn by Frege between the use of 'to be' to mean identity and the use of it to mean the relation of subsumption under a concept at *UBG*, 67–68: "Kann man nicht ebensogut von etwas aussagen, es sei Alexander der Große, oder es sei die Zahl Vier, oder es sei der Planet Venus, wie man von etwas aussagen kann, es sei grün, oder es ein Säugetier? Wenn man so denkt, unterscheidet man nicht die Gebrauchsweisen des Wortes 'ist'. In den letzten beiden Beispielen dient es als Kopula, als bloßes Formwort der Aussage. Als solches kann es zuweilen durch die bloße Personalendung vertreten werden. Man vergleiche z.B. 'dieses Blatt ist grün' und 'dieses Blatt grünt'. Wir sagen dann, daß etwas unter einen Begriff falle, und das grammatische Prädikat bedeutet dabei diesen Begriff. In den ersten drei Beispielen wird dagegen das 'ist' wie in der Arithmetik das Gleichheitszeichen gebraucht, um eine Gleichung auszusprechen." TRANSLATION: "Surely one can just as well assert of a thing that it is Alexander the Great, or is the number four, or is the planet Venus, as that it is green or is a mammal? If anybody thinks this, he is not distinguishing the modes

fer to the same individual, the planet Venus. However, each does so under a different description; in Frege's later terminology, each expression is said to have a different sense (*Sinn*), although both have the same reference (*Bedeutung*). An identity statement thus expresses a relation between an object and itself, namely that of being self-same, of being-not-other-than (*PW*, 91 n./*NS*, 100 n.). There are uninformative identity statements, such as 'a = a', and there are informative ones, such as the two we have just been examining. Briefly, identity statements may be informative because the same object may have more than one name or definite description and we may not know that until, precisely, an identity statement informs us of such; or we may ourselves discover that two singular terms refer to the same object and convey that new piece of knowledge to others, and we do so by means of a sentence of identity.[53] The sentence under consideration, 'The morning star is the evening star', reports such a discovery: it records the fact that what had appeared to be two different heavenly bodies, which were being picked out by the two definite descriptions 'the morning star' and 'the evening star', were in reality two aspects of one and the same body, or two perspectives under which the one planet Venus appeared in the sky to star-gazers, on one occasion in the morning sky, on the other in the evening sky.

GLOSSARY

the morning star: a

the evening star: b

. . . is identical with . . . : . . . = . . .

IDEOGRAPHIC TRANSLATION

a = b

The last statement of identity under the fourth rubric, '2457 + 186 = 2643', is already in ideographic form and requires no further attention. When an identity obtains between numbers, we call it equality (see *PW*, 86/*NS*, 95; and 226/244: "we maintain our position that the equals sign in mathematics is to be construed as a sign of identity").

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of use of the word 'is'. In the last two examples, it serves as a copula, as a mere form-word of the statement. As such, it can sometimes be represented by a simple finite verb-ending: compare 'this leaf is green' and 'this leaf turns green'. In that case, we say that something falls under a concept, and the grammatical predicate signifies this concept. In the first three examples, on the other hand, 'is' is used as the sign of equality is in arithmetic, to express an equation."

53. My discussion of "Über Sinn und Bedeutung" (*USB*) in the next section (§ 3.1) will deal with Frege's account of the informative character of (some) identity statements.

§ 3.1 Appearance and Reality

Just as the hyper-mesocosmic and therewith counterintuitive orientation of modern inquiry gave rise to a world of mere appearances, so too Frege's ideography elicits a rift between reality and appearance within the province of natural language:

> In the sentence 'There are men' we *seem* to be speaking of individuals [*scheint von Individuen gesprochen zu werden*] that fall under the concept 'man', whereas it is *only* about the concept 'man' we are talking. [. . .] From this one sees how easily one can be led astray by language to false conceptions, and what value it must have for philosophy to escape the dominion of language [*Herrschaft der Sprache*]. (*PW*, 67/*NS*, 74; my emphasis)

This passage shows that the motifs of

1) the discernment of semblances within natural language and of a linguistic reality (a semantic one) below this superficies;
2) deceptive linguistic appearances (which are often syntactical); and
3) the critique of natural language

form a whole, and that the microstructures examined above in § 3 concretely illustrate what Frege has in mind when he brings up these motifs or engages in polemics against natural language; they show why he deems the latter unfit for the conduct of rigorous logical work and therefore why he discards the traditional analysis of sentences into subject, copula, and predicate. These three moments are at work in *Concept-Script*. The last two occur very straightforwardly in the preface, when Frege says that

> it is a task of philosophy to break the dominion of the word [*Herrschaft des Wortes*] over the human spirit by exposing the misconceptions regarding the relations among concepts that the use of ordinary language often nearly unavoidably occasions and by freeing thought from that with which the sole makeup of the linguistic means of expression afflicts it. . . . (*BS*, xii–xiii)[54]

54. The polemics against natural language run through Frege's work from beginning to end. Here are a few citations to illustrate this:

- *PMC*, 68: "In an earlier stage of language formation, so it seems, there took place an excessive degree of exuberant growth of linguistic forms. A later time again had to lay aside and simplify many of them. The main task of the logician consists in liberating himself from language and in simplifying it. Logic will be the judge of languages. One should do away, in logic, with the subject and the predicate. . . ."
- *PW*, 6–7/*NS*, 7: "it is the business of the logician to conduct an unceasing struggle against the psychological and partly against language and grammar insofar as they fail to give untrammelled expression to the logical";
- *PW*, 143/*NS*, 155: "Instead of following grammar blindly, the logician ought rather to see his task as that of freeing us from the fetters of language"; and

The first part of *Concept-Script* also cautions readers against the deceptive aspects of natural language:

> Let us warn here against an error easily occasioned by linguistic usage. If we compare the two propositions:
>
> "the number 20 can be represented as the sum of four squares"
>
> and
>
> "each positive integer can be represented as the sum of four squares"
>
> then it seems possible to regard '. . . can be represented as the sum of four squares' as a function that in one case has the argument 'the number 20' and in the other 'each positive integer'. (*BS*, § 9, p. 17)

Frege here alerts us to the confusion between the semantic types of singular term and of quantifier term (see the remainder of the paragraph in his text). This again exemplifies his point that the subject-predicate distinction conceals semantic distinctions. The opposition between reality and appearance is somewhat more difficult to spot in *Concept-Script*. Yet it is undoubtedly there; for instance, it is embodied in the opposition between the phrase "type of signs suited to the things themselves [*Sachen selbst*]," and the comment that "the distinction between categorical, hypothetic, and disjunctive judgments seems to me to have *merely gram-*

◆ *PW*, 149/*NS*, 160–61: "It is the task of this science [logic] to purify the logical of all that is alien and hence of all that is psychological, and to free thinking from the fetters of language by pointing up the logical imperfections of language."

See also *PW*, 252/*NS*, 272; *GA*, § 39, p. 51, and "Der Gedanke," 40: "Ich muß mich begnügen, den an sich unsinnlichen Gedanken in die sinnliche sprachliche Form gehüllt dem Leser darzubieten. Dabei macht die Bildlichkeit der Sprache Schwierigkeiten. Das Sinnliche drängt sich immer wieder ein and macht den Ausdruck bildlich und damit uneigentlich. So entsteht ein Kampf mit der Sprache, und ich werde genötigt, mich noch mit der Sprache zu befassen, obwohl das ja hier nicht meine eigentliche Aufgabe ist." TRANSLATION: "I must rest content with presenting the reader with the non-sensory thought clad in sensory linguistic form. In this respect, the imagistic character of language gives rise to difficulties. The sensory always forces its way in and renders expression imagistic and thereby improper. So there arises a struggle with language, and I find myself compelled to deal with language, even though it is not my proper concern here."

Lastly, note the striking text at *PW*, 269–70/*NS*, 288–89, where we find together the topics of appearance as semblance within (natural) language, of the occluding and deceptive character of language, of the tendency of language to form pseudo-referential expressions ("One feature of language that threatens to undermine the reliability of thinking is its tendency to form proper names to which no objects correspond" and "the fatal tendency of language to form apparent proper names [*scheinbare Eigennamen*]"), and of the revelatory function of ideography ("In the formalized language of mathematics an important difference comes to light [*ans Licht getreten*] that lies concealed in phonetic language [*in der Wortsprache verdeckt*]").

matical [*nur grammatisch*] significance" (*BS*, Preface, xi and § 4, p. 4, respectively; my emphasis). In Frege's texts, the pairs *Sachen–nur grammatisch* and *sachlich–sprachlich* express the opposition between reality and appearance, as is shown by this citation: "Is it *A* or the idea of A that is the real [*sachliche*], as opposed to the grammatical [*sprachliche*] subject in the sentence '*A* is something that can be experienced'" (*PW*, 53/*NS*, 60); and by this text from "Über Begriff und Gegenstand":

> In the sentence 'the morning star is Venus', we have two proper names, 'the morning star' and 'Venus', for the same object. In the sentence 'the morning star is a planet', we have a proper name, 'the morning star', and a concept word, 'planet'. Linguistically [*sprachlich*], no more has happened than that 'a planet' has replaced 'Venus'; but in reality [*sachlich*] the relation has become wholly different. An identity is a symmetrical relation, whereas an object's falling under a concept is an asymmetrical relation. The 'is' in the sentence 'the morning star is Venus' is obviously not the mere copula, but contentually [*inhaltlich*], too, it is an essential part of the predicate, so that the word 'Venus' does not contain the whole of the predicate.[55]

Concept-Script also contains the more general distinction between appearance as semblance or seeming (as *Schein*) and appearance as manifestation (as *Erscheinen*):

> Moreover, through the present example we see how pure thought, *disregarding any content given by the senses or even by an a priori intuitioi*, may, solely from the content resulting from its own constitution, bring forth judgments that at first glance seem [*scheinen*] to be possible only on the basis of some intuition. This may be compared with condensation, which transforms the air that to a child's consciousness appears [*erscheinende*] as nothing into a visible fluid that forms drops.[56]

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55. *UBG*, 68: "Im Satze 'der Morgenstern ist die Venus' haben wir zwei Eigennamen 'Morgenstern' und 'Venus' für denselben Gegenstand. In dem Satze 'der Morgenstern ist ein Planet' haben wir einen Eigennamen: 'der Morgenstern' und ein Begriffswort: 'ein Planet'. Sprachlich zwar ist nichts geschehen, als daß 'die Venus' ersetzt ist durch 'ein Planet'; aber sachlich ist die Beziehung eine ganz andere geworden. Eine Gleichung ist umkehrbar; das Fallen eines Gegenstandes unter einen Begriff ist eine nicht umkehrbare Beziehung. Das 'ist' im Satze 'der Morgenstern ist die Venus' ist offenbar nicht die bloße Kopula, sondern auch inhaltlich ein wesentlicher Teil des Prädikats, so daß in den Worten: 'die Venus' nicht das ganze Prädikat enthalten ist."

56. *BS*, § 23, p. 55: "Ausserdem sieht man an diesem Beispiele, wie das von jedem durch die Sinne oder selbst durch eine Anschauung a priori gegebenen Inhalte absehende reine Denken allein aus dem Inhalte, welcher seiner eigenen Beschaffenheit entspringt, Urtheile hervorzubringen vermag, die auf den ersten Blick nur auf Grund irgendeiner Anschauung möglich zu sein *scheinen*. Man kann dies mit der Verdichtung vergleichen, mittels deren es gelungen ist, die dem kindlichen Bewusstsein als Nichts *erscheinende* Luft in eine sichtbare tropfenbildende Flüssigkeit zu verwandeln." (Emphasis mine.)

In Frege's "Über Sinn und Bedeutung," the distinction between reality and its appearance is present in a threefold way:[57]

1) In a sentence Frege adduces towards the beginning of the long development on the semantic behavior of subordinate clauses (*USB*, 51–64), to illustrate the phenomenon of the significance and sense of entire clauses in certain oblique contexts, the distinction occurs as that between the apparent motion of the Sun in the Earth's skies on the one hand and the real motion of the Earth on the other—that is to say, it is instantiated precisely in the form characteristic of it at the birth of modern science, in the heliocentric theory of Copernicus, as we saw in § 2.1 above.[58]
2) In the warning against the presence of seemingly singular terms (*scheinbare Eigennamen*) in natural language and in the concomitant requirement that no pseudo-referential singular term be allowed into the concept-script, the distinction appears as the opposition between *grammatisch richtiger Weise* (grammatically correct fashion) and *in der Tat* (in reality).[59]
3) It is operative in the discussion of the semantic behavior of singular terms (in relations of identity, primarily) and of subordinate clauses that occupies Frege in the article, from beginning to end.

I shall attend to the last of these three occurrences of the distinction, especially as it concerns the analysis of subordinate clauses. A brief description of the subject

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57. See also the late paper "Gedankengefüge" (*GF*, 79): "Wenn behauptet wird, '5 ist kleiner als 4 oder 5 ist grösser als 4', hat jeder der Teilsätze die sprachliche Form, die er auch hätte, wenn er einzeln mit behauptender Kraft ausgesprochen würde, während *in der Tat* nur das ganze Gefüge als wahr hingestellt werden soll." (Emphasis mine.)

58. *USB*, 52: "'Kopernikus glaubte, daß der Schein der Sonnenbewegung durch die wirkliche Bewegung der Erde hervorgebracht werde.'" TRANSLATION: "'Copernicus believed that the apparent motion of the sun is produced by the real motion of the Earth.'" Because of the hybrid character of natural language, the relation between reality and appearance within language is not causal.

59. See *USB*, 55–56: "Von einer logisch vollkommenen Sprache (Begriffsschrift) ist zu verlangen, daß jeder Ausdruck, der aus schon aufgeführten Zeichen in *grammatisch richtiger Weise* als Eigenname gebildet ist, auch *in der Tat* einen Gegenstand bezeichne, und daß kein Zeichen als Eigenname neu eingeführt werde, ohne daß ihm eine Bedeutung gesichert sei. Man warnt in den Logiken vor der Vieldeutigkeit der Ausdrücke als einer Quelle von logischen Fehlern. Für mindestens eben angebracht halte ich die Warnung vor scheinbaren Eigennamen, die keine Bedeutung haben." (Emphasis mine.) TRANSLATION: "Of a logically perfect language (concept-script) one must require that each expression that has been formed, according to correct grammar, as a proper name out of already introduced signs in reality designate an object and that no new sign be introduced as a proper name without being secured a reference. Logic books warn against logical errors that arise from the equivocity of expressions. I consider as at least as pertinent a warning against seeming proper names, which have no reference."

matter of "Über Sinn und Bedeutung" and its main divisions is necessary to usher in my point. The text begins with the famous puzzle regarding statements of identity: how can they be informative? In other words, how can one account for the fact that 'a = b', unlike 'a = a', which is redundant and trivially true, expands our knowledge and is not trivially true, if it is true? The key to the account will consist in taking into consideration both the sense (*Sinn*) and the reference (*Bedeutung*) of expressions such as 'a' and 'b', which, as we have seen, are singular terms.

Frege appeals to the telescope as an analogy in order to introduce his threefold distinction of significance (*Bedeutung*), sense (*Sinn*), and representation (*Vorstellung*) and to elucidate the relationship among those three factors:

> The significance of a proper noun is the object itself to which we refer by means of the noun; the representation that we have on that occasion is entirely subjective; in between lies the sense, which is no longer subjective as the representation is, yet is also not the object itself. The following comparison may be suited to clarify these relationships. Someone observes the Moon through a telescope. I compare the Moon itself to the significance; it is the object of observation, which is mediated by the real image—which is formed by the lens within the telescope—and by the retinal image of the observer. The real image I compare to the sense, and the retinal image, to the representation or intuition. (*USB*, 44–45)[60]

Consider the case of someone's gazing at the Moon through a telescope. The comparison drawn by Frege is as follows: just as the Moon is to the real image formed on the instrument's lens, and as the real image is to the retinal image of the viewer, so too the significance (*Bedeutung*) of an expression (e.g. an object in the case of a singular term) is to the sense, and the sense to the representation. The sense mediates, or acts as a mediation (*vemittelt*), between the significance and the representation. The sense, albeit perspectival,[61] is objective, mind-independent, and, as such, capable of being shared by a multiplicity of minds. The representation is born by an individual; insofar as it is understood primarily as a sensory given (as is suggested by Frege's analogy), it is not a common possession, as the sense is, but rather a subjective, or private, feature of mind.

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60. For the German text, see n. 35 above.

61. *USB*, 42: "damit ist die Bedeutung aber, falls sie vorhanden ist, doch immer nur einseitig beleuchtet. Zu einer allseitigen Erkenntnis der Bedeutung wurde gehören, daß wir von jedem gegebenen Sinn sogleich angeben könnten, ob er zur ihr gehöre. Dahin gelangen wir nie." TRANSLATION: "the sense, however, illuminates what is meant [*Bedeutung*] always only one-sidedly—should it so happen that what is meant is indeed present at hand. To a knowledge of all sides of what is meant would belong the ability to say at once of any given sense whether it is pertinent to what is meant. That, we never attain." And *USB*, 45: "Das Bild im Fernrohre ist zwar nur einseitig; es ist abhängig vom Standorte; aber es ist doch objektiv, insofern es mehreren Beobachtern dienen kann." TRANSLATION: "The image in the telescope is admittedly only one-sided; it is dependent upon the standpoint of observation; yet it is still objective insofar as it can serve several observers."

Signs (be they single phonetic words, phrases, sentences, or ideograms) can thus be considered in three respects: in respect of their sense, their significance, or the representations that become associated with them over the course of the history and particular circumstances (various cultural and psychological features) of sign users. The representations, being largely subjective, are not relevant to the semantic behavior and use of signs. The semantic behavior of signs is a common possession of the sign users (*USB*, 44 and 46).

After treating the significance and sense of singular terms (40–46), Frege turns to an examination of the sense and significance of entire *assertoric* sentences. An assertion contains a thought, and this thought is its sense. The significance (its *Bedeutung* or truth-value potential) of an assertoric sentence having a (whole) thought as its sense is its truth-value. Frege deems the latter thesis to be an assumption (*Vermutung*) that must be tested.[62] The remainder of the paper consists in variously trying this conjecture. The testing procedure, already applied in the discussion of singular terms, consists in substituting, within a sentence (which may be complex), an expression (which may be an entire sentence) for another expression of *like* significance though different sense in order to ascertain whether the substitution leaves the sentence's significance (its truth-value) unchanged.[63] Since the assumption requiring confirmation states that the significance of an assertoric sentence is its truth-value, the expressions to be replaced will be entire sentences or clauses. If the assumption is correct, then the truth-value of a complex sentence will remain unchanged when one of its component sentences or clauses is replaced by a sentence or clause of like significance, albeit different sense. There are two exceptions to this generalization: direct quotation and indirect speech. In the first, a sentence signifies another sentence, and in the second, a thought. Since the behavior of entire sentences or clauses within complex sentences is to be examined, subordinate clauses are apt candidates for this investigation. Except for the last paragraph, wherein Frege returns to the topic of informative identities and pres-

62. *USB*, 48: "So werden wir dahin gedrängt, den *Wahrheitswert* eines Satzes als seine Bedeutung auzuerkennen." *USB*, 49: "Wenn unsere Vermutung richtig ist, daß die Bedeutung eines Satzes sein Wahrheitswert ist, so muß dieser unverändert bleiben, wenn ein Satzteil durch einen Ausdruck von derselben Bedeutung, aber anderem Sinne ersetzt wird." And *USB*, 50: "Es soll nun die Vermutung, daß der Wahrheitswert eines Satzes dessen Bedeutung ist, weiter geprüft werden." TRANSLATIONS: "We thus see ourselves forced to recognize the *truth-value* of a sentence as being its significance." "If our supposition is correct that the significance of a sentence is its truth-value, then the latter must remain unchanged when a part of the sentence is replaced by an expression of same significance, yet different sense." "The supposition that the truth-value of a sentence is its significance is now to be further tested."

63. Frege uses several verbs to characterize this operation of substitution: *ersetzen* (*USB*, 47, 49, 50, 59, 60, 61, 63) *einsetzen* (51, 52), *eintreten* (62), *vertreten* (64).

ents his account of how they can increase our knowledge, the rest of the text is devoted to this inquiry into the semantic behavior of subordinate clauses (51–64).

The connection between the lengthy treatment of subordinate clauses and the initial and closing reflections on identity and singular terms lies 1) in the fact that an account of each of the two syntactical phenomena involves the semantic factors of both sense and significance, *and* 2) in the fact that the relation of informative identity is one precisely between the significances of the terms that stand in it and, as such, is itself an instance of the procedure of substitution: an informative identity substitutes or, more precisely, licenses the substitution of a singular term for another singular term of like significance, yet of different sense, the singular terms differing, that is, in the mode of presentation[64] or in the description under which their referent is given (the significance of a singular term being its referent). All the substitutions performed in the text presuppose a relation of sameness between the items that are being substituted for each other.

Where, then, is the distinction between reality and appearance in this text? It is again a cleft between a homogeneous syntactical surface and a heterogeneous semantic depth, the latter being what is determinative of logical form and thus decisive for logic. What Frege's inquiry into subordinate clauses shows is that there is not one semantic behavior that corresponds to the syntactical category of subordinate clause, but rather a number of complex behaviors. "From a semantic point of view" (*dem Sinne nach*, 59) or "considered logically" (*logisch betrachtet*, 57), the syntactic category of subordinate clause is thus not a unified or univocal category.[65]

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64. *USB*, 41: "Wenn sich das Zeichen '*a*' von dem Zeichen '*b*' nur als Gegenstand (hier durch die Gestalt) unterscheidet, nicht als Zeichen; das soll heißen: nicht in der Weise, wie es etwas bezeichnet: so würde der Erkenntniswert von $a = a$ wesentlich gleich dem von $a = b$ sein, falls $a = b$ wahr ist. Eine Verschiedenheit kann nur dadurch zustande kommen, daß der Unterschied des Zeichens einem Unterschiede in der Art des Gegebenseins des Bezeichneten entspricht." TRANSLATION: "If the sign '*a*' is distinguished from the sign '*b*' only as an object (here by means of its shape), not as a sign—namely, not in the way it designates something—then the cognitive value of $a = a$ becomes essentially equal to that of $a = b$, provided $a = b$ is true. A distinction can arise only if the difference between the signs corresponds to a difference in the mode of givenness of that which is designated." For the aspectival character of the givenness of objects, see the passages quoted in n. 61 above.

65. Note the opposition between the grammatical and the logical levels at *PW*, 253–54/*NS*, 274: "Where we have a compound sentence consisting of an antecedent and a consequent, there are two main cases to distinguish. The antecedent and consequent may each have a complete thought as its sense. Then, over and above these, we have the thought expressed by the whole compound sentence. By recognizing the thought as true, we recognize neither the thought in the antecedent as true nor that in the consequent as true. A second case is where neither antecedent nor consequent has a sense in itself, but where nevertheless the whole compound sentence does express a thought—a thought that is general in character. In such a case, we have a relation not between judgments or thoughts but between concepts, the relation, namely, of subordination. The antecedent and consequent

The following classifies the results of the inquiry into the semantic behavior of subordinate clauses:

A) In most cases, the sense of a subordinate clause is not a whole thought but only a part of a thought, and its significance is not a truth-value. This is due to either one of two reasons:

1) The words in the subordinate clause have oblique significance: this is to say that the subordinate clause *does not have a thought as its sense* but rather as its significance.
   Category A1 comprises the following cases canvassed by Frege: that-clauses and oblique contexts (51–52); doxic contexts (52–53); inferential contexts (53); final clauses (53); clauses subordinated to verbs of command, request, or prohibition (53); epistemic contexts (53); interrogative contexts (53–54); noun clauses (54); and adjectival clauses (56–57).

2) Owing to the presence in it of an indefinite indicator, the subordinate clause has *a partial thought as its sense*. It expresses a complete thought only in tandem with the main clause.
   Category A2 includes the following cases examined by Frege: quantified conditional sentences (57–58); quantified conditional sentences, with time indicators (58); 'who' or 'what' noun clauses (59); and 'where', 'when', 'whenever' adverbial clauses (59).

B) Some subordinate clauses do have a truth-value as their significance, but they have *more than one thought as their sense*, for the latter includes a part of another thought. Frege examines the examples illustrating Category B on pages 61–64 of his paper.

The procedure of substitution of a clause by another one of equal truth-value thus cannot be applied to the subordinate clauses belonging to Categories A and B.

C) Some subordinate clauses, however, have a complete thought as their sense and thus a truth-value as their significance. Category C is illustrated by the following cases: a main and a subordinate clause having a singular term as a common element (59); clauses of concession (59–60); and the antecedent and consequent clauses of conditional sentences, that is, what are now often called 'material conditionals' (60). The principle of the substitutivity of identicals applies to the clauses of Category C.

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are here sentences only in the grammatical, not in the logical, sense." The first case distinguished by Frege in this citation belongs to what I have called Category C below: it is a truth-functional conditional sentence. The second case belongs to Category A2: it is a universally quantified conditional sentence. Clearly, the opposition between the grammatical and logical conceptions of sentence is one between a syntactic notion of sentence and a semantic one. See also *PW*, 190/*NS*, 207: "I call something a *quasi-sentence* if it has the grammatical form of a sentence and yet is not an expression of a thought, although it may be part of a sentence that does express a thought, and thus part of a sentence proper."

The upshot of this lengthy test of the assumption that the significance of an assertoric sentence having a whole thought as its sense is its truth-value is that those subordinate clauses to which the procedure of substitution cannot be applied do not falsify the assumption, for they are such as do not have a thought as their sense but rather as their significance (Category A1); or such as have less than a thought as their sense (Category A2); or such as have more than one thought as their sense (Category B). In other words, the clauses from categories A and B do not even qualify to take the test.

§ 4. The Complexity of the Singular Terms of Natural Language and the Syntax-Semantics Divide

The following aperçu of the types of singular term used in natural language shows how words belonging to *different parts of speech* may play the *same semantic role*. This account will be more systematic than Frege's, and hence goes beyond him, although it is an outgrowth of his pioneering semantics.

We saw that singular predicative sentences are illustrative of the relation of subsumption and that this relation obtains between objects and concepts or relations or, more generally stated, between objects and functions. Since singular terms are those words or phrases that are used to refer to (or to stand for or pick out) objects (in the wide sense of the term, i.e. individual things), all the sentences being considered here contain singular terms. There are concrete and abstract singular terms. Concrete singular terms stand for things individuated in space and time. Abstract singular terms stand for abstract objects. The following are instances of abstract objects: 1) attributes, that is, higher-order objects (the referents of nominalized predicates[66]); 2) states of affairs (the referents of nominalized assertoric sentences); 3) sign types, in contrast to their tokens or occurrences; 4) geometrical figures, tones, pieces of music, melodies; 5) institutions and their parts (e.g. games and their moves); 6) classes, sets; and 7) numbers.[67]

There are three types of singular term in natural language: deictic expressions, definite descriptions, and proper names:

1) *Deictic expressions* (i.e. expressions used deictically), that is, demonstrative pronouns, such as 'this', 'that'; adverbs such as 'here' and 'now'; personal pronouns such as 'I', 'you', 'he', 'she', 'it', 'we', 'they', 'me', 'him', 'her', 'us', 'them'; expressions composed of a demonstrative, or a possessive, adjective and a common noun, such as 'this pencil', 'their car'; as well as expressions consisting of a common noun and a

66. Nominalization consists in turning a predicate or a sentence into a noun. For example, 'blueness' is the nominalized form of the predicate 'is blue' and 'that I told her about the mortgage' (as in 'that I told her about the mortgage should not make any difference') is the nominalized form of the sentence 'I told her about the mortgage'.

67. This list is drawn from Tugendhat, *VEP*, 500; see the discussion at 500–2. The account in this section is generally indebted to *VEP*.

definite article, such as we find in 'the cat just sat on the mat' and 'the man is watching us'.

The deictic use of the compound expressions belonging to the last subclass is to be contrasted with their sortal use; for example, in 'the airplane is a recent invention' or 'the jaguar is not a domestic animal', 'the airplane' and 'the jaguar', despite their syntactic similarity to 'the cat' and 'the man' in the two prior examples, do not refer to an individual and thus are not singular but general terms (this point was already made in the previous section, regarding the sentence 'The horse is an herbivorous animal', although the notion of sortal was not introduced on that occasion).[68] It should be noted as well that not all occurrences of personal pronouns are cases of deictic use: personal pronouns also admit of anaphoric use.[69]

Deictic expressions are characterized by the fact that their reference is dependent upon their context of use—upon the occasion of speech. What objects they pick out is contingent upon the occasion of their use in that they identify an object not only relatively to the speech situation but also as being in immediate relation to the speech situation. These expressions are said to be deictic, for what they refer to is often established by an act of pointing (we also call them 'demonstratives').[70]

Deictic singular terms offer an excellent example of the type of discrepancy that may obtain between the syntax and the semantics of words in natural language: syntactically speaking, there are seven different parts of speech or categories of words (personal and demonstrative pronouns, adverbs, demonstrative and possessive adjectives, definite articles, and common nouns) that may perform the one semantic function of picking out objects *in a context*. This is *one* of the reasons why the account of singular terms in a semantics of natural language is so labored and so difficult (more difficult than the account of the semantic behavior of predicates).

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68. The class of predicates can be divided into at least two subclasses: the class of sortals and all other predicates. A sortal is a predicate that definitely delimits what it sorts or classifies and does not permit an arbitrary division or partition of that to which it applies. For example, '. . . is a cat' is a sortal, for one cat is definitely demarcated from another, and a part of a cat cannot itself be called a cat. On the other hand, '. . . is blue' is not a sortal. If two objects are of the same hue and value of blue and are definitely delimited from each other, their demarcation from each other will not be due to their being blue. Furthermore, a blue surface can be arbitrarily divided: every part of the surface is itself blue.

69. An anaphoric pronoun is not used to pick out an object on a particular occasion of use, but rather to relay, so to speak, a word or phrase occurring before it. For example, in 'Every person is such that he or she is a vegetarian', the pronouns 'she' and 'he' are anaphoric; they point back to the word 'person'.

70. 'Deictic' stems from the Greek adjective *deiktikos*, which means 'able to show', 'demonstrative', and is related to the verb *deiknunai*, 'to show', 'to point out'. See *PW*, 91/*NS*, 100: "For the word 'that', together with an appropriate pointing gesture, must here be construed as a proper name (in the logical sense) i.e. as a sign for an object." The qualifier 'in the logical sense' is obviously meant to introduce the semantic point of view, just as 'considered logically' (quoted from *USB*, 57) was a few pages above.

2) *Definite descriptions*, such as 'the president of the United States in the year 1959', 'the author of *Hamlet*', 'the leaning tower at Pisa', 'the Greek sculpture of a goddess, with broken arms, at the Louvre Museum in Paris', 'the head of the Palestine Liberation Organization', and 'the discoverer of function-theoretic logic'.

The phrases in this class pick out an object by means of a unique description of that individual thing (the description often being further specified by the context of use).

3) *Proper names*, such as 'Manhattan', 'Sarah', and 'Pablo Picasso'.

Regarding the semantics of singular terms in natural language, there is much to be said, and much has in fact been said in the literature.[71] Suffice it to say here that a careful consideration of the use of proper names reveals them to be higher-level singular terms, which is to say that the semantics of proper names presupposes that of the other two classes of singular terms, which are thereby more fundamental and primitive. Without these two classes of singular terms, proper names alone would not enable us to gain access to the world and its contents.[72]

I now provide the renditions of the sentences given as instances of the relation of subsumption, the first microstructure cited in the initial pages of § 2. This will serve to illustrate the types of singular term.

- Galileo is Tuscan.

GLOSSARY

Galileo: g

. . . is Tuscan: T(. . .)

IDEOGRAPHIC TRANSLATION

Tg

- The highest mountain on Earth is on the border of Nepal and Tibet.

GLOSSARY

The highest mountain on Earth (definite description containing a proper name, similar to an expression such as 'the capital of China'): h

. . . is on the border of . . . and . . . (triadic predicate): B(. . .)(. . .)(. . .)

Nepal (proper name): n

Tibet (proper name): t

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71. See Tugendhat's elaborate treatment of the topic in *VEP*, chap. 21–27. For a more recent contribution to the topic, see Truls Wyller, *Indexikalische Gedanken* (Freiburg/Munich: Alber, 1994).

72. For a short presentation of this relation of presupposition, see Ernst Tugendhat and Ursula Wolf, *Logisch-semantische Propädeutik* (Stuttgart: Reclam, 1983), 146–67.

IDEOGRAPHIC TRANSLATION

Bhnt

- This man robbed the liquor store on Astor Place.

  'this man' is a deictic expression consisting of a demonstrative adjective and a common noun; strictly speaking, there is no room for this sort of singular term in the ideography of logic. In a logical context, it must be replaced by a non-deictic singular term.

  GLOSSARY

  'This man' is to be replaced by the name of the man, namely by 'George Shrub'.

  George Shrub: g

  . . . robbed . . . (dyadic predicate): R(. . .)(. . .)

  the liquor store on Astor Place (definite description, inclusive of a proper name): l

  IDEOGRAPHIC TRANSLATION

  Rgl

- Julia loves Robert.

  GLOSSARY

  Julia: j

  Robert: r

  . . . loves . . . : L(. . .)(. . .)

  IDEOGRAPHIC TRANSLATION

  Ljr

- The Earth has one satellite.

  GLOSSARY

  The Earth: e

  . . . has one satellite: S(. . .)

  IDEOGRAPHIC TRANSLATION

  Se

Although the last sentence illustrative of subsumption, namely 'The horse is in the backyard', was the first sentence to be rendered by ideograms in § 2, it will be of interest to compare the initial formulation of the glossary entries for the

sentence's components with the manner in which the semantic ground covered thus far would enable one to gloss the entries at this point. Here is what it would look like.

GLOSSARY

'The horse', being a deictic expression, must be replaced by the name of the horse. In this case, given the context of utterance, the horse's name is 'Bailey'.

Bailey: b

Since '. . . is in the backyard' is a monadic predicate containing a deictic expression, the demonstrative must be replaced by a proper name or a definite description, and since the backyard does not have a name of its own, we shall use a definite description containing the name of the person owning the backyard.

. . . is in the Adlers' backyard: Y(. . .)

IDEOGRAPHIC TRANSLATION

Bb

This concludes the discussion of microstructures begun at the inception of § 2.

§ 5. Conclusion

§ 5.1 Semantic and Pragmatic Limitations of the Concept-Script, and the Task of Logic

Frege's ideography or universal characteristic aims at universal applicability, namely, applicability to any region of things inquiry into which requires proof construction and testing of the binding force or validity of the inferences constitutive of a proof. This requires clarification, as the linguistic status that is claimed on behalf of the ideography may occasion a certain perplexity (e.g. how could "this" be a language, even if only a written one?) and cause one to lose sight of the task, or the purpose, of logic.[73]

Since Fregean logic is said to be a language (a set of signs, rules for the formation of strings of signs, and rules for the transformation of the strings) and since our closest model of language is naturally supplied by the tongue we speak, the concept-script looks rather impoverished when one compares it to one's natural language.

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73. *PW*, 3/*NS*, 3: 'Psychology . . . does not study the property 'true' as, in its field, physics focuses on the properties 'heavy', 'warm', etc. This is what logic does. It would not perhaps be beside the mark to say that the laws of logic are nothing other than an unfolding of the content of the word 'true'. Anyone who has failed to grasp the meaning of this word—what marks it off from others—cannot attain to any clear idea of what the task of logic is."

Many may feel that it barely even deserves to be called a language. Indeed, considered in separation from its task and contrasted with natural language, the script exhibits two limitations, one concerning what it can say, the other regarding what it can do. Both become visible when the script is examined in light of the distinction between form and content (or force and sense, respectively) that is constitutive of natural language.

What is meant by the form-content difference of natural language? Content is what is said by an utterance, that is, the matter it conveys and entertains—its propositional content, as it is also called. Form is what the utterance does, the act it performs, what is also called its illocutionary force;[74] an important aspect of illocutionary force is the illocutionary point of the utterance.[75] An utterance's force may be to issue a command, to state a truth, to formulate a wish, to baptize a child, to marry two people, to ask a question, to welcome someone, to apologize, to congratulate, to toast, to bless, to curse, to veto, to declare open, to repeal, to urge, to dedicate, and so forth. Two utterances may have the same content, yet different forms: an example being the two utterances 'It is raining' and 'Is it raining?' Although both sentences entertain the same content (the fact of raining) and, although each says the same thing and, in this case, even uses the same words, each does something different, the word order and the punctuation being what makes the difference in this case: the first utterance makes an assertion or statement, whereas the second one raises a question. Each, in other words, performs a different speech act; each has a different force (the word order, the punctuation, and the mood of the verb being the force indicators in this example).[76]

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74. The distinction between form and content being introduced at this point is different from the one put forth in the Semantics subsection of the Introduction above. The concept of semantic or logical form characterizes a feature of the propositional content of an utterance, whereas the concept of pragmatic form (the illocutionary force) specifies the type of relation the propositional content of an utterance entertains with the world and other speakers.

75. John Searle rightly discerns several strands in the fabric of any illocutionary force, the three most important ones being illocutionary point, direction of fit, and sincerity condition. See *TIA*, 2–12. Regarding illocutionary point, he remarks: "The point or purpose of a type of illocution I shall call its *illocutionary point*. Illocutionary point is part of but not the same as illocutionary force. Thus, e.g., the illocutionary point of requests is the same as that of commands: both are attempts to get hearers to do something. But the illocutionary forces are clearly different. In general, one can say that the notion of illocutionary force is the resultant of several elements of which illocutionary point is only one, though, I believe, the most important one" (*TIA*, 3).

76. Natural languages utilize a number of devices—some being syntactical—to indicate illocutionary force: e.g. mood (English counts three moods: the indicative, the subjunctive, and the imperative; many languages have more), word order (used to indicate wishes and questions, for example), stress, and intonation, and in writing punctuation (e.g. the question mark).

As regards content, there are certain things that the concept-script is constitutionally unable to say. Firstly, it cannot refer to itself and, as such, is incapable of self-reflection. Briefly stated, the reason for this limitation lies in the fact that the structural correspondence between the syntactical and semantic roles of the ideography's signs is tighter than in natural language.[77]

One may feel that citing this limitation is out of place, for it does not concern the fields of possible application of the ideography, or the extension of the script's universe of discourse, envisaged by Frege in the preface to *Concept-Script*, namely the various domains of (natural and ideal) things, their respective sciences and, in some cases at least, their burgeoning ideographies. Granted, the script need not be able to speak about itself in order to exhibit complex logical forms and, on their basis, to make possible the construction of gapless (in Frege's terms) valid proofs. However, if one considers self-reference to be an important aspect of philosophical discourse, and if one observes that Frege states, again in the preface to *Concept-Script*, that the script may be fruitfully applied in philosophy, then it may seem less inappropriate to cite this limitation in the present context. Moreover, Frege himself was not indifferent to self-referential consistency, as evidenced by the following text: "If anyone tried to contradict the statement that what is true is true independently of our recognizing it as such, he would by his very assertion contradict what he had asserted: he would be in a similar situation to the Cretan who said that all Cretans are liars."[78]

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77. *WBB*, 94: "Wir schaffen uns künstliche Hände, Werkzeuge für besondere Zwecke, die so genau arbeiten, wie die Hand es nicht vermöchte. Und wodurch wird diese Genauigkeit möglich? Durch eben die Starrheit, die Unveränderlichkeit der Theile, deren Mangel die so vielseitig geschickt macht. So genügt auch die Wortsprache nicht." TRANSLATION: "We fashion artificial hands for ourselves, tools for specific purposes, which work with a precision of which the hand would not be capable. And by what means is this precision attained? By the fixity, the unchangeable character of the parts, the lack of which makes the hand so severally adroit. In the same way, natural language does not suffice."

78. *PW*, 132/*NS*, 144. Frege is pointing here to very difficult logical problems. The particular phenomenon he is touching upon with the example of the assertion the propositional content of which denies that it is true or that there is truth now has a name: it is called 'performative contradiction'. A performative or pragmatic contradiction is an utterance that contradicts itself, i.e. in which the semantic and pragmatic aspects stand in a relation of contradictory opposition: for example, the propositional content of the assertion 'There is no truth' denies its illocutionary force. In the assertoric utterance 'There is no truth', what is said by the utterance contradicts or denies what the utterance, qua speech act, does, namely state a truth. It is what some also describe as a self-refuting utterance.

Note that formalizing the propositional content of 'There is no truth' will not reveal the contradiction under discussion. Unlike a semantic or analytic contradiction, a pragmatic one is not self-exhibiting (which is not to say that a pragmatic contradiction cannot be transformed into an analytic one).

It is erroneous, however, to think that the liar's paradox is similar in structure to 'There is no truth'. If 'There is no truth' is true, then it is false, and if it is false, then it is true, and so

More immanently, it must be noted that not only can Frege not dispense with natural language in either scientific, or logical, or philosophical matters, but that he also relies on its self-referential ability in order to expose and to warn us against its misleading and falsifying character, to wage polemics against and cast suspicion on it, all the while expressing irritation at its self-referentiality and the problems it causes (witness the famous sentence 'the concept *horse* is not a concept' at *UBG*, 71).[79] On the other hand, one must recognize that although natural language is enormously flexible[80] and thus gives the impression that it can speak about nearly

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on: in other words, its truth-value oscillates endlessly. This is not the case with the sentence 'All Cretans are liars' as uttered by the Cretan Epimenides. If the sentence is true, then it is false; but if it is false, then it is not the case that all Cretan are liars, and if not all Cretans are liars, then at least one is telling the truth. But if at least one Cretan is telling the truth, then the sentence cannot be true.

The interested reader will find an excellent discussion of the phenomenon of performative contradiction in François Récanati, "Pragmatic Paradoxes," *Graduate Faculty Philosophy Journal* 17 (1994), 289–98. For a formal logician's view on self-reference, see Frederic B. Fitch, "Self-Reference in Philosophy," *Mind* 55 (1946), 64–73. Fitch speaks of self-referential inconsistency instead of performative contradiction. Referring to Descartes's hyperbolic doubt, he points out the positive role that performative contradictions can play in philosophy (67). The reader will find further examples of the use of performative contradictions in indirect philosophical arguments in Vittorio Hösle, "Foundational Issues of Objective Idealism," *Graduate Faculty Philosophy Journal* 17 (1994), 245–87, here 267–77. To my knowledge, most work in this area of logic is being done by German philosophers and logicians. For a far more extended discussion and further references, see Dieter Wandschneider, *Grundzüge einer Theorie der Dialektik* (Stuttgart: Klett-Cotta, 1995), and Bernd Brassel, *Das Programm der idealen Logik* (Würzburg: Königshausen & Neumann, 2005).

79. Bluntly stated, natural language is good enough to say how bad it is. In more nuanced fashion, *Concept-Script* acknowledges the indispensable character of natural language: "A number of fundamental principles of thought has already been introduced in the first part, with a view to transforming them into rules for the application [*Anwendung*] of our signs. These rules and the principles of which the rules are copies [*Abbilder*] cannot be expressed in the ideography because they form its basis [or justification] [*ihr zu Grunde liegen*]" (*BS*, § 13, p. 25). As a result, the "fundamental principles of thought" that constitute the originals from which the copies are drawn must be expressed in natural language. In the *Posthumous Writings*, Frege likewise recognizes the fact that there is no dispensing with natural language. For example, at *PW*, 266/*NS*, 285: "These investigations are especially difficult because in the very act of conducting them we are easily misled by language: by language that is, after all, an indispensable tool for carrying them out. Indeed, one might think that language would first have to be freed from all logical imperfections before it was employed in such investigations. But, of course, the work necessary to do this can itself only be done by using this tool, for all its imperfections." And at *PW*, 37/*NS*, 42: "Because modes of inference must be elucidated in words"; and *PW*, 39/*NS*, 44: "In my *Concept-Script* I laid down nine axioms, to which we must add the rules set out in words".

80. See *WBB*, 110: "Die hervorgehobenen Mängel haben ihren Grund in einer gewissen Weichheit und Veränderlichkeit der Sprache, die andrerseits Bedingung ihrer Ent-

anything, it does in fact encounter limitations when it attempts to express complex logical or mathematical relations.[81] Historically, the study of the behavior of numbers in their relations to each other, which is, of course, none other than mathematics, left natural language behind and sought ideographic means of expression as soon as it reached a certain degree of complexity. This exit out of natural language was unavoidable insofar as ideograms alone afford semiotic means sufficiently perspicuous to allow the manifestation of the very fine distinctions and complex relations taken into view by mathematics, for these new semiotic means (the ideographic ones) are free of the burden of phonetic phenomena, their irrelevant (to the task at hand, to be sure) psychological and historical evocations, and the ambiguities, vague expressions, and pseudo-referential terms that litter natural language. In fact, natural language's difficulties with mathematical matters began well before the development of complex mathematics, namely at the more elementary level of the fashioning of a user-friendly and effective system of numeration for various practical (e.g. bookkeeping and accounting procedures) and theoretical purposes (e.g. the recording of astronomical observations). The place-value system of numeration is the most efficient, economical, and clearest one civilization has devised. In this respect, one should note that the Arabic-Hindu ciphers or numerals that stand at the center of our place-value system are already ideographic, not phonetic signs. That is why they do not need to be translated when used in the various natural languages.[82]

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wickelungsfähigkeit und vielseitigen Tauglichkeit ist. Die Sprache kann in dieser Hinsicht mit der Hand verglichen werden, die uns trotz ihrer Fähigkeit, sich den verschiedensten Aufgaben anzupassen, nicht genügt." TRANSLATION: "The flaws underscored above are due to a certain softness and adaptability of [natural] language, which on the other hand is the condition of its capacity to develop and its many-sided usefulness. In this respect, language may be compared with the hand, which, despite its ability to adapt to the most varied tasks, does not suffice."

81. Frege mentions this, too, at *BS*, x: "Damit sich hierbei nicht unbemerkt etwas Anschauliches eindrängen könnte, musste Alles auf die Lückenlosigkeit des Schlusskette ankommen. Indem ich diese Forderung auf das strengste zu erfüllen trachtete, fand ich ein Hindernis in der Unzulänglichkeit der Sprache, die bei aller entstehenden Schwerfälligkeit des Ausdruckes doch, je verwickelter die Beziehungen wurden, desto weniger die Genauigkeit erreichen liess, welche mein Zweck verlangte. Aus diesem Bedürfnisse ging der Gedanke der vorliegenden Begriffsschrift hervor." TRANSLATION: "To bar the unnoticed intrusion of any intuitive factor, it was all a question of the inferential chain's being free of gaps. In attempting to satisfy this requirement in the strictest possible way, I encountered an obstacle in the shortcomings of *language*, which, for all the resulting ponderousness of expression, *allowed for the precision demanded by my purpose all the less as the relations increased in complexity*. Out of this want arose the thought of the present concept-script." (Emphasis mine.)

82. For an informative discussion of systems of numeration and their respective merits, see John A. Peterson and Joseph Hashisaki, *Theory of Arithmetic* (New York: Wiley, 2d ed., 1967), chap. 1.

Secondly, as we saw above in § 4, the concept-script does not countenance deictic singular terms. *Because the reference of deictic terms depends on the situation of speech, there are no equivalents in the ideography of logic for deictic words*. In logic, deictic expressions are systematically replaced by non-deictic singular terms. Insofar as proper names gain access to the world only via deictic terms, the script depends on natural language to secure its relation to the world.

Thirdly, the script has no room for fuzzy predicates such as '. . . is a heap' (see *BS*, § 27, where Frege bans them and therewith sorites from the script), which are, however, omnipresent in ordinary language.

The fourth limitation lies on the side of form. The script admits of two kinds of speech act: 1) one may assert, or make truth claims, and 2) one may stipulate the content or sense of a new sign. The issuance of truths is by far the most frequent activity, the act of stipulation being used to achieve concision in the expression of frequently employed complex notions. In keeping with this twofold ability, the script has two force indicators: the assertion-stroke ('⊢') and the stipulation-stroke ('⊩'). The latter is used to indicate that a new sign is being defined; it introduces a new sign into the ideography. The new sign abbreviates a complex concatenation of signs (for the first such definition in *BS*, see formula 69 in § 24). Among the force indicators, one might include the single and double horizontal lines that Frege uses as argument indicators (see *BS*, § 6, where they are introduced). The two argument indicators, however, *qualify* the assertion-stroke, and therefore they presuppose it (i.e. they cannot occur in separation from it). Indeed, they indicate that the content being asserted is so on the strength of prior assertions (i.e. insofar as it follows from and is justified by prior assertions). There are thus numerous speech acts that the script cannot perform (or, more accurately, that a subject cannot perform within the script): the entire range of acts that permeates practical everyday life (commands, wishes, promises, questions, to name but a few) is not available to the ideography (a brief outline of a broader and more systematic perspective on speech acts will close these concluding remarks). The illocutionary range of a language consists in the greater or lesser variety of forces expressible in it, and, thereby, in the variety of speech acts performable within it. The breadth of illocutionary acts performable within natural language is as wide as it is because of the many diverse tasks it is called upon to carry out in average daily life: a preponderance of these tasks is practical and intersubjectively oriented, which is to say that utterances are issued with the intent to communicate with others and to effect changes in the surrounding world.

By contrast, the concept-script's pragmatic aim is less to communicate and elicit beliefs in the reader than it is first of all to advance truths and to offer their arrangement in an inferential sequence for the reader's consideration (see Frege's proposal for the ideal of mathematics at *PW*, 157/*NS*, 171). As far as Frege's primary project is concerned, however, it is true that the latter end is itself to serve the further one of testing the assumption that the propositions of arithmetic are logical

theorems, in other words, that they are derivable from axioms (logical truths) and logical definitions alone. Hence, the script's long-term pragmatic aim is to win over the reader to the thesis of the logical reducibility of arithmetic and thus to give rise to a belief.[83] These goals account for the narrow illocutionary range of the script. *The types of force expressible in the script are consonant with its theoretical office*: to offer a highly controllable medium for proof construction and conceptual analysis (a logically analyzed concept, if it has a certain degree of complexity, may be better represented by a new, simpler sign, which is introduced by stipulation). What makes the medium highly controllable is the mode of display of the proofs and the requirements that preside over it. To achieve the above goals, two basic features of the deductive sequences that make up proofs must be amenable to rational controls: their correctness and completeness.

1) The assessment of correctness depends on the formal intelligibility of the propositions that make up the argument or the proof, which itself depends on the perspicuity, disambiguated character, and syntactical regimentation of the semiotic means. Syntactical regimentation consists of two things: the framing of rules of well-formedness of propositions, and the closer alignment of syntax upon semantics. Insofar as the looseness of the relation between syntax and semantics within natural language is productive of ambiguities, there is a certain overlap between two of the above conditions.
2) An inferential chain is complete when there are no gaps in it, when none of its links are missing. The assessment of completeness depends upon there being rules of inference, which are nothing other than rules for the production of the links that make up an inferential chain. If every link in a given chain has been derived from one or more previous links in accordance with a rule of inference, then the concatenation is complete.

The fourth limitation of the script was characteristic of systems of logic until the advent of extended logics in the twentieth century; it is the expression of logic's age-old option for truth-claiming discourse, whether or not the latter includes modalities, as does, for instance, Aristotle's modal syllogistic (*Prior Analytics*, Book A, chap. 8–22).

As the preface to *Concept-Script* shows, Frege was entirely aware of this:

> I believe that the relation of my concept-script to the language of life [*Sprache des Lebens*] can be most clearly brought out if I compare it to the microscope's relation to the eye. Because of the range of its uses and the versatility with which it can adapt to the most diverse circumstances, the eye is far superior to the microscope. It is true that when considered as an optical

83. See *PW*, 204/*NS*, 220: "A proof does not merely serve to convince us of the truth of what is proved; it also serves to reveal logical relations between truths."

> instrument, it shows many imperfections, which ordinarily go unnoticed only as a result of its intimate connection with our mental life. However, as soon as scientific purposes require greater sharpness of discrimination, the eye proves to be insufficient. The microscope, on the other hand, is perfectly suited to precisely such purposes, but that is just why it is useless for all others. This ideography, likewise, is a device invented for certain scientific purposes, and one must not condemn it because it is not suited to others. (*BS*, xi)

(This passage and the historical context to which it points were discussed in §§ 2 and 3 above.)

The purpose of pointing out these four limitations is thus not to find fault with the ideography but rather to demarcate it explicitly, which, it is to be hoped, should dispel any misgivings or qualms the reader may have. To be sure, in certain contexts these two limitations could be cited as points of criticism, for example, if unduly general claims were made on behalf of the script or if certain normative claims were made that would presuppose that the script offers an exhaustive model of language and of its logical intricacies. I hasten to add that this last remark should not be taken to mean that I subscribe to the view that there is nothing normative about logic: I am agreement with Frege that logic has a normative dimension.[84]

Now the ideography precisely makes the distinction between form and content semiotically manifest. Indeed, since the script has a sign of assertion (the vertical stroke that I cited above and that is placed at the leftmost end of a formula) and a sign of content (the horizontal stroke that is attached to the immediate right of the vertical stroke, also as shown), it, unlike natural language, distinguishes *in its very signs* between assertion and assertible or judgeable content,[85] that is, between illocutionary force and propositional content. To say that it distinguishes *in its signs* between the force of assertion and the content that is asserted is to say that the distinction is made visible in its signs, which is only the case for some illocutionary forces in natural language: in particular, natural language does not have an unequivocal sign of assertion, for even the indicative mood is not an entirely reliable sign of assertion.

§ 5.2 On the Classification of Speech Acts

The following is meant to give a broader perspective on the phenomenon of linguistic force by briefly discussing the attempts made by philosophers at gradually organizing the theory of speech acts into a systematic whole.

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84. For example, *PW*, 128/*NS*, 139: "Like ethics, logic can also be called a normative science." See also 4/4, 145/157, and 149/161.

85. Note the following distinction at *PW*, 2/*NS*, 2: "Inwardly to *recognize something as true* is to *make a judgment*, and to give expression to this judgement is to make an assertion." See also 139/150.

Although, as we have just seen, the distinction between force and content is present in Frege's work *ab ovo*, it is also true that Frege never produced a fuller theory of illocutionary forces.[86] The author of the English translation of Frege's *Foundations of Arithmetic*, John Langshaw Austin, was to do just that in his watershed lectures on pragmatics, delivered in 1955 and posthumously published as *How to Do Things with Words*. Austin generalized Frege's notion of assertoric force as the concept of illocutionary force. His work has stimulated the growth of a large body of complex theory, described as pragmatics or speech-act theory.

Austin introduces the notion of force as follows: "Admittedly we can use 'meaning' also with reference to illocutionary force—'He meant it as an order', et cetera. But I want to distinguish *force* and meaning, in the sense in which meaning is equivalent to sense and reference, just as it has become essential to distinguish sense and reference."[87] Austin estimates the number of English illocutionary verbs at about one thousand.[88] He then goes on to give a fivefold classification of these verbs.[89]

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86. The distinction between force and content is also to be found in *PW*, e.g., 168/*NS*, 183, 177/192, and 185/201, and in "Der Gedanke," 35: "In einem Behauptungssatz ist also zweierlei zu unterscheiden: der Inhalt, den er mit der entsprechenden Satzfrage gemein hat und die Behauptung. Jener ist der Gedanke or enthält wenigstens den Gedanken." TRANSLATION: "Therefore, in an assertoric sentence two things are to be distinguished: the content, which it has in common with the corresponding interrogative sentence, and the act of assertion. The former is the thought or at least contains the thought."

The text also notes the connection between force and form: "In der Form des Behauptungssatzes sprechen wir die Anerkennung der Wahrheit aus. Wir brauchen dazu das Wort 'wahr' nicht. Und selbst, wenn wir es gebrauchen, liegt die eigentlich behauptende Kraft nicht in ihm, sondern in der Form des Behauptungssatzes . . ." (ibid.). TRANSLATION: "In the form of the assertoric sentence we express the recognition of truth. We do not need the word 'true' for this. And even when we do use it, the properly assertoric force does not lie in it, but in the form of the assertoric sentence. . . ." There are contexts, however, that neutralize force with respect to the actual world (in contrast to some other possible world): all contexts of fictional discourse—novels, plays, operas—are such (see ibid., 35–36).

87. J. L. Austin, *How to Do Things with Words* (Cambridge, Mass.: Harvard University, 1975), 100. The Egyptologist Alan Gardiner laid bare the performative aspects of speech before Austin, in his great but unfortunately insufficiently acknowledged work *The Theory of Speech and Language* (1932) (Oxford: Clarendon, 2d ed., 1951). Like Austin, Gardiner draws a distinction between form and content: words and sentences have both form and content, and both their form and content contribute to their meaning. For instance, a singular term (say, a proper name) picks out an object, and that is its content. At the same time, however, it *shows* that it is a name; it displays its form. Since a sentence's form is its "illocutionary force," as Austin will say, Gardiner already distinguishes between force and content. What Austin will call 'illocutionary force' or simply 'force', Gardiner calls 'special sentence-quality' (see pp. 181–90 of Chapter IV, "The Sentence and its Form").

88. Austin, *How to Do Things with Words*, 150.

89. Ibid., 151–64.

John Searle's 1975 paper, "A Taxonomy of Illocutionary Acts," offers a more refined sorting of illocutionary phenomena. When discussing Austin's classification (*TIA*, 8–12), Searle rightly observes that Austin's five lists of verbs "are not classifications of illocutionary acts but of English illocutionary verbs" and that "Austin seems to assume that a classification of different verbs is *eo ipso* a classification of kinds of illocutionary acts, that any two non-synonymous verbs must mark different illocutionary acts" (9). The paper also introduces Searle's powerful distinction between the word-to-world direction of fit and the world-to-word direction of fit of speech acts. The first direction of fit (the fitting of word to world) is characteristic of all truth-claiming discourse: the illocutionary point of aletheic speech acts is to get their propositional content into agreement with the world. The second direction of fit (i.e. the fitting of world to word) characterizes acts such as vows, promises, commands, orders, requests, and wishes: the illocutionary point of these and other practical speech-acts is to get the world into agreement with their propositional content. *The illocutionary force is the aspect of the speech act that determines how the propositional content is to relate to the world: it specifies the direction of fit between words and world*. The distinction is powerful, for it brings much order to the seemingly endless profusion of uses of language (or language games) that came to characterize natural language on Wittgenstein's view. Searle is right, contra Wittgenstein, to say that

> the illusion of limitless uses of language is engendered by an enormous unclarity about what constitutes the criteria for delimiting one language game or use of language from another. If we adopt illocutionary point as the basic notion on which to classify uses of language, then there are a rather limited number of basic things we do with language. . . . (29)

The year 1976 saw the publication of Ernst Tugendhat's introductory lectures to the philosophy of language.[90] In the book, Searle's contributions to speech pragmatics in *Speech Acts* often come up for discussion. Having had no knowledge of Searle's then just published paper on taxonomy, Tugendhat went on in the final chapter of the book to put forth a unifying concept of illocutionary forces that is the same as Searle's concept of fit (*VEP*, 508–10), namely that of an agreement between sentence and reality (*Übereinstimmung von Satz und Wirklichkeit*). There are only two ways, Tugendhat writes, in which agreement between sentence and reality may obtain: 1) in the case of assertions, reality being the standard of measure, if there is agreement, the sentence corresponds to reality, or—to use Searle's term—

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90. Ernst Tugendhat, *Vorlesungen zur Einführung in die sprachanalytische Philosophie* (Frankfurt a. M.: Suhrkamp, 1976); English translation: *Traditional and Analytical Philosophy: Lectures on the Philosophy of Language*, trans. P. A. Gorner (Cambridge: Cambridge University, 1982). See Richard Rorty's enthusiastic and informative review of the book in *The Journal of Philosophy* 82 (1985), 720–29.

fits reality; 2) in the case of sentences such as commands, wishes, or requests, the sentence being the measure, reality is to correspond to the sentence (or is to be made, by appropriate human or other agency, to correspond to, or fit, the sentence), if there is to be agreement (510). "As there are two and only two modes of agreement, one would have to recognize," Tugendhat says, "that there are only two basic sentential forces: agreement conditions are either truth conditions or fulfillment conditions" (510). In other words, assertions have truth conditions, and non-assertions have fulfillment conditions. (As far as questions are concerned, there are two kinds of questions: questions that request sentences as answers, even if it is only in the implicit form of a 'yes' or a 'no', and questions that expect singular terms as answers, the so-called what-, who-, where-, which-, when-questions.) Tugendhat then immediately takes issue with Searle in terms that are analogous to Searle's critique of Wittgenstein's proliferation of fuzzily related language games, which admits only of an organization according to family resemblances: "Whereas Searle multiplies the 'illocutionary forces' far beyond the number of grammatical forces, here a diversity of grammatical forces would be ranged under one basic force" (510). Searle lists 12 criteria in his taxonomy paper for distinguishing illocutionary acts (*TIA*, 2–8), but he nonetheless deems three more important than the others (illocutionary point, direction of fit, and sincerity condition), which at least mitigates Tugendhat's reproach of needless multiplication of forces.

In accordance with this unifying concept of agreement or fit, the classification of the data of speech-act theory undergoes great simplification, and greater systematicity accrues to this part of the philosophy of language. Tugendhat indeed proposes to divide all sentences into two large classes: 1) theoretical or aletheic sentences and 2) practical or non-aletheic sentences. Understanding the first involves understanding their truth-conditions; understanding the second requires understanding their fulfillment conditions. This is not to deny that there are many illocutionary forces; rather, the point is that many are specifications of the two main genera, and that, unlike the two main forces, these do not contribute to the sense of the sentences but instead supplement their communicative and expressive aspects. In addition, there are utterances lacking in, or of unsaid propositional content, such as 'thanks!', 'good day', goodbye', 'hurray', 'wow', or 'outch'. Tugendhat thinks that there are also hybrid sentences, that is, sentences having both a practical and a theoretical aspect.

I conclude with François Récanati's treatment of the topic of the classification of illocutionary forces in his contribution to the theory of performative utterances.[91] His discussion begins with Searle's taxonomy paper. He agrees with Searle

91. François Récanati, *Les énoncés performatifs* (Paris: Minuit, 1981), hereinafter cited as *EP* with page reference; English translation: *Meaning and Force: The Pragmatics of Performative Utterances* (Cambridge: Cambridge University, 1987).

that some criteria of classification are more important than others. Considering the three main criteria elevated by Searle—namely, illocutionary point, sincerity condition or psychological state expressed, and direction of fit—Récanati argues that the first two implicitly presuppose a classification into types and that the third one, the direction of fit, is what makes possible this classification of illocutionary points and psychological states (*EP*, 178–79). Without having any knowledge of Tugendhat's book, he thus agrees with Tugendhat that the direction of fit offers the "fundamental criterion" (179) for organizing the manifold of forces.[92] But he qualifies this last assertion by saying that the criterion in question applies only to illocutionary acts endowed with a referential aspect, namely a propositional content, thereby concurring again with Tugendhat. The first two divisions in the classificatory tree of illocutionary acts thus consist of those acts that have propositional content (say, the left branch of the tree) and of those that are without content (the right branch of the tree).[93] The group of acts with propositional content in turn divides into performatives and constatives. This division overlaps with Tugendhat's division into practical and theoretical sentences. It may even be that Récanati's overall very subtle analysis has room for Tugendhat's notion of hybrid sentences. It is a very important issue since normative sentences fall into that category.

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92. However, in a footnote on p. 157 of the English translation of *EP*, Récanati acknowledges his agreement with Tugendhat on this point.

93. One may take issue with ranging acts of thanking and of congratulating, e.g., in the class of acts that have no content, on the ground that we do express thanks or congratulations *for something*. Although that point is well taken, it is also true that the occasion for the issuance of thanks or congratulations need not be specified in the utterance; it is precisely the occasion of the act, not its content.

Bibliography

Conventions of Citation: All abbreviations of the titles of Frege's works used in the text are placed to the left of the bibliographic entry below for easy reference. Translations are listed immediately after the originals. (Existing translations were modified as deemed necessary, and without notice—this is particularly true in the case of *Begriffsschrift*.) Translations of works cited for which no English translations exist are my own. All instance of emphasis in quotations stem for the cited author unless otherwise noted. When a work is cited repeatedly and in uninterrupted succession within the body of the text, its abbreviation is cited only in the first instance. Page numbers are given without the abbreviation of the word 'page', except where confusion with a section number might occur, as in the case of citations from *BS* or *GA*.

Works by authors other than Frege are listed in the bibliography only if frequency of citation or convenience requires such. As in the case of Frege's writings, abbreviations of the titles of writings by authors other than Frege are located in the left-hand margin alongside the relevant bibliographic entry. All other works and abbreviations are referenced in the footnotes.

Works by Gottlob Frege

BLHP "Ueber den Briefwechsel Leibnizens und Huygens mit Papin" (1881). In Gottlob Frege, *Begriffsschrift und andere Aufsätze*, ed. Ignacio Angelelli (Darmstadt: Wissenschaftliche Buchgesellschaft, 1977), 93–96; English translation: "On the Correspondence of Leibniz and Huygens with Papin," trans. Marcus Brainard and Pierre Adler, *New Yearbook for Phenomenology and Phenomenological Philosophy* VIII (2008), 321–23.

BS *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens* (1879). In Gottlob Frege, *Begriffsschrift und andere Aufsätze*, ed. Ignacio Angelelli (Darmstadt: Wissenschaftliche Buchgesellschaft, 1977), v–xvi, 1–88; translations:

1) *"Begriffsschrift," a formula language, modeled upon that of arithmetic, for pure thought*, trans. Stephan Bauer-Mengelberg. In Jean van Heijenoort, ed., *From Frege to Gödel: A Source Book in Mathematical Logic* (Cambridge, Mass.: Harvard University, 1972), 1–82;
2) *Conceptual Notation: A Formula Language of Pure Thought Modelled Upon the Formula Language of Arithmetic*. In Gottlob Frege, *Conceptual Notation and Related Articles*, trans. Terrell Ward Bynum (Oxford: Clarendon, 1972);
3) *Idéographie*, trans. Corine Besson (Paris: Vrin, 1999).

CP *Collected Papers on Mathematics, Logic, and Philosophy*, ed. Brian McGuinness, trans. Max Black et al. (Oxford: Blackwell, 1984).

FB "Function und Begriff" (1891). In Gottlob Frege, *Funktion, Begriff, Bedeutung*, ed. Günther Patzig (Göttingen: Vandenhoeck und Ruprecht, 1986), 18–39; English translations: "Function and Concept," in *TPW*, 21–41, and *CP*, 137–56.

GA *Die Grundlagen der Arithmetik, eine logisch-mathematische Untersuchung uber den Begriff der Zahl* (1884); bilingual edition: *The Foundations of Arithmetic, a Logico-mathematical Inquiry into the Concept of Number*, trans. J. L. Austin (Evanston, Ill.: Northwestern University, 1980).

Gedanke "Der Gedanke. Eine logische Untersuchung" (1918). In Gottlob Frege, *Logische Untersuchungen*, ed. Günther Patzig (Göttingen: Vandenhoeck & Ruprecht, 1986), 30–53; English translation: "Thoughts," in *CP*, 351–72.

GF "Gedankengefüge" (1923). In Gottlob Frege, *Logische Untersuchungen*, ed. Günther Patzig (Göttingen: Vandenhoeck & Ruprecht, 1986), 72–91; English translation: "Compound Thoughts," in *CP*, 390–406

NS *Nachegelassene Schriften*, ed. Hans Hermes, Friedrich Kambartel, and Friedrich Kaulbach (Hamburg: Meiner, 1983), 2 vols.; English translations in *PW*, *PMC*, and *TPW*.

PMC *Philosophical and Mathematical Correspondence*, ed. Brian McGuinness, trans. Hans Kaal (Chicago: University of Chicago, 1980).

PW *Posthumous Writings*, trans. Peter Long and Roger White (Chicago: University of Chicago, 1979).

TPW *Translations from the Philosophical Writings of Gottlob Frege*, ed. Peter Geach and Max Black (Lanham, Md.: Rowman and Littlefield, 3d ed., 1980).

UBG "Über Begriff und Gegenstand" (1892). In Gottlob Frege, *Funktion, Begriff, Bedeutung*, ed. Günther Patzig (Göttingen: Vandenhoeck & Ruprecht, 1986), 66–80; English translation: "On Concept and Object," in *TPW*, 42–55.

USB "Über Sinn und Bedeutung" (1892). In Gottlob Frege, *Funktion, Begriff, Bedeutung*, ed. Günther Patzig (Göttingen: Vandenhoeck & Ruprecht, 1986), 40–65; English translation: "On Sense and Meaning," in *TPW*, 56–79.

WBB "Über die wissenschaftliche Berechtigung einer Begriffsschrift" (1882). In Gottlob Frege, *Begriffsschrift und andere Aufsätze*, ed. Ignacio Angelelli (Darmstadt: Wissenschaftliche Buchgesellschaft, 1977), 106–14; English translation: "On the Scientific Justification of a Conceptual Notation," in *CN*, 83–89.

WF "Was ist eine Funktion?" (1904). In Gottlob Frege, *Funktion, Begriff, Bedeutung*, ed. Günther Patzig (Göttingen: Vandenhoeck & Ruprecht, 1986), 81–90; English translations: "What Is a Function?" in *CP*, 285–92, and in *TPW*, 107–16.

Works by Others

FPL Dummett, Michael. *Frege Philosophy of Language* (Cambridge, Mass.: Harvard University, 1981).

LU II/1 Husserl, Edmund. *Logische Untersuchungen. Zweiter Teil: Untersuchungen zur Phänomenologie und Theorie der Erkenntnis*, vol. 1, ed. Ursula Panzer, Husserliana XIX/1 (The Hague: Nijhoff, 1984); English translation: *Logical Investigations*, trans. J. N. Findlay (London: Routledge and Kegan Paul, 1970).

EP Récanati, François. *Les énoncés performatifs* (Paris: Minuit, 1981); English translation: *Meaning and Force: The Pragmatics of Performative Utterances* (Cambridge: Cambridge University, 1987).

TIA Searle, John. "A Taxonomy of Illocutionary Acts," in *Expression and Meaning* (Cambridge: Cambridge University, 1979), 1–29.

VEP Tugendhat, Ernst. *Vorlesungen zur Einführung in die sprachanalytische Philosophie* (Frankfurt a. M.: Suhrkamp, 1976); English translation: *Traditional and Analytical Philosophy: Lectures on the Philosophy of Language*, trans. P. A. Gorner (Cambridge: Cambridge University, 1982).

PA Tugendhat, Ernst. *Philosophische Aufsätze* (Frankfurt a. M.: Suhrkamp, 1992).

BAB Tugendhat, Ernst. "Die Bedeutung des Ausdrucks 'Bedeutung' bei Frege," in *PA*, 230–50; English translation: "The Meaning of 'Bedeutung' in Frege," *Analysis* 30 (1970), 177–89.