

# The Ontic Probability Interpretation of Quantum Theory – Part I

## The Meaning of Einstein’s Incompleteness Claim

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### ABSTRACT

Ignited by Einstein and Bohr a century ago, the philosophical struggle about Reality is yet unfinished, with no signs of a swift resolution. Despite vast technological progress fueled by the iconic Einstein/Podolsky/Rosen paper (EPR) [1] [2] [3], the intricate link between ontic and epistemic aspects of Quantum Theory (QT) has greatly hindered our grip on Reality and further progress in physical theory. Fallacies concealed by tortuous logical negations made EPR comprehension much harder than it could have been had Einstein written it himself in German. EPR is plagued with preconceptions about what a physical property is, the 'Uncertainty Principle', and the Principle of Locality. Numerous interpretations of QT vis à vis Reality exist and are keenly disputed [4] [5] [6] [7] [8] [9] [10] [11]. This is the first of a series of articles arguing for a novel physical interpretation I call ‘The Ontic Probability Interpretation’ (TOPI). A gradual explanation of TOPI is given intertwined with a meticulous logico-philosophical scrutiny of EPR, with Part I focusing on the meaning of Einstein’s ‘incompleteness’ claim for QT: a conceptual confusion, a preconception about Reality, and a flawed dichotomy are shown to be severe obstacles for the EPR argument to succeed. Part II completes the analysis, proving EPR claim of ‘incompleteness’ for QT is fallacious [12]. Part III further develops TOPI, while scrutinizing the mythical ‘Schrödinger’s Cat’, as well as the ‘Basis’ and ‘Measurement’ pseudo-problems [13]. Part IV introduces QR/TOPI: a new theory that solves the century-old problem of integrating Special Relativity with Quantum Theory [14].

### List of Acronyms

QT	Quantum Theory	EPR	The Einstein/Podolsky/Rosen Paper
TOPI	The Ontic Probability Interpretation	PD	Probability Distribution
PI	Physical Interaction	GI	Gauge Interaction
TM	True Measurement	TRC	The Reality Criterion
TCC	The Conceptual Confusion	SD	Standard Deviation of a PD
TRP1	The Reality Preconception 1	TFD	The Fallacious Dichotomy

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## 1. Introduction

As a *realist*, Einstein wrote: “there is something like the ‘real state’ of a physical system, which independent of any observation or measurement exists objectively and which can in principle be described by means of physical terms”. However, *probability*-wise, Einstein was a *subjectivist* – blaming the stochastic makeup of QT on its *incompleteness*. But more than *chance* as Nature’s modus operandi, he obstinately detested its “spooky action at a distance” – blaming again such “telepathy” predicted by QT on its *incompleteness* [15].

Poorly understood even today, EPR [1] and Bohr’s response [2] were published on May 15 and October 15, 1935 with identical titles: “Can Quantum Mechanics Description of Physical Reality be Considered Complete?”. Prior to his formal response, Bohr had sent a letter [3] to *Nature*. EPR discussed thought experiments where the position and momentum of two correlated ‘particles’ were predicted by QT and ‘measured’. I put ‘particles’ and ‘measured’ in quotes because: (a) quantum objects are neither particles nor waves; and (b) most physical interactions are *not* measurements. John Bell advised for the word ‘measurement’ to “be banned altogether in quantum mechanics” [16]. Most physicists and philosophers did not listen.

## 2. Elements of a Physical Theory

Against the Logical Positivism in vogue at the time, EPR states:

**EPR1:** *Any serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.*

A factual theory is an explanatory/predictive logico-mathematical formalism whose ultimate referent is Reality; ergo, it must be put to the empirical test. A theory consists of *Ontology*, *Foundation*, *Structure*, *Evidence*, and *Interpretation*. The *Ontology* includes the presumed *real* entities plus known *facts* about their properties and behavior. The *Foundation* comprises: a) *abstract* entities/attributes; and b) unexplained explainers: principles, postulates, hypotheses, etc. The *Structure* entails: a) non-factual formalisms (e.g. Logic, Calculus, Geometry); b) other factual theories (e.g. Space/Time, Relativity, Electromagnetism); and c) *laws* and *theorems* about the *abstract* entities. The *Evidence* incorporates the *empirical* support the theory possesses to claim its verisimilitude; measurements and observers are necessary for the *Evidence* but are not, and must not be, part of the theory. The elusive *Interpretation* attempts to grasp Reality by proposing *semantic rules* via which the *abstract* entities/attributes represent the *real* ones.

## 3. Elements of ‘The Ontic Probability Interpretation’ (TOPI)

An interpretation endows the *Foundation*, *Structure*, and *Evidence* with *physical* meaning, thereby characterizing the *Ontology*. Numerous interpretations/formulations of QT exist and are widely disputed [4] [5] [6] [7] [8] [9] [10] [11]. Like Bunge [17], I will refer to the abstract/real entities of QT as ‘quantons’. Per TOPI, an *abstract* quanton interacts with its *abstract milieu* and has: a) a *current abstract* state that corresponds to the *real* quanton’s state attained from the *last* interaction with its *physical milieu*; b) *current abstract* attributes that parallel *physical* properties of the *real* quanton in its *current real* state; and c) a probability distribution (PD) for the

*transition* to its **next abstract** state/attributes, which is the predicted **ontic** PD for the **real** quanton to transition to its **next real** state/properties. There are attributes a quanton does not possess (e.g. size, shape), i.e. they are not *defined* at all; and others that are *defined* only for some states (like the azimuthal angle is defined(undefined) off(on) the polar axis). Quanton's are not punctiform objects. A property which is defined(undefined) for the *current* state can be undefined(defined) for the *next* state. If, for any state/property, the **PD** is always as narrow as to effectively assign a single next state/property, the theory is classically *deterministic*; otherwise, it is *stochastic*. **QT** is partly stochastic, partly deterministic – I call it ‘quantically deterministic’. *Classical determinism* is a degenerate type of *quantic determinism*. **TOPI** is applicable to Classical Physics [18] [19] [20].

A *composite* quanton can be in *product-states*, for which all sub-quantons are isolated; and in *entangler* states, for which the sub-quantons’ states are not defined per se but as co-states. The same current state is expressible via different linear combinations of eigenstates (different bases for the State-Space). For a given quanton, its *current* milieu defines a basis which, when used to represent the *current* state, results in a linear combination that encrypts (via Born’s Rule) the **PD** for the *next* state/properties [20]. Per **TOPI**, **QT** claims neither explicative nor predictive power between *current* and *next* states. Discrete and continuous systems are covered by **QT/TOPI**.

A ‘Physical Interaction’ (PI) between a quanton and its milieu is -generally- reciprocal, i.e. both change states. A PI implemented by us to acquire *knowledge* will be called a ‘Gauge Interaction’ (GI); GIs were called ‘measurements’ by **QT** pioneers and, ignoring Bell’s advice, they still are by most researchers. If a GI is such that the milieu (the ‘measurer’) changes state and the quanton (the ‘measured’) does not, I call it a ‘True Measurement’ (TM). From a strictly physical viewpoint, the anthropic **GIs** and **TMs** occur all the time without human intervention.

Only some properties may be experimentally accessible, creating the empirical *Evidence*. The *operationalist* believes a physical property has no meaning but the one given by its *measurement* protocol. This is not true because we must understand the **real** property before we can conceive a gauging technique and build and/or select the proper instrumentation [20].

### 3.1 Heisenberg’s Inequalities vis à vis **TOPI**

Orthodox **QT** predicts probabilities, **not** values. Per **QT/TOPI**, probability is not epistemic but ontic [17] [18] [19] [20] [21] [22] [23]. Heisenberg’s inequalities have had more misinterpretations than any other formula in history. Under **QT/TOPI**, given two properties with **noncommutative** operators  $\mathcal{P}_1$  and  $\mathcal{P}_2$  ( $[\mathcal{P}_1, \mathcal{P}_2] = \mathcal{P}_1\mathcal{P}_2 - \mathcal{P}_2\mathcal{P}_1 \neq 0$ ), and depending on the quanton’s *current* state, only one of the properties may have a single value while the other is **undefined**. As for the *next* state and properties, only their **PDs** are univocally determined. Thus, for any current state of the quanton, it is impossible to jointly assign determinate current/next values to both properties. Per **TOPI**, it is the **probability distribution** for the values, not the values themselves, that constitutes the **physical** property of a quanton/milieu system and, hence, **no** single instance of a **GI** can characterize the property. **Inequalities (1)** express the so-called ‘Uncertainty Principle’ for generic properties  $\mathcal{P}_1$  and  $\mathcal{P}_2$  and for momentum  $\mathcal{P}$  and position  $Q$ :

$$\Delta\mathcal{P}_1\Delta\mathcal{P}_2 \geq (1/2)|\langle[\mathcal{P}_1, \mathcal{P}_2]\rangle| \quad \Rightarrow \quad \Delta\mathcal{P}\Delta Q \geq \hbar/2 \quad (1)$$

Per **TOPI**, these inequalities neither express a ‘principle’ nor involve ‘uncertainty’. They do not entail ‘measurements’ either. They constitute a *theorem* of **QT** relating the **SDs** of two conjugate random variables (properties) for the *next* state. The narrower one **PD** is, the broader is the other. This is only true when the quanton’s *current* state is the same for both properties [20].

#### 4. Correctness/Completeness/Elements of Reality

**EPR** asserts how to judge the correctness of a physical theory:

**EPR2**: *The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement.*

A theory is *correct* because none of its central predictions has yet been empirically nullified. Prima facie, **EPR** appears to recognize the *correctness* of **QT**. A *correct* theory may be *incomplete* because it does not predict aspects of Reality (facts) we expected it to predict. Despite being its leitmotif, **EPR** does not assign a meaning to *completeness*, proposing only a *necessary* condition:

**EPR3**: *Whatever the meaning assigned to the term complete, the following requirement for a complete theory seems to be a necessary one: every element of the physical reality must have a counterpart in the physical theory.*

Being **EPR3** just *necessary*, only *incompleteness* can be proven. To do so, an element in the *Ontology* must have no counterpart in the *Foundation/Structure*, viz we must identify a *fact* the theory can neither incorporate as a postulate nor *predict*. **EPR** admits it is us who identify the *ontic* entities/properties/facts (“elements of the physical reality”) which we *expect* the theory to describe/explain/predict by means of our conceived *Foundation/Structure*. Thus, *completeness* relates to both accessible Reality (facts) and our *expectations*, the latter of which could be rooted in prejudices and/or a priori philosophical views. Unexplained explainers (e.g. a *principle*) in the *Foundation* and laws/theorems in the *Structure* belong to neither the *Ontology* nor the *Evidence*: if unprobed predictions defy our prejudices, *experiment* must rule. **EPR** agrees:

**EPR4**: *The elements of the physical reality cannot be determined by a priori philosophical considerations, but must be found by an appeal to results of experiments and measurements.*

To identify an ‘element of physical reality’, **EPR** proposes ‘The Reality Criterion’ (TRC):

**EPR5**: *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity... Regarded not as a necessary, but merely as a sufficient condition of reality, this criterion is in agreement with classical as well as quantum-mechanical ideas of reality.*

##### 4.1 The Conceptual Confusion (TCC)

Palpably against **EPR4**, **EPR5** says that for a property to be real, it is enough that we can *predict* its value “with certainty” and “without in any way disturbing” the system. First, it is hard to understand what the *reality* of a physical quantity has to do with our theoretical ability to predict its value. Its mere accurate direct *measurement* (if possible) would be enough – as long as

the specific theory behind the *measurement* (not the theory that predicts its result) were reliable. And were its direct *measurement* not possible, then its *reality* could be **indirectly** inferred from its being part of (now yes) a theory which successfully predicts other quantities which are directly *measurable*. In any case, *prediction* of the putative physical quantity is **not** necessary while (in accordance with **EPR4**) *measurement* is vital. Second, a mere *prediction cannot* disturb anything physical, and -as I said- the only way to know how *certain* our prediction was is to accurately *measure* the property itself or another physical quantity reliably related to it. Thus, **EPR5** contradicts **EPR4** because “experiments and measurements” are absent so... I surmise **EPR** forgot to include ‘when measured’ after “the value of a physical quantity”.

But here is the striking **EPR** confusion: the text in parentheses shows that **EPR5** conflated three distinct concepts: **(a)** the *prediction certainty* (predicted vs. real); **(b)** the *measurement accuracy* (measured vs. real); and **(c)** the *probability* for a property to assume one of its values. It is crucial to understand that it is **(c)** what **QT** is all about, **not (a)**; and that **(b)** is outside **QT**, serving only to test its *correctness*. Predicting something with a probability is not the same as predicting a probability for something. Predicting a value “with probability equal to unity” amounts to a perfect prediction (predicted = real), and it is utterly different to predicting ‘a probability equal to unity for a value’. Whether *correct* or not, if the theory is classically *deterministic*, all *predicted* probabilities are equal to unity or to zero. Instead, for a quanton in a given state, **QT** predicts a **PD** over the next states/property values, i.e. they are all random variables. Predicting a probability *less* than one can be as accurate (vis à vis Reality) as predicting a probability *equal* to one. I call this muddle ‘The Conceptual Confusion’ (TCC). It could be cogently argued that TCC invalidates **EPR** arguments and conclusions at the outset. That would be unfair – given the enormous technological and philosophical impact **EPR** has had.

**QT** predicts a unity probability **only** when the quanton is in an eigenstate of the property’s *operator*. Only then does an ideal **GI** deliver the value the property had pre-**GI**, i.e. the **GI** is an *ideal TM*. But a *real TM*, if repeated, never delivers a single value but a distribution of them – for classical and quantic systems. In the former, we use a single value because the error-distribution can be consistently made exceptionally narrow. However, most **GIs** are not **TMs**, i.e. the initial state is not an eigenstate, with **QT** predicting a broad **PD** for the next state/properties. Ergo, estimating the *prediction accuracy* (“certainty”) requires comparing two **PDs**: predicted vs. *real*, with the latter assessed by the statistical analysis of repeated experiments. In sum, **EPR** confuses the nil **SD** of the *predicted PD* for a property (when the system is in an eigenstate) with the *prediction and measurement accuracies* for its single value.

How do we then interpret **EPR5**? It cannot be literally, i.e. per **(a)**, because **QT** does **not** predict the *certainty* of its predictions. Clearly, it must be **(c)** for *prediction* plus **(b)** for Reality. Thus, from now on, **TRC** means **EPR5** so interpreted and, if I refer to **(a)** to apply **EPR** rationale, I will use quotes, viz “with certainty”. Only doing so can we be fair to **EPR**, despite **TCC**. With this caveat, and a negligible *experimental error*, **TRC** implies that if a ‘particle’ is in a *momentum* eigenstate, the ‘momentum is real’ and if it is in a *position* eigenstate, the ‘position is real’. Otherwise, **TRC** is mute. Under **TRC**, Reality might oddly depend on the ‘particle’ state.

## 5. The Reality Preconception 1 (TRP1)

**EPR** verbalizes **Heisenberg’s Inequalities** using the operationalist language:

**EPR6:** *A definite value of the coordinate [position], for a particle in the state given by Eq. (2) [an eigenstate of the momentum operator], is thus not predictable, but may be obtained only by a direct measurement. Such a measurement however disturbs the particle and thus alters its state. After the coordinate is determined, the particle will no longer be in the state given by Eq. (2). The usual conclusion from this in quantum mechanics is that **when the momentum of a particle is known, its coordinate has no physical reality.***

Under **QT**, because *position* and *momentum* operators do not commute, the *momentum* eigenstate is **not** a *position* eigenstate; hence, *position* in such a state is *undefined* while a **PD** is predicted for its *next* value under a *position-GI*. By stating that a definite value of the coordinate is “thus not predictable, but may be obtained only by direct measurement”, **EPR** reveals an a priori belief in *classical determinism*: such a *position* must exist and, had not the previous ‘measurement’ of the *momentum* changed the system’s state, it could have been provided by its direct ‘measurement’.

When a ‘particle’ is in a *momentum* eigenstate, a *momentum-GI* is a **TM** so, per **TRC**, the *momentum* is **real**. As for a *position-GI*, being the prediction a **PD**, **TRC** is mute so it is a non sequitur to infer that if the *momentum* is **real** the *position* is **not**. **EPR6** recites the Copenhagen Interpretation of **QT**. **TRC** was purposely devised as “merely” *sufficient* lest, having assumed **QT** correct, **TRC** would imply that the “coordinate [position] has no physical reality” at all. **EPR** believed the *position* was **real** but only if it had a definite value, which is nothing but an a priori philosophical belief (violating **EPR4**). For Einstein, using *probability* amounted to confessing *ignorance* of the underpinning *causal* processes (as he understood them). I call this ‘The Reality Preconception 1’ (TRP1).

## 6. The Fallacious Dichotomy (TFD)

Endeavoring to prove **QT** *incomplete*, **EPR** condenses **EPR5**, **TCC**, and **TRP1** into a dichotomy:

**EPR7:** *From this follows that either (1) the quantum-mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality. For if both of them had simultaneous reality—and thus definite values—these values would enter into the complete description, according to the condition of completeness. If then the wave function provided such a complete description of reality, it would contain these values; these would then be predictable. This not being the case, we are left with the alternatives stated.*

The phrase “For if both of them had simultaneous reality—and thus definite values—...” is now unequivocally asserting **TRP1**: only attributes with definite values are **real**, so two conjugate properties cannot be “simultaneously real” (unless **QT** is *incomplete*). **EPR7** also says that the definite value of a real property must be “predictable”: the “mere” *sufficient* character of **TRC** has now become also *necessary*. Thus, for **EPR**, a theory cannot be *complete* if, in most cases, it predicts a mere **PD**. **EPR7** dogmatically removes *probability* from the *Ontology* and, inevitably, preordains **QT**’s *incompleteness*: *Petitio Principii* at work. It is baffling why Reality was not so ‘defined’ at the outset. A plethora of convoluted logic could have been saved: **QT** would be *incomplete* simply because only rarely does it predict definite values. However, the

inclusion of a priori philosophical considerations into the *Ontology* (against **EPR4**) would have been embarrassingly obvious.

**EPR7** dichotomy boils down to: either (1) the two quantities *do* have “simultaneous reality” (determinate values) and **QT** is *incomplete* because it does not *predict* them, or (2) the quantities *do not* have “simultaneous reality” (at least one has a **PD**) and **QT** is *complete* because it predicts so. **EPR** conflates the joint reality of two physical properties with joint predictability and measurability of *single* values for them. This dichotomy is fallacious because it is predicated on a priori philosophical beliefs regarding Reality. It has only *analytic* value (as opposed to *synthetic*) because **QT** completeness or incompleteness depends on the *ad hoc* definition of “simultaneous reality”, *not* on experimental evidence.

As for **EPR7** phrase “..., it would contain these values; these would then be predictable”, it is obviously intimating the well-known idea of ‘hidden variables’ which, having zero dispersion would presumably restore *Classical Determinism* to Physics, reaffirming **TRP1**. Part IV of this series deals with hidden-variable theories and other **QT** interpretations/formulations [14].

## Conclusions

To honor the spirit of **EPR**, because of the conceptual confusion (**TCC**), I reinterpreted its reality criterion (**TRC**). In violation of its own dictum for identifying the ‘elements of reality’ (**EPR4**), **EPR** revealed its commitment to *classical determinism*, associating *probability* only with human *ignorance* and, thereby, relying on a Reality preconception (**TRP1**). Combining **TRC**, **TCC**, and **TRP1**, **EPR** proposed a mutually exclusive disjunction (**TFD**), whose truth value is only analytic (*not synthetic*) because it depends upon an *ad hoc* ‘definition’ of Reality.

Despite all those logical flaws, **EPR** strived to prove that option (1) in **TFD** was true, i.e. that the two quantities did have “simultaneous reality”. But, because (in most cases) a ‘measurement’ (**GI**) disturbs the state and **TRC** was mute regarding the property’s reality, **EPR** needed to conceive a way of ‘measuring’ without “in any way disturbing the system”. In our **TOPI** lexicon: a way of making a **GI** to effectively work as a **TM**. Such a scheme to prove **QT**’s *incompleteness* was proposed by **EPR**, and it is dissected and proven also inadequate in Part II [12].

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