Towards a stronger concept of argument*

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1 Arguments and contentions

Consistent with most handbooks on informal logic, arguments may be defined as series of at least two statements, where exactly one of them (the conclusion) is *allegedly* justified by the others (the premises). The truth or acceptability of the premises and their logical connection with the conclusion determine whether its conclusion is true or acceptable. For example:

Argument 1 (Simpsons)**.** (1) All Simpsons are yellow. (2) Lisa Simpson is a Simpson. (3) Thus, Lisa Simpson is yellow.

This definition is problematic for at least two reasons.

First, the concept of argument refers more to a chain of reasoning than to series of logically connected sentences. In fact, the word argument comes from the Latin verb *arguo*, *arguer*, *argui*, *argutus*, which means *to put in clear light*, which comes from *ἀργός*, the Greek adjective for *bright*. In some way, an argument is something that makes clear enough why its conclusion is true or is to be believed, and not just a series that happens to justify its conclusions for reasons that are not fully stated. In this way, a valid argument would be more like a rigorous proof, as in the latter we do find each step justified.

I will not deal with this first objection because, although important, it will not affect my treatment, which will consist of examples whose validity or non-validity will be sufficiently manifest (like in our argument 1). Hence,

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in this paper I will not require that each step of an argument has a proof-like justification.

The second objection is the motive of this paper, and it deals with how we normally differentiate an argument from series of statements in general. The way this happens is the Simpsons arguments is through the conclusion marker 'thus,' which turns the otherwise unconnected series '1.1, 1.2, 1.3' into an argument with '1.3' as a conclusion. A *conclusion marker* is a word or expression that indicates—in the relevant contexts—that the sentence that follows is the conclusion of an argument [see 5, ch. 3]. Other conclusion markers are 'hence,' 'therefore,' and 'ergo.'[1] This implies, though, that the following series should also be considered an argument:

Contention 2 (Cookies)**.** (1) I like cookies. (2) Therefore, Ana is French.

That this fits the definition of an argument is seldom deemed as a flaw in the definition of what an argument is. After all, the premises are so manifestly irrelevant for the conclusion that it can be immediately dismissed.

However, I do not see how can anyone rationally *allege* that 2.1 justifies its conclusion. Nothing in the premise affects the plausibility of the conclusion, which is why I do not think this particular contention should qualify as a proper argument. That the conclusion of a series of statements be preceded by a (formal or informal) conclusion marker is, in my consideration, far from being a sufficient condition for that series to be called an argument. Instead, I will call a *contention* to any series where exactly one of them (the conclusion) is preceded by a conclusion marker, and try to clarify the special characteristics of arguments. Arguments are, of course, also contentions, but of a very special kind: they satisfy some conditions that make their premises relevant for their conclusion, even when the argument is not valid.

2 Validity

When we asses the quality of an argument, we are normally concerned about two things. First, whether its premises are true and, second, whether the conclusion follows from the premises. When an argument fulfils the later condition we say that it is a *valid* one, and when it also fulfils the former, we say that it is a *sound* one. The Simpsons argument is valid, as it has the

[1]In other contexts, we can indirectly indicate the conclusion by signalling the premises through 'reason markers,' which indicate that the following sentence(s) are the premises of an argument: these are words like 'because,' 'since,' and 'for.' So our argument could also be stated as: 'Lisa Simpson is yellow because all Simpsons are yellow and she is a Simpson.' For simplicity's sake, though, I will only work with conclusion markers.

structure of a well-known syllogism, but whether it is sound depends on how we interpret the predicate Simpsons: if Simpsons denotes the set comprising the Simpson family, then 1.1 is true and our argument is sound, but if it denotes the set of all characters of that show, premise 1.1 can no longer be true since Apu Nahasapeemapetilon is not yellow.

This is not the place for explaining in detail what makes a contention or an argument valid or not. However, it is very common to differentiate between deductive and inductive standards of validity. An *deductively valid* argument is one whose conclusion cannot possibly be false if its premises are true. This is obviously the case for argument 1.

An *inductively valid* argument, instead, is an argument whose premises support the conclusion with a sufficiently high *probability*. This probabilistic support is to be understood informally or pre-theoretically throughout this paper, although the theoretical sense of mathematical probability is a special case of this informal sense. Hence, under some conditions, the following is a classical example of an inductively valid argument.

Argument 3. (1) The swan s_1 is white. (2) The swan s_2 is white. ... (n) The swan s_n is white. (n') Therefore, all swans are white.

Note that deductive validity is just a special case of inductive validity. After all, a deductively valid argument supports its conclusion with absolute probability, which definitely is a sufficiently high probability.

A very important notion related to deductive validity is that of logical entailment. Where $\sigma_1, ..., \sigma_n \vDash \phi$ states that the series $\sigma_1, ..., \sigma_n$ logically entails ϕ, I define (logical) entailment as follows:

Definition 4 (Entailment)**.** $\sigma_1, ..., \sigma_n \vDash \phi$ iff ϕ is true if all $\sigma_1, ..., \sigma_n$ are true.

It is very tempting to state that a deductively valid argument is just one whose conclusion is entailed by its premises. However, to do so would be problematic as it would make deductively valid any argument with a logically absurd premise (assuming *ex falso quodlibet*) or with a logically true conclusion regardless of what they say. This would make valid any argument of the form '$\phi_1, ..., \phi_n$, thus, *it is possible that* ψ', unless ψ is a logical absurdity.

This clearly disregards what I deem as Hamblin's most important criterium for identifying good arguments:

> *The conclusion must be such that, in the absence of the argument, it would be less accepted than in its presence.* [2, p. 245]

This criterium accounts, in my consideration, for the very extended demand that the premises of a good arguments be relevant for its conclusion. A probabilistic rephrasing of Hamblin's criterium would be:

> *The conclusion must be such that, in the absence of the argument, it would be less* probable *than in its presence.*

This leads to the following redefinition of deductive validity, where $\sigma_1, ..., \sigma_n :. \phi$ stands for a contention (or an argument) from the series of premises $\sigma_1, ..., \sigma_n$ to the conclusion ϕ, $\mathrm{P}(\phi)$, for the probability of ϕ, and $\mathrm{P}(\phi \mid \sigma_1, ..., \sigma_n)$, the for conditional probability of ϕ given all of $\sigma_1, ..., \sigma_n$:

Definition 5 (Deductive validity)**.** A contentions $p_1, ..., p_n :. q$ is *deductively valid* iff (1) $p_1, ..., p_n \vDash q$, (2) $\mathrm{P}(q) < \mathrm{P}(q \mid p_1, ..., p_n)$, and (3) the set $\{p_1, ..., p_n\}$ is not trivial (it does not entail every statement).

The third condition avoids that an argument be valid just because, by the principle *ex falso quodlibet*, it logically follows from its (logically) false premises. This would be most undesirable as the premises would not be very relevant for our belief in the conclusion. Furthermore, it is difficult to see how a necessarily unsound argument ever should be deemed as a valid one. On the other hand, note that 5.2 prevents that there be a valid argument whose conclusion is a logical truth. If this was the case, it would be impossible for the conditional probability of q on any set of premises whatsoever to be greater than its probability tout court, as it would already equal the maximum probability.

Now, not only can definition 5 be generalised for inductive validity but, in doing so, we will also discover some of the logical ground of inductive validity. Where τ is our *inductive threshold*, i.e. the smallest probability value *sufficiently high* for accepting a proposition, inductive validity can be defined as follows.

Definition 6 (Inductive validity)**.** A contention $p_1, ..., p_n :. q$ is *inductively valid* iff (1) $p_1, ..., p_n \vDash \mathrm{P}(q) \geq \tau$, (2) $\mathrm{P}(q) < \mathrm{P}(q \mid p_1, ..., p_n)$, and (3) the set $\{p_1, ..., p_n\}$ is not trivial.

This definition suggests that inductive validity works because any inductively valid argument can be easily transformed into a deductively valid one [cf. 1, ch. II], as the following *deductivisation* of argument 3 shows:

Argument 7. (1) The swan s_1 is white. (2) The swan s_2 is white. ... (n) The swan s_n is white. (n') Therefore, *there is a sufficiently high probability that* all swans are white.

The emphasised string from $7.n'$ is what effectively turns 3 into a deductive argument. The meta-sentential predicate 'there is a sufficiently high probability that' and a basic notion of probability guarantee that $7.n'$ is (at least momentarily) worth of our belief (given n is large enough, the premises are true, and some methodological requirements were fulfilled in gathering and analysing the data).

Now, before explaining how a special class of meta-sentential predicates can help us improve our current theories of arguments, let us expand our study of arguments and contentions into the structural study of the schemes that characterise their structure.

3 Schemes

In the abstract field of argumentation theory, it does not suffice to study isolated or specific arguments. If we want general results, which is the point of an abstract study field, we need to asses *argument schemes*. Nevertheless, the concept of argument scheme requires a clear concept of argument, which we do not have yet. Hence, we will work first with (*contention*) *schemes*, which are series of formulae where exactly one of them (the conclusion) is an open formula preceded by a conclusion marker, and at least one of the others (the premises) must be an open formula too.

Where open formulae can have free variables of any type (sentential, predicative, individual, ...), I formally define schemes as follows:

Definition 8 (Scheme)**.** The series $\sigma_1, ..., \sigma_n \therefore \phi$ is a (contention) scheme iff ϕ and at least one of $\sigma_1, ..., \sigma_n$ are open formulae.

Now, when evaluating schemes, we are only concerned with whether that argument is generally valid: we want to know if all possible arguments with the structure of that scheme are valid regardless of how we interpret their free variables. We are not concerned with their soundness because open formulae are normally neither true nor false—in which case their schemes could not have true premises—and we are not interested in the cases where they are (cf. section 5).[2]

An example of a generally valid argument scheme, modelled after argument 1, would be:

[2]Some philosophers have regarded open formulae as somewhat troublesome. Despite not seeming to be proper truth-bearers, they often are part of our formal chains of deduction. This lead Francisco Miró Quesada [4] to propose a system of first order logic that avoids the use of free variables. For its part, Jan Łukasiewicz [3] was the first to propose a method for assigning logical values to open formulae, although not precisely the typical values 'true' and 'false,' but fractional ones in the closed interval $[0, 1] \subset \mathbb{N}$.

Scheme 9. (1) A is B. (2) x is A. (3) Thus, x is C.

As we already advanced, the structure of argument 1, which is represented by scheme 9, is that of a well-known valid syllogism. Since an argument is deductively valid by virtue of its form or structure, we may transfer the validity of such an argument to another one sharing its structure.

All the above suggests extending the domain of the concept of validity into schemes as follows:

Definition 10 (Scheme validity)**.** A scheme $\sigma_1, ..., \sigma_n :. \ \phi$ is deductively or inductively valid iff it is deductively or inductively valid, accordingly, for all $\sigma_1, ..., \sigma_n$, and ϕ.

That the conclusion of a valid scheme cannot be a logical absurdity—which follows from condition 5.2—, is particularly crucial here. To see why this is important, we need to introduce the concept of *sub-scheme*, which is just an scheme that results from saturating or binding some of the free variables of its corresponding super-scheme.

Definition 11 (Sub-scheme)**.** $\pi_1, ..., \pi_n :. \ \psi$ is a sub-scheme of $\sigma_1, ..., \sigma_n :. \ \phi$ iff (1) $\pi_1, ..., \pi_n :. \ \psi$ is a scheme, (2) $\pi_1 \vDash \sigma_1, ..., \pi_n \vDash \sigma_n$, and $\psi \vDash \phi$, and (3) the set of free variables of ψ is a subset of that of ϕ, and similarly for each π_i with respect to σ_i.

Now, consider the following argument scheme modelled after the *modus ponendo ponens*:

Scheme 12 (MPP)**.** (1) If ϕ, then ψ. (2) ϕ. (3) Thus, ψ.

The following sub-scheme of 12 that is not valid in the sense of definition 10.

Scheme 13. (1) If ϕ, then ψ or $\neg\psi$. (2) ϕ. (3) Thus, ψ or $\neg\psi$.

Since the conclusion is tautological, assuming the *tertium non datur*, its probability value is the maximum one and it equals its conditional probability on the premises. Had we allowed for the conclusion of a valid argument to be a logical absurdity, the general validity of scheme 12 would have been passed to its sub-scheme 13, which would be a most undesirable result for the reasons we have already seen.

It is important to point out here that definition 10 still preserves the validity of all schemes $\sigma_1, ..., \sigma_n :. \ \phi$, such that $\sigma_1, ..., \sigma_n \vDash \phi$, but neither ϕ is a tautology nor any of $\sigma_1, ..., \sigma_n$ is a logical absurdity. This theorem assumes some quite conventional properties of the probability relation.

Theorem 14. The scheme $\sigma_1, ..., \sigma_n :. \phi$ is valid iff $\sigma_1, ..., \sigma_n \vDash \phi$ holds for all $\sigma_1, ..., \sigma_n$, and ϕ (where ϕ is a not tautology and $\{\sigma_1, ..., \sigma_n\}$ is not trivial).

Proof. The left-to-right side follows immediately. The right-to-left side easily obtains if we assume that (1) the probability of any logical absurdity is less than the maximum probability, and that, (2) whenever $\sigma_1, ..., \sigma_n \vDash \phi$, the probability of ϕ given all of $\sigma_1, ..., \sigma_n$ is the maximum probability.

Hence, our definition of deductive validity discards a great amount of arguments that it does not make sense to call valid—those with logically absurd premises or tautological conclusions—while preserving all the rest.

The concept of scheme inductive validity, of course, can also be reduced to that of deductive scheme validity as follows:

Definition 15 (Scheme inductive validity)**.** A scheme $\sigma_1, ..., \sigma_n :. \phi$ is inductively valid iff the scheme $\sigma_1, ..., \sigma_n :. \mathrm{P}(\phi) \geq \tau$ is deductively valid for all $\sigma_1, ..., \sigma_n$, and ϕ.

In the following sections we are going to see that, by the hand of meta-sentential predicates, we can use our concept of deductive validity for studying other non-typical types of 'validity.'

4 Meta-sentential predicates

I will call a *meta-sentential predicate* to any predicative expression that takes a sentence (that can be the conclusion of an argument) as one of its arguments and returns a meta-sentence (that can still be the conclusion of an argument).

A meta-sentential predicate can express, for example, an epistemic attitude towards a sentence like 'x is certain that ϕ' or 'there is good evidence that ϕ,' in which case we may call it an *epistemic fragment* [see 6]. It can also express an (allegedly) objective property of a sentence, like the truth predicate 'it is true that ϕ' or 'it is the case that ϕ.' Furthermore, they can also be axiological predicates such as 'it is good that ϕ' and 'it would be disgusting that ϕ.' Other meta-sentential predicates are not easy to classify, as they could be considered to express an epistemic or an objective property, as in modal predicates like 'it is possible that ϕ' or probabilistic predicates such as 'the probability of ϕ is n.[3]

The following condition must be satisfied by all such predicates:

[3]If we characterise probabilistic predicates as 'epistemic,' then we may be committed to a subjective interpretation of probability. Although no such characterisation will be made, it would bear no problem as this interpretation of probability is sufficiently adequate for my purposes in this work.

Postulate 16 (Meta-sentential predicate)**.** A meta-sentential predicate C is an n-place predicate, where at least one of its arguments is a formula and returns a formula, all instances of these formulae can appear as the conclusion of an argument.

Whenever a given sentence or formula is modified by appending a meta-sentential predicate before it, we say that it has undergone a meta-sentential predication. Hence, the formula $\mathsf{C}(\phi)$ will be called the 'meta-sentential predication of ϕ by C.' I will extend this operation into contentions themselves, so that the 'meta-sentential predication of a contention by C' will be the argument resulting from the meta-sentential predication of its conclusion by C. Thus, $7.n'$ is the meta-sentential predication of $3.n'$ by 'there is a sufficiently high probability that' and 7 is the meta-sentential predication of 3 by that same meta-sentential predicate.

As we have already seen, inductive validity makes sense because there is an intuitive way to link Sit to deductive validity, which consists in a meta-sentential predication of its conclusion that makes the argument deductively valid. But meta-sentential predications also open new ways for evaluating and comparing the strength of arguments, which can be also used for more appropriate characterisations of arguments.

5 Contextual validity

Some arguments clearly fail to meet both inductive and deductive standards and, yet, are still considerably better than contention 2 (I like cookies...).

Argument 17. (1) Some Simpsons are yellow. (2) Lisa Simpson is a Simpson. (3) Therefore, Lisa Simpson is yellow.

Whether this argument is rationally accepted by someone will depend on the underlying conventions and aims that shares with the arguer. If they only aim at showing that the conclusion is possible, then it is fair to say that the acceptance of the argument was not irrational. But if they were aiming at proving that the conclusion is very likely, then this argument, as it stands, absolutely fails.

Let us notice, though, that argument 17 can also be deductivised by appending an modal (meta-sentential) predicate into its conclusion.

Argument 18. (1) Some Simpsons are yellow. (2) Lisa Simpson is a Simpson. (3) Therefore, *it is possible that* Lisa Simpson is yellow.

This may suggest that, in principle, almost any argument could be deductivised through its meta-sentential predication by the modal predicate of possibility. In fact, let us consider the following version of contention 2.

Contention 19. (1) I like cookies. (2) Thus, *it is possible that* Ana is French.

If we can conceive of a possible world where Ana is French, then the conclusion 19.2 may be regarded as a logical true.

However, this would be contrary to the way in which I would like to propose that a meta-sentential predicate should transform a non-valid argument into a valid one. Lucky me, definition 5 already disqualifies this argument as valid one since, (a) unlike 18.1 to 18.3, premise 19.1 is irrelevant to its conclusion with or without the meta-sentential predicate, and (b) this definition unequates deductive validity with entailment and excludes arguments with logically true conclusions.

Hence, the following extensions of the concept of validity will preserve the properties we already gained in definitions 5 and 10. I will, accordingly, call 'validity with respect to C' or just 'C-validity' to the condition whereby a contention becomes valid after being predicated by C.

Definition 20 (C-validity)**.** An argument $p_1, ..., p_n \ \therefore\ q$ is C-valid iff $p_1, ..., p_n \therefore \mathsf{C}(q)$ is (inductively or deductively) valid.

This immediately translates into my general definition of scheme C-validity:

Definition 21 (Scheme C-validity)**.** A scheme $\sigma_1, ..., \sigma_n \therefore \phi$ is C-valid iff all of its instances are C-valid.

Thus defined, meta-sentential predicates are the key to my first condition of a stronger definition or argument.

Postulate 22 (Argument)**.** In order for a contention $p_1, ..., p_n \therefore q$ to be an argument, it is necessary that there be a meta-sentential predicate C that makes it C-valid.

It has to be noted, though, that this is but a first idea for a necessary (not sufficient) condition of the definition of an argument, and that no complete definition has been formulated here. One important point is related to the fact that definitions 20 and 21 may define an infinite class of validities since there can be as many types of C-valid arguments as meta-sentential predicates are.

This leads us to the question of which meta-sentential predicates should be part of the domain of postulate 22, which I cannot possibly address here.

This investigation is for establishing the bases for the formal study of arguments and fallacies, and it deals with those parts of argumentation theory that can be formalised. Although I am open to be convinced otherwise, I consider that the deciding what kind of linguistic expression qualifies as a contextually acceptable meta-sentential predicate or not largely depends upon informal and somewhat arbitrary conventions. For instance, it may be suggested that $\mathrm{P}(q \mid p_1, ..., p_n) \leq \mathrm{P}(\mathsf{C}(q) \mid p_1, ..., p_n)$ must be satisfied by any meta-sentential predicate worthy of that name, but I myself am not convinced about this.

In any case, once an appropriate definition of argument is reached, the concept of argument scheme would be defined as follows:

Definition 23 (Argument scheme)**.** A scheme $\sigma_1, ..., \sigma_n \therefore \phi$ is an argument scheme iff all its instances are arguments.

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