International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 3 Issue 5, May – 2019, Pages: 34-42

Note on Fractional Triple Aboodh Transform and Its Properties

S. Alfaqeih, T.ÖZIS¸

Faculty of Science Department of Mathematics, 35100, Ege University, Bornova – Izmir – TURKEY
sfaqeih@yahoo.com, turgut.ozis@ege.edu.tr

Abstract: *In this paper, the definition of triple Aboodh transform of fractional order α, where α ϵ [0, 1], is introduced for functions which are fractional differentiable. We also present several properties of this transform. Furthermore, some main theorems and their proofs are discussed.*

Keywords— Aboodh transform, Rieman-Lioville derivative, factional double Aboodh transform , Convolution.

1. Introduction

Differential equations both ordinary and partial including fractional have a lot of applications in real life science such as mathematics, physics, statistics, engineering ,and son. However, we do not have general method to solve these equations. One of most popular and rather method for solving differential equations is the transform method. In literature, several different transforms are introduced and applied to find the solution of differential equations such as Laplace transform [1,2], Natural transform [3], Sumudu transform [4], Ezaki transform[5], and so on.

A new one of them is Aboodh transform [6,7] which was introduced by Khalid Aboodh in 2013, the transform has deeper connection with Laplace and Sumudu transforms [8]. Aboodh in 2014 introduced the double Aboodh transform which is a higher version of the simple Aboodh transform[9], S.alfaqeih and T.Ozis in 2019 introduced the triple Aboodh transform and use this method to solve integral, partial and fractional differential equations[10].

In this paper, we extend the work done by S.Alfqeih and T.Ozis [11,12] , by introducing the definition of the fractional triple Aboodh transform and its inverse, then we discuss several main properties and theorems related to this transform. Also we find the fractional triple Aboodh transform for some fractional partial derivatives.

This article is organized as follows:

In Section (2), we give some notations about triple Aboodh transform, first and double fractional Aboodh transforms, Mittag-Leffler function and modified fractional Rieman-Lioville derivative. In section (3), the definition of fractional triple Aboodh transform is introduced**.** In section (4), we present and prove some properties of the triple fractional Aboodh transform, in section (5), the convolution theorem of the triple fractional Aboodh transform and its proof are stated. In section (6), we present the inversion formula and inversion theorem and its proof. Finally, the conclusion follows in section (7).

2. Preliminaries

In this section , the definitions of triple Aboodh transform, simple and double fractional Aboodh transforms, and the fractional derivative via fractional difference are presented.

Defention 2.1 The triple Aboodh transfom of a continuous function $f\ (x,t)$ is defined by:

$$K\left(s,p,q\right)=A_{txy}\left(f\ \left(t,x,y\right)\right)=\frac{1}{spq}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(st+px+qy)}f\ \left(t,x,y\right)dtdxdy \qquad (1)$$

And, the inverse of triple Aboodh transform is given by:

$$f\ (t,x,y)=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}se^{st}\left[\frac{1}{2\pi i}\int_{\beta-i\infty}^{\beta+i\infty}pe^{px}\left[\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}qe^{qy}K\,(s,p,q)dq\right]dp\right]ds \qquad (2)$$

For more details see[10]

Defintion 2.2: [11] The fractional Aboodh transfrom of function $f(t)$ is given by:

$$A_\alpha[f(t)] = K_\alpha(s) = \frac{1}{s}\int_0^\infty E_\alpha(-(st)^\alpha) f(t)(dt)^\alpha, s \in \mathbb{C}, t > 0. \qquad (3)$$

Definition 2.3: [12] the fractional double Aboodh transform of function $f(t,x)$ is defined by :

$$A_\alpha^2(f(t,x)) = K_\alpha(s,p) = \frac{1}{sp}\int_0^\infty\int_0^\infty E_\alpha\left(-(st+px)^\alpha\right) f(t,x)(dt)^\alpha (dx)^\alpha. \qquad (4)$$

Defintion 2.4: [13] The Mittag-Leffler function is defined by as follows:

$$E_\alpha(t) = \sum_{k=0}^\infty \frac{t^k}{\Gamma(\alpha k+1)}, \ t \in \mathbb{C}, \ \Re(\alpha) > 0. \qquad (5)$$

Defintion 2.5: (Fractional Derivative via Fractional Difference) let $f(t)$ be a continuous function and not necessarily differentiable, then the fractional difference of $f(t)$ is defined by:

$$\Delta^\alpha f(t) = (FW - h)^\alpha f(t) = \sum_{j=0}^\infty (-1)^j \binom{\alpha}{j} f(\gamma + (\alpha - k)h) \qquad (6)$$

Where $FW(h)$ is a forward operater defined by:

$$FW(h) f(t) = f(t+h)$$

And $h \in \Re^+$ is a constant discretization span .
And its α-derivative is defined by :

$$D^\alpha f(t) = \lim_{h\to 0}\frac{\Delta^\alpha f(t)}{h^\alpha}$$

For more details see [14,15].

Definition 2.6: (Modified Fractional Rieman-Lioville Derivative) Let $f(t)$ be the function that defined in definition 2.5, then

a. If $f(t) = b$, where b is constant , then its α-derivative is given by :

$$D_t^\alpha f(t) = \begin{cases} \dfrac{c}{\Gamma(1-\alpha)t^\alpha}, & \alpha \le 0 \\ 0, & otherwise \end{cases}$$

b. If $f(t)$ is not constant , hence

$$f(t) = f(0) + (f(t) - f(0)),$$

and the fractional derivative given by:

$$f^\alpha(t) = D_t^\alpha f(0) + D_t^\alpha(f(t) - f(0)).$$

Now,for α >0, we put

$$D_t^\alpha(f(t) - f(0)) = D_t^\alpha f(t) = D_t\left(f^{\alpha-1}(t)\right)$$

And if, $m < \alpha < m+1$, we put

$$f^{\alpha}(t)=\left(f^{\alpha-m}(t)\right)^{m}, m\le\alpha\le m+1, m\ge 1$$

Theorem 2.7: The solution of fractional differential equation $dx=f(t)(dt)^{\alpha}, t>0, x(0)=0,$ is given by:

$$x(t)=\int_0^t f(v)(dv)^{\alpha} \qquad ,x(0)=0$$

$$=\alpha\int_0^t (t-v)^{\alpha-1} f(v)dv \qquad 0<\alpha<1$$

Where the integration with respect to $(dt)^{\alpha}$. for more result see [16,17] .

3. Fractional triple Aboodh Transform

Definition 3.1: Let $f(t,x,y)$ be a function where $t,x,y>0$, then the fractonal triple Aboodh transform of order α is defined by :

$$A_{\alpha}^{3}(f(t,x,y))=K_{\alpha}(s,p,q)=\frac{1}{spq}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}E_{\alpha}\left(-(st+px+qy)^{\alpha}\right)f(t,x,y)(dt)^{\alpha}(dx)^{\alpha}(dy)^{\alpha},$$

$$=\lim_{u,v,w\to\infty}\frac{1}{spq}\int_0^{u}\int_0^{v}\int_0^{w}E_{\alpha}\left(-(st+px+qy)^{\alpha}\right)f(t,x,y)(dt)^{\alpha}(dx)^{\alpha}(dy)^{\alpha},\ \text{s,p,q}\in\square . \qquad (7)$$

By using the multiplication property of Mittag-Leffler function, we can write (9) as follows:

$$A_{\alpha}^{3}(f(t,x,y))=K_{\alpha}(s,p,q)=\frac{1}{spq}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}E_{\alpha}\left(-(st)^{\alpha}\right)E_{\alpha}\left(-(px)^{\alpha}\right)E_{\alpha}\left(-(qy)^{\alpha}\right)f(t,x,y)(dt)^{\alpha}(dx)^{\alpha}(dy)^{\alpha} \qquad (8)$$

Remark 1: When $\alpha=1,$, Fractional triple Aboodh transfrom (7) turns to tiple Aboodh transform (1).

4. Properties Of fractional triple Aboodh transform

1) Linearity Property.

Let $f(t,x,y),\ g(t,x,y)$ be two functions of three variables, then:

$$A_{\alpha}^{3}\{c_1 f(t,x,y)+c_2 g(t,x,y)\}=c_1 A_{\alpha}^{3}\{f(t,x,y)\}+c_2 A_{\alpha}^{3}\{g(t,x,y)\},$$

where c_1,c_2 are constants.

Proof:

we can simply get the proof, by applying the definition ()

2) Changing of Scale.

If $A_{\alpha}^{3}\{f(t,x,y)\}=k_{\alpha}(s,p,q)$, then: $A_{\alpha}^{3}\{f(at,bx,cy)\}=\frac{1}{a^{\alpha}b^{\alpha}c^{\alpha}}k_{\alpha}\left(\frac{s}{a},\frac{p}{b},\frac{q}{c}\right),$

Where a,b,c are constants.

Proof:

$$A_{\alpha}^{3}\{f(at,bx,cy)\}=\frac{1}{spq}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}E_{\alpha}(-(st+px+qy)^{\alpha})g(at,bx,cy)(dt)^{\alpha}(dx)^{\alpha}(dy)^{\alpha}.$$

International Journal of Engineering and Information Systems (IJEAIS)
ISSN: 2643-640X
Vol. 3 Issue 5, May – 2019, Pages: 34-42

By letting $u = at, v = bx, w = cy$, we have:

$$A_\alpha^3\{f(at,bx,cy)\} = \frac{1}{a^\alpha b^\alpha c^\alpha}\frac{1}{spq}\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha\left(-\left(\frac{s}{a}u + \frac{p}{b}v + \frac{q}{c}w\right)^\alpha\right) f(u,v,w)(du)^\alpha (dv)^\alpha (dw)^\alpha$$

$$= \frac{1}{a^\alpha b^\alpha c^\alpha} k_\alpha\left(\frac{s}{a},\frac{p}{b},\frac{q}{c}\right).$$

3) Shifting property.

If $A_\alpha^3\{f(t,x,y)\} = k_\alpha(s,p,q)$, then $A_\alpha^3\left\{E_\alpha(-at-bx-cy)^\alpha f(t,x,y)\right\} = k_\alpha(s+a,p+b,q+c)$, where a,b,c are constants.

$$A_\alpha^3\left\{E_\alpha\left(-(at+bx+ct)^\alpha\right) f(t,x,y)\right\}$$

Proof :

$$= \frac{1}{spq}\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha\left(-(st+px+qy)^\alpha\right) E_\alpha\left(-(at+bx+cy)^\alpha\right) f(t,x,y)(dt)^\alpha (dx)^\alpha (dy)^\alpha,$$

Depending on the following property of the Mittag-Leffler function,

$$E_\alpha\left(-(st+px+qt)^\alpha\right) E_\alpha\left(-(at+bx+cy)^\alpha\right) = E_\alpha\left(-((s+a)x+(p+b)x+(q+c)y)^\alpha\right)$$

we have:

$$A_\alpha^3\left\{E_\alpha(-at-bx-cy)^\alpha f(t,x,y)\right\}$$

$$= \frac{1}{spq}\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha\left(-((s+a)t+(p+b)x+(s+c)y)^\alpha\right) f(t,x,y)(dt)^\alpha (dx)^\alpha (dy)^\alpha$$

$$= k_\alpha(s+a,p+b,q+c)$$

4) Multiplication by $t^\alpha x^\alpha y^\alpha$.

If $A_\alpha^3\{f(t,x,y)\} = k_\alpha(s,p,q)$, then $A_\alpha^3\left(t^\alpha x^\alpha y^\alpha f(t,x,y)\right) = \frac{1}{spq} D_s^\alpha D_p^\alpha D_q^\alpha\left(spqk_\alpha(s,p,q)\right)$.

Proof:

$$A_\alpha^3\left(t^\alpha x^\alpha y^\alpha f(t,x,y)\right) = \frac{1}{spq}\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha\left(-(st+px+qy)^\alpha\right) t^\alpha x^\alpha y^\alpha f(t,x,y)(dt)^\alpha (dx)^\alpha (dy)^\alpha$$

$$= \frac{1}{spq}\int_0^\infty\int_0^\infty\int_0^\infty D_s^\alpha D_p^\alpha D_q^\alpha\left[E_\alpha\left(-(st+px+qy)^\alpha\right)\right] f(t,x,y)(dt)^\alpha (dx)^\alpha (dy)^\alpha$$

$$= \frac{1}{spq} D_s^\alpha D_p^\alpha D_q^\alpha\left[\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha\left(-(st+px+qy)^\alpha\right)(dt)^\alpha (dx)^\alpha (dy)^\alpha\right]$$

$$= \frac{1}{spq} D_s^\alpha D_p^\alpha D_q^\alpha\left(spq\, k_\alpha(s,p,q)\right).$$

5) Fractional triple Aboodh transform of some fractional partial derivatives:

a) The fractional triple Aboodh transform of fractional first partial derivatives is given by:

1. respect to t

$$A_{\alpha}^{3}\left(\frac{\partial^{\alpha}}{\partial t^{\alpha}} g(t,x,y)\right) = s^{\alpha} K_{\alpha}(s,p,q) - \frac{\Gamma(1+\alpha)}{s} K_{\alpha}(0,p,q).$$

2. respect to x

$$A_{\alpha}^{3}\left(\frac{\partial^{\alpha}}{\partial x^{\alpha}} g(t,x,y)\right) = p^{\alpha} K_{\alpha}(s,p,q) - \frac{\Gamma(1+\alpha)}{p} K_{\alpha}(s,0,q).$$

3. respect to y

$$A_{\alpha}^{3}\left(\frac{\partial^{\alpha}}{\partial y^{\alpha}} g(t,x,y)\right) = q^{\alpha} K_{\alpha}(s,p,q) - \frac{\Gamma(1+\alpha)}{q} K_{\alpha}(s,p,0).$$

Proof:

(1) By using the fractional integration by part formula with respect to t , we obtain:

$$\frac{1}{spq}\int_0^{\infty}\int_0^{\infty} E_{\alpha}\left(-(px+qy)^{\alpha}\right)\left[\Gamma(1+\alpha) f(t,x,y) E_{\alpha}\left(-(st)^{\alpha}\right)\Big|_0^{\infty} - \int_0^{\infty} D_t^{\alpha} E_{\alpha}\left(-(st)^{\alpha}\right) f(t,x,y)(dt)^{\alpha}\right](dx)^{\alpha}(dy)^{\alpha}$$

$$= \frac{1}{spq}\int_0^{\infty}\int_0^{\infty} -g(0,x,y)\Gamma(1+\alpha)(dx)^{\alpha}(dy)^{\alpha} + s^{\alpha}\frac{1}{spq}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} E_{\alpha}\left(-(st)^{\alpha}\right)E_{\alpha}\left(-(px)^{\alpha}\right)E_{\alpha}\left(-(qy)^{\alpha}\right) f(t,x,y)(dt)^{\alpha}(dx)^{\alpha}(dy)^{\alpha}$$

$$= -\frac{\Gamma(1+\alpha)K_{\alpha}(0,p,q)}{s} + s^{\alpha} A_{\alpha}^{3}\left(f(t,x,y)\right)$$

$$= s^{\alpha} K_{\alpha}(s,p,q) - \frac{\Gamma(1+\alpha)}{s} K_{\alpha}(0,p,q).$$

The proof of part 2 and 3 is similar to part 1.

b) The fractional triple Aboodh transform of a mixed fractional partial derivative is given by:

$$A_{\alpha}^{3}\left[\frac{\partial^{3\alpha}}{\partial t^{\alpha}\partial x^{\alpha}\partial y^{\alpha}} f(t,x,y)\right] = (spq)^{\alpha} K_{\alpha}(s,p,q) - \frac{(sp)^{\alpha}}{q}(\alpha!)K_{\alpha}(s,p,0) - \frac{(sq)^{\alpha}}{p}(\alpha!)K_{\alpha}(s,0,q)$$

$$-\frac{(pq)^{\alpha}}{s}(\alpha!)K_{\alpha}(0,p,q) + \frac{s^{\alpha}}{pq}(\alpha!)^{2}K_{\alpha}(s,0,0) + \frac{p^{\alpha}}{sq}(\alpha!)^{2}K_{\alpha}(0,p,0) + \frac{q^{\alpha}}{sp}(\alpha!)^{2}K_{\alpha}(0,0,q) - \frac{1}{spq}(\alpha!)^{3} f(0,0,0).$$

Proof:

Depending on part (a) , we get the result.

Remark 2: $K_{\alpha}(s,0,0) = K_{\alpha}(s),\ K_{\alpha}(0,p,q) = K_{\alpha}(p,q),$ where $K_{\alpha}(s)$ and $K_{\alpha}(p,q)$, denote the fractional first and double Aboodh transforms given by equation (3),(4) respectivly.

Remark 3 :For $\alpha = 1$, all above results are sutable for triple Aboodh transform.

5. Convolution Theorem

Theorem 5.1 : The α- order triple convolution of functions $f\left(t,x,y\right)$ and $g\left(t,x,y\right)$ defined by the following expression:

$$\left(f\left(t,x,y\right)***_{\alpha}g\left(t,x,y\right)\right)=\int_{0}^{t}\int_{0}^{x}\int_{0}^{y}f\left(t-u,x-v,y-w\right)g\left(u,v,w\right)\left(du\right)^{\alpha}\left(dv\right)^{\alpha}\left(dw\right)^{\alpha}. \quad (9)$$

thus, the fractional triple Aboodh transform of (9) is given by:

$$A_{\alpha}^{3}\left(f\left(t,x,y\right)***_{\alpha}g\left(t,x,y\right)\right)=spqA_{\alpha}^{3}\left\{f\left(t,x,y\right)\right\}A_{\alpha}^{3}\left\{g\left(t,x,y\right)\right\}$$

Proof:

$$A_{\alpha}^{3}\left(f\left(t,x,y\right)***_{\alpha}g\left(t,x,y\right)\right)$$

$$=\frac{1}{spq}\int_{0}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}E_{\alpha}\left(-(st+px+qy)^{\alpha}\right)\left[\int_{0}^{t}\int_{0}^{x}\int_{0}^{y}f\left(t-u,x-v,y-w\right)g\left(u,v,w\right)\left(du\right)^{\alpha}\left(dv\right)^{\alpha}\left(dw\right)^{\alpha}\right]\left(dt\right)^{\alpha}\left(dx\right)^{\alpha}\left(dy\right)^{\alpha} \quad (10)$$

by letting $\phi=t-u,\varphi=x-v,\gamma=y-w$ and the limit from zero to infinity, (10) becomes:

$$=\frac{1}{spq}\int_{0}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\left[\begin{array}{l}E_{\alpha}\left(-(s(\phi+u))^{\alpha}\right)E_{\alpha}\left(-(p(\varphi+v))^{\alpha}\right)E_{\alpha}\left(-(q(\gamma+w))^{\alpha}\right)\\ \times\left(\int_{0}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f\left(\phi,\varphi,\gamma\right)g\left(u,v,w\right)\left(du\right)^{\alpha}\left(dv\right)^{\alpha}\left(dw\right)^{\alpha}\right)\left(d\phi\right)^{\alpha}\left(d\varphi\right)^{\alpha}\left(d\gamma\right)^{\alpha}\end{array}\right]$$

$$=\left[\begin{array}{l}\left(\frac{1}{spq}\int_{0}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}E_{\alpha}\left(-(su)^{\alpha}\right)E_{\alpha}\left(-(pv)^{\alpha}\right)E_{\alpha}\left(-(qw)^{\alpha}\right)g\left(u,v,w\right)\left(du\right)^{\alpha}\left(dv\right)^{\alpha}\left(dw\right)^{\alpha}\right)\\ \times\left(\int_{0}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}E_{\alpha}\left(-(s\phi)^{\alpha}\right)E_{\alpha}\left(-(p\varphi)^{\alpha}\right)E_{\alpha}\left(-(q\gamma)^{\alpha}\right)f\left(\phi,\varphi,\gamma\right)\left(d\phi\right)^{\alpha}\left(d\varphi\right)^{\alpha}\left(d\gamma\right)^{\alpha}\right)\end{array}\right]$$

$$=spqA_{\alpha}^{3}\left\{f\left(t,x,y\right)\right\}A_{\alpha}^{3}\left\{g\left(t,x,y\right)\right\}$$

6. Inversion equation of triple fractional Aboodh transform

Definition 6.1:[17] The Dirac's distripution $\delta_{\alpha}\left(t,x,y\right)$ of order α , where $\alpha\in\left(0,1\right)$ is defined by:

$$\int_{\Re}\int_{\Re}\int_{\Re}f\left(t,x,y\right)\delta_{\alpha}\left(t-\phi,x-\varphi,y-\gamma\right)\left(dt\right)^{\alpha}\left(dx\right)^{\alpha}\left(dy\right)^{\alpha}=\alpha^{3}f\left(\phi,\varphi,\gamma\right). \quad (11)$$

Example : The triple fractional Aboodh transform of $\delta_{\alpha}\left(t-\phi,x-\varphi,y-\gamma\right)$ can be given by:

$$A_{\alpha}^{3}\left\{\delta_{\alpha}\left(t-\phi,x-\varphi,y-\gamma\right)\right\}=\frac{1}{spq}\int_{0}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}E_{\alpha}\left(-(st+px+qy)^{\alpha}\right)\delta_{\alpha}\left(t-\phi,x-\varphi,y-\gamma\right)\left(dt\right)^{\alpha}\left(dx\right)^{\alpha}\left(dy\right)^{\alpha}.$$

$$=\frac{\alpha^{3}}{spq}E_{\alpha}\left(-(s\phi+p\varphi+q\gamma)^{\alpha}\right)$$

In particular,

$$A_\alpha^3\left\{\delta_\alpha\left(t,x,y\right)\right\}=\frac{\alpha^3}{spq}\ .$$

The relation between $\delta_\alpha\left(t,x,y\right)$ and $E_\alpha(t+x+y)^\alpha$ is clarified by the following lemma.

Lemma 6.2 : [18] The following formula holds

$$\frac{\alpha^3}{\left(\mu_\alpha\right)^{3\alpha}}\int\limits_{\Re}\int\limits_{\Re}\int\limits_{\Re}E_\alpha\left(i\left(-ut\right)^\alpha\right)E_\alpha\left(i\left(-vx\right)^\alpha\right)E_\alpha\left(i\left(-wy\right)^\alpha\right)\left(du\right)^\alpha\left(dv\right)^\alpha\left(dw\right)^\alpha=\delta_\alpha\left(t,x,y\right). \qquad (12)$$

Where μ_α , is the period of the complex-valued Mittag-leffer function and satisfy

$$E_\alpha\left(i\left(\mu_\alpha\right)^\alpha\right)=1.$$

Proof:

We test the consistency between (12) and

$$\int\limits_{\Re}\int\limits_{\Re}\int\limits_{\Re}E_\alpha\left(i\left(\xi t\right)^\alpha\right)E_\alpha\left(i\left(\psi x\right)^\alpha\right)E_\alpha\left(i\left(\zeta y\right)^\alpha\right)\delta_\alpha\left(t,x,y\right)\left(dt\right)^\alpha\left(dx\right)^\alpha\left(dy\right)^\alpha=\alpha^3 \qquad (13)$$

By substituting (13), in (12), we get:

$$\alpha^3=\int\limits_{\Re}\int\limits_{\Re}\int\limits_{\Re}\left(dt\right)^\alpha\left(dx\right)^\alpha\left(dy\right)^\alpha\left[\begin{array}{l}E_\alpha\left(i\left(\xi t\right)^\alpha\right)E_\alpha\left(i\left(\psi x\right)^\alpha\right)E_\alpha\left(i\left(\zeta y\right)^\alpha\right)\\ \dfrac{\alpha^3}{\left(\mu_\alpha\right)^{3\alpha}}\int\limits_{\Re}\int\limits_{\Re}\int\limits_{\Re}E_\alpha\left(i\left(-ut\right)^\alpha\right)E_\alpha\left(i\left(-vx\right)^\alpha\right)E_\alpha\left(i\left(-wy\right)^\alpha\right)\left(du\right)^\alpha\left(dv\right)^\alpha\left(dw\right)^\alpha\end{array}\right]$$

$$=\int\limits_{\Re}\int\limits_{\Re}\int\limits_{\Re}\left(dt\right)^\alpha\left(dx\right)^\alpha\left(dy\right)^\alpha\frac{\alpha^3}{\left(\mu_\alpha\right)^{3\alpha}}\int\limits_{\Re}\int\limits_{\Re}\int\limits_{\Re}E_\alpha\left(i\left((\xi-u)t\right)^\alpha\right)E_\alpha\left(i\left((\psi-v)x\right)^\alpha\right)E_\alpha\left(i\left((\zeta-w)y\right)^\alpha\right)\left(du\right)^\alpha\left(dv\right)^\alpha\left(dw\right)^\alpha$$

$$=\int\limits_{\Re}\int\limits_{\Re}\int\limits_{\Re}\delta_\alpha\left(t,x,y\right)\left(dt\right)^\alpha\left(dx\right)^\alpha\left(dy\right)^\alpha$$

$$=\alpha^3$$

Theorem 6.2:(Inversion Theorem)

The inverse of the triple Fractional Aboodh transform (7) can be defined as follows:

$$f\left(t,x,y\right)=\frac{1}{\left(\mu_\alpha\right)^{3\alpha}}\int\limits_{-i\infty}^{i\infty}\int\limits_{-i\infty}^{i\infty}\int\limits_{-i\infty}^{i\infty}spq\,E_\alpha\left(st\right)^\alpha E_\alpha\left(px\right)^\alpha E_\alpha\left(qy\right)^\alpha k_\alpha\left(s,p,q\right)\left(ds\right)^\alpha\left(dp\right)^\alpha\left(dq\right)^\alpha \qquad (14)$$

Proof:

Subsitute (14) into (7) and depending on (12) and (13) we get:

$$f(t,x,y)=\frac{1}{(\mu_\alpha)^{3\alpha}}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}\Big[spq\,E_\alpha(st)^\alpha E_\alpha(px)^\alpha E_\alpha(qt)^\alpha (ds)^\alpha (dp)^\alpha (dq)^\alpha$$

$$\times\frac{1}{spq}\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha(-su)^\alpha E_\alpha(-pv)^\alpha E_\alpha(-qw)^\alpha f(u,v,w)(du)^\alpha (dv)^\alpha (dw)^\alpha\Big]$$

$$=\frac{1}{(\mu_\alpha)^{3\alpha}}\int_0^\infty\int_0^\infty\int_0^\infty\Big[f(u,v,w)(du)^\alpha (dv)^\alpha (dw)^\alpha$$

$$\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}E_\alpha(p(t-u))^\alpha E_\alpha(q(x-v))^\alpha E_\alpha(p(y-v))^\alpha (ds)^\alpha (dp)^\alpha (dq)^\alpha\Big]$$

$$=\frac{1}{(\mu_\alpha)^{3\alpha}}\int_0^\infty\int_0^\infty\int_0^\infty\frac{(\mu_\alpha)^{3\alpha}}{\alpha^3}f(u,v,w)\delta_\alpha(u-t,v-x,w-y)(du)^\alpha (dv)^\alpha (dw)^\alpha$$

$$=\frac{1}{\alpha^3}\int_0^\infty\int_0^\infty\int_0^\infty f(u,v,w)\delta_\alpha(u-t,v-x,w-y)(du)^\alpha (dv)^\alpha (dw)^\alpha$$

$$=f(t,x,y).$$

7. CONCLUSION

In this article, we have extended the work of [11,12] to the triple fractional Aboodh transform. Several main properties and theorems related to fractional triple Aboodh transform are discussed and proved. We also, implemented the introduced transform to some partial fractional derivatives. The triple convolution theorem of fractional order are presented and proved. Finally, we defined the inverse of the fractional triple Aboodh transform. Our results are in agreement with the triple Aboodh transform when $\alpha=1$.

REFERENCES

1. Khalaf R.F. and Belgacem F.B.M., Extraction of the Laplace, Fourier, and Mellin Transforms from the Sumudu transform, AIP Proceedings, 1637, 1426 (2014)
2. Thakur, A. K. and Panda, S. ‘’ Some Properties of Triple Laplace Transform’’, Journal of Mathematics and Computer Applications Research (JMCAR), 2250-2408, (2015).
3. Khan Z.H., and Khan W.A., Natural transform-properties and applications, NUST Journal of Engineering Sciences, 1(2008) 127-133
4. Belgacem F.B.M. and Karaballi A.A., Sumudu transform fundamental properties investigations and applications, Journal of applied mathematics and stochastic analysis, (2006)
5. Tarig M. Elzaki & Eman M. A.Hilal, Solution of Telegraph Equation by Modified of Double Sumudu Transform "Elzaki Transform" Mathematical Theory and Modeling, ISSN 2224-5804(Paper), 1(2011), ISSN 2225-0522 (Online), Vol.2, No.4, 2012.
6. Khalid Suliman Aboodh, The New Integral Transform" Aboodh Transform "Global Journal of Pure and Applied Mathematics ISSN 0973-1768 Volume 9, Number 1 (2013), pp. 35-43.
7. Mohand, M, Khalid Suliman Aboodh,& Abdelbagy, A. On the Solution of Ordinary Differential Equation with Variable Coefficients using Aboodh Transform, Advances in Theoretical and Applied Mathematics ISSN 0973-4554 Volume 11, Number 4 (2016), pp. 383-389.
8. A. K. H. Sedeeg and M. M. A. Mahgoub, Comparison of New Integral Transform Aboodh Transform and Adomian Decomposition Method, Int. J. Math. And App., 4 (2–B) (2016), 127-135.

9. Khalid Suliman Aboodh, Application of New Transform "Aboodh Transform" to Partial Differential Equations, Global Journal of Pure and Applied Mathematics, ISSN 0973-1768 Volume 10, Number 2 (2014), pp.249-254.
10. S. Alfaqeih, T.Ozis¸ Note on Triple Aboodh Transform and Its Application, International Journal of Engineering and Information Systems (IJEAIS)ISSN: 2000-000X Vol. 3 Issue 3, March – 2019, Pages: 41-50
11. S. Alfaqeih, T.Ozis First Aboodh Transform of Fractional Order and Its Properties International Journal of Progressive Sciences and Technologies (IJPSAT) ISSN: 2509-0119.Vol. 13 No. 2 March 2019, pp. 252-256
12. S. Alfaqeih, T.Ozis, Note on Double Aboodh Transform of Fractional Order and Its Properties, OMJ, 01 (01): 1–14, ISSN: 2672-7501.
13. I. Podlubny, Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications, Vol. 198 (Academic press, 1998).
14. G. Jumarie, Table of some basic fractional calculus formulae derived from a modified Riemann–Liouville derivative for non-differentiable functions, Appl.Math.Lett.22(2009)378–385.
15. R. Hilfer, Applications of fractional calculus in physics (World Scientific, 2000).
16. K. Oldhman and J. Spanier, The fractional calculus: Theory and applications of differentiation and integration to arbitrary order (Academic Press, New York, 1974).
17. Guy Jumarie Laplace's transform of fractional order via the Mittag–Leffler function and modified Riemann–Liouville derivative.