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10.1 Introduction

Bertrand Russell was neither the first nor the last philosopher to engage in serious theorizing about propositions. But his work between 1903, when he published *The Principles of Mathematics*, and 1919, when his final lectures on logical atomism were published, remains among the most important on the subject. And its importance is not merely historical. Russell's rapidly evolving treatment of propositions during this period was driven by his engagement with – and discovery of – puzzles that either continue to shape contemporary theorizing about propositions, or *ought* to do so. Russell's creative responses to these puzzles also laid the foundation for many later accounts (most obviously, contemporary 'Russellian' accounts of propositions).¹

In this entry we provide an opinionated overview of Russell's influential treatment of propositions, with a focus on the evolution of his views from 1903 to 1919. A growing secondary literature is dedicated to Russell's changing views during this period, and their often complex or opaque motivations. We do not intervene overmuch in this ongoing scholarly discussion. Instead, our aim is to trace some of the central motivations for Russell's evolving views, and highlight the extent to which these motivations remain relevant to contemporary theorizing about propositions.

10.2 Propositional Roles

Russell's conception of the explanatory role of propositions changed between 1903 and 1919. In 1903 his position was recognizably orthodox. He took propositions to be the objects of judgment and assertion and ultimate bearers of truth and falsity. For example, the proposition expressed by an utterance of 'Lyra loves Will' is also what Maggie believes when she believes that Lyra loves Will, and both the utterance and the belief inherit their truth from the truth of the proposition expressed and believed. Finally, Russell took propositions to be what figure in logical relations of implication. Thus, the *proposition* expressed by 'Lyra loves Will and Maggie swims' entails the *proposition* expressed by 'Lyra loves Will'.

As we will see, however, by 1913 Russell abandons this conception of the explanatory role of propositions, and thereby departs quite substantially from current orthodoxy regarding propositional roles.² On the 1913 account, which we examine below, propositions are no longer entities in their own right. Rather, truth or falsity ultimately derives from the truth or falsity of *judgments*.

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10.3 1903: The Principle of Mathematics

10.3.1 Terms, Things, and Concepts

On Russell's (1903) view, propositions are neither linguistic nor psychological. They are structured complexes that exist in the mind-independent world. So the proposition expressed by 'Maggie swims', for example, is a structured complex consisting of Maggie herself and *swimming*. Semantically significant expressions in sentences 'indicate' the entities that constitute the proposition.³ Russell calls these entities 'terms':

Whatever may be an object of thought, or may occur in any true or false proposition, or can be counted as *one*, I call a *term*. This, then, is the widest word in the philosophical vocabulary. I shall use as synonymous with it the words unit, individual and entity.

(Russell 1903: 44, italics in original.)

Russell draws a distinction between two kinds of terms: *things* and *concepts*. The former are indicated by proper names, whereas the latter are indicated by all other (non-syncategorematic) expressions. So 'Maggie' indicates one sort of term – a thing – while 'swimming' indicates another sort of term – a concept.⁴

10.3.2 Propositional Aboutness⁵

Russell also draws a distinction between the terms of a proposition, which are terms that the proposition is *about*, and terms that are *not* what the proposition is about:

I shall speak of the *terms* of a proposition as those terms, however numerous, which occur in a proposition and may be regarded as subjects about which the proposition is.

(Russell 1903: 46, italics in original)

Things can only ever occur as subjects (or the terms of the proposition), whereas concepts have two-fold occurrence: they can occur either as subjects or as part of what is asserted about the subject(s).

10.3.3 Denoting Concepts

However, the category of entities that a proposition is 'about' is not exhausted by entities that occur as terms of the proposition. Rather, there are entities that a proposition can be about that do not occur as constituents in the proposition at all. These entities are not directly represented, since they are not propositional constituents, but are instead *denoted* by a special sort of propositional constituent. These special propositional constituents are *denoting concepts*.

Recall the proposition encoded by 'Maggie swims'. That consists of Maggie herself and the property of swimming, and is true iff Maggie instantiates *swimming*. Now consider the proposition encoded by 'The Olympic and world champion and Americas record holder in the women's 100 metre butterfly swims'. The definite description does not contribute *Maggie* to the proposition encoded.⁷ Rather, it contributes something like a uniquely instantiable concept. But now we cannot say that the proposition is true iff that uniquely instantiable concept instantiates *swimming!* So Russell's solution circa 1903 is to take a step in Gottlob Frege's direction by introducing *denoting concepts.*⁸

To fix ideas, consider a 'toy' version of the theory, according to which expressions of the form The FT occurring outside the scope of quotation marks and similar devices contributes a concept that, if it denotes anything, denotes a unique entity. Furthermore, any proposition containing a denoting concept in 'subject' position and a concept in 'predicate' position is true iff whatever that denoting concept denotes, if anything, also instantiates the concept in 'predicate' position.

For Russell, the denoting relation is a 'logical', rather than psychological relation. By contrast, the 'indicating' relation – the relation that holds between words and the terms they stand for – is a psychological relation that holds in virtue of psychological facts. The occurrence of a denoting concept is *denotative*¹⁰ when the proposition is not 'about the concept, but about a term connected in a certain peculiar way with the concept' (Russell 1903: 54). Thus, the central role played by denoting concepts is that of 'aboutness-shifters': when denoting concepts occur in propositions, they make those propositions about the entities denoted, rather than about the denoting concepts themselves. Denoting concepts thereby make it possible for a proposition to be about an entity that is not among its constituents.¹¹

10.3.3.1 Informative Identities

With denoting concepts firmly in hand, Russell deploys them to address two other related questions: the question of how definitions could be informative, and the question of why identity statements are significant. We briefly outline his solution to the latter problem.¹²

Why is it ever worth while to affirm identity? This question is answered by the theory of denoting. If we say "Edward VII is the King," we assert an identity; the reason why this assertion is worth making is, that in the one case the actual term occurs, while in the other a denoting concept takes its place.

(Russell 1903: 65)

The puzzle about what makes an identity informative recurs in 'On Denoting' in 1905, and echoes the opening passage of Frege's 'On Sense and Bedeutung' in 1892. As Russell famously discusses, George IV wants to know whether Scott is the author of *Waverly*, but does not thereby want to know whether Scott is Scott. Denoting concepts are meant solve this puzzle. That George IV can have different wants reflects a difference between the propositions expressed by 'Scott is Scott' and 'Scott is the author of *Waverley*'. On Russell's view, 'the author of *Waverly*' indicates a denoting concept that denotes Scott. And so, while it does not make much sense to ask whether Scott is Scott, it does make sense to ask whether the author of *Waverly* is Scott, since that is, in effect, to ask whether Scott is identical to whatever is denoted by the denoting concept indicated by 'the author of *Waverley*'.

10.3.4 Propositions, Truth, and Correspondence

Russell's (1903) account was also shaped in part by his views about truth. Recall that during this period, Russell took propositions to be the ultimate bearers of truth values. And he recognized that the right account of propositions will be constrained by the right account of the nature of truth. But as we will see, circa 1903 Russell had ample reason to reject a *correspondence* theory of truth for propositions, which in turn informed his early account of propositions.

In short, neither Russell, nor his comrade G.E. Moore, saw room for daylight between propositions – understood as complexes composed of worldly constituents – and facts. As Russell puts it:

The fundamental objection may be simply expressed by saying that true propositions express *facts*, while false ones do not. This at once raises the problem: What is a fact? And the difficulty of the problem is this, that a fact appears to be merely a true proposition, so that what seemed a significant assertion becomes a tautology.

(Russell 1904b: 523)

Yet this appears to be a succinct restatement of an argument made a few years earlier by Moore:

It is commonly supposed that the truth of a proposition consists in some relation which it bears to reality; and falsehood in the absence of this relation. The relation in question is generally called a 'correspondence' or 'agreement', and it seems to be generally conceived as one of partial similarity; but it is to be noted that only propositions can be said to be true in virtue of their partial similarity to something else, and hence that it is essential to the theory that a truth should differ in some specific way from the reality, in relation to which its truth is to consist ... It is the impossibility of finding any such difference between a truth and the reality to which it is supposed to correspond which refutes the theory ...

(Moore 1901–1902: 20–21)

The precise reasons for Russell's rejection of the correspondence theory of truth are a matter of debate. Regardless, Russell's stance left him with a challenge: provide an account of propositions – and a complementary account of truth – that could make sense of propositions as ultimate bearers of truth and falsity, while avoiding commitment to a correspondence theory of truth. As we will see, his solution circa 1903 was to deny that propositions represent the world: true propositions *just are* facts. This was Russell's 'identity theory of truth'. 15

Unlike Russell's commitment to the direct view of representation, which becomes only more thoroughgoing between 1903 and 1919, his views on truth and its relationship to propositions evolve significantly during the same period. He eventually abandons both his commitment to propositions as ultimate bearers of truth and falsity, and his wholesale rejection of the correspondence theory. We will see how this evolution proceeded in tandem with shifts in his views about the nature of propositions.

10.3.5 Some Problems

Russell was often his own fiercest critic. So it should come as no surprise than many of the most pressing problems for his 1903 account of proposition were raised by Russell himself. We canvas three central problems in this section. These are the problem posed by propositional unity and 'false facts' (§3.5.1); the complications introduced by denoting concepts (§3.5.2); and the Appendix B Paradox (i.e., the 'Russell-Myhill Antinomy' (RM)) (§3.5.3).

None of these problems was initially sufficient to dislodge Russell's commitment to the 1903 account. Indeed, we will see how his discovery of the Theory of Descriptions permitted him to avoid apparent complications introduced by denoting concepts. However, Russell would later cite – albeit occasionally indirectly – versions of unity problems and RM when explaining his eventual abandonment of core elements of his 1903 view and, indeed, his abandonment of propositions altogether.

10.3.5.1 Propositional Unity and the Problem of 'False Facts'

A central problem for Russell's (1903) view, and one that would remain persuasive for him through 1919, is the problem of accommodating false propositions. It is a problem that is deceptively simple to state in its basic form – if true propositions are facts, then what are false propositions? False facts? – but devilishly difficult to nail down in detail.

Russell's commitment to a 'direct' view of representation requires that the proposition expressed by a sentence, or which is judged by a subject, must include the worldly entities that form the subject matter of the relevant sentence or judgment (rather than merely Fregean senses, or some similar intermediate entity). For instance, the proposition expressed by 'Lyra loves Will' must include Lyra, Will, and the relation of loving. Yet the proposition is not a mere collection of terms. What unifies them into a proposition? This is a version of what is now known as the 'problem of propositional unity'.

The problem of propositional unity and the problem of accommodating false propositions are coordinate: a straightforward solution to one problem seems to preclude a satisfying solution to the other, and so one problem disappears seemingly only if the other bites.

In 1903, it is the problem of propositional unity that receives a straightforward solution. When 'Lyra loves Will' is true, Lyra loves Will. And that is because Russell thought that Lyra loving Will *just is* a true proposition. Hence, there is no special unity *problem* with true propositions for Russell, since whatever unifies propositions is just whatever unifies facts.

Yet by giving the problem of propositional unity a straightforward solution, Russell faces a serious problem of accommodating falsehood. For Russell, the problem is this: what could false propositions be, given that true propositions are facts? False facts? While Russell flirts with that view in 1903, incoherence seems to threaten. And indeed, in later work Russell repeatedly raises the problem of 'false facts' or 'objective falsehoods' as a reason (though perhaps not the only one) for abandoning not only the account of propositions developed in the *Principles*, but a commitment to propositions as entities.¹⁷

Russell avoids outright incoherence in 1903 by following Moore in taking truth to be a *simple* and *unanalysable* property of propositions. ¹⁸ This view of truth permits him to treat the division between true and false propositions as primitive, and introduce facts as true propositions, while remaining quiet about what further category false propositions might belong to. However, whatever the merits of this proposal about truth, a lingering problem remains: what explains propositional unity for *false* propositions, given how the unity is secured for true propositions? Once again, Russell finds himself squeezed between a desire to explain propositional unity and the need to accommodate falsehood.

One response to the lingering problem is to extend Russell's straightforward answer to the problem of unity. This requires insisting upon a disjunctive account of propositional unity: when 'Lyra loves Will' is true, it expresses a proposition whose unity consists in Lyra loving Will; when the sentence is false, it expresses a proposition whose unity requires the relation of loving to play some other unifying role. However, while there is nothing incoherent about treating a relation as playing different roles in different contexts, introduction of the second (putative) role for the 'relating relation' represents a dilution of sorts of Russell's original solution to the problem. ²⁰

Another option, more radical by Russell's lights, would be to adopt a correspondence theory of truth for propositions. In effect, this is the route pursued by some proponents of contemporary Russellian accounts of propositions. Propositions become intermediaries between judgments (or sentences) and the facts. 'Lyra loves Will', for instance, expresses a proposition which is true in virtue of there being a fact – namely Lyra loving Will – which corresponds to that proposition; if the proposition were false, it would be because there is no corresponding fact. The problem of unity then becomes a metaphysical problem concerning entities that are composed of worldly constituents but are not themselves facts. And the problem of accommodating falsehood collapses into a more general challenge to explain how propositions – understood as entities distinct from the represented facts and from judgments – can have non-derivative truth-conditions and other remarkable features.²¹

Russell was not tempted by such a view. And this remained true even when he later relented and accepted a correspondence theory of truth. While Russell would later come to accept a correspondence theory of truth for *beliefs* – and judgments more generally – he never accepted a correspondence theory of truth for *propositions*. As noted above, his early writings are peppered with attacks on the correspondence theory of truth for propositions and, in particular, his intermittent hostility toward the notion of objective falsehoods.²² Russell expresses this position in his *Problems of Philosophy*:

It might be said that [a subject's] belief is a relation to a different object, namely 'that Desdemona loves Cassio'; but it is almost as difficult to suppose that there is such an object as this, when Desdemona does not love Cassio, as it was to suppose that there is 'Desdemona's love for Cassio'. Hence it will be better to seek for a theory of belief which does not make it consist in a relation of the mind to a single object.

(Russell 1912, p. 124)

Russell's dismissal here is striking in part because he was no stranger to seemingly extravagant commitments. Nevertheless, it seems clear that throughout this period Russell viewed correspondence as a non-starter when it came to solving the problems of unity and propositional falsehood.²³

10.3.5.2 The Problem of Denoting Concepts

At this point you may be wondering why Russell was not more sanguine about a correspondence theory of truth for propositions, given the central role denoting concepts play in his account of propositions in his 1903. Apart from their other peculiar characteristics, denoting concepts require a representational relationship between a constituent of a proposition and things in the external world. Including denoting concepts in some propositions therefore generates a disunity in Russell's account of propositions, and his complementary account of truth. Propositions that do not contain denoting concepts as constituents are, if true, identical to facts. Yet true propositions containing denoting concepts are not themselves facts, but instead stand in a representation relation to a fact.²⁴

As previously noted, Russell's introduction of denoting concepts marked a concession to Frege's theory and its concomitant commitment to 'indirect' cognitive contact. Russell seemed at various turns to criticize Frege's theory on not only semantic grounds, but also epistemological and metaphysical grounds. But he struggled to make clear the source of his unease. For example, in his December 12, 1904 letter to Frege, he writes:

Concerning sense and denotation, I see nothing but difficulties which I cannot overcome ... I believe that in spite of all its snowfields Mont Blanc itself is a component part of what is actually asserted in 'Mont Blanc is more than 4,000 metres high'. We do not assert the thought, for this is a private psychological matter: we assert the object of the thought, and this is, to my mind, a certain complex (an objective proposition, one might say) in which Mont Blanc is itself a component part. If we do not admit this, then we get the conclusion that we know nothing at all about Mont Blanc. This is why for me the *denotation* of a proposition is not the True, but a certain complex which (in the given case) is true. In the case of a simple proper name like 'Socrates', I cannot distinguish between sense and denotation; I see only the idea, which is psychological, and the object. Or better: I do not admit the sense at all, but only the idea and the denotation.

(Letter to Frege, 12 December 1904)²⁵

Russell clearly thought Frege's theory was deeply mistaken, and his own 1903 theory is tarnished by implication. Russell would not find a satisfactory resolution until his discovery of the Theory of Descriptions in 1905.

10.3.5.3 Appendix B Paradox, a.k.a. the Russell-Myhill Antinomy

In Appendix B of *Principles of Mathematics*, Russell discovered a paradox which threatened not only his 1903 account of propositions, but potentially any account of propositions that treats them as entities. It is now known as the 'Russell-Myhill Antinomy' (RM).²⁶

For our purposes, it makes most sense to provide a brief, informal statement of the paradox that skips over some of Russell's original reasoning²⁷.

Suppose propositions are entities. Then propositions can form classes, and we can quantify over them. And for each class of propositions, we can introduce a proposition that is true iff every proposition in the class is true. Assuming that such propositions must differ depending on the propositions belonging to the corresponding class, it follows that each class of propositions corresponds to a distinct proposition. So there are at least as many propositions as there are classes of propositions. But this violates a theorem

from Cantor – his power class theorem – which says that the number of sub-classes of a domain will be strictly larger than the number of entities in that domain. Paradox!

We will not run through the full-dress version of the paradox, or discuss in detail Russell's commitment to the various principles that contribute to generate the paradox. Our informal presentation suffices to bring out a crucial component of the paradox, namely the supposition that propositions are entities. There are nice and hard questions about which other principles at work in the paradox might plausibly be rejected in order to preserve a commitment to propositions, and what a commitment to propositions might look like once we abandon this or that principle. And these questions have become newly relevant, and consequently the focus of sustained attention, in response to recent work on unrestricted quantification and higher order logic. However, our interest lies with the role that the Russell-Myhill Antimony played in the development of Russell's own views about propositions.

What is perhaps most surprising about Russell's inclusion of the paradox in Appendix B is that it did not cause him to immediately repudiate his account of propositions. It had no obviously palatable solution.³⁰ So why did he remain committed to the existence of propositions as entities beyond 1903? Indeed, he would not question the status of propositions as entities until 1906, and then not seriously until several years later with his shift to his Multiple Relation Theory of Judgment. However, we will leave the issue to others, and instead hew to our main topic.

10.4 1903-1906: Intermediate Progress and Russell's Theory of Descriptions

A residual problem with Russell's (1903) account was his reliance upon denoting concepts, which sow disunity within his accounts of propositions and truth, and moves him closer to Frege's theory, which he considered deeply mistaken. However, given that the theory of denoting concepts solved some of the central puzzles Russell was concerned with, why exactly did he eventually abandon denoting concepts?³¹

As we suggested above, the answer may lie in the notoriously difficult 'Gray's *Elegy*' passage in 'On Denoting'. While a detailed discussion of this passage goes beyond the scope of this entry, the difficult discussion in that passage concerns the core role of denoting concepts as aboutness-shifters: if denoting concepts are devices that shift what propositions they occur in are about – if they make those propositions about what those concepts denote – how could there be a proposition about a denoting concept? The worry is not merely that such a proposition would be inexpressible, but that such propositions would be incoherent.³²

Whatever the exact motivation for Russell's abandonment of his theory of denoting concepts, his solution was the Theory of Descriptions introduced in 'On Denoting'. Recall that definite descriptions exhibit some features of singular terms (e.g., singular definite descriptions denote at most one thing), and some features of quantified noun phrases (i.e., determiner phrases of syntactic form [DET + NP], with 'the' behaving essentially as a 'uniqueness' quantifier). Russell in 1903 assimilates them to singular terms, thus pressuring him to introduce denoting concepts. Russell in 1905, however, assimilates definite descriptions to quantified noun phrases to profound effect. Thus, the new theory permits Russell to deny that expressions like 'the last Tudor monarch' are genuine referring expressions. Instead, these expressions become mere 'incomplete symbols' – they superficially appear as singular terms, but 'vanish' upon analysis.³³

Incomplete symbols have three distinctive features: (i) they do not correspond to any constituent of propositions expressed by sentences in which the symbols occur; (ii) they can be eliminated: any proposition expressed by a sentence containing an incomplete symbol can be expressed by a sentence without that symbol (or any corresponding expression); and (iii) incomplete symbols have no meaning in isolation.³⁴

It is easy to see how definite descriptions satisfy these conditions under Russell's Theory of Descriptions. 'The last Tudor monarch was a woman' becomes equivalent to 'There was exactly one last Tudor monarch, and that person was a woman', satisfying (ii). And that second sentence does not express

a proposition with a constituent corresponding to 'The last Tudor monarch', satisfying (i). Finally, the meaning of 'The last Tudor monarch' cannot be given by specifying a referent (e.g., an object or property it picks out), but must be explained in terms of its contribution to the truth conditions of sentences like 'The last Tudor monarch was a woman'.

Definite descriptions were not the only incomplete symbols Russell recognized. But classing them as incomplete symbols allowed him to dispense with denoting concepts. His original motivation for denoting concepts disappears, since putative complex referring expressions turn out to be incomplete symbols, demanding no corresponding propositional constituent.

10.5 1906-1910: Rejection of Propositions

It is perhaps ironic that Russell's notion of an incomplete symbol would later come to play an instrumental role in his eventual abandonment of propositions. On Russell's (1903) view, propositions are entities with the same standing as other particulars: they could be named, quantified over, etc. Prior to Russell's recognition of a category of incomplete symbols, these (apparent) features of propositions posed a major obstacle to any attempt to avoid commitment to propositions as entities. Yet once Russell recognized that some expressions are incomplete symbols, he possessed the resources to formulate a view that accommodates apparent proposition talk, and yet which avoids treating propositions as entities. After 1905, Russell could – and eventually did – hold that sentences which appear to name propositions, or otherwise treat them as entities, are really equivalent to sentences that do not include expressions for which there must be a corresponding proposition. On this sort of view, expressions which might otherwise be thought of as names for propositions, such as 'that Lyra loves Will', turn out to be incomplete symbols.³⁵

Russell did not abandon propositions all at once. He floats the option in 1906, in his paper 'On the Nature of Truth', but does not take the plunge until 1910 with 'On the Nature of Truth and Falsehood'. And it is far from clear exactly which reasons were most decisive in motivating and justifying his shift. ³⁶ On the more formal end, remember that Russell had discovered a paradox that threatens the coherence of views on which propositions are entities, namely the Russell-Myhill Antinomy. On the less formal end, the advances made possible by his Theory of Descriptions did not immediately free Russell from his lingering worries about how to accommodate false propositions. Russell's remarks on the shift suggest that he was sensitive to both problems, and that he saw the abandonment of propositions as the first step in a solution to them.

There are nice and hard philosophical questions about the extent to which the problems that Russell raises for his earlier views generalize. Contemporary theorizing about propositions has often – though not always – been marked by a complacent disregard for some of Russell's central worries. But Russell was no slouch. While some aspects of his views were doubtless idiosyncratic, it is far from obvious that his main complaints about these views were entirely parochial. For instance, the problems posed by false propositions were in part due to Russell's insistence – following Moore – that if propositions are the ultimate truth bearers, truth and falsity are themselves simple unanalysable properties of propositions. If Russell is right, but Russellian propositions are not the ultimate bearers of truth and falsity, what, if anything, then is their connection to truth and falsity? This question becomes pressing when Russell abandons propositions, and helps shape his positive accounts in 1910 and 1913. Yet if Russell is wrong, and truth can be an analyzable but non-derivative property of propositions, what might an analysis even look like? While contemporary theorists have sought to answer this question, especially in recent years, it is far from clear that any proposed answers are plausible and well-motivated.³⁷

10.5.1 Constraints on Abandonment

Abandoning propositions, or at least the claim that propositions are entities, required Russell to embark upon a constructive task of several parts. He first had to justify his claim that expressions which we might have taken (and he earlier took) to stand for propositions are in fact incomplete symbols. He then

had to make fresh sense out of the relationship between Russellian propositions and truth, since he could no longer treat propositions as the ultimate bearers of truth and falsity.

Russell's core innovation was to transform what had been a theory of propositions into a theory of judgment. Remember that on his 1903 view, judgments were explained by appeal to propositions. A subject's judgment that p is taken to consist in the subject standing in a dual relation to a specific kind of complex particular, namely a proposition. By contrast, Russell's abandonment of propositions leads him to reverse the order of explanation. Sentences that involve apparent commitment to propositions (e.g., 'Either Lyra loves Will or she does not') are then understood, as is appropriate for sentences including incomplete symbols, in terms of a right account of judgment (in much the same way that the Theory of Descriptions permitted sentences involving definition descriptions to be understood in terms of those involving only quantifiers, variables, and predicates). This approach also opens the door for Russell to use the truth or falsity of judgments to underwrite the apparent attribution of truth or falsity to propositions (e.g., 'it is true that Lyra loves Will, but false that she only loves him').

The problem for Russell, then, is to provide an account of judgment that includes no fundamental role for propositions, and yet which permits an adequate theory of truth. Russell's attempt to solve this problem is the 'Multiple Relation Theory of Judgment' (hereafter 'MRTJ').³⁹ He formulates the theory at least four times, but we focus on the version he develops in 1910 when he first adopts it, and turn in the next section to the most sophisticated version of the MRTJ, which Russell develops in his (famously abandoned) 1913 manuscript *Theory of Knowledge*.⁴⁰

10.5.2 MRTJ: 1910 Version

Russell's (1910) version of the MRTJ introduces the major defining features of the view, and so brings out most clearly its continuity with his earlier work on propositions.

Despite abandoning many of the doctrines that defined his 1903 account of propositions, in 1910 Russell still accepts a direct view of representation. He still thinks, as Pincock (2008: 107) puts it, that 'in a propositional attitude, the entities that are the subject-matter of this attitude are also parts of the propositional attitude'. On the 1903 view, the represented entities are part of the propositional attitude in virtue of being part of a proposition to which the subject was related by a two-place judgment relation. By contrast, Russell's MRTJ jettisons the 1903 role for propositions, and treats judgment as consisting in a multiple relation between the judging subject and the entities that constitute the judgment's subject matter. For example, Maggie's belief that Lyra loves Will is now a complex that consists in Maggie standing in the believing relation to Lyra, Will, and the loving relation. Instead of this judgment having the form B(Maggie, p), it has the form B(Maggie, Lyra, loving, Will).

With an account of judgment in view, the next step is to explain how the account might be leveraged to eliminate propositions, and to provide an adequate account of truth.

In order to eliminate propositions, Russell needs the MRTJ to underwrite an account of both the truth of ordinary declarative sentences (e.g., 'Lyra loves Will') and the truth of sentences that include apparent reference to propositions (e.g., 'It's true that Lyra loves Will'). Landini (1991: 45–48) meticulously reconstructs how this account might work within Russell's system, and how it might be extended from atomic sentences (e.g., 'Lyra loves Will') to molecular sentences (e.g., 'Either Lyra loves Will or she does not'). The account provides accounts of the truth or falsity of atomic and molecular sentences in terms of the obtaining or not of judgments (e.g., Maggie's believing that Lyra loves Will).

This account is only half the battle, and far from the most interesting half. Russell will have managed to provide an adequate account of the truth of atomic and molecular propositions, and thus completed his elimination of propositions, only if he can provide an account of the truth or falsity of judgments themselves. It is at this point that Russell seems to make a drastic departure from his earlier views on truth, for he explicitly endorses a *correspondence* theory of truth.

This departure is not as drastic as it looks initially. From a certain point of view, Russell held a correspondence theory of truth *for belief* even in 1903. In the case of belief, a correspondence theory requires

the specification of an appropriate relation between a belief and whatever fact or state of affairs makes it true. And that is precisely what Russell's (1903) view seems to provide: true beliefs are true in virtue of having facts (i.e., true propositions) as their objects. Of course, this was not a theory of truth, properly speaking, since at the time Russell thought that propositions – not beliefs – are the ultimate bearers of truth and falsity.

Yet what is it for a belief to correspond with a fact? It cannot be that the belief has a fact as its object: that would just run us back into Russell's (1903) account of propositions and all its problems. Here is what Russell says in 1910:

We may therefore state the difference between truth and falsehood as follows: Every judgement is a relation of a mind to several objects, one of which is a relation; the judgement is *true* when the relation which is one of the objects relates the other objects, otherwise it is false.

(Russell 1910, p. 180)

He expresses the same position, even more succinctly, two years later in the Problems of Philosophy:

When the belief is *true*, there is another complex unity, in which the relation which was one of the objects of the belief relates the other objects.

(Russell 1912, p. 128)

Of greatest import, however, is how Russell then elaborates this correspondence theory of truth for belief using an example:

Thus, e.g., if Othello believes *truly* that Desdemona loves Cassio, then there is a complex unity, 'Desdemona's love for Cassio', which is composed exclusively of the *objects* of belief, in the same order as they had in the belief, with the relation which was one of the objects occurring now as the cement that binds together the other objects of the belief. On the other hand, when a belief is false, there is no such complex unity composed only of the objects of the belief.

(Russell 1912, p. 128)

As this passage makes clear, Russell's correspondence theory of truth for belief generates an immediate explanatory burden, *viz.*, what it is for objects of belief to have an *order* or (more perspicuously) a *structure*. Russell was well aware of this burden. It was his need to meet it which partly motivated his development of increasingly sophisticated versions of the MRTJ. He takes his first unsuccessful stab at meeting the burden in 1910.⁴² Dissatisfaction with this initial attempt leads Russell to develop the account he defends in his abandoned 1913 manuscript *Theory of Knowledge*.

10.6 1913: Russell's Most Sophisticated MRTJ

Theory of Knowledge, despite going unpublished in Russell's lifetime, has become the central focus for those interested in tracing Russell's evolving accounts of propositions and judgment. Russell abandoned the incomplete manuscript, and with it his pursuit of a theory of judgment that could underwrite an abandonment of propositions, in response to a set of famously obscure criticisms raised by Ludwig Wittgenstein. Much of the secondary literature on *Theory of Knowledge* concerns the increasingly scholastic dispute over exactly what Wittgenstein's objections to the theory were, which parts of Russell's account were most responsible for its supposed vulnerability, and whether (and why) the theory might be ultimately salvageable.⁴³ Our interest in *Theory of Knowledge* is largely limited to some problems it highlights for anyone who wishes to pursue either a version of the MRTJ (or some descendent of it).

The 1913 multiple relation theory of judgment, just like the 1910 version, holds that belief is a multiple relation between a subject and the entities that together constitute the subject matter of the belief.

Its main innovation – and it turns out to be significant – is the addition of a new relatum: a logical form. For example, Maggie's belief that Lyra loves Will becomes a five-place relation between Maggie, Lyra, Will, the relation of loving, and the logical form of dual complexes.⁴⁴ So when S believes that aRb, this judgment has the structure $B(S, a, R, b, \gamma)$, where γ is the form of dual complexes.

Russell introduces logical forms (hereafter 'forms') into the MRTJ to solve several problems. These problems arise for Russell because of the explanatory burden sketched at the end of the previous section; namely, the burden of explaining what it is for objects of belief to have the sort of order or structure that can underwrite a correspondence theory of truth for belief, i.e., which can generate the result that there will be a unique fact whose obtaining is necessary and sufficient for the truth of a given belief. Pincock distinguishes two parts to this burden:

This problem is to say, for each propositional attitude, what its essential features are. To do this, Russell must first say what each propositional attitude's composition is: what are its parts and how are these parts related. I call this *the proposition problem*. Second, Russell must say what relations a given propositional attitude must stand in if it is to be that kind of propositional attitude. For some propositional attitudes, like judgement or belief, these relations include correspondence relations which fix the truth-value of the propositional attitude. I call this aspect of the propositional attitude problem *the correspondence problem*.

(Pincock 2008: 112, emphasis added)

He eventually argues (rightly, in our view) that while Russell in 1913 had the resources to provide *an* account of judgment's structure, that account could not actually underwrite an adequate correspondence theory of truth for belief.⁴⁵

The most prominent of the problems Russell sought to address in the new theory are what Griffin (1985) dubs the 'narrow' and 'wide' direction problems. Put briefly, the *narrow direction problem* demands an account of the difference between, for example, judging that Lyra loves Will and judging that Will loves Lyra. And the *wide direction problem* is that of explaining why it is impossible to believe nonsense (e.g., to believe that loves Lyra Will). Most attribute the latter problem to Wittgenstein. We elaborate these problems, and why Russell thought logical forms might help solve them, in the rest of this section.

10.6.1 Direction Problems

Russell was already aware of the narrow direction problem in 1910, but *Theory of Knowledge* contains his final and most sophisticated treatment of it. To see why the problem arises, it is worth examining why the problem does not straightforwardly undermine Russell's (1903) view of propositions. On that view, the difference between Maggie believing that Lyra loves Will and Maggie believing that Will loves Lyra is just a difference between the objects of belief, namely the propositions believed. These propositions will be facts when true. And while there are doubtless metaphysical challenges that face any account of the difference between the fact that Lyra loves Will and the fact that Will loves Lyra, these are not challenges that concern judgment itself (or propositions, for that matter, except insofar as facts are true propositions). In effect, Russell in 1903 could prevent the narrow direction problem from arising by simply helping himself to whatever metaphysical account someone might develop of the structure and unity of facts.

Russell's abandonment of propositions, and embrace of the MRTJ, prevents him from exploiting this straightforward solution to the narrow direction problem. On the MRTJ, the only unity enjoyed by the constituents of a subject's judgment is the unity they inherit from the judging itself. As a result, Russell could not assume that the difference between Lyra loving Will and Will loving Lyra would be inherited by the corresponding judgments. Russell therefore had to look elsewhere for a solution to the (newly pressing) narrow direction problem.

Unlike the narrow direction problem, the wide direction problem first makes an appearance in 1913. The problem has its source in Wittgenstein's objections to Russell in the *Tractatus*, especially⁴⁷:

The correct explanation of the form of the proposition, 'A makes the judgement p', must show that it is impossible for a judgement to be a piece of nonsense.

(Russell's theory does not satisfy this requirement.)⁴⁸

We will call Wittgenstein's claim the Nonsense Criterion.

Russell's MRTJ appears fail the nonsense criterion. Central to Russell's theory is the claim that it is the belief relation that unites the constituents to form a judgment. But nothing in the belief relation appears to exclude such patent nonsense as that seemingly expressed by 'Love Lyra Will'. All that is required for Maggie to believe that love Lyra Will is for the objects and relation to be brought into psychological relations to the form of dual complexes by the belief relation. And why should that be impossible?⁴⁹

We now wish to highlight a distinct but oft-neglected problem that Russell also sought to resolve with the introduction of logical forms, and which acquires its force indirectly from the wide direction problem. Cognitive contact, or acquaintance, with the (non-formal) constituents of complexes is insufficient to give us knowledge of how these constituents should be combined to form a judgment. Yet such knowledge is plausibly necessary if a subject wishes to make a judgment composed of these constituents. Compare:

... in order to understand "A and B are similar", we must know what is supposed to be done with A and B and similarity, i.e. what it is for two terms to have a relation

(Russell 1913: 116)

Russell's central claim from the MRTJ, namely that judgment is a multiple relation between a subject and the relevant worldly entities that constitute the judgment's subject matter, plausibly entails that the subject must know how to bring these constituents together to constitute a genuine judgment. Because subjects must assemble judgments from diverse constituents, they must, if they wish to judge, at least know how to assemble such judgments from the relevant constituents. Russell appears to claim that the constituents must provide all the ingredients necessary for a subject to grasp the judgment. So the problem for Russell is to give an account of how cognitive contact with the constituents can be sufficient for knowledge of how the constituents should be combined. This we dub the *Problem of Combination*.

The problem of combination is pressing for any account of propositions that takes inspiration from Russell's MRTJ. The problem acquires real force when combined with the wide direction problem. Theorists must explain not only why it is impossible to judge nonsense, but how the knowledge manifested in judging permits a subject to both recognize nonsense and avoid attempting to think it.⁵¹

10.6.2 Forms and Direction

As we have seen, logical forms are introduced via their role in complexes. The form of a complex is the structure that captures how the constituents are combined in the complex. Russell views forms as the 'residue' that remains after the removal of the constituents of a complex. For example, the form of dual complexes, such as that expressed by 'Lyra loves Will', is the fact expressed by 'something stands in some relation to something'. We get the form by first 'removing' Lyra, (yielding the fact expressed by 'Something loves Will'), then removing Will, and then finally removing the relation of loving. That forms are simple then follows: the residue that remains when the constituents have been removed cannot itself have constituents. A form is thus no more than a structure.⁵²

Russell appeals to the structural character of forms to solve the narrow direction problem. He first considers the narrow direction problem as it arises for ordinary complexes: in virtue of what does the

complex A precedes B differ from the complex B precedes A? Because the forms and the constituents in these complexes are identical, Russell posits *positions* within a complex to explain the difference between the two complexes.⁵³ Positions are relations between the constituents of a complex and the complex itself: relation-positions relate the relating relation, and object-positions the terms of the relating relation, to the complex at issue. In the case of the dual complex A precedes B, where the relating relation is asymmetric, two distinct object-positions can be distinguished: (P1) ξ is *earlier in* α and (P2) ξ is *later in* α (where ξ is an object and α a complex). The distinct complexes that A precedes B and that B precedes A thus differ because each complex assigns different objects to the object-positions (P1) and (P2).⁵⁴

There are two pathological cases of position. First, when the relating-relation of a complex is *symmetric*, the two object-position relations will be identical, as in the subordinate relation expressed in 'Maggie believes that Lyra is as tall as Lyra'.⁵⁵ Second, there are non-symmetrical complexes where there is only one possible assignment of constituents to positions (so-called *heterogeneous* complexes), as in the subordinate relation expressed in 'Maggie understands that Lyra is a member of the class of humans'.⁵⁶ In both cases the constituents can combine to form only one distinct complex since, in the latter case, Lyra and the class of humans are of different 'logical types'. Heterogeneous complexes turn out to be important for Russell.⁵⁷ For instance, in the above example concerning the dual complex that A precedes B, the object-position relation (P1) (ξ is *earlier in* α) can combine with an object ξ and a complex α to form *only* a single complex, since (Russell maintains) the substitution of a (simple) object for α and a complex for ξ in (P1) is not 'logically possible'.

Russell took the nature of these positional relations to be determined by the nature of a complex's relating-relation. ⁵⁸ For example, a symmetrical, two-place, relating-relation R determines the requirement that the two object-positions within the form of any complex in which R is the relating-relation must be identical. Similarly, the nature of the object-positional relation (P1) is constrained (again, by a complex's relating-relation) to be such that, *if* the object-position ξ contains an object of logical type t, then the complex-position α must itself contain a *complex* of objects of logical type t. ⁵⁹ And, as in the case of symmetric complexes, such type-restrictions are the kind of structural constraint that the form can capture.

Unsurprisingly, given the complexity of Russell's solution to the narrow direction problem as it arises for ordinary complexes, extending the idea to *judgments* is not straightforward. Russell could not simply appeal to the form of the relevant judging relation. First, note that, for Russell in 1913, the most basic of these multiple relations is a relation of *understanding*. And understanding-complexes – which result when a subject stands in the multiple relation of understanding to an appropriate collection of relata – cannot always include understanding-*relations* that are sufficient to determine a unique complex given a set of relata. For example, in the complexes picked out by 'Maggie understands that Lyra loves Will' and 'Maggie understands that Will loves Lyra', specifying an understanding-relation and the relevant relata does *not* suffice to discriminate between the complexes. And that is because Maggie is doing "the same thing" in each case – namely, entering into a relation of understanding with a pair of objects, a relation, and the form of dual complexes. Thus there appears to be no room to distinguish the understanding-relations involved, and so no room to thereby distinguish the complexes.⁶⁰

It is at this point that positions within forms play an important role. Russell proposes – though never manages to work out in detail, before abandoning *Theory of Knowledge* – that in the relevant cases, it would always be possible to analyze the complex into a heterogeneous complex:

The proposition "a is before b" must be interpreted as meaning "there is a complex in which a is earlier and b is later"

(Russell 1913, p. 135)

Here, Russell seems to think that heterogeneous complexes – such as that involved in the dual complex that A precedes B – will inevitably involve relations between the relata of the complex to positions within a form. ⁶¹ Returning to the case of *understanding*, that form could not be the form of the understanding complex *itself*, but must instead be the form that acts as a relatum of the understanding

relation. That is because what Russell ultimately seeks to explain is the difference between two ways in which (e.g.) Will and Lyra are related by the relation of loving, when one agent understands that Lyra loves Will, and another understands that Will loves Lyra.⁶²

10.7 1919: Despair!

Despite the many sophisticated maneuvers that unfold in the extant portions of *Theory of Knowledge*, it is clear that Russell came to recognize that the project was ultimately doomed.⁶³ Indeed, by the time Russell gave the lectures that became *The Philosophy of Logical Atomism*, he had no new theory to offer. Having convinced himself that propositions could not be entities in their own right, and that the multiple relation theory of judgment was unworkable, he despaired of finding a workable view of judgment (and consequently, of propositions), writing that

... one has to be content on many points at present with pointing out difficulties rather than laying down quite clear solutions

(Russell 1919, p. 61)

Summarizing the main challenges, Russell writes:

There are really two main things that one wants to notice on this matter that I am treating of just now. The *first* is that the impossibility of treating the proposition believed as an independent entity, entering as a unit into the occurrence of the belief, and the *other* is the impossibility of putting the subordinate verb on a level with its terms as an object term in the belief.

(Russell 1919, p. 61)

Russell's first constraint is familiar at this point.⁶⁴ But while his second constraint on a right account of judgment may seem more obscure, it really just reflects his struggles with the MRTJ. When Russell writes here of the 'subordinate verb', he seeks to pick out the relation that a subject judges to obtain amongst other relata – what we above called a 'subordinate relation'. Recall that when Maggie believes that Lyra loves Will, the relation of loving serves as the 'subordinate verb' or relating-relation. What might it mean to place the relating-relation 'on a level' with those entities a judging subject takes the relation to relate? It is just to adopt the MRTJ, and thereby treat the relating-relation as an object of judgment in exactly the same sense as its relata. Russell's second constraint thus expresses his despair at the prospects of his own MRTJ, a despair occasioned by the seemingly intractable objections raised by Wittgenstein.

Nevertheless, Russell's aporetic position in 1919 has come to frame later theorizing about propositions. It is Russell to whom we originally owe the sense that it is just plain *hard* to know what to say about propositions. While various 'Russellian' proposals have sought to navigate or dismiss the various challenges he raised against his 1903 view and its immediate successors, the jury remains out about whether (and why) any such account might have satisfied Russell. At the same time, the MRTJ was Russell's best attempt to make sense of judgment, meaning, and truth without positing propositions; its apparent failure is therefore an indirect argument for propositions. Hence, Russell's work on propositions remains an unavoidable touchstone for later work on propositions.

Notes

1 Contemporary views along these lines also sometimes go by the name 'Millianism', after J.S. Mill, a prominent advocate of the view that proper names are simply 'tags' for their bearers ('expressions without connotation'). See, in particular, Mill (1843) (though the view undoubtedly had ancient origins). We prefer to reserve 'Millianism' for a thesis about semantic contents of proper names. For contemporary proponents of descendants of Russell's (1903) views, see, e.g., Salmón (1986), Soames (1987), King (2007), King et al. (2014), and this volume (passim).

- 2 Alternatively, Russell can be read as maintaining his early conception of the explanatory role of propositions, and later concluding that nothing satisfies it. Either way, it was open to Russell to accept that the complexes he considered propositions circa 1903 (the *facts*, at least) still *exist* according to his later views, they just don't enjoy the pride of place Russell thought they did in 1903. In a similar vein, Richard Cartwright (1997: 76) thinks a complex with Maggie and *swimming* as constituents *exists*, but is agnostic about whether 'Maggie swims' expresses it. Similarly, Stephen Schiffer (2003: 96) denies such complexes have any important role to play in semantics, but countenances them nonetheless for the work they can do in the metaphysics of modality.
- 3 As Russell writes: 'But a proposition, unless it happens to be linguistic (like the proposition expressed by "Plum-pudding is a word" eds.) does not itself contain words: it contains the entities indicated by words' (1903: 48).
- 4 While the choice of 'concept' suggests something psychological, Russell instead uses it for something 'logical', namely, a category of entity with parallels to our contemporary category of property.
- 5 A tension immediately presents itself. Russellian propositions are complexes of things and concepts, or just concepts, which are represented by our thought and talk, but are not themselves representational in the right way. While 'Maggie swims' represents Maggie as a swimmer, Maggie herself and swimming itself, as it were, do not represent Maggie as a swimmer (in the right way). But here, and elsewhere, Russell talks about what propositions (or their constituents) are about, which seems to imply that they are representational entities, after all. Combined with the observation in fn. 2, this might lead one to accept that (non-representational) Russellian propositions are 'about' their constituents only in the attenuated sense (if there is such a sense) that propositions are constituted by them, and propositional constituents ('terms') like Maggie herself are only 'about' themselves in the sense (if there is such a sense) of enjoying numerical self-identity. This interpretation is reinforced somewhat by the air of stipulation in the above quote, reminiscent of (e.g.) the mereologist's (otherwise harmless) insistence that things are (improper) parts of themselves.
- 6 Russell's original motivation for introducing denoting concepts is primarily mathematical. Russell needed a device that would allow him given this 1903 theory of propositions to make sense of our capacity to think about an infinitely complex object, as well as about the null class. As he writes in the *Principles of Mathematics*:

An infinitely complex object, though there may be such, can certainly not be manipulated by the human intelligence; but infinite collections, owing to the notion of denoting, can be manipulated without introducing any concept of infinite complexity (*Principles*, section 72).

This passage suggests that infinitely complex objects are ineligible to serve as propositional constituents of propositions we can grasp, because of their inaccessibility to human intelligence, and thus our inability to stand in direct cognitive contact with them. A central example of a denoting concept is thus *all numbers*, which denotes, for Russell, an infinitely complex object. Absent this denoting concept, any proposition that was about all numbers would consist of infinitely many constituents, and thus fail to be an entity that could be 'manipulated by the human intelligence'.

- 7 In addition to the familiar reasons Russell adduces in his 1905 and the arguments offered in Kripke (1980), the proposal that a definite description 'indicates' what it denotes is *also* unworkable since that would require (e.g.) assigning incompatible semantic compositional rules to English. See Salmón (1986: 20–21) for further discussion.
- 8 Along with denotative occurrences (see below). Introducing denoting concepts is a step in Frege's direction in the following sense: First, although definite descriptions bear hallmarks of both singular terms (like proper names) and quantified noun phrases (like 'all plum-puddings'), Frege (1892) assimilates definite descriptions to singular terms, giving them an exactly similar semantic treatment. And, second, Frege's general theory of singular terms, as expressing a sense that is distinct from, but related to, their referent, is completely anathema to Russell's preference for 'direct cognitive contact' (his famous 1910–11 doctrine of 'acquaintance'). See Proops (2014) for a nuanced tracing of Russell's evolving treatment of acquaintance, and its connection to his views about propositional constituency; and see Amijee (2013) for a discussion of how Russell's notion of acquaintance relates to propositional knowledge. The concessions to Frege never sat well with Russell, and, as we will see, he completely abandons them by 1905.
- 9 Note that true propositions with denoting concepts (with a denoting occurrence) are not Russellian *facts*; rather, they stand in some representational relation to facts. More on this below.
- 10 This qualification is required because denoting concepts can occur in propositions and yet not have a denotative occurrence, i.e., when the proposition is *about* the denoting concept. However, as is noted by Cartwright (1987b: 103), the notion of a denotative occurrence is primitive and more basic than the notion of a denoting concept.
- 11 This yields the desired result: 'The Olympic and world champion and Americas record holder in the women's 100 metre butterfly' expresses a complex of concepts, but since the occurrence of the relevant denoting concept is denotative in the sentence 'The Olympic and world champion and Americas record holder in the women's 100 metre butterfly swims', it is true iff the denotation of the denoting concept *Olympic and world champion and Americas record holder in the women's 100 metre butterfly*, viz., Maggie, swims.

- 12 Very roughly, the solution to the former paves the way for the solution to the latter.
- 13 The theory according to which a proposition is true iff it somehow corresponds with the facts.
- 14 Russell (1904b) (above) may also appear to invite an argument against correspondence from parsimony: Russell may have found it distasteful to countenance both true propositions and facts, as one might find countenancing pervasive, undetectably coincident dopplegängers distasteful. That is, of course, rather speculative. What's not is the fact that, in addition to the Moore-style argument quoted above, in his 1905 paper 'The Nature of Truth', Russell provides a different argument against the correspondence theory:

But even supposing some other definition of correspondence with reality could be found, a more general argument against definitions of truth would still hold good. An idea is to be true when it corresponds with reality, i.e., when it is true that it corresponds with reality, i.e., when the idea that it corresponds with reality corresponds with reality, and so on. This will never do. In short, if we don't know the difference between a proposition's being true and not being true, we don't know the difference between a thing's having a property and not having it, and therefore we can't define a thing as true when it has a certain property such as corresponding with reality ... We may therefore abandon the notion that truth consists in a relation between thought and things, or between ideas and reality (Russell 1906, pp. 493–4).

Russell's point here seems to be that we cannot define truth in terms of a property such as correspondence without presupposing the notion of truth, and thus rendering the definition circular. However, there remain good questions about how this circularity argument is really supposed to work. And further questions arise about whether the argument was even available to Russell in 1903, or whether it instead presupposes a view he held only briefly, namely his 'substitutional theory'. (For discussion of the substitutional theory, which Russell held only briefly between 1905 and 1907, see Hylton 1980 and Landini 1998.) We will not explore these issues here.

- 15 Denoting concepts, of course, complicate this picture. See Cartwright (1987a) and Baldwin (1991) for influential treatments of the identity theory of truth in Russell and Moore.
- 16 See fn. 8 above.
- 17 Russell's concern with accommodating false propositions persists through 1919. For instance:

To suppose that in the actual world of nature there is a whole set of false propositions going about is to my mind monstrous. I cannot bring myself to suppose it. I cannot believe that they are in the sense in which facts are there (Russell 1919: 57).

See also Russell (1910, 1912). See Cartwight (1987a) for an influential discussion of the problems Russell faces with false facts.

- 18 Moore (1899: 180).
- 19 Cf. Proops (2011: 200) and Landini (1996: 323).
- 20 Walter Burley, a fourteenth century Scholastic who anticipated many of Russell's views on semantics circa 1903, countenances *two* 'relating relations' unifying propositions in his (1999): one for the true ones, and a distinct relating relation for the false ones (see. In his more radically realist mood, Russell (1906: 48) countenances facts as well as *fictions* objective falsehoods that very much resemble non-obtaining states of affairs. In a less realist mood, Russell finds it 'monstrous' to admit objective falsehoods, as we have seen. Indeed, Russell (1912):

'If we imagine a world of mere matter, there would be no room for falsehoods in such a world, and although it would contain what may be called "facts", it would not contain any truths, in the sense in which truths are things of the same kind as falsehoods' (Russell 1912: 70).

- 21 This challenge has been the focus of sustained attention in the contemporary literature on propositions. See especially King (2007), Soames (2010), King et al. (2014), and Hanks (2015), each of whom provide a different account of the metaphysics of propositions, and consequently a different account of how propositions possess truth conditions. See also Caplan et al., Gaskin, Hodgson, and Simchen (this volume).
- 22 Proops (2011: 201) suggests that Russell's refusal to extend his later correspondence theory of truth to propositions was grounded in his acceptance of the Principle of Sufficient Reason. Proops reads Russell as objecting to the brute difference between true propositions and facts that a correspondence theory would need to endorse: a brute difference in how the relating relation combines with the same set of constituents to generate a fact or true proposition.
- 23 Hanks (2021) presses a similar point against contemporary 'Russellian' accounts of singular propositions. Also, those like Merricks (2015) who take propositions to have their truth-conditions primitively face a challenge: if a main motivation for abandoning Russell's original theory, which does not treat propositions as representational, is a demand to provide an analysis of the truth or falsity of propositions, why not think that a similar demand simply recurs for any view that treats propositions as representational? And if a similar demand does not arise, why think that the original demand was legitimate? Moreover, for those skeptical of the contemporary project of explaining the truth-conditions of propositions, the foregoing considerations suggest that Russell's identity theory is a more stable resting point than it might at first appear.

- 24 For instance, recall that, according to Russell's (1903) view, the proposition expressed by 'The last Tudor monarch was a woman' is true, but it does not have Elizabeth I the last Tudor monarch as a constituent. Instead, 'The last Tudor monarch' is a complex referring term that introduces a denoting concept into the proposition, and this denoting concept in turn denotes Elizabeth I. So 'The last Tudor monarch was a woman' is true not because it expresses a proposition identical to Elizabeth I being a woman, but because it expresses a proposition that somehow represents Elizabeth I being a woman.
- 25 Russell originally composed the letter in German. Here we have followed Hans Kaal's translation (in Frege 1980), with two exceptions. First, we follow Salmón and Soames (1988: 56) in translating *Bedeutung* as 'denotation'. Second, we translate *der Wahre* as 'the True', to reflect Frege's conviction that true declarative sentences (ultimately) denote a special sort of *entity* the True. Finally, by 'proposition' Russell intended what is expressed in the context of this volume by 'sentence' or 'declarative sentence'.
- 26 The Russell-Myhill Antimony is commonly pitched as a special problem for Russellian theories of propositions, including loose contemporary descendants of Russell's (1903) theory of propositions. But that's misleading at best. What Myhill (1958) discovered was that Russell's paradox of propositions can be formulated in Church's (1951) logic of sense and denotation. (See Klement 2001 and Uzquiano 2015: 342 for further discussion, including a derivation of a version of the paradox in the former within a logic even closer to Frege's than Church's. Frege had ignored RM because he believed it could not arise within his own system, unlike the more famous of Russell's paradoxes.) This underscores the threat RM poses not just to theories in the vein of Russell (1903) but also (quasi-)Fregean theories of propositions, regardless of the manifest differences between typical Russellian and Fregean views of propositions. It is worth noting that unstructured views of propositions along the lines of those defended in Keller (this volume) may be vulnerable to RM as well.
- 27 Here is Russell's original derivation of the paradox:
 - Let us state this new contradiction more fully. If m be a class of propositions, the proposition 'every m is true' may or may not be itself an m. But there is a one-one relation of this proposition to m; if n be different from m, 'every n is true' is not the same proposition as 'every m is true'. Consider now the whole class of propositions of the form of the form 'every m is true', and having the property of not being members of their respective m's. Let this class be w and let p be the proposition 'every w is true'. If p is a w, it must possess the defining property of w; but this property demands that p should not be a w. On the other hand, if p be not a w, then p does possess the defining property of w, and therefore is a w. Thus the contradiction appears unavoidable. (Russell 1903: 539)
- 28 For a full-dress version of the reasoning that generates the paradox, see Klement (2001), as well as Simchen (this volume). See also Deutsch (2008, 2014, and this volume) for a treatment of the Russell-Myhill antimony alongside some other well-known paradoxes of propositions.
- 29 See, for example, Uzquiano (2015), Dorr (2016), Goodman (2017), and Kment (2022).
- 30 Indeed, as Klement (2001): 15 notes, Russell was aware that the paradox could not be resolved by the simple theory of types he had just formulated. See also Deutsch (2014) (and this volume) for discussion.
- 31 Contra one popular line of argument, Russell's shift was not motivated by the need to reject Meinongian subsisting entities, which Meinong introduced so that he could explain the meaningfulness of empty or fictional names. See, for example, Quine (1966: 291–2): 'In *Principles of Mathematics*, 1903, Russell's ontology was unrestrained. Every word referred to something ... By contrast, in OD, a reformed Russell emerges ... fed up with Meinong's impossible objects'. Russell circa 1903 allowed that denoting concepts like that expressed by 'the present King of France' may denote nothing. Hence, an expression like 'the present King of France' could indicate a propositional constituent and so remain meaningful despite not indicating (in this case) an actual man.
- 32 For a meticulous reconstruction of Russell's argument along these lines, see Salmón (2005a). See also Kremer (1994). Confidence in any interpretation of the argument, however, is tempered by Kaplan's (1969: 144) observation after surveying a number of (jointly incompatible) accounts of Russell's use of 'denoting complex' that 'all these (indeed all possible) views regarding the meaning of "denoting complex" are supported by the text'.
- 33 See Hylton (2005) for a nice overview of the significance of 'On Denoting' for Russell's abandonment of his theory of denoting concepts, and (e.g.) Neale (2005) for critical assessment.
- 34 For a textual defense of this characterization of Russell's notion of 'incomplete symbol', see Pickel (2013). Controversy persists over whether (i)–(iii) are all necessary, and whether any are individually sufficient.
- 35 More work remains, of course, since linguistically simple singular terms, like 'Logicism', also appear to name propositions. But an assimilation of linguistically simple singular terms that fail to be 'logically proper names' was already outlined and defended in his (1905) and (1910–11).
- 36 For example, some follow Landini (1996: 323) in seeing Russell's concerns about paradox as decisive in explaining his abandonment of propositions. (Interestingly, Russell considers a response to the *Liar* paradox not RM that eschews propositions in his 1906: 46.) Others, like Proops (2011), argue that we must look elsewhere for a complete explanation of Russell's shift in position.
- 37 Once again, see the ongoing debate between, amongst others, King (2007, and this volume), Soames (2010, and this volume), King et al. (2014), and Hanks (2015, and this volume).

- 38 Compare Landini (1991: 45) (edited): 'This is corroborated in the *Principia* when Russell writes that 'a proposition, like such phrases as "the so and so", where grammatically it appears as subject, must be broken up into its constituents if we are to find the true subject or subjects' (p. 48). Taking this lead, we construe the multiple relation theory as a direct application of the theory of incomplete symbols to the contexts in which (propositions) occur as grammatical objects'.
- 39 For a closely related construal of the problem that the MRTJ is meant to solve, see Pincock (2008: 112).
- 40 See Griffin (1985) for an early enumeration and explication of these different versions of the MRTJ, including the embryonic version from Russell (1906) that lacks some of the central features of later versions.
- 41 Pincock *ibid*. dubs the resulting metaphysical constraint on judgement 'PART', and describes it as follows: 'In a propositional attitude, the entities that are the subject-matter of this attitude are also parts of the propositional attitude'. See also Russell (1903, Ch. 16), Russell (1904a), Hylton (2005), Gilmore (2014, *this volume*), Caplan et al. (*this volume*), and elsewhere.
 - See, e.g., Frege (1892), Frege (1997), and Textor (this volume).
- 42 He distinguishes two 'senses' for the relating relation (e.g., the relation of loving in 'Maggie believes that Lyra loves Will), hoping that these senses will distinguish (for example) Maggie believing that Lyra loves Will and Maggie believing that Will loves Lyra. See Griffin (1985: 219ff) for discussion.
- 43 See, for example, Lebens (2017), Zalabardo (2015), Connelly (2014), MacBride (2013), Pincock (2008), Hanks (2007), Stevens (2003), Weiss (1995), Landini (1991), and Griffin (1985).
- 44 A dual complex is a complex in which a relation relates two relata.
- 45 Here, Pincock builds upon Landini (1991) and Hanks (2007).
- 46 See Fine (2000) for an influential treatment of some of the relevant metaphysical problems.
- 47 As we noted, there has been much scholarly debate over exactly what Wittgenstein's objections were, why Russell might have thought them fatal to his account, and whether he was right to see them as fatal. We will not wade any further distance into that knotty debate.
- 48 Wittgenstein (1921: 5.5422).
- 49 On the other hand, one might baulk at the suggestions that something *in the relation* must exclude something like this. If the claim is that nothing about the belief relation *itself* prevents one from bearing it to the content (if any) of 'Love Lyra Will', then one might simply demur. Belief cannot relate one to love Lyra Will for the same reason it cannot relate one to plum-pudding. It's perhaps just a brute fact about belief that it's not the sort of relation that has plum-pudding in its range.
- 50 Cf. Russell (1913: 116-17).
- 51 While Landini (1991: 66–7) argues that Wittgenstein rightly took exception to the positing of forms to solve this problem judging it to be an attempt by Russell to say what can only be shown we are less pessimistic. It seems open to Russell to model acquaintance with a form on broadly inferentialist accounts of our grasp of logical constants. These are accounts according to which grasp of a logical constant consists in a disposition to infer according to the basic inferential rules that govern the constant (see Boghossian 2003 for a representative account of this kind). A first move for Russell, in response to the Problem of Combination, might be to insist that a thinker is acquainted with a form γ only if she is disposed to combine constituents in judgment according to the composition rules determined by γ.
- 52 Compare Caplan et al. (this volume).
- 53 Russell (1913: 88). Compare Gilmore (2013).
- 54 In the first complex A occupies P1 and B occupies P2, while the second complex reverses the assignment. Compare Crimmins (1992).
- 55 Here, the 'subordinate relation' is expressed by 'is as tall as'. Fine (2000) questions the coherence of this account of symmetric complexes. Take the symmetric complex expressed by 'a is adjacent to b'. Fine argues that a and b must occupy positions that are in some sense distinct (cf. pp. 17–18 and fn. 10). But if the positions are distinct then Russell's account of symmetric complexes seems incoherent, for how could two distinct positions be identical? The answer to Fine is to distinguish two senses in which positions can be distinct. First, positions can be distinct in that they have exclusive occupants: if a occupies position ξ , then ξ cannot also be occupied by b. Second, positions can be distinct in the manner in which they contribute to the identity of the complex. Russell's account requires that positions be identical only in the second respect: if all that distinguishes the positions ξ and ζ is the fact that they have distinct occupants then the assignment of a to ζ and b to ξ will generate a complex indistinguishable from the assignment of a to ξ and b to ζ .
- 56 Here, the 'subordinate relation' is expressed by 'is a member of'.
- 57 For example, Russell relies upon the notion of heterogeneous complex to elaborate his conception of truth later in *Theory of Knowledge* (cf. 1913: 144–48).
- 58 Cf. Russell (1913: 146–147). This claim that the relating relation determines the positions, and thus the appropriate form, makes some sense of Russell's remark in his lectures on logical atomism that the relating relation operates as a 'verb', and does not stand on par with the other constituents.

- 59 The constraint thus functions as a restriction on the relative logical complexity of the objects assigned to the two positions.
- 60 Note that this problem does not arise when a specification of the subordinate relating relation and its relata suffices to fix only one possible combination (e.g., when these constituents would form a heterogeneous complex, if actually combined with one another).
- 61 Russell argues that if positions were relations to the relating relation then an object assigned to a position relative to the relating relation would always have to stand in that position to the relating relation. But this would result in absurdities, since Lyra can occupy different positions in the complexes expressed by 'Lyra loves Will' and 'Asriel loves Lyra', complexes whose relating relations are identical. So Russell concludes that positions must be relative to particular complexes, and so must be features of the form (1913: 111).
- 62 Given what he can say about the narrow direction problem, a first step in a solution to the wide direction problem is also available to Russell at this stage, as he likely recognized. This is speculation, to an extent, but is supported by what Russell says in *Theory of Knowledge* about logical possibility. For Russell, a complex is *logically possible* iff there can exist a 'proposition' with the same form, constituents, and assignment of constituents to positions within the form. The combination of Russell's commitment to heterogeneous complexes with the definition of logical possibility entails that Russell is committed to the possibility of heterogeneous judgments. And those are just judgments whose constituents admit of only limited permutations. Using the resources made available by his form-involving account of ordinary complexes, he could give an account on which certain combinations of understanding relations and various relata are impossible, one that appeals to restrictions on what entities could be related to positions within the form of the subordinate relating relation. Yet no such broadly metaphysical account of the impossibility of, for example, Mary understanding that loving Will Lyra, can alone solve the direction problems. For remember that those problems ultimately concern the demands of a correspondence theory of truth for belief.
- 63 For a compelling account of Russell's struggles with, and eventual abandonment of, the project of *Theory of Knowledge*, see Pincock (2008). Our discussion owes much to Pincock's innovative and sympathetic reconstruction of Russell's *Theory of Knowledge* account.
- 64 While we have seen that Russell had several arguments available to him for the first constraint namely the constraint that propositions not be treated as independent entities his discussion in 1919 focuses almost exclusively on the problem of accommodating falsehood. Of course, this lends some support to those who do not see the Russell-Myhill Antimony, as well as other paradoxes, as the primary motivation for Russell's shift to the MRTJ.

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