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Neutrosophic LA-Semigroup Rings

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Abstract. Neutrosophic LA-semigroup is a midway structure between a neutrosophic groupoid and a commutative neutrosophic semigroup. Rings are the old concept in algebraic structures. We combine the neutrosophic LA-semigroup and ring together to form the notion of neutrosophic LA-semigroup ring. Neutrosophic LA-semigroup ring is defined analogously to neutrosophic group ring and neutrosophic semigroup ring.

Keywords: Neutrosophic LA-semigroup, ring, neutrosophic LA-semigroup ring.

1. Introduction

Smarandache [13] in 1980 introduced neutrosophy which is a branch of philosophy that studies the origin and scope of neutralities with ideational spectra. The concept of neutrosophic set and logic came into being due to neutrosophy, where each proposition is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F. This mathematical tool is used to handle problems with imprecise, indeterminate, incomplete and inconsistent etc. Kandasamy and Smarandache apply this concept in algebraic structures in a slight different manner by using the indeterminate/unknown element I, which they call neutrosophic element. The neutrosophic element I is then combine to the elements of the algebraic structure by taking union and link with respect to the binary operation * of the algebraic stucutre. Therefore, a neutrosophic algebraic structure is generated in this way. They studied several neutrosophic algebraic structure [3,4,5,6]. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids, and neutrosophic bigroupoids and so on.

A left almost semigroup denoted as LA-semigroup is an algebraic structure which was studied by Kazim and Naseeruddin [7] in 1972. An LA-semigroup is basically a midway structure between a groupoid and a commutative semigroup. It is also termed as Able-Grassmann's groupoid shortly AG -groupoid [11]. LA-semigroup is a non-associative and non-commutative algebraic structure which closely matching with commutative semigroup. LA-semigroup is a generalization to semigroup theory which has wide applications in collaboration with semigroup. Mumtaz et al.[1] introduced neutrosophic left almost semigroup in short neutrosophic LA-semigroup which is basically generated by an LA-semigroup and the neutrosophic element I. Mumtaz et al.[1] also studied their generalization and other properties. Neutrosophic group rings [5] and neutrosophic semigroup rings [5] are defined analogously to group rings and semigroup rings respectively. In fact these are generalization of group ring and semigroup ring ring. The notion of neutrosophic matrix ring have been successfully applied and used in the neutrosophic models such as neutrosophic cognitive maps (NCMs), neutrosophic relational maps (NRMs) etc.

In this paper, we introduced neutrosophic LA-semigroup rings owing to neutrosophic semigroup rings. Neutrosophic LA-semigroup rings are generalization of neutrosophic semigroup rings. These neutrosophic LA-semigroup rings are defined analogously to neutrosophic group rings and neutrosophic semigroup rings. We also studied some of their basic properties and other related notions in this paper. In section 2, we after reviewing the literature, we presented some basic concepts of neutrosophic LA-semigroup and rings. In section 3, neutrosophic LA-semigroup rings are introduced and studied some of their properties.

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2. Basic Concepts

Definition 2.1 [1]: Let $(S,*)$ be an LA-semigroup and let $\langle S \cup I \rangle = \{a + bI : a,b \in S\}$. The neutrosophic

LA-semigroup is generated by S and I under the operation $*$ which is denoted as $N(S)=\{\langle S\cup I\rangle,*\}$, where I is called the neutrosophic element with property $I^2=I$. For an integer n, $n+I$ and nI are neutrosophic elements and $0.I=0$.

I^{-1}, the inverse of I is not defined and hence does not exist.

Definition 2.2 [1]: Let $N(S)$ be a neutrosophic LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$ is called a neutrosophic sub LA-semigroup if $N(H)$ itself is a neutrosophic LA-semigroup under the operation of $N(S)$.

Definition 2.3 [1]: Let $N(S)$ be a neutrosophic LA-semigroup and $N(K)$ be a subset of $N(S)$. Then $N(K)$ is called Left (right) neutrosophic ideal of $N(S)$ if

$$N(S)N(K)\subseteq N(K),\ \{N(K)N(S)\subseteq N(K)\}.$$

If $N(K)$ is both left and right neutrosophic ideal, then $N(K)$ is called a two sided neutrosophic ideal or simply a neutrosophic ideal.

Definition 2.4 [5]: Let $(R,+,\cdot)$ be a set with two binary operations $+$ and $\cdot$. Then $(R,+,\cdot)$ is called a ring if the following conditions are hold.

1. $(R,+)$ is a commutative group under the operation of $+$.
2. $(R,\cdot)$ is a semigroup under the operation of $\cdot$.
3. $a(b+c)=ab+ac$ and $(b+c)a=ba+ca$ for all $a,b,c\in R$.

Definition 2.5 [5]: Let $(R,+,\cdot)$ be a ring and $(R_1,+,\cdot)$ be a proper subset of $(R,+,\cdot)$. Then $(R_1,+,\cdot)$ is called a subring if $(R_1,+,\cdot)$ itself is a ring under the operation of R.

Definition 2.6 [5]: Let R be a ring. The neutrosophic ring $\langle R\cup I\rangle$ is also a ring generated by R and I under the operation of R, where I is called the neutrosophic element with property $I^2=I$. For an integer n, $n+I$ and nI are neutrosophic elements and $0.I=0$. I^{-1}, the inverse of I is not defined and hence does not exist.

Example 2.8: Let $\mathbb{Z}$ be the ring of integers. Then $\langle \mathbb{Z}\cup I\rangle$ is the neutrosophic ring of integers.

Definition 2.8 [5]: Let $\langle R\cup I\rangle$ be a neutrosophic ring. A proper subset P of $\langle R\cup I\rangle$ is called a neutosophic subring if P itself a neutrosophic ring under the operation of $\langle R\cup I\rangle$.

Definition 2.9 [5]: Let $\langle R\cup I\rangle$ be a neutrosophic ring. A non-empty set P of $\langle R\cup I\rangle$ is called a neutrosophic ideal of $\langle R\cup I\rangle$ if the following conditions are satisfied.

1. P is a neutrosophic subring of $\langle R\cup I\rangle$, and
2. For every $p\in P$ and $r\in\langle R\cup I\rangle$, pr and $rp\in P$.

3. Neutrosophic LA-semigroup Rings

In this section, we introduced neutosophic LA-semigroup rings and studied some of their basic properties and types.

Definition 3.1: Let $\langle S\cup I\rangle$ be any neutrosophic LA-semigroup. R be any ring with 1 which is commutative or field. We define the neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$ of the neutrosophic LA-semigroup $\langle S\cup I\rangle$ over the ring R as follows:

1. $R\langle S\cup I\rangle$ consists of all finite formal sum of the form $\alpha=\sum_{i=1}^{n} r_i g_i$, $n<\infty$, $r_i\in R$ and $g_i\in\langle S\cup I\rangle$ $\left(\alpha\in R\langle S\cup I\rangle\right)$.
2. Two elements $\alpha=\sum_{i=1}^{n} r_i g_i$ and $\beta=\sum_{i=1}^{m} s_i g_i$ in $R\langle S\cup I\rangle$ are equal if and only if $r_i=s_i$ and $n=m$.
3. Let $\alpha=\sum_{i=1}^{n} r_i g_i, \beta=\sum_{i=1}^{m} s_i g_i\in R\langle S\cup I\rangle$; $\alpha+\beta=\sum_{i=1}^{n}(\alpha_i+\beta_i)g_i\in R\langle S\cup I\rangle$, as $\alpha_i,\beta_i\in R$, so $\alpha_i+\beta_i\in R$ and $g_i\in\langle S\cup I\rangle$.
4. $0=\sum_{i=1}^{n} 0g_i$ serve as the zero of $R\langle S\cup I\rangle$.
5. Let $\alpha=\sum_{i=1}^{n} r_i g_i\in R\langle S\cup I\rangle$ then $-\alpha=\sum_{i=1}^{n}(-\alpha_i)g_i$ is such that

$$\alpha+(-\alpha)=0$$
$$=\sum_{i=1}^{n}(\alpha_i+(-\alpha_i))g_i$$
$$=\sum 0g_i$$

Thus we see that $R\langle S\cup I\rangle$ is an abelian group under $+$.

6. The product of two elements α,β in $R\langle S\cup I\rangle$ is follows:

Let $\alpha=\sum_{i=1}^{n}\alpha_i g_i$ and $\beta=\sum_{j=1}^{m}\beta_j h_j$. Then

$$\alpha.\beta=\sum_{\substack{1\le i\le n\\ 1\le j\le m}}^{n}\alpha_i.\beta_j g_i h_j \;=\sum_k y_k t_k$$

where $y_k=\sum\alpha_i\beta_j$ with $g_i h_j=t_k$, $t_k\in\langle S\cup I\rangle$ and $y_k\in R$.

Clearly $\alpha.\beta\in R\langle S\cup I\rangle$.

7. Let $\alpha=\sum_{i=1}^{n}\alpha_i g_i$ and $\beta=\sum_{j=1}^{m}\beta_j h_j$ and $\gamma=\sum_{k=1}^{p}\delta_k l_k$.

Then clearly $\alpha(\beta+\gamma)=\alpha\beta+\alpha\gamma$ and $(\beta+\gamma)\alpha=\beta\alpha+\gamma\alpha$ for all $\alpha,\beta,\gamma\in R\langle S\cup I\rangle$, that is the distributive law holds.

Hence $R\langle S\cup I\rangle$ is a ring under the binary operations $+$ and $\bullet$. We call $R\langle S\cup I\rangle$ as the neutrosophic LA-semigroup ring.

Similarly on the same lines, we can define neutrosophic Right Almost semigroup ring abbrivated as neutrosophic RA-semigroup ring.

Example 3.2: Let $\mathbb{R}$ be the ring of real numbers and let $N(S)=\{1,2,3,4,1I,2I,3I,4I\}$ be a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

Then $\mathbb{R}\langle S\cup I\rangle$ is a neutrosophic LA-semigroup ring.

Theorem 3.3: Let $\langle S\cup I\rangle$ be a neutrosophic LA-semigroup and $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring such that $R\langle S\cup I\rangle$ is a neutrosophic LA-semigroup ring over R. Then $\langle S\cup I\rangle\subseteq R\langle S\cup I\rangle$.

Proposition 3.4: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring over the ring R. Then $R\langle S\cup I\rangle$ has non-trivial idempotents.

Remark 3.5: The neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$ is commutative if and only if $\langle S\cup I\rangle$ is commutative neutrosophic LA-semigroup.

Remark 3.6: The neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$ has finite number of elements if both R and $\langle S\cup I\rangle$ are of finite order.

Example 3.7: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring in Example (1). Then $R\langle S\cup I\rangle$ is a neutrosophic LA-semigroup ring of infinite order.

Example 3.8: Let $\langle S\cup I\rangle=\{1,2,3,4,5,1I,2I,3I,4I,5I\}$ with left identity 4, defined by the following multiplication table.

.	1	2	3	4	5	1I	2I	3I	4I	5I
1	4	5	1	2	3	4I	5I	1I	2I	3I
2	3	4	5	1	2	3I	4I	5I	1I	2I
3	2	3	4	5	1	2I	3I	4I	5I	1I
4	1	2	3	4	5	1I	2I	3I	4I	5I
5	5	1	2	3	4	5I	1I	2I	3I	4I
1I	4I	5I	1I	2I	3I	4I	5I	1I	2I	3I
2I	3I	4I	5I	1I	2I	3I	4I	5I	1I	2I
3I	2I	3I	4I	5I	1I	2I	3I	4I	5I	1I
4I	1I	2I	3I	4I	5I	1I	2I	3I	4I	5I
5I	5I	1I	2I	3I	4I	5I	1I	2I	3I	4I

Let $\mathbb{Z}_2$ be the ring of two elements. Then $\mathbb{Z}_2\langle S\cup I\rangle$ is a neutrosophic LA-semigroup ring of finite order.

Theorem 3.9: Every neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$ contains atleast one proper subset which is an LA-semigroup ring.

Proof: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. Then clearly $RS\subseteq R\langle S\cup I\rangle$. Thus $R\langle S\cup I\rangle$ contains an LA-semigroup ring.

Definition 3.10: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring and let P be a proper subset of $R\langle S\cup I\rangle$. Then P is called a subneutrosophic LA-semigroup ring of $R\langle S\cup I\rangle$ if $P=R\langle H\cup I\rangle$ or $Q\langle S\cup I\rangle$ or $T\langle H\cup I\rangle$. In $P=R\langle H\cup I\rangle$, R is a ring and $\langle H\cup I\rangle$ is a proper neutrosophic sub LA-semigroup of $\langle S\cup I\rangle$ or in $Q\langle S\cup I\rangle$, Q is a proper subring with 1 of R and $\langle S\cup I\rangle$ is a neutrosophic LA-semigroup and if $P=T\langle H\cup I\rangle$, T is a subring of R with unity and $\langle H\cup I\rangle$ is a proper neutrosophic sub LA-semigroup of $\langle S\cup I\rangle$.

Example 3.11: Let $\langle S\cup I\rangle$ and $\mathbb{R}\langle S\cup I\rangle$ be as in Example 3.2. Let $H_1=\{1,3\}, H_2=\{1,1I\}$ and $H_3=\{1,3,1I,3I\}$ are neutrosophic sub LA-semigroups. Then $\mathbb{Q}\langle S\cup I\rangle$, $\mathbb{R}H_1$, $\mathbb{Z}\langle H_2\cup I\rangle$ and $\mathbb{Q}\langle H_3\cup I\rangle$ are all subneutrosophic LA-semigroup rings of $\mathbb{R}\langle S\cup I\rangle$.

Definition 3.12: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S\cup I\rangle$ is called a neutrosophic subring if $P=\langle S_1\cup I\rangle$ where S_1 is a subring of RS or R.

Example 3.13: Let $R\langle S\cup I\rangle=\mathbb{Z}_2\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring in Example 3.8. Then clearly $\langle \mathbb{Z}_2\cup I\rangle$ is a neutrosophic subring of $\mathbb{Z}_2\langle S\cup I\rangle$.

Theorem 3.14: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring of the neutrosophic LA-semigroup over the ring R. Then $R\langle S\cup I\rangle$ always has a nontrivial neutrosophic subring.

Proof: Let $\langle R\cup I\rangle$ be the neutrosophic ring which is generated by R and I. Clearly $\langle R\cup I\rangle\subseteq R\langle S\cup I\rangle$ and this guaranteed the proof.

Definition 3.15: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset T of $R\langle S\cup I\rangle$ which is a pseudo neutrosophic subring. Then we call T to be a pseudo neutrosophic subring of $R\langle S\cup I\rangle$.

Example 3.16: Let $\mathbb{Z}_6\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring of the neutrosophic LA-semigroup $\langle S\cup I\rangle$ over $\mathbb{Z}_6$. Then $T=\{0,3I\}$ is a proper subset of $\mathbb{Z}_6\langle S\cup I\rangle$ which is a pseudo neutrosophic subring of $\mathbb{Z}_6\langle S\cup I\rangle$.

Definition 3.17: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S\cup I\rangle$ is called a sub LA-semigroup ring if $P=R_1H$ where R_1 is a subring of R and H is a sub LA-semigroup of S. SH is the LA-semigroup ring of the sub LA-semigroup H over the subring R_1.

Theorem 3.18: All neutrosophic LA-semigroup rings have proper sub LA-semigroup rings.

Definition 3.19: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S\cup I\rangle$ is called a subring but P should not have the LA-semigroup ring structure and is defined to be a subring of $R\langle S\cup I\rangle$.

Definition 3.20: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S\cup I\rangle$ is called a neutrosophic ideal of $R\langle S\cup I\rangle$,

1. if P is a neutrosophic subring or subneutrosophic LA-semigroup ring of $R\langle S\cup I\rangle$.
2. For all $p\in P$ and $\alpha\in R\langle S\cup I\rangle$, αp and $p\alpha\in P$.

One can easily define the notions of left or right neutrosophic ideal of the neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$.

Example 3.21: Let $\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup with the following table.

$*$	1	2	3	$1I$	$2I$	$3I$
1	3	3	3	$3I$	$3I$	$3I$
2	3	3	3	$3I$	$3I$	$3I$
3	1	3	3	$1I$	$3I$	$3I$
$1I$	$3I$	$3I$	$3I$	$3I$	$3I$	$3I$
$2I$	$3I$	$3I$	$3I$	$3I$	$3I$	$3I$
$3I$	$1I$	$3I$	$3I$	$1I$	$3I$	$3I$

Let $R=\mathbb{Z}$ be the ring of integers. Then $\mathbb{Z}\langle S\cup I\rangle$ is a neutrosophic LA-semigroup ring of the neutrosophic LA-semigroup over the ring $\mathbb{Z}$. Thus clearly $P=2\mathbb{Z}\langle S\cup I\rangle$ is a neutrosophic ideal of $R\langle S\cup I\rangle$.

Definition 3.22: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S\cup I\rangle$ is called a pseudo neutrosophic ideal of $R\langle S\cup I\rangle$

1. if P is a pseudo neutrosophic subring or pseudo subneutrosophic LA-semigroup ring of $R\langle S\cup I\rangle$.
2. For all $p\in P$ and $\alpha\in R\langle S\cup I\rangle$, αp and $p\alpha\in P$.

Definition 3.23: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring and let R_1 be any subring (neutrosophic or otherwise). Suppose there exist a subring P in $R\langle S\cup I\rangle$ such that R_1 is an ideal over P i.e,

$rs, sr \in R_1$ for all $p \in P$ and $r \in R$. Then we call R_1 to be a quasi neutrosophic ideal of $R\langle S \cup I \rangle$ relative to P.

If R_1 only happens to be a right or left ideal, then we call R_1 to be a quasi neutrosophic right or left ideal of $R\langle S \cup I \rangle$.

Definition 3.24: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup ring. If for a given R_1, we have only one P such that R_1 is a quasi neutrosophic ideal relative to P and for no other P. Then R_1 is termed as loyal quasi neutrosophic ideal relative to P.

Definition 3.25: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup. If every subring R_1 of $R\langle S \cup I \rangle$ happens to be a loyal quasi neutrosophic ideal relative to a unique P. Then we call the neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$ to be a loyal neutrosophic LA-semigroup ring.

Definition 3.26: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup ring. If for R_1, a subring P is another subring $(R_1 \neq P)$ such that R_1 is a quais neutrosophic ideal relative to P. In short P happens to be a quasi neutrosophic ideal relative to R_1. Then we call (P, R_1) to be a bounded quasi neutrosophic ideal of the neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$.

Similarly we can define bounded quasi neutrosophic right ideals or bounded quasi neutrosophic left ideals.

Definition 3.27: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup ring and let R_1 be any subring (neutrosophic or otherwise). Suppose there exist a subring P in $R\langle S \cup I \rangle$ such that R_1 is an ideal over P i.e, $rs, sr \in R_1$ for all $p \in P$ and $r \in R$. Then we call R_1 to be a quasi neutrosophic ideal of $R\langle S \cup I \rangle$ relative to P. If R_1 only happens to be a right or left ideal, then we call R_1 to be a quasi neutrosophic right or left ideal of $R\langle S \cup I \rangle$.

Definition 3.28: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup ring. If for a given R_1, we have only one P such that R_1 is a quasi neutrosophic ideal relative to P and for no other P. Then R_1 is termed as loyal quasi neutrosophic ideal relative to P.

Definition: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup. If every subring R_1 of $R\langle S \cup I \rangle$ happens to be a loyal quasi neutrosophic ideal relative to a unique P. Then we call the neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$ to be a loyal neutrosophic LA-semigroup ring.

Definition 3.29: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup ring. If for R_1, a subring P is another subring $(R_1 \neq P)$ such that R_1 is a quais neutrosophic ideal relative to P. In short P happens to be a quasi neutrosophic ideal relative to R_1. Then we call (P, R_1) to be a bounded quasi neutrosophic ideal of the neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$.

Similarly we can define bounded quasi neutrosophic right ideals or bounded quasi neutrosophic left ideals.

One can define pseudo quasi neutrosophic ideal, pseudo loyal quasi neutrosophic ideal and pseudo bounded quasi neutrosophic ideals of a neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$.

4. LA-semigroup Neutrosophic Ring

In this section, LA-semigroup Neutrosophic ring is introduced and studied some of their basic properties.

Definition 4.1: Let S be an LA-semigroup and $\langle R \cup I \rangle$ be a commutative neutrosophic ring with unity.

$\langle R \cup I \rangle [S]$ is defined to be the LA-semigroup neutrosophic ring which consist of all finite formal sums of the form $\sum_{i=1}^{n} r_i s_i$; $n < \infty$, $r_i \in \langle R \cup I \rangle$ and $s_i \in S$. This LA-semigroup neutrosophic ring is defined analogous to the group ring or semigroup ring.

Example 4.2: Let $\langle \mathbb{Z}_2 \cup I \rangle = \{0, 1, I, 1+I\}$ be the neutrosophic ring and let $S = \{1, 2, 3\}$ be an LA-semigroup with the following table:

*	1	2	3
1	1	1	1
2	3	3	3
3	1	1	1

Then $\langle \mathbb{Z}_2 \cup I \rangle [S]$ is an LA-semigroup neutrosophic ring.

Definition 4.3: Let $\langle S \cup I \rangle$ be a neutrosophic LA-semigroup and $\langle K \cup I \rangle$ be a neutrosophic field or a commutative neutrosophic ring with unity. $\langle K \cup I \rangle [\langle S \cup I \rangle]$ is defined to be the neutrosophic LA-semigroup neutrosophic ring which consist of all finite formal sums of the form $\sum_{i=1}^{n} r_i s_i$; $n < \infty$, $r_i \in \langle K \cup I \rangle$ and $s_i \in S$.

Example 4.4: Let $\langle \mathbb{Z} \cup I \rangle$ be the ring of integers and let $N(S) = \{1, 2, 3, 4, 1I, 2I, 3I, 4I\}$ be a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

Then $\langle \mathbb{Z} \cup I \rangle \langle S \cup I \rangle$ is a neutrosophic LA-semigroup neutrosophic ring.

Theorem 4.5: Every neutrosophic LA-semigroup neutrosophic ring contains a proper subset which is a neutrosophic LA-semigroup ring.

Proof: Let $\langle R \cup I \rangle \langle S \cup I \rangle$ be a neutrosophic LA-semigroup neutrosophic ring and let $T = R\langle S \cup I \rangle$ be a proper subset of $\langle R \cup I \rangle \langle S \cup I \rangle$. Thus clearly $T = R\langle S \cup I \rangle$ is a neutrosophic LA-semigroup ring.

Conclusion

In this paper, we introduced neutosophic LA-semigroup rings which are more general concept than neutrosophic semigroup rings. These neutrosophic LA-semigroup rings are defined analogously to neutrosophic semigroup rings. We have studiesd several properties of neutrosophic LA-semigroup rings and also define different kind of neutrosophic LA-semigroup rings.

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