



Soft Neutrosophic Bi-LA-semigroup and Soft Neutrosophic N-LA-seigroup

Mumtaz Ali¹, Florentin Smarandache², Muhammad Shabir³

^{1,3}Department of Mathematics, Quaid-i-Azam University, Islamabad, 44000, Pakistan. E-mail: mumtazali770@yahoo.com, mshbirbhatti@yahoo.co.uk

²University of New Mexico, 705 Gurley Ave., Gallup, New Mexico 87301, USA E-mail: fsmarandache@gmail.com

Abstract. Soft set theory is a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. In this paper we introduced soft neutrosophic bi-LA-semigroup, soft neutrosophic sub bi-LA-semigroup, soft neutrosophic N -LA-semigroup with the discussion of some of their characteristics. We also introduced a

new type of soft neutrosophic bi-LA-semigroup, the so called soft strong neutrosophic bi-LA-semigroup which is of pure neutrosophic character. This is also extend to soft neutrosophic strong N-LA-semigroup. We also given some of their properties of this newly born soft structure related to the strong part of neutrosophic theory.

Keywords: Neutrosophic bi-LA-semigroup, Neutrosophic N -LA-semigroup, Soft set, Soft neutrosophic bi-LA-semigroup. Soft Neutrosophic N -LA-semigroup.

1 Introduction

Florentine Smarandache for the first time introduced the concept of neutrosophy in 1995, which is basically a new branch of philosophy which actually studies the origin, nature, and scope of neutralities. The neutrosophic logic came into being by neutrosophy. In neutrosophic logic each proposition is approximated to have the percentage of truth in a subset T , the percentage of indeterminacy in a subset I , and the percentage of falsity in a subset F . Neutrosophic logic is an extension of fuzzy logic. In fact the neutrosophic set is the generalization of classical set, fuzzy conventional set, intuitionistic fuzzy set, and interval valued fuzzy set. Neutrosophic logic is used to overcome the problems of impreciseness, indeterminate, and inconsistencies of date etc. The theory of neutrosophy is so applicable to every field of algebra. W.B. Vasantha Kandasamy and Florentin Smarandache introduced neutrosophic fields, neutrosophic rings, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups and neutrosophic N -groups, neutrosophic semigroups, neutrosophic bisemigroups, and neutrosophic N -semigroups, neutrosophic loops, neutrosophic biloops, and neutrosophic N -loops, and so on. Mumtaz ali et. al. introduced neutrosophic LA-semigroups. Soft neutrosophic LA-semigroup has been introduced by Florentin Smarandache et.al.

Molodtsov introduced the theory of soft set. This mathematical tool is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. This theory has been applied successfully in many fields such as smoothness of functions, game theory, operation research, Riemann integration, Perron integration, and probability. Recently soft set theory attained much attention of the researchers since its appearance and the work based on several operations of soft set introduced in [2, 9, 10]. Some properties and algebra may be found in [1]. Feng et al. introduced soft semirings in [5]. By means of level soft sets an adjustable approach to fuzzy soft set can be seen in [6]. Some other concepts together with fuzzy set and rough set were shown in [7, 8].

In this paper we introduced soft neutrosophic bi-LA-semigroup and soft neutrosophic N -LA-semigroup and the related strong or pure part of neutrosophy with the notions of soft set theory. In the proceeding section, we define soft neutrosophic bi-LA-semigroup, soft neutrosophic strong bi-LA-semigroup, and some of their properties are discussed. In the last section soft neutrosophic N -LA-semigroup and their corresponding strong theory have been constructed with some of their properties.

2 Fundamental Concepts

Definition 1. Let $(BN(S), *, \circ)$ be a nonempty set with two binary operations $*$ and \circ . $(BN(S), *, \circ)$ is said to be a neutrosophic bi-LA-semigroup if $BN(S) = P_1 \cup P_2$ where atleast one of $(P_1, *)$ or (P_2, \circ) is a neutrosophic LA-semigroup and other is just an LA-semigroup. P_1 and P_2 are proper subsets of $BN(S)$.

If both $(P_1, *)$ and (P_2, \circ) in the above definition are neutrosophic LA-semigroups then we call $(BN(S), *, \circ)$ a strong neutrosophic bi-LA-semigroup.

Definition 2. Let $(BN(S) = P_1 \cup P_2; *, \circ)$ be a neutrosophic bi-LA-semigroup. A proper subset $(T, \circ, *)$ is said to be a neutrosophic sub bi-LA-semigroup of $BN(S)$ if

1. $T = T_1 \cup T_2$ where $T_1 = P_1 \cap T$ and $T_2 = P_2 \cap T$ and
2. At least one of (T_1, \circ) or $(T_2, *)$ is a neutrosophic LA-semigroup.

Definition 3. Let $(BN(S) = P_1 \cup P_2, *, \circ)$ be a neutrosophic bi-LA-semigroup. A proper subset $(T, \circ, *)$ is said to be a neutrosophic strong sub bi-LA-semigroup of $BN(S)$ if

1. $T = T_1 \cup T_2$ where $T_1 = P_1 \cap T$ and $T_2 = P_2 \cap T$ and
2. (T_1, \circ) and $(T_2, *)$ are neutrosophic strong LA-semigroups.

Definition 4. Let $(BN(S) = P_1 \cup P_2, *, \circ)$ be any neutrosophic bi-LA-semigroup. Let J be a proper subset of $BN(S)$ such that $J_1 = J \cap P_1$ and $J_2 = J \cap P_2$ are ideals of P_1 and P_2 respectively. Then J is called the neutrosophic biideal of $BN(S)$.

Definition 5. Let $(BN(S), *, \circ)$ be a strong neutrosophic bi-LA-semigroup where $BN(S) = P_1 \cup P_2$ with $(P_1, *)$ and (P_2, \circ) be any two neutrosophic LA-semigroups. Let J be a proper subset of $BN(S)$ where $I = I_1 \cup I_2$

with $I_1 = I \cap P_1$ and $I_2 = I \cap P_2$ are neutrosophic ideals of the neutrosophic LA-semigroups P_1 and P_2 respectively. Then I is called or defined as the strong neutrosophic biideal of $BN(S)$.

Definition 6. Let $\{S(N), *, \dots, *_2\}$ be a non-empty set with N -binary operations defined on it. We call $S(N)$ a neutrosophic N -LA-semigroup (N a positive integer) if the following conditions are satisfied.

- 1) $S(N) = S_1 \cup \dots \cup S_N$ where each S_i is a proper subset of $S(N)$ i.e. $S_i \subset S_j$ or $S_j \subset S_i$ if $i \neq j$.
- 2) $(S_i, *_i)$ is either a neutrosophic LA-semigroup or an LA-semigroup for $i = 1, 2, 3, \dots, N$.

If all the N -LA-semigroups $(S_i, *_i)$ are neutrosophic LA-semigroups (i.e. for $i = 1, 2, 3, \dots, N$) then we call $S(N)$ to be a neutrosophic strong N -LA-semigroup.

Definition 7. Let

$S(N) = \{S_1 \cup S_2 \cup \dots \cup S_N, *_1, *_2, \dots, *_N\}$ be a neutrosophic N -LA-semigroup. A proper subset $P = \{P_1 \cup P_2 \cup \dots \cup P_N, *_1, *_2, \dots, *_N\}$ of $S(N)$ is said to be a neutrosophic sub N -LA-semigroup if $P_i = P \cap S_i, i = 1, 2, \dots, N$ are sub LA-semigroups of S_i in which atleast some of the sub LA-semigroups are neutrosophic sub LA-semigroups.

Definition 8. Let

$S(N) = \{S_1 \cup S_2 \cup \dots \cup S_N, *_1, *_2, \dots, *_N\}$ be a neutrosophic strong N -LA-semigroup. A proper subset $T = \{T_1 \cup T_2 \cup \dots \cup T_N, *_1, *_2, \dots, *_N\}$ of $S(N)$ is said to be a neutrosophic strong sub N -LA-semigroup if each $(T_i, *_i)$ is a neutrosophic sub LA-semigroup of $(S_i, *_i)$ for $i = 1, 2, \dots, N$ where $T_i = S_i \cap T$.

Definition 9. Let

$S(N) = \{S_1 \cup S_2 \cup \dots \cup S_N, *_1, *_2, \dots, *_N\}$ be a neutrosophic N -LA-semigroup. A proper subset $P = \{P_1 \cup P_2 \cup \dots \cup P_N, *_1, *_2, \dots, *_N\}$ of $S(N)$ is said to be a neutrosophic N -ideal, if the following conditions are true,

1. P is a neutrosophic sub N -LA-semigroup of

- $S(N)$.
- Each $P_i = S \cap P_i, i = 1, 2, \dots, N$ is an ideal of S_i .

Definition 10. Let

$S(N) = \{S_1 \cup S_2 \cup \dots \cup S_N, *_1, *_2, \dots, *_N\}$ be a neutrosophic strong N -LA-semigroup. A proper subset $J = \{J_1 \cup J_2 \cup \dots \cup J_N, *_1, *_2, \dots, *_N\}$ where $J_t = J \cap S_t$ for $t = 1, 2, \dots, N$ is said to be a neutrosophic strong N -ideal of $S(N)$ if the following conditions are satisfied.

- Each it is a neutrosophic sub LA-semigroup of $S_t, t = 1, 2, \dots, N$ i.e. It is a neutrosophic strong N -sub LA-semigroup of $S(N)$.
 - Each it is a two sided ideal of S_t for $t = 1, 2, \dots, N$.
- Similarly one can define neutrosophic strong N -left ideal or neutrosophic strong right ideal of $S(N)$.

A neutrosophic strong N -ideal is one which is both a neutrosophic strong N -left ideal and N -right ideal of $S(N)$.

Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U , and $A, B \subset E$. Molodtsov defined the soft set in the following manner:

Definition 11. A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \rightarrow P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U . For $a \in A$, $F(a)$ may be considered as the set of a -elements of the soft set (F, A) , or as the set of a -approximate elements of the soft set.

Example 1. Suppose that U is the set of shops. E is the set of parameters and each parameter is a word or sentence. Let

$$E = \left\{ \begin{array}{l} \text{high rent, normal rent,} \\ \text{in good condition, in bad condition} \end{array} \right\}.$$

Let us consider a soft set (F, A) which describes the attractiveness of shops that Mr. Z is taking on rent. Suppose that there are five houses in the universe

$U = \{s_1, s_2, s_3, s_4, s_5\}$ under consideration, and that $A = \{a_1, a_2, a_3\}$ be the set of parameters where a_1 stands for the parameter 'high rent', a_2 stands for the parameter 'normal rent', a_3 stands for the parameter 'in good condition'. Suppose that

$$\begin{aligned} F(a_1) &= \{s_1, s_4\}, \\ F(a_2) &= \{s_2, s_5\}, \\ F(a_3) &= \{s_3\}. \end{aligned}$$

The soft set (F, A) is an approximated family $\{F(a_i), i = 1, 2, 3\}$ of subsets of the set U which gives us a collection of approximate description of an object. Then (F, A) is a soft set as a collection of approximations over U , where

$$\begin{aligned} F(a_1) &= \text{high rent} = \{s_1, s_4\}, \\ F(a_2) &= \text{normal rent} = \{s_2, s_5\}, \\ F(a_3) &= \text{in good condition} = \{s_3\}. \end{aligned}$$

Definition 12. For two soft sets (F, A) and (H, B) over U , (F, A) is called a soft subset of (H, B) if

- $A \subseteq B$ and
- $F(a) \subseteq H(a)$, for all $x \in A$.

This relationship is denoted by $(F, A) \subset (H, B)$. Similarly (F, A) is called a soft superset of (H, B) if (H, B) is a soft subset of (F, A) which is denoted by $(F, A) \supset (H, B)$.

Definition 13. Two soft sets (F, A) and (H, B) over U are called soft equal if (F, A) is a soft subset of (H, B) and (H, B) is a soft subset of (F, A) .

Definition 14. Let (F, A) and (K, B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by $(F, A) \cap_R (K, B) = (H, C)$ where (H, C) is defined as $H(c) = F(c) \cap K(c)$ for all $c \in C = A \cap B$.

Definition 15. The extended intersection of two soft sets (F, A) and (K, B) over a common universe U is the soft set (H, C) , where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B, \\ G(c) & \text{if } c \in B - A, \\ F(c) \cap G(c) & \text{if } c \in A \cap B. \end{cases}$$

We write $(F, A) \cap_{\epsilon} (K, B) = (H, C)$.

Definition 16. The restricted union of two soft sets (F, A) and (K, B) over a common universe U is the soft set (H, C) , where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as $H(c) = F(c) \cup G(c)$ for all $c \in C$. We write it as $(F, A) \cup_R (K, B) = (H, C)$.

Definition 17. The extended union of two soft sets

(F, A) and (K, B) over a common universe U is the soft set (H, C) , where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B, \\ G(c) & \text{if } c \in B - A, \\ F(c) \cup G(c) & \text{if } c \in A \cap B. \end{cases}$$

We write $(F, A) \cup_{\epsilon} (K, B) = (H, C)$.

3 Soft Neutrosophic Bi-LA-semigroup

Definition 18. Let $BN(S)$ be a neutrosophic bi-LA-semigroup and (F, A) be a soft set over $BN(S)$. Then (F, A) is called soft neutrosophic bi-LA-semigroup if and only if $F(a)$ is a neutrosophic sub bi-LA-semigroup of $BN(S)$ for all $a \in A$.

Example 2. Let $BN(S) = \{\langle S_1 \cup I \rangle \cup \langle S_2 \cup I \rangle\}$ be a neutrosophic bi-LA-semigroup where $\langle S_1 \cup I \rangle = \{1, 2, 3, 4, 1I, 2I, 3I, 4I\}$ is a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

$\langle S_2 \cup I \rangle = \{1, 2, 3, 1I, 2I, 3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

Let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Then clearly (F, A) is a soft neutrosophic bi-LA-semigroup over $BN(S)$, where

$$F(a_1) = \{1, 1I\} \cup \{2, 3, 3I\},$$

$$F(a_2) = \{2, 2I\} \cup \{1, 3, 1I, 3I\},$$

$$F(a_3) = \{4, 4I\} \cup \{1I, 3I\}.$$

Proposition 1. Let (F, A) and (K, D) be two soft neutrosophic bi-LA-semigroups over $BN(S)$. Then

1. Their extended intersection $(F, A) \cap_E (K, D)$ is soft neutrosophic bi-LA-semigroup over $BN(S)$.
2. Their restricted intersection $(F, A) \cap_R (K, D)$ is soft neutrosophic bi-LA-semigroup over $BN(S)$.
3. Their AND operation $(F, A) \wedge (K, D)$ is soft neutrosophic bi-LA-semigroup over $BN(S)$.

Remark 1. Let (F, A) and (K, D) be two soft neutrosophic bi-LA-semigroups over $BN(S)$. Then

1. Their extended union $(F, A) \cup_E (K, D)$ is not soft neutrosophic bi-LA-semigroup over $BN(S)$.
2. Their restricted union $(F, A) \cup_R (K, D)$ is not soft neutrosophic bi-LA-semigroup over $BN(S)$.
3. Their OR operation $(F, A) \vee (K, D)$ is not soft neutrosophic bi-LA-semigroup over $BN(S)$.

One can easily proved (1), (2), and (3) by the help of examples.

Definition 19. Let (F, A) and (K, D) be two soft neutrosophic bi-LA-semigroups over $BN(S)$. Then (K, D) is called soft neutrosophic sub bi-LA-semigroup of (F, A) , if

1. $D \subseteq A$.
2. $K(a)$ is a neutrosophic sub bi-LA-semigroup of $F(a)$ for all $a \in A$.

Example 3. Let (F, A) be a soft neutrosophic bi-LA-semigroup over $BN(S)$ in Example (1). Then clearly (K, D) is a soft neutrosophic sub bi-LA-semigroup of (F, A) over $BN(S)$, where

$$K(a_1) = \{1, 1I\} \cup \{3, 3I\},$$

$$K(a_2) = \{2, 2I\} \cup \{1, 1I\}.$$

Theorem 1. Let (F, A) be a soft neutrosophic bi-LA-semigroup over $BN(S)$ and $\{(H_j, B_j) : j \in J\}$ be a non-empty family of soft neutrosophic sub bi-LA-semigroups of (F, A) . Then

- 1) $\bigcap_{j \in J} (H_j, B_j)$ is a soft neutrosophic sub bi-LA-semigroup of (F, A) .
- 2) $\bigwedge_R (H_j, B_j)$ is a soft neutrosophic sub bi-LA-semigroup of (F, A) .
- 3) $\bigcup_{j \in J} (H_j, B_j)$ is a soft neutrosophic sub bi-LA-semigroup of (F, A) if $B_j \cap B_k = \emptyset$ for all $j, k \in J$.

Definition 20. Let (F, A) be a soft set over a neutrosophic bi-LA-semigroup $BN(S)$. Then (F, A) is called soft neutrosophic biideal over $BN(S)$ if and only if $F(a)$ is a neutrosophic biideal of $BN(S)$, for all $a \in A$.

Example 4. Let $BN(S) = \{\langle S_1 \cup I \rangle \cup \langle S_2 \cup I \rangle\}$ be a neutrosophic bi-LA-semigroup, where $\langle S_1 \cup I \rangle = \{1, 2, 3, 1I, 2I, 3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

And $\langle S_2 \cup I \rangle = \{1, 2, 3, I, 2I, 3I\}$ be another neutrosophic LA-semigroup with the following table.

.	1	2	3	I	2I	3I
1	3	3	2	3I	3I	2I
2	2	2	2	2I	2I	2I
3	2	2	2	2I	2I	2I
I	3I	3I	2I	3I	3I	2I
2I	2I	2I	2I	2I	2I	2I
3I	2I	2I	2I	2I	2I	2I

Let $A = \{a_1, a_2\}$ be a set of parameters. Then (F, A) is a soft neutrosophic biideal over $BN(S)$, where

$$F(a_1) = \{1, 1I, 3, 3I\} \cup \{2, 2I\},$$

$$F(a_2) = \{1, 3, 1I, 3I\} \cup \{2, 3, 2I, 3I\}.$$

Proposition 2. Every soft neutrosophic biideal over a neutrosophic bi-LA-semigroup is trivially a soft neutrosophic bi-LA-semigroup but the converse is not true in general.

One can easily see the converse by the help of example.

Proposition 3. Let (F, A) and (K, D) be two soft neutrosophic biideals over $BN(S)$. Then

- 1) Their restricted union $(F, A) \cup_R (K, D)$ is not a soft neutrosophic biideal over $BN(S)$.
- 2) Their restricted intersection $(F, A) \cap_R (K, D)$ is a soft neutrosophic biideal over $BN(S)$.
- 3) Their extended union $(F, A) \cup_\varepsilon (K, D)$ is not a soft neutrosophic biideal over $BN(S)$.
- 4) Their extended intersection $(F, A) \cap_\varepsilon (K, D)$ is a soft neutrosophic biideal over $BN(S)$.

Proposition 4. Let (F, A) and (K, D) be two soft neutrosophic biideals over $BN(S)$. Then

1. Their *OR* operation $(F, A) \vee (K, D)$ is not a soft neutrosophic biideal over $BN(S)$.
2. Their *AND* operation $(F, A) \wedge (K, D)$ is a soft neutrosophic biideal over $BN(S)$.

Definition 21. Let (F, A) and (K, D) be two soft neutrosophic bi-LA-semigroups over $BN(S)$. Then

(K, D) is called soft neutrosophic biideal of (F, A) , if

- 1) $B \subseteq A$, and
- 2) $K(a)$ is a neutrosophic biideal of $F(a)$, for all $a \in A$.

Example 5. Let (F, A) be a soft neutrosophic bi-LA-semigroup over $BN(S)$ in Example (*). Then (K, D) is a soft neutrosophic biideal of (F, A) over $BN(S)$, where

$$K(a_1) = \{1I, 3I\} \cup \{2, 2I\},$$

$$K(a_2) = \{1, 3, 1I, 3I\} \cup \{2I, 3I\}.$$

Theorem 2. A soft neutrosophic biideal of a soft neutrosophic bi-LA-semigroup over a neutrosophic bi-LA-semigroup is trivially a soft neutrosophic sub bi-LA-semigroup but the converse is not true in general.

Proposition 5. If (F', A') and (G', B') are soft neutrosophic biideals of soft neutrosophic bi-LA-semigroups (F, A) and (G, B) over neutrosophic bi-LA-semigroups $N(S)$ and $N(T)$ respectively. Then

$(F', A') \times (G', B')$ is a soft neutrosophic biideal of soft neutrosophic bi-LA-semigroup $(F, A) \times (G, B)$ over $N(S) \times N(T)$.

Theorem 3. Let (F, A) be a soft neutrosophic bi-LA-semigroup over $BN(S)$ and $\{(H_j, B_j) : j \in J\}$ be a non-empty family of soft neutrosophic biideals of (F, A) . Then

- 1) $\bigcap_{j \in J} \mathcal{R}(H_j, B_j)$ is a soft neutrosophicbi ideal of (F, A) .
- 2) $\bigwedge_{j \in J} (H_j, B_j)$ is a soft neutrosophic biideal of (F, A) .
- 3) $\bigcup_{j \in J} \mathcal{E}(H_j, B_j)$ is a soft neutrosophic biideal of (F, A) .
- 4) $\bigvee_{j \in J} (H_j, B_j)$ is a soft neutrosophic biideal of (F, A) .

4 Soft Neutrosophic Stornq Bi-LA-semigroup

Definition 22. Let $BN(S)$ be a neutrosophic bi-LA-semigroup and (F, A) be a soft set over $BN(S)$. Then (F, A) is called soft neutrosophic strong bi-LA-semigroup if and only if $F(a)$ is a neutrosophic strong sub bi-LA-semigroup for all $a \in A$.

Example 6. Let $BN(S)$ be a neutrosophic bi-LA-semigroup in Example (1). Let $A = \{a_1, a_2\}$ be a set of parameters. Then (F, A) is a soft neutrosophic strong bi-LA-semigroup over $BN(S)$, where

$$F(a_1) = \{1I, 2I, 3I, 4I\} \cup \{2I, 3I\},$$

$$F(a_2) = \{1I, 2I, 3I, 4I\} \cup \{1I, 3I\}.$$

Proposition 6. Let (F, A) and (K, D) be two soft neutrosophic strong bi-LA-semigroups over $BN(S)$. Then

1. Their extended intersection $(F, A) \cap_E (K, D)$ is soft neutrosophic strong bi-LA-semigroup over $BN(S)$.
2. Their restricted intersection $(F, A) \cap_R (K, D)$ is soft neutrosophic strong bi-LA-semigroup over $BN(S)$.
3. Their AND operation $(F, A) \wedge (K, D)$ is soft neutrosophic strong bi-LA-semigroup over

$BN(S)$.

Remark 2. Let (F, A) and (K, D) be two soft neutrosophic strong bi-LA-semigroups over $BN(S)$. Then

1. Their extended union $(F, A) \cup_E (K, D)$ is not soft neutrosophic strong bi-LA-semigroup over $BN(S)$.
2. Their restricted union $(F, A) \cup_R (K, D)$ is not soft neutrosophic strong bi-LA-semigroup over $BN(S)$.
3. Their OR operation $(F, A) \vee (K, D)$ is not soft neutrosophic strong bi-LA-semigroup over $BN(S)$.

One can easily proved (1),(2), and (3) by the help of examples.

Definition 23. Let (F, A) and (K, D) be two soft neutrosophic strong bi-LA-semigroups over $BN(S)$. Then (K, D) is called soft neutrosophic strong sub bi-LA-semigroup of (F, A) , if

1. $B \subseteq A$.
2. $K(a)$ is a neutrosophic strong sub bi-LA-semigroup of $F(a)$ for all $a \in A$.

Theorem 4. Let (F, A) be a soft neutrosophic strong bi-LA-semigroup over $BN(S)$ and $\{(H_j, B_j) : j \in J\}$ be a non-empty family of soft neutrosophic strong sub bi-LA-semigroups of (F, A) . Then

1. $\bigcap_{j \in J} \mathcal{R}(H_j, B_j)$ is a soft neutrosophic strong sub bi-LA-semigroup of (F, A) .
2. $\bigwedge_{j \in J} (H_j, B_j)$ is a soft neutrosophic strong sub bi-LA-semigroup of (F, A) .
3. $\bigcup_{j \in J} \mathcal{E}(H_j, B_j)$ is a soft neutrosophic strong

sub bi-LA-semigroup of (F, A) if

$$B_j \cap B_k = \phi \text{ for all } j, k \in J .$$

Definition 24. Let (F, A) be a soft set over a neutrosophic bi-LA-semigroup $BN(S)$. Then (F, A) is called soft neutrosophic strong biideal over $BN(S)$ if and only if $F(a)$ is a neutrosophic strong biideal of $BN(S)$, for all $a \in A$.

Example 7. Let $BN(S)$ be a neutrosophic bi-LA-semigroup in Example (*). Let $A = \{a_1, a_2\}$ be a set of parameters. Then clearly (F, A) is a soft neutrosophic strong biideal over $BN(S)$, where

$$F(a_1) = \{1I, 3I\} \cup \{1I, 2I, 3I\},$$

$$F(a_2) = \{1I, 3I\} \cup \{2I, 3I\}.$$

Theorem 5. Every soft neutrosophic strong biideal over $BN(S)$ is a soft neutrosophic biideal but the converse is not true.

We can easily see the converse by the help of example.

Proposition 7. Every soft neutrosophic strong biideal over a neutrosophic bi-LA-semigroup is trivially a soft neutrosophic strong bi-LA-semigroup but the converse is not true in general.

Proposition 8. Every soft neutrosophic strong biideal over a neutrosophic bi-LA-semigroup is trivially a soft neutrosophic bi-LA-semigroup but the converse is not true in general.

One can easily see the converse by the help of example.

Proposition 9. Let (F, A) and (K, D) be two soft neutrosophic strong biideals over $BN(S)$. Then

1. Their restricted union $(F, A) \cup_R (K, D)$ is not a soft neutrosophic strong biideal over $BN(S)$.
2. Their restricted intersection $(F, A) \cap_R (K, D)$ is a soft neutrosophic strong biideal over $BN(S)$.
3. Their extended union $(F, A) \cup_\varepsilon (K, D)$ is not

a soft neutrosophic strong biideal over $BN(S)$.

4. Their extended intersection $(F, A) \cap_\varepsilon (K, D)$ is a soft neutrosophic strong biideal over $BN(S)$.
5. Their *OR* operation $(F, A) \vee (K, D)$ is not a soft neutrosophic biideal over $BN(S)$.
6. Their *AND* operation $(F, A) \wedge (K, D)$ is a soft neutrosophic biideal over $BN(S)$.

Definition 25. Let (F, A) and (K, D) be two soft neutrosophic strong bi-LA-semigroups over $BN(S)$. Then (K, D) is called soft neutrosophic strong biideal of (F, A) , if

1. $D \subseteq A$, and
2. $K(a)$ is a neutrosophic strong biideal of $F(a)$, for all $a \in A$.

Theorem 6. A soft neutrosophic strong biideal of a soft neutrosophic strong bi-LA-semigroup over a neutrosophic bi-LA-semigroup is trivially a soft neutrosophic strong sub bi-LA-semigroup but the converse is not true in general.

Proposition 10. If (F', A') and (G', B') are soft neutrosophic strong biideals of soft neutrosophic bi-LA-semigroups (F, A) and (G, B) over neutrosophic bi-LA-semigroups $N(S)$ and $N(T)$ respectively. Then $(F', A') \times (G', B')$ is a soft neutrosophic strong biideal of soft neutrosophic bi-LA-semigroup $(F, A) \times (G, B)$ over $N(S) \times N(T)$.

Theorem 7. Let (F, A) be a soft neutrosophic strong bi-LA-semigroup over $BN(S)$ and $\{(H_j, B_j) : j \in J\}$ be a non-empty family of soft neutrosophic strong biideals of (F, A) . Then

1. $\bigcap_{j \in J} (H_j, B_j)$ is a soft neutrosophic strong bi

ideal of (F, A) .

2. $\bigwedge_{j \in J} (H_j, B_j)$ is a soft neutrosophic strong biideal of (F, A) .
3. $\bigcup_{\mathcal{E}} \bigcup_{j \in J} (H_j, B_j)$ is a soft neutrosophic strong biideal of (F, A) .
4. $\bigvee_{j \in J} (H_j, B_j)$ is a soft neutrosophic strong biideal of (F, A) .

5 Soft Neutrosophic N-LA-semigroup

Definition 26. Let $\{S(N), *_1, *_2, \dots, *_N\}$ be a neutrosophic N-LA-semigroup and (F, A) be a soft set over $S(N)$.

Then (F, A) is called soft neutrosophic N-LA-semigroup if and only if $F(a)$ is a neutrosophic sub N-LA-semigroup of $S(N)$ for all $a \in A$.

Example 8. Let $S(N) = \{S_1 \cup S_2 \cup S_3, *_1, *_2, *_3\}$ be a neutrosophic 3-LA-semigroup where

$S_1 = \{1, 2, 3, 4, 1I, 2I, 3I, 4I\}$ is a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

$S_2 = \{1, 2, 3, 1I, 2I, 3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

And $S_3 = \{1, 2, 3, I, 2I, 3I\}$ is another neutrosophic LA-semigroup with the following table.

.	1	2	3	I	2I	3I
1	3	3	2	3I	3I	2I
2	2	2	2	2I	2I	2I
3	2	2	2	2I	2I	2I
I	3I	3I	2I	3I	3I	2I
2I	2I	2I	2I	2I	2I	2I
3I	2I	2I	2I	2I	2I	2I

Let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Then clearly (F, A) is a soft neutrosophic 3-LA-semigroup over $S(N)$, where

$$F(a_1) = \{1, 1I\} \cup \{2, 3, 3I\} \cup \{2, 2I\},$$

$$F(a_2) = \{2, 2I\} \cup \{1, 3, 1I, 3I\} \cup \{2, 3, 2I, 3I\},$$

$$F(a_3) = \{4, 4I\} \cup \{1I, 3I\} \cup \{2I, 3I\}.$$

Proposition 11. Let (F, A) and (K, D) be two soft neutrosophic N-LA-semigroups over $S(N)$. Then

1. Their extended intersection $(F, A) \cap_E (K, D)$ is soft neutrosophic N-LA-semigroup over $S(N)$.
2. Their restricted intersection $(F, A) \cap_R (K, D)$ is soft neutrosophic N-LA-semigroup over $S(N)$.
3. Their *AND* operation $(F, A) \wedge (K, D)$ is soft neutrosophic N-LA-semigroup over $S(N)$.

Remark 3. Let (F, A) and (K, D) be two soft neutrosophic N-LA-semigroups over $S(N)$. Then

1. Their extended union $(F, A) \cup_E (K, D)$ is not soft neutrosophic N-LA-semigroup over $S(N)$.
2. Their restricted union $(F, A) \cup_R (K, D)$ is not soft neutrosophic N-LA-semigroup over $S(N)$.
3. Their *OR* operation $(F, A) \vee (K, D)$ is not soft neutrosophic N-LA-semigroup over $S(N)$.

One can easily proved (1),(2), and (3) by the help of examples.

Definition 27. Let (F, A) and (K, D) be two soft neutrosophic N-LA-semigroups over $S(N)$. Then (K, D) is called soft neutrosophic sub N-LA-semigroup of (F, A) , if

1. $D \subseteq A$.
2. $K(a)$ is a neutrosophic sub N-LA-semigroup of $F(a)$ for all $a \in A$.

Theorem 8. Let (F, A) be a soft neutrosophic N-LA-semigroup over $S(N)$ and $\{(H_j, B_j) : j \in J\}$ be a non-empty family of soft neutrosophic sub N-LA-semigroups of (F, A) . Then

1. $\bigcap_R (H_j, B_j)$ is a soft neutrosophic sub N-LA-semigroup of (F, A) .

2. $\bigwedge_R (H_j, B_j)$ is a soft neutrosophic sub N-LA-semigroup of (F, A) .
3. $\bigcup_{\mathcal{E}} (H_j, B_j)$ is a soft neutrosophic sub N-LA-semigroup of (F, A) if $B_j \cap B_k = \phi$ for all $j, k \in J$.

Definition 28. Let (F, A) be a soft set over a neutrosophic N-LA-semigroup $S(N)$. Then (F, A) is called soft neutrosophic N-ideal over $S(N)$ if and only if $F(a)$ is a neutrosophic N-ideal of $S(N)$ for all $a \in A$.

Example 9. Consider Example (***) Let $A = \{a_1, a_2\}$ be a set of parameters. Then (F, A) is a soft neutrosophic 3-ideal over $S(N)$, where

$$F(a_1) = \{1, 1I\} \cup \{3, 3I\} \cup \{2, 2I\},$$

$$F(a_2) = \{2, 2I\} \cup \{1I, 3I\} \cup \{2, 3, 3I\}.$$

Proposition 12. Every soft neutrosophic N-ideal over a neutrosophic N-LA-semigroup is trivially a soft neutrosophic N-LA-semigroup but the converse is not true in general.

One can easily see the converse by the help of example.

Proposition 13. Let (F, A) and (K, D) be two soft neutrosophic N-ideals over $S(N)$. Then

1. Their restricted union $(F, A) \cup_R (K, D)$ is not a soft neutrosophic N-ideal over $S(N)$.
2. Their restricted intersection $(F, A) \cap_R (K, D)$ is a soft neutrosophic N-ideal over $S(N)$.
3. Their extended union $(F, A) \cup_{\mathcal{E}} (K, D)$ is not a soft neutrosophic N-ideal over $S(N)$.

4. Their extended intersection $(F, A) \cap_{\varepsilon} (K, D)$ is a soft neutrosophic N-ideal over $S(N)$.

Proposition 15. Let (F, A) and (K, D) be two soft neutrosophic N-ideals over $S(N)$. Then

1. Their *OR* operation $(F, A) \vee (K, D)$ is a not soft neutrosophic N-ideal over $S(N)$.
2. Their *AND* operation $(F, A) \wedge (K, D)$ is a soft neutrosophic N-ideal over $S(N)$.

Definition 29. Let (F, A) and (K, D) be two soft neutrosophic N-LA-semigroups over $S(N)$. Then (K, D) is called soft neutrosophic N-ideal of (F, A) , if

1. $B \subseteq A$, and
2. $K(a)$ is a neutrosophic N-ideal of $F(a)$ for all $a \in A$.

Theorem 8. A soft neutrosophic N-ideal of a soft neutrosophic N-LA-semigroup over a neutrosophic N-LA-semigroup is trivially a soft neutrosophic sub N-LA-semigroup but the converse is not true in general.

Proposition 16. If (F', A') and (G', B') are soft neutrosophic N-ideals of soft neutrosophic N-LA-semigroups (F, A) and (G, B) over neutrosophic N-LA-semigroups $N(S)$ and $N(T)$ respectively. Then $(F', A') \times (G', B')$ is a soft neutrosophic N-ideal of soft neutrosophic N-LA-semigroup $(F, A) \times (G, B)$ over $N(S) \times N(T)$.

Theorem 9. Let (F, A) be a soft neutrosophic N-LA-semigroup over $S(N)$ and $\{(H_j, B_j) : j \in J\}$ be a non-empty family of soft neutrosophic N-ideals of (F, A) . Then

1. $\bigcap_{j \in J} (H_j, B_j)$ is a soft neutrosophic N-ideal of (F, A) .

2. $\bigwedge_{j \in J} (H_j, B_j)$ is a soft neutrosophic N-ideal of (F, A) .

3. $\bigcup_{j \in J} (H_j, B_j)$ is a soft neutrosophic N-ideal of (F, A) .

4. $\bigvee_{j \in J} (H_j, B_j)$ is a soft neutrosophic N-ideal of (F, A) .

6 Soft Neutrosophic Strong N-LA-semigroup

Definition 30. Let $\{S(N), *_1, *_2, \dots, *_N\}$ be a neutrosophic N-LA-semigroup and (F, A) be a soft set over $S(N)$. Then (F, A) is called soft neutrosophic strong N-LA-semigroup if and only if $F(a)$ is a neutrosophic strong sub N-LA-semigroup of $S(N)$ for all $a \in A$.

Example 10. Let $S(N) = \{S_1 \cup S_2 \cup S_3, *_1, *_2, *_3\}$ be a neutrosophic 3-LA-semigroup in Example 8. Let

$A = \{a_1, a_2, a_3\}$ be a set of parameters. Then clearly

(F, A) is a soft neutrosophic strong 3-LA-semigroup over $S(N)$, where

$$F(a_1) = \{1I\} \cup \{2I, 3I\} \cup \{2I\},$$

$$F(a_2) = \{2I\} \cup \{1I, 3I\} \cup \{2I, 3I\},$$

Theorem 10. If $S(N)$ is a neutrosophic strong N-LA-semigroup, then (F, A) is also a soft neutrosophic strong N-LA-semigroup over $S(N)$.

Proposition 17. Let (F, A) and (K, D) be two soft neutrosophic strong N-LA-semigroups over $S(N)$. Then

1. Their extended intersection $(F, A) \cap_E (K, D)$ is soft neutrosophic strong N-LA-semigroup over $S(N)$.

2. Their restricted intersection $(F, A) \cap_R (K, D)$ is soft neutrosophic strong N-LA-semigroup over $S(N)$.
3. Their *AND* operation $(F, A) \wedge (K, D)$ is soft neutrosophic strong N-LA-semigroup over $S(N)$.

Remark 4. Let (F, A) and (K, D) be two soft neutrosophic strong N-LA-semigroups over $S(N)$. Then

1. Their extended union $(F, A) \cup_E (K, D)$ is not soft neutrosophic strong N-LA-semigroup over $S(N)$.
2. Their restricted union $(F, A) \cup_R (K, D)$ is not soft neutrosophic strong N-LA-semigroup over $S(N)$.
3. Their *OR* operation $(F, A) \vee (K, D)$ is not soft neutrosophic strong N-LA-semigroup over $S(N)$.

One can easily proved (1),(2), and (3) by the help of examples.

Definition 31. Let (F, A) and (K, D) be two soft neutrosophic strong N-LA-semigroups over $S(N)$. Then (K, D) is called soft neutrosophic strong sub N-LA-semigroup of (F, A) , if

3. $D \subseteq A$.
4. $K(a)$ is a neutrosophic strong sub N-LA-semigroup of $F(a)$ for all $a \in A$.

Theorem 11. Let (F, A) be a soft neutrosophic strong N-LA-semigroup over $S(N)$ and $\{(H_j, B_j) : j \in J\}$ be a non-empty family of soft neutrosophic strong sub N-LA-semigroups of (F, A) . Then

1. $\bigcap_R (H_j, B_j)$ is a soft neutrosophic strong sub N-LA-semigroup of (F, A) .

2. $\bigwedge_R (H_j, B_j)$ is a soft neutrosophic strong sub N-LA-semigroup of (F, A) .
3. $\bigcup_E (H_j, B_j)$ is a soft neutrosophic strong sub N-LA-semigroup of (F, A) if $B_j \cap B_k = \phi$ for all $j, k \in J$.

Definition 32. Let (F, A) be a soft set over a neutrosophic N-LA-semigroup $S(N)$. Then (F, A) is called soft neutrosophic strong N-ideal over $S(N)$ if and only if $F(a)$ is a neutrosophic strong N-ideal of $S(N)$ for all $a \in A$.

Proposition 18. Every soft neutrosophic strong N-ideal over a neutrosophic N-LA-semigroup is trivially a soft neutrosophic strong N-LA-semigroup but the converse is not true in general. One can easily see the converse by the help of example.

Proposition 19. Let (F, A) and (K, D) be two soft neutrosophic strong N-ideals over $S(N)$. Then

1. Their restricted union $(F, A) \cup_R (K, D)$ is not a soft neutrosophic strong N-ideal over $S(N)$.
2. Their restricted intersection $(F, A) \cap_R (K, D)$ is a soft neutrosophic N-ideal over $S(N)$.
3. Their extended union $(F, A) \cup_E (K, D)$ is also a not soft neutrosophic strong N-ideal over $S(N)$.
4. Their extended intersection $(F, A) \cap_E (K, D)$ is a soft neutrosophic strong N-ideal over $S(N)$.
5. Their *OR* operation $(F, A) \vee (K, D)$ is a not soft neutrosophic strong N-ideal over $S(N)$.
6. Their *AND* operation $(F, A) \wedge (K, D)$ is a

soft neutrosophic strong N-ideal over $S(N)$.

Definition 33. Let (F, A) and (K, D) be two soft neutrosophic strong N-LA-semigroups over $S(N)$. Then (K, D) is called soft neutrosophic strong N-ideal of (F, A) , if

1. $B \subseteq A$, and
2. $K(a)$ is a neutrosophic strong N-ideal of $F(a)$ for all $a \in A$.

Theorem 12. A soft neutrosophic strong N-ideal of a soft neutrosophic strong N-LA-semigroup over a neutrosophic N-LA-semigroup is trivially a soft neutrosophic strong sub N-LA-semigroup but the converse is not true in general.

Theorem 13. A soft neutrosophic strong N-ideal of a soft neutrosophic strong N-LA-semigroup over a neutrosophic N-LA-semigroup is trivially a soft neutrosophic strong N-ideal but the converse is not true in general.

Proposition 20. If (F', A') and (G', B') are soft neutrosophic strong N-ideals of soft neutrosophic strong N-LA-semigroups (F, A) and (G, B) over neutrosophic N-LA-semigroups $N(S)$ and $N(T)$ respectively. Then $(F', A') \times (G', B')$ is a soft neutrosophic strong N-ideal of soft neutrosophic strong N-LA-semigroup $(F, A) \times (G, B)$ over $N(S) \times N(T)$.

Theorem 14. Let (F, A) be a soft neutrosophic strong N-LA-semigroup over $S(N)$ and $\{(H_j, B_j) : j \in J\}$ be a non-empty family of soft neutrosophic strong N-ideals of (F, A) . Then

1. $\bigcap_{j \in J} (H_j, B_j)$ is a soft neutrosophic strong N-ideal of (F, A) .
2. $\bigwedge_{j \in J} (H_j, B_j)$ is a soft neutrosophic strong N-ideal of (F, A) .

3. $\bigcup_{j \in J} (H_j, B_j)$ is a soft neutrosophic strong N-ideal of (F, A) .
4. $\bigvee_{j \in J} (H_j, B_j)$ is a soft neutrosophic strong N-ideal of (F, A) .

Conclusion

This paper we extend soft neutrosophic bisemigroup, soft neutrosophic N -semigroup to soft neutrosophic bi-LA-semigroup, and soft neutrosophic N -LA-semigroup. Their related properties and results are explained with many illustrative examples. The notions related with strong part of neutrosophy also established.

References

- 1) H. Aktas, N. Cagman, Soft sets and soft groups, Inf. Sci. 177 (2007) 2726-2735.
- 2) K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 64(2)(1986) 87-96.
- 3) M. Shabir, M. Ali, M. Naz, F. Smarandache, Soft neutrosophic group, Neutrosophic Sets and Systems. 1(2013) 5-1.
- 4) M. Ali, F. Smarandache, M. Shabir, M. Naz, Soft neutrosophic Bigroup, and Soft Neutrosophic N-group, Neutrosophic Sets and Systems. 2 (2014) 55-81.
- 5) M. I. Ali, F. Feng, X. Liu, W. K. Min, M. Shabir, On some new operations in soft set theory. Comp. Math. Appl., 57(2009), 1547-1553.
- 6) S. Broumi, F. Smarandache, Intuitionistic Neutrosophic Soft Set, J. Inf. & Comput. Sc. 8(2013) 130-140.
- 7) D. Chen, E.C.C. Tsang, D.S. Yeung, X. Wang, The parameterization reduction of soft sets and its applications, Comput. Math. Appl. 49(2005) 757-763.
- 8) F. Feng, M. I. Ali, M. Shabir, Soft relations applied to semigroups, Filomat 27(7)(2013) 1183-1196.
- 9) M.B. Gorzalzany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, Fuzzy Sets Syst. 21(1987) 1-17.

- 10) W. B. V. Kandasamy, F. Smarandache, Basic Neutrosophic Algebraic Structures and their Applications to Fuzzy and Neutrosophic Models, Hexis (2004).
- 11) W. B. V. Kandasamy, F. Smarandache, N-Algebraic Structures and S-N-Algebraic Structures, Hexis Phoenix (2006).
- 12) W. B. V. Kandasamy, F. Smarandache, Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures, Hexis (2006).
- 13) P.K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45(2003) 555-562.
- 14) P. K. Maji, Neutrosophic Soft Sets, Ann. Fuzzy Math. Inf. 5(1)(2013) 2093-9310.
- 15) D. Molodtsov, Soft set theory first results, Comput. Math. Appl. 37(1999) 19-31.
- 16) Z. Pawlak, Rough sets, Int. J. Inf. Comp. Sci. 11(1982) 341-356.
- 17) F. Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic. Rehoboth: American Research Press (1999).
- 18) F. Smarandache, M. Ali, M. Shabir, Soft Neutrosophic Algebraic Structures and Their Generalization. Education Publishing, Ohio 43212, USA (2014).
- 19) L.A. Zadeh, Fuzzy sets, Inf. Cont. 8(1965) 338-353.

Received: July 5, 2014. Accepted: August 10, 2014.