MAXWELL'S PARADOX: THE METAPHYSICS OF CLASSICAL ELECTRODYNAMICS AND ITS TIME-REVERSAL INVARIANCE

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Abstract

In this paper, I argue that the recent discussion on the time-reversal invariance of classical electrodynamics (see (Albert 2000: ch.1), (Arntzenius 2004), (Earman 2002), (Malament 2004), (Horwich 1987: ch.3)) can be best understood assuming that the disagreement among the various authors is actually a disagreement about the metaphysics of classical electrodynamics. If so, the controversy will not be resolved until we have established which alternative is the most natural. It turns out that we have a paradox, namely that the following three claims are incompatible: the electromagnetic fields are real, classical electrodynamics is time-reversal invariant, and the content of the state of affairs of the world does not depend on whether it belongs to a forward or a backward sequence of states of the world.

Keywords: Classical Electrodynamics; Time-reversal Invariance; Field Ontology; Symmetries.

1. Introduction

A recent disagreement among philosophers of physics revolves around the question of whether classical electrodynamics is time-reversal invariant. David Albert (2000:ch.1) argues that classical electrodynamics violates time-reversal invariance, while orthodoxy has been defended, among others, by Frank Arntzenius (2004), John Earman (2002), and David Malament (2004). Paul Horwich (1987: ch.3) instead, has put forward an intermediate position, which has been taken to be incoherent (Arntzenius 2004). In this paper, it is argued that the analysis discussed in the literature does not get us to the heart of the matter, and does not account for Horwich's position. We argue that different judgments about time-reversal invariance of classical electrodynamics rest on different judgments about what the ontology of classical electrodynamics is. If so, we cannot settle the dispute over the time-reversal invariance of classical electrodynamics until we have established which alternative is the most natural way to interpret the formalism of classical electrodynamics. It turns out that each alternative classical electrodynamics is so costly that the situation could be described as a paradox, which we dub

"Maxwell's Paradox.[1]" Assuming that a possible history of the world is a sequence of instantaneous states, it seems to be impossible to simultaneously hold that: 1) electromagnetic fields are just as real as particles; 2) that the ontology of the theory does not depend on whether we are considering the forward or the backward history of the world; and 3) that classical electrodynamics is time-reversal invariant.

In Section 2 Albert's argument against the time-reversal invariance of classical electrodynamics is presented, while in Section 3 we discuss the defense of the traditional position. In Section 4, it is argued that the different proposals are different choices of the ontology of classical electrodynamics. An analysis of the merits and the objections of each of these theories are provided in Section 5, and in Section 6, Maxwell's paradox is formulated and discussed.

1. The Argument against Invariance

According to Albert (2000: ch.1), a complete description of the world, the *instantaneous state* of the world S, has to be genuinely instantaneous (i.e., the descriptions at different times must be independent) and complete. Typically, in classical mechanics the state is taken to be constituted by the couple of positions and velocities. Albert instead thinks that such couple should be called the *dynamical condition* D at an instant: it violates independence (since the velocity depends on the position) and provides all the information required "in order to bring the full predictive resources of the dynamical laws of physics to bear" (Albert 2000: 17). Rephrasing, the instantaneous state S represents what exists in the world at one instant, while the dynamical condition D specifies what is needed at one time to determine the state of the system at another time. According to Albert, thus, in classical mechanics, S is given by the particles' positions, while D by the positions and the velocities.

A time-reversal transformation involves an operator T that transforms a possible temporal sequence of instantaneous states S_1, S_2,...,S_N (a possible history of the world), into the backward sequence $T(S_N)$,$T(S_{N-1})$,...,$T(S_1)$= S_N, S_{N-1},...,S_1. According to the traditional notion of time-reversal invariance, a theory is time-reversal invariant if and only if, for each possible history of the world also the backward sequence is a possible history of the world. A history of the world can be thought as a movie, in which the sequence of the various instantaneous states is the sequence of the different frames. To say that a theory is time-reversal invariant is then to say that the movie projected backward and the movie projected forward both represent possible state of affairs of the world. Albert claims that S should be invariant under T: S represents what there is in the world and a sequence of instantaneous states represents a possible history of the world, so remembering the movie analogy, T is acting on the ordering of the frames, not on their content, which should remain the same.

In classical mechanics, the instantaneous state S remains invariant under T: T(x)=x. The velocity instead flips sign, since it transforms under T according to its definition as a function of x and t: $T(v)=T(dx/dt)=dx/(-dt) =-dx/dt =-v$. Therefore, while S=(x) is left unchanged, D=(x,v)

[1] Note that the name does not refer to anything that Maxwell wrote or implied; Maxwell did not give his name to a paradox. We decided to call it like that simply because the main equations of classical electrodynamics are Maxwell's equations.

transforms as T(x,v)=(x,-v). Classical mechanics is time-reversal invariant because both the forward history and the backward history are possible histories of the world.

When considering classical electrodynamics, Albert believes we need to add to S also "the magnitudes and directions of the electric (E) and magnetic (B) fields at every point in space" (Albert, 2000:14). That is, the instantaneous state is given by the triplet (x,E,B). The fields are, unlike velocities, logically independent of the particles' positions and therefore they should be added to S in order to complete the picture of the world at one time. That is, electromagnetic fields are real just as much as the particles are. According to Albert, as we already saw, the instantaneous state should not change under T. Indeed, velocity is defined as the rate of change of position, so that it would make sense for it to flip sign under T. However, the fields have no such definition: magnetic fields are not the rate of change of anything. Thus, he argues, they should be mathematically represented simply by vector functions, which would not transform under T. Therefore, we would have T(x,E,B)=(x, E,B). As a matter of fact, though, in order for classical electrodynamics to be time-reversal invariant, we need S to change under T: in particular, we need T(E)=E, and T(B)= - B. Given a possible history of the world S_1, S_2,...,S_N then S'_N, S'_{N-1},...,S'_1 (where S'=(x,E,-B) for any time) is also a possible history of the world, while S_N, S_{N-1},...,S_1 is not. This is incompatible with the requirement above, so classical electrodynamics is not time-reversal invariant.

It is useful to spell out Albert's argument schematically as follows:

1. [FIELDS]: In classical electrodynamics, electromagnetic fields belong to the instantaneous state. That is S=(x,E,B);
2. [TIME-REVERSAL]: a time-reversal transformation T is one that turns a sequence of states of the world S_1, S_2,...,S_N into its reverse T(S_N), T(S_{N-1}),...,T(S_1)= S_N, S_{N-1},...,S_1;
3. [T-R INVARIANCE]: A theory is time-reversal invariant just in case the (theoretical) forward and backward sequences of states of the world both correctly describe possible states of affairs of the world;
4. In particular, for classical electrodynamics to be time-reversal invariant we need B to flip sign under T. That is, T(B)= - B;
5. [STATE]: Under a time-reversal transformation, the instantaneous state of the world at one time does not change. That is, T(S)=S;
6. In particular, T(B)=B;
7. (6) contradicts (4);
8. Therefore, classical electrodynamics is not time-reversal invariant.

Albert judges [FIELDS], [TIME-REVERSAL], [T-R INVARIANCE], [STATE] and the fact that there is no reason for B to flip sign under T to be so obviously true that we have to reject that classical electrodynamics is time-reversal invariant[2]. Clearly, this is not the only possibility: one

[2] Let us remind the reader why Albert engages in the issue of time reversibility in a book in which he discusses the foundations of statistical mechanics. Albert, who follows closely Boltzmann, wants to show that the tension between the time irreversibility of macroscopic phenomena and the time reversibility of the underlying microscopic physics is ultimately not problematic if we take into account statistical mechanics. Therefore, in order to set up the discussion, he needs to clarify what it means for a theory to be time-reversal invariant, and to show that the microphysics is time-reversal invariant (in some relevant sense). In developing his account of the notion, he concludes that classical mechanics is time-reversal

could reject one (or more) of the other premises. We argue that the disagreement about the time-reversal invariance of classical electrodynamics can be accounted for focusing on [FIELDS] and [STATE], and that it is fundamentally a disagreement about ontology.

2. The Defense of Invariance

Arntzenius (2004), Earman (2002), and Malament (2004), among others, disagree with Albert's conclusion about the lack of time-reversal invariance of classical electrodynamics. These three accounts differ in the details, but ultimately are very similar: their common goal is to show how one can naturally define the electromagnetic fields so that they transform under T as required in order to make classical electrodynamics time-reversal invariant. Because of this similarity, we will call their account 'AEM' (from their initials). Malament's account seems more complete and detailed than the ones of Earman and Arntzenius, thus we will describe it in a little more detail in the following[3]. In Malament's paper, the electromagnetic fields are defined as follows. Let us consider a smooth, connected 4-dimensional manifold M and a pseudo-Reinmaninan metric g_{ab}

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invariant (since velocities are rates of changes of positions, their time-reversal is given by turning all velocities around, as usually intended), and thus he finds the tension mentioned above. The situation changes, though, if we consider other theories. He writes: "*None* of the fundamental physical theories that anybody has taken seriously throughout the past century and a half is (as I mentioned above) invariant under time-reversal." Nonetheless, this does not mean that the tension has disappeared. In fact, we still have it because "Most of them [a.n. fundamental physical theories] *are* time-reversal invariant, though, insofar as the positions of particles are concerned" (Albert 2000: 15). An anonymous reviewer has pointed out that it is unclear why we should even discuss Albert's view of time-reversal, given that at the end it is not the relevant notion to set up the foundations of statistical mechanics. Another reviewer has also complained that Albert's view has been heavily and devastatingly criticized by many authors (see, for instance, (Arntzenius 2000), (Earman 2002), (Malament 2004)), so it is obviously a dead view. We disagree, since we think it is worthwhile discussing Albert's view and taking it seriously. In fact it seems to be interesting for a variety of reasons. First, because it is not obviously false, and as such deserves consideration. Second, because the objections against it are not controversial either, as we will see. Third, because even if at the end Albert's notion of time-reversal will not change the discussion in the foundations of statistical mechanics, it will change the metaphysical picture we get from classical electrodynamics, and also it will give us insights (he claims) in the foundations of quantum mechanics. Fourth, because the distinction he draws between the instantaneous state and the dynamical condition can clarify the role of the various variables in a fundamental physical theory (see later on the connection with the primitive ontology view, footnote 11).

[3] Roughly put, Earman argues that writing classical electrodynamics in a covariant form in terms of the electromagnetic tensor F, and defining the 4-vecor potential in terms of it and of the Green function, one obtains the correct transformations for E and B under time-reversal, defined as inverting a continuous non-vanishing timelike vector field (given that E and B can be defined in terms of the 4-potential). According to Earman himself, his account differs from the one of Malament only in the fact that Malament assumes rather than derives the transformation for the electric filed, but the overall idea is very similar. Arntzenius (2000) also argues that "the correct conception of the electromagnetic 4-potential is that it is a 4-vector that lives in tangent space, transforms like a tangent vector, and can interact with other tangent vectors, even though is not tangent to any actual worldline […]. But once one accepts this, it is clear that 4-potentials transform non-trivially under (passive and active) time-reversal, and hence that classical electromagnetism is invariant under time-reversal , precisely as orthodoxy would have it." (Arntzenius 2004: 37).

with signature (1,3) on M, i.e. a relativistic space-time. The worldline of any (massive) point-particle can be described therefore as a smooth curve on M. The electromagnetic force can be represented just by a map from the tangent line to the curve to force vectors, regardless any temporal orientation, in any point: $(L,q)\rightarrow F(L,q)$. To choose a temporal direction, we take a direction of the 4-velocity v^a. So the force is represented by the map $v^a \rightarrow F^a(v^a,q)$. In requiring that this map has the desired properties (for instance, that it is linear in q, and that the force is orthogonal to the velocity), we get that it has to be represented by an antisymmetric tensor: $F(v^a,q) = qF^a{}_b v^b$. From $F^a{}_b$ and the currents, we can recover classical electrodynamics (namely, the Lorentz force and Maxwell's equations). Arguably, in Malament's account, the time-reversal operation is understood as flipping the temporal orientation originally chosen. Given that $F(v^a,q) = qF^a{}_b v^b$, it is straightforward that $T(F_{ab}) = -F_{ab}$ because the choice of an orientation is a choice of the direction of v^a. From this, Malament shows that it follows that Maxwell's equations are time-reversal invariant. Suppose we want to know how the fields transform. We can recover Maxwell's equations not in terms of F_{ab}, but in terms of E and B as soon as we specify additional structure (see details in Malament (2004)). We need a spatial structure to specify B, in addition to a temporal orientation. In this way, we see that, under the properly defined time-reversal invariance, we get the correct transformations since E turns out to be a polar vector and B an axial vector, rather than a scalar or a vector field. This explains why B flips sign under T and E does not: by definition, under reflection a polar (or true) vector will match its mirror image, while an axial (pseudo) vector will match its mirror image in magnitude, but it will point in the opposite direction.

To be able to compare properly Albert's approach to the one of AEM, we need to clarify what AEM take to be the components of the instantaneous state S, and what their concept of time-reversal transformation is. The first point is easy: AEM seem to take the fields as part of the instantaneous state4. Regarding the notion of time-reversal transformation, the situation is a little more complex. Jill North (North 2008:211) has argued that Malament's conception of time-reversal transformation is different from the one of Albert. According to North, Albert's idea of time-reversal transformation T_A "mirrors the material content of space-time across a time slice," while Malament's time-reversal transformation T_M "inverts the temporal orientation." The difference between the two, she argues, is that in Malament's account the temporal orientation is considered alike to a physical field, so that some field τ should also be added to S. If North is correct, then AEM reject [TIME-REVERSAL] (namely, that $T(S_1),T(S_2),\ldots,T(S_N)=S_N,S_{N-1},\ldots,S_1$), and [STATE] (i.e., the claim that $T(S)=S$) fails by definition: τ is in fact in S, and T_M flips it. We think that, regardless of whether there is a difference in the definition of the time-reversal transformation, the disagreement between Albert and Malament is fundamentally about [STATE]. In fact, if they both accept [TIME-REVERSAL], Malament, who defines B as an axial vector, has to deny [STATE] to get time-reversal invariance of classical electrodynamics. If instead Malament denies [TIME-REVERSAL], then he would deny [STATE] as well, this time as a consequence of his choice of time-reversal: τ is in S, and τ flips sign under T (by definition of $T=T_M$). Therefore, if we generalize this to AEM's position, we can conclude that classical

[4] Earman and Arntzenius explicitly hold that the fields are part of the instantaneous state, while Malament is not so explicit and straightforward about it. Furthermore, Malament is not generally willing to commit himself to a given ontology of classical electrodynamics (private communication).

electrodynamics is time-reversal invariant because they assume [FIELDS] but reject that there is no reason for B to flip sign under T, since B is an axial vector. Because of this, AEM reject [STATE]: B belongs to S, and S will not remain invariant under T (see later for more about this).

In contrast to the analysis above, Stephen Leeds (2006) has argued that the disagreement between Albert and Malament is due to a mathematical mistake on Malament’s part. Malament's analysis is in terms of passive transformations, while Albert's account is in terms of active ones. When we consider the active counterparts of the passive transformations used by Malament, he writes, we discover they do not provide the correct transformation for B. We think this conclusion is wrong: the problem is that in his derivation of the active counterparts of Malament's transformations, Leeds uses the constraint that S remains invariant under T, which, as we just saw, Malament denies. Because of this, we believe that Leeds’ argument fails.

Frank Arntzenius and Hilary Greaves (2009) agree with AEM that we should reject [STATE], arguing as follows. They introduce the notion of ‘geometrical’ transformation: given a particular mathematical object, there is a natural way for it to transform under a particular transformation, which depends on its geometrical definition. The electromagnetic fields are intrinsically defined as to transform under T to make classical electrodynamics invariant, so Albert is wrong and Malament right. We think Arntzenius and Greaves are correct in their claim that each mathematical object has a natural, or geometrical, way to transform under a given transformation. But we think they are begging the question against Albert: since we are talking about mathematical objects as representing physical objects, Albert could agree with them that objects transform under a given symmetry transformation according to their geometrical definition, and still disagree about which one is the correct way of mathematically representing B. That is, he would take it to be a vector function, which would be compatible with his view, and not an axial vector, as AEM suggest instead. One needs in fact to provide additional independent reasons to believe that the true nature of the fields is the one captured by the mathematics of an axial vector. What Malament and the others have done is to show how another definition can be provided, in addition to the one proposed by Albert, which is compatible with the invariance of classical electrodynamics under T. Nonetheless, no one has shown yet that this is the correct one. Arntzenius and Greaves realize this, since they acknowledge that Albert's position is a sensible one, and dismiss it for different reasons (see Section 5).

Last but not least, we have Horwich (1987: ch.3), who seems to accept both [STATE] (that is, he accepts the instantaneous state will not change under T), and the time-reversal invariance of classical electrodynamics. He develops this position in just a few sentences, so it is up for debate how one could have an invariant classical electrodynamics with an invariant S. Arntzenius thinks that Horwich is simply incoherent (Arntzenius 2004), presumably because he assumes Horwich accepts [FIELDS], namely that the instantaneous state is (x,E,B). Instead, we will argue at the end of the next section that Horwich accepts [STATE] but he *rejects* [FIELDS], so that he can consistently claim that classical electrodynamics is time-reversal invariant.

3. The Possible Metaphysics of Classical Electrodynamics

We believe people disagree about the time-reversal invariance of classical electrodynamics because they disagree about ontology. The idea is that classical electrodynamics can be consistently interpreted as depicting different worlds, depending on whether we endorse

Albert's view or the one of AEM. The world is made of particles and fields in both cases, but such fields are mathematically represented by different objects, vector functions for Albert, axial vectors for the others[5].

The symmetry properties of a theory depend on the ontology of the theory: different ontologies are captured by different mathematical objects, which have different properties. Since AEM and Albert disagree about what kind of object fields are, they disagree about what symmetry properties the theory has. Contrary to Albert, AEM can accept the time-reversal invariance of classical electrodynamics because they reject [STATE]: an axial vector will not stay invariant under T. Therefore, when they argue about the time reversibility of classical electrodynamics, they actually talk past each other: they are talking about two different theories of classical electrodynamics, and it is not unexpected that different theories have different symmetry properties.

Note that it is not surprising that people turn out to disagree on how to interpret a theory: any physical theory is expressed in terms of mathematical relations among different variables. In order to interpret a theory as somehow depicting reality, one needs to take at least some of these variables as representing physical objects. However, the very same mathematical framework could be interpreted in different ways, and certain interpretations might be more natural than others: there is, so to speak, a sort of underdetermination of the ontology by the formalism. Given its definition, the instantaneous state S captures the metaphysics of the theory, while the dynamical condition D contains also the variables needed to implement the dynamics for the stuff in the instantaneous state S. To make this point also graphically, we can use the symbol ";". It will separate in D the elements that belong to S, which we will put on the left of the semicolon, from the rest of the variables in the theory. Interpreting the mathematical object in S in the 'most natural way' will give us what there is physically in the world. For example, in classical mechanics, D=(x;v), and S=(x), which naturally represents point-particles. In classical electrodynamics, one could argue about which is the most appropriate way of mathematically describing E and B, and each choice has consequences: the symmetry properties of a theory are the symmetry properties of the objects in its ontology, and different mathematical objects will behave differently under a given transformation. As a result, depending on what mathematical objects we take to represent the fields, classical electrodynamics will or will not be time-reversal invariant.

Therefore, here is how one can summarize Albert's position and the one of AEM respectively[6]:

- Albert-CED=(x,E,B;) -- The world is constituted by particles and fields, the latter being represented by vector functions. Therefore, [FIELDS] (i.e. S=(x,E,B)) and [STATE] (i.e. T(S)=S) hold, but classical electrodynamics is not time-reversal invariant.

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[5] It is unclear what Malament would say, since he is always resistant to commit himself to a given ontology for classical electrodynamics. Presumably, though, he would be happy to think that, as suggested by an anonymous reviewer, the energy tensor belongs to the state, rather than the fields. I do not see a contradiction with what we claim here, though: even if he would take the electromagnetic tensor as part of the instantaneous state, this would still make E and B the kind of objects that properly transform to make classical electrodynamics invariant under T.

[6] For sake of simplicity, we will not mention velocities.

- AEM-CED=(x,E,B';) -- The world is made of particles and fields, the latter being represented by polar and axial vectors respectively. Therefore, [FIELDS] (i.e. S=(x,E,B)) is true, but [STATE] (i.e. T(S)=S) is not, allowing the theory to be time-reversal invariant[7].

The disagreement here is therefore about which is the most suitable way of mathematically describing a piece of furniture of the world, namely the electromagnetic fields: are they true vectors, or are they axial vectors?

Notice that if North's interpretation of Malament's notion of time-reversal is correct, we would have to introduce the specific time-reversal transformation we are dealing with (either T_A or T_M), which in turn would have us consider another theory of classical electrodynamics. Therefore, in addition to the two positions stated above, namely Albert-CED and AEM-CED (in which the time-reversal transformation is implemented by T_A, which "mirrors the material content of space-time across a time slice" (North 2008:216)), we would also have MN-CED (for 'Malament's classical electrodynamics with North's interpretation of Malament's notion of T"):

- MN-CED -- The world is made of particles, electromagnetic fields, and a temporal field. Mathematically, E is represented by a polar vector, and B by an axial vector, while the mathematical representation of time-reversal transformation is given by T_M (which "inverts the temporal orientation"). As a consequence, [FIELDS] is true (i.e. S=(x,E,B)) ,but [STATE] (i.e. T(S)=S) and [T-R INVARIANCE] are not[8]. Thus, classical electrodynamics is time-reversal invariant.

Of course, one could also disagree about what belongs to S. By moving the semicolon, we can generate different theories assigning a different role to the mathematical object of the same mathematical formalism. For instance, one could consider classical mechanics to be (x,v;), or (v;x), in addition to the more natural (x;v): (x,v;) describes a world with particles' positions and velocities, (v;x) a world with only velocities, and (x;v) a world with only particles' positions[9]. We will assume that velocities are not in the instantaneous state. A theory that does not assume that, dubbed `the Feynman proposal,' will be discussed later. Note that the fact that there are different ways of reading the formalism does not mean that they are all sensible. For instance, classical mechanics is usually taken to be (x;v), and the reasons for this are the ones presented by Albert: (x,v;) is not instantaneous, while (x;v) is not complete[10]. Observe that if we start moving the semicolon in the dynamical condition of classical electrodynamics we obtain, among others, the following position:

- Horwich-CED=(x;E,B') -- The world is made of particles, while the fields do not compose matter: they are mathematical fictions to correctly describe the behavior of the particles. That is, [FIELDS] fails (i.e. S=(x)), but [STATE] holds (i.e., T(S)=S), since B does not

[7] Here B' indicates that the mathematical object representing a magnetic field in this theory is different from the mathematical object that represents it in the theory above.

[8] This is because [T-R INVARIANCE] is defined in terms of T_A.

[9] Note that also (; x; v) is possible but it does not really seem to describe a satisfactory physical theory since there is nothing in the theory that specifies what there is in the world.

[10] Arguably, an Aristotelian about properties would like (x,v;) very much, since it describes a world in which we have both the objects *and* the velocities. This seems to be discussed also, for instance, in (Arntzenius 2000), (Forrest 1998), and (Tooley 1988).

belong to S, and therefore what B does under T is irrelevant, and classical electrodynamics is time-reversal invariant.

Horwich has been accused by Arntzenius to be confused: "Horwich infers that in the reversed history the magnetic field points in the opposite direction. But this is inconsistent with his claim that 'basic' or 'fundamental' quantities should not undergo time-reversal operations" (Arntzenius 2004: 35 fn4). Note, though, that if instead Horwich is taken to endorse the theory discussed last, the confusion dissolves, and there is no inconsistency in this position: the fields do not belong to the instantaneous state S, and therefore they are not part of the basic and fundamental quantities. Note that the important point here is not so much to recover the actual position of Horwich: it might well be that he might not hold this position, and that he really is incoherent. What is important, though, is that the position above seems to be a possible position, independently of whether it is a faithful representation of Horwich's own take on classical electrodynamics[11].

There are many more ways of interpreting the formalism of classical electrodynamics than just the ones presented so far, some more interesting than others. For example, we could also have:

- <u>Einstein-CED=(E,B; x)</u> -- This could describe something close to Einstein's position, in which matter is made exclusively of fields, while particles are conceived as singularities in the fields[12].

This theory has not been elaborated by anyone yet, and it is rarely considered. Perhaps it requires more attention than the one it has received so far, but for the time being we will leave it just as a possible suggestion.

Another proposal has been recently put on the table: what Arntzenius and Greaves call the `Feynman Proposal.' The idea can be summarized as follows: the world is made of fields described as Malament does, and of particles, but their worldlines are intrinsically directed

[11] In this regard, it is interesting to note that we can draw a parallel between Horwich-CED and a view that recently has gained some attention in the foundation of quantum mechanics, namely the one based on the notion of primitive ontology (see, for instance, Allori, Goldstein, Tumulka, and Zanghi 2008). According to this view, the primitive ontology of a theory is what in the theory represents matter, while the rest of the ontology helps implementing the law of temporal evolution of matter. Thus, in quantum theories, the primitive ontology is some object in three-dimensional space or four-dimensional space-time (particles, matter density fields, events in space-time called flashes, and so on), while the wave function is the object in the theory that allows to generate the dynamics for the primitive ontology. In the terminology used in this paper, the state S would be the primitive ontology, while the dynamical condition D would be composed of the primitive ontology and the wave function. Similarly, we would say that the primitive ontology of Horwich-CED is particles, while the one of Albert-CED and AEM-CED is particles and fields, where Albert-CED and AEM-CED differ in what kind of mathematical objects the fields are supposed to be represented by. Given that the wave function is not part of the primitive ontology of quantum theories, it is allowed to change in ways that the primitive ontology could not, in order for the theory to possess the symmetry properties that we want it to have. The situation is similar in Horwich-CED: in other words, the role of the fields in Horwich-CED and the role of the wave function in quantum theories is similar, since they do not represent matter, while they allow to implement the dynamics for matter.

[12] The notion of time-reversal transformation assumed here is the one proposed by Albert, T_A.

depending on whether they are worldlines of particles or of antiparticles. The name comes from the following quote from Richard Feynman: “Every particle in Nature has an amplitude to move backwards in time, and therefore has an anti-particle" (Feynman 1985:95). Assuming Malament's definition of electromagnetic fields in terms of the antisymmetric tensor F, it turns out that the theory is time-reversal invariant[13], but E flips sign under T instead of B. We might be tempted to call the theory (x,E’,B;), but we think this would not be correct, since it does not capture the intrinsic directedness of wordlines. A better characterization would probably be (x,v,E’,B;) to be contrasted with (x,E’,B;) in which velocities do not seem to be part of the instantaneous state of the world. Schematically, then, we would have:

- Feynman-CED=(x,v,E’,B;) -- The world is made of particles, with velocities and positions, and electromagnetic fields represented *a la* Malament, but where T(E)=-E and T(B)=B.

4. Which Metaphysics?

All the proposed theories seem to be possible ways of metaphysically interpreting the formalism of classical electrodynamics. They provide different pictures of the world, and accordingly have different symmetry properties. Leaving the last three theories, MN-CED, Einstein-CED, and Feynman-CED, to be discussed at the end of the section, we have the following situation. Albert, considering classical electrodynamics to be Albert-CED=(x,E,B;), judges it to break time-reversal invariance; AEM, considering classical electrodynamics to be AEM-CED=(x,E,B’;), conclude the contrary; Horwich, arguably considering classical electrodynamics to be Horwich-CED==(x;E,B’), considers classical electrodynamics to be time-reversal invariant but for a different reason. However, which of the above is the ‘true’ classical electrodynamics?

The main argument to favor AEM-CED over Albert-CED lies on the importance of symmetry properties: AEM-CED is better than Albert-CED because the former has more symmetry properties, and physicists always put a lot of weight on symmetries, since they seem to construct theories around symmetry groups[14]. To defend Albert’s position, the only choice seems to be to question the importance of symmetry properties. It seems a difficult position to defend, though, since symmetries have always had an important role in theory construction and theory evaluation[15]. In this regard, some have argued that the time-reversal symmetry in particular is important because its failure indicates that time has an objective direction according to that theory. This is controversial, though: Arntzenius and Greaves have argued that this is not the case, since it could also be due to the existence of an objective space-time handedness[16]. However, another line of argument for AEM-CED over Albert-CED appeals to Ockham's razor (Arntzenius and Greaves 2009): Albert-CED needs a standard absolute rest and

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[13] Here we also assume that the notion of time-reversal transformation is the one proposed by Albert, T_A.

[14] Note that the transformation (E,B)→ (E,-B) will take solutions into solutions and thus will be a symmetry of the theory. Nonetheless, it is not going to be a “time-reversal” transformation in Albert’s view, since “[m]agnetic fields are not - neither logically nor conceptually- the *rates of change* of anything” (Albert 2000:20). Thus, it is hard to see how this transformation could possibly have any physical meaning. This is the sense in which one should understand the claim that Albert-CED has less symmetry properties than the alternatives.

[15] See (Brading and Castellani 2003) for the importance and the role of symmetries in physics.

[16] See (Arntzenius 2000) and (Arntzenius and Greaves 2009) for details.

an objective temporal orientation, while AEM-CED does not. Again, this argument does not work against Horwich-CED since the two theories share the same definition of B. In any case, the potential problem with this is that one could argue that simplicity is not really a virtue.

There are other reasons to favor AEM-CED, even if they are in a sense weaker than the one we just saw. One could argue that AEM-CED is coherent with history: we started describing matter as made of particles; we then discovered electromagnetic phenomena and added fields to S; only in the relativistic framework, did we realize that the magnetic fields are mathematically represented by axial vectors, that is we arrived at AEM-CED. A problem with this argument could be that when we discover a theory should not matter; maybe there are reasons to take the older theory more seriously than the most recent one. It seems to us, though, that this reply misses the point; the suggestion is that the most recent theory is best not in virtue of being recent, but because it represents our true understanding of nature.

Turning to the evaluation of AEM-CED, it seems to us that it faces a serious challenge: in this theory, [STATE] is false, that is the instantaneous state of the world S changes under T. That amounts to saying that the content of a state of the world could change depending on whether it comes from the forward or the backward movie of the world. This seems at best counterintuitive, and does not happen in Albert-CED or Horwich-CED: how is it possible that the magnetic fields in the state of the forward movie point in one direction, but in the states of the backward one they point to the opposite direction? One could take this to be a *reductio ad absurdum* for the theory. As we saw earlier, Arntzenius and Greaves try to justify the rejection of [STATE] in terms of what they call a 'geometrical' transformation: given a particular mathematical object, there is a natural way for it to transform under a particular transformation, which depends on its geometrical intrinsic definition. That is, the electromagnetic fields are intrinsically defined as to transform under T to make the theory invariant, which makes [STATE] false (that is, T(S) is not S). Even if correct, though, this does not help at all, especially since these fields are real: it still does not seem sensible that the field would be pointing in two different directions in the backward and forward movie. Arntzenius explicitly argues that there is no problem with allowing S to change under T: S changes under T because we should not expect mathematical objects to 'forget' what kinds of mathematical objects they are under symmetry transformations. We find this reply very unconvincing for someone who interprets the instantaneous state S as Albert does: after all, the content of a snapshot should not change depending on whether one puts it at the beginning or at the end of a sequence, independently of how one chooses to represent what is depicted in the snapshot! Another possible reply to this challenge, which unfortunately strikes us as weak, is to argue that this is counterintuitive, but still less counterintuitive than what the alternative positions propose.

Let us now turn to Horwich-CED. The main arguments to prefer this theory to the alternatives are the following. First, the symmetry argument could be used to favor Horwich-CED over Albert-CED. In addition, a simplicity argument could be used to favor Horwich-CED over AEM-CED: they both have symmetries, but the former is better because it has a simpler ontology. If we can account for everything just assuming there are particles, the argument says, why also assume there are fields? The key, the argument continues, is that we do not need them in S: they can simply be regarded as playing a role in determining the motion of the particles, the argument concludes. In addition to the problem of using simplicity as a virtue, the worry with this kind of argument is that it does not seem to be true that we just need particles to explain the phenomena. In fact, there seems to be energy associated to the fields, and how can Horwich-CED explain such energy if there are no fields? Another argument for Horwich-CED over AEM-CED is that in the former S is invariant under T while in the latter S is not. This is the same as what happens in Albert-CED, but here we also have more symmetry properties.

As far as problems for Horwich-CED go, a first worry would be that asserting that there are no fields is contrary to our intuitions and our ordinary beliefs. That is, Horwich-CED makes many of our beliefs false, and theories like that, as skepticism for instance, are not to be favored. One could respond by rejecting such a criterion of theory choice: after all, many of the modern scientific theories ask us to revise many of our ordinary beliefs, but we do not reject them because of this. At the same time, though, to select a counterintuitive theory is always a hard choice to make. In addition to this, there are other so-called worries that instead just seem simple misunderstandings. First, one could think that in this theory S is not a true instantaneous state of the world: there are no fields, so it is incomplete. This is not so, though. In fact, the instantaneous state S is complete, since the theory says there are really no fields in the world.

Another complaint against Horwich-CED is that there are no free fields, i.e. that all fields are `generated' by particles. However, if the fields are not supposed to represent physical objects, then the solutions of Maxwell's equations, including free solutions, never have any physical meaning. Another alleged challenge to Horwich-CED rests on the fact that the theory is completely indeterministic[17]: if in S we just have particles' positions, then in the instantaneous state at a given time we do not have all that we need to determine the instantaneous state at a later time. However, when we talk about determinism what we really have in mind involves D instead of S. That is, determinism as we understand it should be defined in terms of D: a theory is deterministic just in case, given D at one time, one can determine D at a different time. According to this definition, neither classical mechanics nor Horwich-CED turn out to be indeterministic theories.

When considering the last three theories mentioned above, let us focus on MN-CED and Feynman-CED, given that the Einstein proposal is too sketchy to be evaluated. North argues that the notion of time-reversal T_M involved in MN-CED is to be preferred to the one proposed by Albert, what she calls the standard view T_A: T_A flips objects to their mirror image; T_M just flips the temporal orientation. Since all the other proposals we have discussed here are all involving T_A, we can read her arguments to favor MN-CED over all of them. A first concern with this approach is to figure out whether this characterization is correct[18]. Assuming that there is nothing wrong with it, North compares T_A and T_M on six points. Here is a short summary of her arguments for T_M in a four-dimensional setting and some potential difficulties with them. First, according to North, T_A lacks a profound justification, while T_M does not: 4-velocities invert under T_M because of their definition. It is unclear, though, why there is no justification for T_A, since North herself acknowledges that this is the straightforward intuitive

[17] See (Arntzenius 2000) for a similar worry in the framework of classical mechanics.

[18] Malament (private communication) has declared that he feels uncomfortable with this characterization of his view. Nonetheless, since the view is coherent, it is a live possibility and indeed can be held by somebody else without any problem. For sake of simplicity, it seems harmless to us to continue to call this view with his name.

view of time-reversal. The second point of comparison relies on the distinction between active and passive transformation: the charge is that T_A seems to have no corresponding passive transformation, while T_M does. Nevertheless, Earman (2002), as North herself acknowledges, proposes a possible passive transformation of T_A, and in that case the alleged advantage of T_M over T_A disappears[19]. A third advantage of T_M over T_A as identified by North is that T_A takes for granted that these theories are time-reversal invariant, while T_M does not. In this way, Malament's view possesses a greater amount of symmetries. This seems to be the strongest argument for T_M, but one can reply to it denying that having more symmetries is a valuable property. Fourth, North claims, following Arntzenius (1997), that Malament's view can be more easily adapted to indeterministic theories. Nevertheless, this seems at best controversial, since there is an ongoing controversy regarding which is the best way of characterizing indeterministic theories. In addition, the charge is not that T_A cannot be applied to indeterministic theories; it is rather that it is more contrived. As such, one could object that to rely on vague notions such as 'straightforward,' 'natural,' or 'contrived' is always dangerous. Fifth, North claims that T_M generalizes naturally to curved space-time, while T_A does not. In addition to the problem already identified above, this does not seem to be true, especially in the framework of AEM-CED: arguably, it is the definition of electromagnetic fields as Malament defines them that guarantees the generalization to curved space-time, rather than the choice of time-reversal transformation. Lastly, T_M provides a better test for temporal orientation because it only flips the temporal orientation. In any case, a fundamental drawback of this account is the existence of a temporal field in addition to the other elements in the ontology: is the addition of such a metaphysical baggage worth the alleged advantages?

Regarding the Feynman proposal, Arntzenius and Greaves (2008) argue that it might be appealing because it is arguably more compatible with quantum field theories. On our part, we do not think this means a lot: after all, how to interpret the formalism of quantum field theory is even more controversial than how to interpret the one of classical electrodynamics, and so it is extremely unclear what is supposed to be compatible with what. In any case, Arntzenius and Greaves point to a potential weakness of the theory: it might not be adequate to describe neutral particles, since there is nothing to determine what the orientation of the particle's worldline should be. In addition, this picture has all the problems of AEM's view over Horwich's and over Albert's (mainly that S changes under T), but someone might consider that it is less natural than both of them due to the intrinsic directedness of worldlines.

5. Maxwell's Paradox

We have argued so far that there are several distinct and consistent theories of classical electrodynamics. Since symmetry properties are determined by the ontology of the theory, we need to determine which of the previous proposals provides the most natural ontology before addressing the question of which symmetry properties a given theory has: until we have solved the controversy about which alternative is the most natural and should be adopted, we will not be able to solve the disagreement about the symmetry properties of classical electrodynamics.

Unfortunately, as this overview has just shown, the discussion is far from being settled: each proposal seems to have costs so high that we might even claim that we are in a presence of

[19] See (North 2008) for further discussion of the difficulties of Earman's proposal.

a paradox. Let us come back to Albert's original argument. If the definitions of time-reversal transformation and invariance have not been truly questioned, then it can be re-written as follows:

1. [FIELDS]: Electromagnetic fields in classical electrodynamics are just as real as particles;
2. [STATE]: The content of a state of the world does not depend on whether this state is taken from a forward or a backward sequence of states;
3. Therefore, [T-R INVARIANCE of CED] is false: classical electrodynamics is not time-reversal invariant.

If we go with AEM (and similarly for the Feynman proposal), what we get is:

1. [FIELDS]: Electromagnetic fields in classical electrodynamics are just as real as particles;
2. [T-R INVARIANCE of CED]: classical electrodynamics is time-reversal invariant;
3. Therefore [STATE] is false: the content of a state of the world depends on whether this state is taken from a forward or a backward sequence of states.

Finally, if Horwich is correct we have:

1. [STATE]: The content of a state of the world does not depend on whether this state is taken from a forward or a backward sequence of states;
2. [T-R INVARIANCE of CED]: classical electrodynamics is time-reversal invariant;
3. Therefore [FIELDS] is false: electromagnetic fields in classical electrodynamics are not just as real as particles.

Note that if the notion of time-reversal transformation is changed, as North proposes, the situation would not change much: we would have MN-CED, in which we add to the ontology a temporal field, instead of AEM-CED, but we would still have that [FIELDS] and [T-R INVARIANCE of CED] hold, while [STATE] would not, just like in AEM-CED. That means that we cannot take [FIELDS], [STATE] and [T-R INVARIANCE of CED] to be true for the same theory. Therefore, however we turn it, we are in presence of a very counterintuitive conclusion derived from apparently acceptable reasoning from apparently acceptable premises. The debaters thus can be taken to be struggling with this paradox: we have only three choices, which correspond to the accounts of our three players. Albert solves the paradox rejecting the time-reversal invariance of classical electrodynamics, motivated by the intuitions that electromagnetic fields are real and that the ordering of a sequence of states should not change the content of such states. However, this is not a small cost: symmetries are important! AEM (and also MN and Feynman) instead reject [STATE]. They want electromagnetic fields to constitute matter like Albert does, but they also want symmetries, thus they have to give up the idea that the content of an instantaneous state of the world does not change depending on whether this state is taken from the forward or the backward story of the world. This, again, strikes us as counterintuitive: how can the content of the state change under T? Finally, Horwich wants to keep the time-reversal symmetry and the invariance of the content of the states with respect to time-reversal, so he has to deny the reality of fields. However, is it really an acceptable choice to say that there are no fields?

The lesson to be drawn from this discussion therefore seems to be that classical electrodynamics involves a paradox, which we dub 'Maxwell's paradox.' That is, the following very plausible claims are incompatible:

- Electromagnetic fields in classical electrodynamics are just as real as particles;
- The content of a state of the world does not depend on whether this state is taken from a forward or a backward sequence of states;
- Classical electrodynamics is time-reversal invariant.

No matter how much we would like all of them to be true at the same time, they simply cannot be. This is an interesting conclusion, drawn from a theory that many of us thought could not surprise us any longer.

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