NECESSITY, THEISM, AND EVIDENCE

MICHAEL ALMEIDA

ABSTRACT

The minimal God exemplifies essential omnipotence, omniscience, and moral perfection, but none of the other properties of the traditional God. I examine the metaphysical and epistemological consequences of the minimal God in augmented S5, S4, and \( K_{\sigma\tau} \). The epistemological consequences in S5 include the impossibility of agnosticism—it is impossible that \( P(\Box F_G \mid E) = P(\neg \Box F_G \mid E) \). Indeed, even weaker forms of agnosticism are impossible—it is impossible that \( P(\Box F_G \mid E) = n, \quad n(0 < n < 1) \). The metaphysical consequences include the impossibility that God—or any other object—might acquire, lose, or exchange an essential property. It is impossible, for instance, that an essentially divine being might become essentially human. The metaphysical and epistemological consequences for the minimal God in S4 are equally untenable. I show that we can avoid all of these unwelcome epistemological consequences for the minimal God in \( K_{\sigma\tau} \). But \( K_{\sigma\tau} \) has some extraordinary metaphysical consequences for the minimal God. All of the problems for the minimal God generalize to the maximal God.

Keywords: Essentialism, S5, S4, \( K_{\sigma\tau} \), Theism

1. Introduction

If we let \( Fx \) conjoin the properties of omnipotence, omniscience, and moral perfection, then \( \Box F_G \) expresses the minimal view that God is essentially omnipotent, omniscient, and morally perfect.\(^1\) \( \Box F_G \) does not express the proposition that God exemplifies all of the essential properties found in the traditional conception of God. But even the minimal God has several

\(^1\) Strictly \( \langle \lambda x, Fx \rangle \) is the conjunctive property of omnipotence, omniscience, and moral perfection and \( \langle \lambda x, \Box Fx \rangle \langle \sigma \rangle \) expresses the minimal view that God is essentially omnipotent, omniscient, and morally perfect. In the interest of simplifying, \( \Box F_G \) is used, always intended to be read de re.
unexpected metaphysical and epistemological consequences in the logics S5, S4, and K\(\omega\).

In section (2) I consider the consequences of the minimal God in the logic S5. To express the metaphysical and epistemological consequences the language is augmented with epistemic probabilities of the form \(P(_-|-)\) and quantifiers \(\forall S\) and \(\exists S\) ranging over all possible states of affairs \(S\). Sentences of the form \(\exists S P(\square F_G | S) = n\) state that there is some state of affairs \(S\) such that the probability of \(\square F_G\) given \(S\) is \(n\). And sentences of the form \(\forall S P(\square F_G | S) = n\) state that every state of affairs \(S\) is such that the probability of \(\square F_G\) given \(S\) is \(n\).

It is an epistemic consequence of augmented S5 that there are no states of affairs \(S\) and \(S'\) in any world such that \(P(\square F_G | S) > 0\) and \(P(\neg \square F_G | S') > 0\). We cannot have evidence for and against even a minimal God. This is not an epistemic consequence in S4, though there is the weaker consequence that it is impossible that \(P(\neg \square F_G | E) > 0\) & \(P(\square F_G | E) > 0\). On the total evidence \(E\), it is impossible that the epistemic probability for and against the minimal God is greater than 0.

In section (3) I consider the consequences for the minimal God in S4. Since it is not possible in S4 that \(P(\neg \square F_G | S) > 0\) unless it is true that \(\neg \square F_G\), no theist could assign the non-existence of the minimal God any positive probability at all, for any \(S\) whatsoever. And among the unexpected

---

2 Unless otherwise indicated. ‘God’ refers throughout to the minimal God. The forthcoming discussion primarily concerns the broad and narrow predicative versions in (1)–(2).

(1) \(x\) is broadly essentially \(F\) iff. \(x\) is necessarily \(F\).

(2) \(x\) is narrowly essentially \(F\) iff. \(F\) is a property in the nature of \(x\) and \(x\) is necessarily \(F\).


4 I take states of affairs to be abstract objects existing in every possible world and obtaining in some possible worlds.
metaphysical consequences in S4, it is perfectly possible that a contingent property of the minimal God should become an essential property. God could survive that sort of change. It is also possible in S4 that the minimal God might acquire the essential properties of some exotic species and even acquire an *haecceity* it does not exemplify.

In section (4) I consider the consequences of the minimal conception of God in the logic $K_{\varphi \sigma}$. The unwelcome epistemological consequences of S4 and S5 are avoidable in $K_{\varphi \sigma}$. So $K_{\varphi \sigma}$ has some important advantages over S4 and S5. But $K_{\varphi \sigma}$ has some extraordinary metaphysical consequences for the minimal God. In $K_{\varphi \sigma}$, essential properties are not necessarily essential and contingent properties are not necessarily contingent. So the minimal God, and of course everything else, can survive the loss, gain, or exchange of an essential property. The minimal God can even survive the gain, loss, or exchange of individual essences or haecceities. God might exemplify essential omnipotence and omniscience in one world, for instance, and contingent omnipotence and omniscience in another. The minimal God might uniquely exemplify an haecceity in one world and not exemplify that haecceity in another. It is true in $K_{\varphi \sigma}$ that God could survive the exchange of the complete profile of essential properties for another.

In section (5) I consider some of the consequences of $K_{\varphi \sigma}$ for philosophical theology. Among the more fascinating consequence of $K_{\varphi \sigma}$ is that there might be entire regions of metaphysical space governed by S5 and other regions of metaphysical space governed by, say, T or still weaker logics. In some of these regions the S5 theorems are contingently necessary and in others the T theorems are contingently necessary. Regions might feature local consequence relations $\vdash_{S5}$ and $\vdash_{T}$ that govern inference and argumentation in those areas only. And valid reasoning in philosophical theology will depend on the particular region we happen to inhabit.

2. On the Consequences of S5

The evidence against the existence of the minimal God typically includes intrinsically bad states of affairs such as the suffering of sentient creatures and the evidence for the existence of God typically includes intrinsically good states of affairs such as the flourishing of sentient creatures. There is,

---

5 $K_{\varphi \sigma}$ is normal, symmetric, and reflexive. It is equivalent to the Brouwer system B.

6 Strictly, the consequences to follow are for augmented S5, S4, and $K_{\varphi \sigma}$ for the proposition $\Box F_G$. The logics must be augmented with the epistemic probability operator $P(\square F_G)$. The metaphysical consequences of these logics for philosophical theology of course do not depend on the epistemic probability operator. I thank a referee for *Logique et Analyse* for noting this.
however, a proof that there cannot be any states of affairs $S$ and $S'$ such that $S$ constitutes evidence for the minimal God and $S'$ constitutes evidence against the minimal God.

Let’s assume there are states of affairs $S$ and $S'$ such that $S$ constitutes some evidence for $\Box F_G$ and $S'$ constitutes some evidence for $\neg \Box F_G$. $\Box F_G$ expresses the proposition that God is essentially omnipotent, omniscient, and morally perfect. So, we should expect that the epistemic probabilities of $\Box F_G$ and $\neg \Box F_G$ are both positive. But it is a consequence of S5 that positive epistemic probabilities for both $\Box F_G$ and $\neg \Box F_G$ are impossible. Let $P(\Box F_G | S)$ be the probability of $\Box F_G$ on the evidence $S$ and assume that $\Diamond (S \land S')$ or that $S$ and $S'$ are compossible. Our initial assumption is in (1).\(^7\)

\[(1) \quad P(\Box F_G | S) > 0 \land P(\neg \Box F_G | S') > 0\]

The position described in (1) is what most religious believers and non-believers maintain.\(^8\) There is some evidence that raises the probability that the minimal God exists above 0 and there is some evidence that raises the probability that the minimal God does not exist above 0. Almost no one believes that $P(\Box F_G | S) = 1$ or that $P(\neg \Box F_G | S') = 1$. But if $S'$ is a possible state of affairs, then we know that $\Box (S' \rightarrow \Box F_G) \rightarrow P(\Box F_G | S') = 1$.\(^9\) The state of affairs $S'$ entails $\Box F_G$ only if the probability of $\Box F_G$ on $S'$ is 1.

\[(2) \quad \Box (S' \rightarrow \Box F_G) \rightarrow P(\Box F_G | S') = 1\]

And so if it is true that $P(\neg \Box F_G | S') > 0$, then we know from (2) that $\neg \Box (S' \rightarrow \Box F_G)$.

\[(3) \quad P(\neg \Box F_G | S') > 0 \rightarrow \neg \Box (S' \rightarrow \Box F_G)\]

\(^7\) The forthcoming argument does not depend on an objectivist interpretation of $P(\Box F_G | S)$. On an epistemic interpretation, agents that assign $P(\Box F_G | S) = n, n(0 < n < 1)$ will find that the probability assignment entails a contradiction, since it follows from $P(\Box F_G | S) = n, n(0 < n < 1)$ that $\Box (S \rightarrow \neg \Box F_G)$. And if $\Box (S \rightarrow \neg \Box F_G)$ then $P(\neg \Box F_G | S) = 1$, on any interpretation of probability. But the propositions $P(\Box F_G | S) = n, n(0 < n < 1)$ and $P(\neg \Box F_G | S) = 1$ are inconsistent. On the most natural interpretation of $P(\Box F_G | S)$ in this context, since we are concerned with evidence $S$ for $\Box F_G$, $P(\Box F_G | S)$ represents epistemic probability. For a discussion of epistemic probability, objective probability, and credence, see D.H. Mellor, *Probability, A Philosophical Introduction* (New York: Routledge, 2005).

\(^8\) The probability axioms alone do not entail that $P(\Box F_G | S)$ has any particular value and so do not rule out (1). It is false, for instance, that $P(\Box F_G | S) = 1$ by axiom (2).

Axiom (1): $P(X) \geq 0$ for all proposition $X$.

Axiom (2): $P(T) = 1$, where $T$ is a logically necessary proposition.

Axiom (3): If propositions $X$ and $Y$ are logically incompatible, then $P(X \lor Y) = (P(X) + P(Y))$.

\(^9\) If $\Box (S \rightarrow \Box F_G)$ then $P(\Box F_G | S) = 1$, since $P(\Box F_G | S) = P((\Box F_G \land S) | P(S) = 1$. 
But it is a theorem in S5 that S' entails $\Box F_G$ or S' entails $\sim \Box F_G$, and so (4) is also an S5 theorem.\(^{10}\)

\[(4) \sim \Box (S' \rightarrow \Box F_G) \rightarrow \Box (S' \rightarrow \sim \Box F_G)\]

And from (1), (2), and (4) it follows that S' entails $\sim \Box F_G$.

\[(5) \Box (S' \rightarrow \sim \Box F_G)\]

Since strengthening antecedents is valid for strict conditionals, the proposition in (5) entails (6).

\[(6) \Box ((S' \& S) \rightarrow \sim \Box F_G)\]

And from (6), (2), and $\Diamond (S \& S')$, it follows that the probability that God does not exist on S and S' is 1.

\[(7) P(\sim \Box F_G \mid S' \& S) = 1\]

So it follows from the fact that S' provides some evidence against the minimal God that S & S' provides conclusive evidence against the minimal God. So, it is a consequence of S5 that $P(\sim \Box F_G \mid S') > 0 \rightarrow P(\sim \Box F_G \mid S' \& S) = 1$. We could also show that $P(\Box F_G \mid S') > 0 \rightarrow P(\Box F_G \mid E) = 1$. So if there is any evidence at all against the minimal God, then $\sim \Box F_G$ is certain on the total evidence E.

But it is also true, analogous to (3) above, that if the probability that God exists on evidence S is greater than 0, then S does not entail that God does not exist.

\[(8) P(\Box F_G \mid S) > 0 \rightarrow \sim \Box (S \rightarrow \Box F_G)\]

But as noted above it is a theorem of S5 that S entails $\Box F_G$ or S entails $\sim \Box F_G$. And so (9) is true.

\[(9) \sim \Box (S \rightarrow \sim \Box F_G) \rightarrow \Box (S \rightarrow \Box F_G)\]

And from (1), (8), and (9) it follows that (10).

\[(10) \Box (S \rightarrow \Box F_G)\]

---

\(^{10}\) The theorem in (4) can seem false. But if we assume $\sim \Box (S' \rightarrow \Box F_G)$ then $\Diamond (S' \& \sim \Box F_G)$. But if $\Diamond \sim \Box F_G$, then $\Box \sim \Box F_G$, since every world is modally equivalent in S5. But if $\Box \sim \Box F_G$ then $\Box (S' \rightarrow \sim \Box F_G)$. So, $\sim \Box (S' \rightarrow \Box F_G) \rightarrow \Box (S' \rightarrow \Box F_G)$. 
The proposition in (11) follows from (10) and strengthening antecedents.

\[(11) \Box(\mathbf{S}' \land \mathbf{S}) \rightarrow \Box \mathbf{F}_G\]

And from (11), (2) and ◊(S & S') we can conclude that the probability that the minimal God exists given S and S' is 1.

\[(12) P(\Box \mathbf{F}_G \mid \mathbf{S} \land \mathbf{S}') = 1\]

Since it follows from (7) that \(P(\Box \mathbf{F}_G \mid \mathbf{S}' \land \mathbf{S}) = 0\), it follows from (7) and (12) that (13).

\[(13) P(\Box \mathbf{F}_G \mid \mathbf{S} \land \mathbf{S}') = 1 \land P(\Box \mathbf{F}_G \mid \mathbf{S} \land \mathbf{S}') = 0\]

The proposition in (13) is impossible, so we can validly conclude that the assumption in (1) is necessarily false.\(^\text{11}\) But if (1) is necessarily false, then it is impossible that there is some state of affairs \(\mathbf{S}\) that constitutes some evidence for \(\Box \mathbf{F}_G\) and some state of affairs \(\mathbf{S}'\) that constitutes some evidence against \(\Box \mathbf{F}_G\). That is, there cannot be some evidence in favor of the minimal God and some evidence against the minimal God.

The conclusion that (1) is necessarily false is surprising. It is little more than a truism that some state of affairs \(\mathbf{S}\) constitutes at least some evidence for the existence of the minimal God and some state of affairs \(\mathbf{S}'\) constitutes at least some evidence against the existence of the minimal God.

\(^{11}\) There is a non-conditional version of the argument as well. Here’s the proof followed by a justification for each line.

1. \(P(\Box \mathbf{F}_G \mid \mathbf{S}) > 0\)
2. \(P(\Box \mathbf{F}_G \mid \mathbf{S}) > 0 \rightarrow \Diamond \Box \mathbf{F}_G\)
3. \(\Diamond \Box \mathbf{F}_G \rightarrow \Box \Box \mathbf{F}_G\)
4. \(\Box \Box \mathbf{F}_G \rightarrow P(\Box \mathbf{F}_G) = 1\)
5. \(\Box \Box \mathbf{F}_G \rightarrow \forall S P(\Box \mathbf{F}_G \mid S) = 1\)
6. \(\therefore \forall S P(\Box \mathbf{F}_G \mid S) = 1\)

Let \(\Gamma\) be an epistemic agent who believes that \(P(\Box \mathbf{F}_G \mid \mathbf{S}) > 0\). Here is the justification for each line of the proof.

For (1) \(\Gamma\) believes that \(P(\Box \mathbf{F}_G \mid \mathbf{S}) > 0\).

For (2), if \(\Gamma\) believes that \(P(\Box \mathbf{F}_G \mid \mathbf{S})\) is positive, then \(\Gamma\) believes that \(\Box \mathbf{F}_G\) is at least possible. \(\Gamma\) believes that impossible propositions get assigned probability 0 ~◊ \(\mathbf{F}_G\) \(\rightarrow P(\Box \mathbf{F}_G) = 0\) and (2) is just the contrapositive of that proposition.

For (3), \(\Gamma\) believes that, in S5, possibility entails necessity for true attributions of essential properties such as \(\Box \mathbf{F}_G\).

For (4), \(\Gamma\) believes that necessary truths are assigned probability 1 and that \(\Box \mathbf{F}_G\) is a necessary truth.

For (5), \(\Gamma\) infers from (4) that (5) \(\Box \Box \mathbf{F}_G \rightarrow P(\Box \mathbf{F}_G \mid \mathbf{S}) = 1\) for any \(\mathbf{S}\) at all.

For (6), \(\Gamma\) infers \(\forall S P(\Box \mathbf{F}_G \mid \mathbf{S}) = 1\) from (1) – (5).

To get to the explicit contradiction we need only iterate the proof.
The consequences of S5 for the minimal God all generalize to the maximal conception of God. The maximally great God in Hartshorne, Malcolm, and Plantinga exemplifies essential omnipotence, omniscience, moral perfection in every possible world. But the consequences in S5 require only that the minimal God exemplify essential omnipotence, omniscience, and moral perfection.

On the plausible assumption that our total evidence E provides some evidence for $\neg \Box F_G$ and some evidence for $\Box F_G$ we could have reached the contradiction in (14).

(14) $P(\Box F_G | E) = 1 \& P(\neg \Box F_G | E) = 0$

In addition to the contradictions following from (1), it is evident that we can derive a contradiction from the agnostic proposition that $P(\Box F_G | S) = .5 \& P(\neg \Box F_G | S') = .5$ or from the agnostic proposition in (15).

(15) $P(\Box F_G | E) = .5 \& P(\neg \Box F_G | E) = .5$

According to (15) the total evidence E for and against the existence of the minimal God is about equal. When we consider all of the arguments advanced in favor of the existence of God and against the existence of God we might find that (15) is a reasonable position. The probability that the minimal God exists on the total evidence is roughly equal to the probability that the minimal God does not exist on the total evidence. But from the left conjunct in (15) it follows that $\Box (E \rightarrow \neg \Box F_G)$ and from the right conjunct in (15) it follows that $\Box (E \rightarrow \Box F_G)$. And from $\Box (E \rightarrow \neg \Box F_G)$ and $\Box (E \rightarrow \Box F_G)$ together we can derive the contradiction in (14). So the proposition in (15) is also necessarily false. It is not possible that the probability of $\Box F_G$ on the total evidence is about the same as the probability of $\neg \Box F_G$ on the total evidence.

In general, it is impossible that the probability of $P(\Box F_G | E) = n$ for any n between 0 and 1. (16) is necessarily false for any n ($0 < n < 1$).

(16) $P(\Box F_G | E) = n \& P(\neg \Box F_G | E) = (1 - n)$

---


It is common for theists to assign $P(\square F_G | E)$ roughly [.7, .9] and assign $P(\neg \square F_G | E)$ roughly [.1 , .3] so the probability that God exists is roughly in the interval .7 to .9 and the probability that God does not exist is roughly in the interval .1 to .3.\footnote{This is based on my experience that theists (i.e., serious-minded theists) almost never assign certainty to their theistic beliefs. It might be partly due to the Bayesian consequence that the higher one’s probability for theism, the less significant all contravening evidence becomes. As we approach certainty, Bayesians urge that we arrive at the incredible epistemic position in which even very serious contravening evidence counts for very little against theistic belief.} But it is impossible that the probabilities of $\square F_G$ and $\neg \square F_G$ take any of these values.

In S5, the only possible assignment of values to $P(\square F_G | E)$ are 0 and 1. (17) is necessarily true. But virtually no one assigns certainty to the existence or non-existence of the minimal God.

(17) $P(\square F_G | E) = 1 \lor P(\neg \square F_G | E) = 1$

And, for all possible states of affairs $S$, (18) is also a theorem.

(18) $\forall S P(\square F_G | S) = 1 \lor \forall S P(\neg \square F_G | S) = 0$

Either every state of affairs in every possible world constitutes conclusive evidence for $\square F_G$ or every state of affairs in every possible world constitutes conclusive evidence against $\square F_G$. (18) has the bizarre consequence that either $\square F_G$ or $\neg \square F_G$ is certain given the state of affairs of there being six trees in your front yard. This is necessarily true despite the fact that the number of trees in your front yard is obviously irrelevant to the existence of the minimal God. (18) follows from (19) and the S5 theorem that $\square \square F_G \lor \square \neg \square F_G$.

(19) $\square F_G \rightarrow \forall S P(\square F_G | S) = 1 \land \neg \square F_G \rightarrow \forall S P(\neg \square F_G | S) = 1$

The theorem in (19) states that, if $\square F_G$ is true, then every possible state of affairs constitutes conclusive evidence for $\square F_G$ and if $\neg \square F_G$ is true, then every possible state of affairs constitutes conclusive evidence that $\neg \square F_G$. (19) entails that, for every possible state of affairs $S$, whether $S$ is evidence for or against $\square F_G$ depends entirely on whether $\square F_G$ is true. Apart from the truth of $\square F_G$ there is no state of affairs that constitutes any evidence at all for or against God’s existence.

3. On the Consequences of S4

The logic of metaphysical necessity in S5 places important restrictions on the essential properties that God, or anything else, might exemplify. Since the logic includes the theorems $\forall x (\Diamond \square Fx \rightarrow \square Fx)$ and $\forall x (\square Fx \rightarrow \square \square Fx)$, there are no contingent essential properties or contingent contingent
properties in S5. It is a consequence of S5 that essential properties are in general necessarily essential and contingent properties are in general necessarily contingent. It is impossible in S5 that God, or anything else, gains, loses, or exchanges an essential property.\(^{15}\)

Of course, S5 does not rule out different objects exemplifying the same essential property since it is possible that \(\Box F_G \& \Box F_C\), for instance. And S5 does not rule out that an essential property of one object might be a contingent property of another, since it is possible that \(\Box F_G \& F_C\). But it is impossible that \(\Box F_G \& \Diamond \neg F_G\) or \(\neg \Box \Box F_G \& \Diamond \Box F_G\). So, if omnipotence is an essential property of God, then it is necessarily an essential property of God. But it might be a contingent property of some other object. And if impassibility is contingent property of God, then impassibility could not be an essential property of God. But it might be an essential property of some other object.

The S5 theorems also restrict individual essences or haecceities. If \(\Box F_x\) is an individual essence of God then God exemplifies \(F_x\) in every possible world and nothing other than God exemplifies \(F_x\)—either contingently or essentially—in any possible world. If we introduce haecceities into S5—so that \(\Box F_x\) is an individual essence or haecceity of God just if \(\Box (F_G \& \forall x (F_x \leftrightarrow x = G))\)—then it is evident that, as with essential properties generally, God cannot survive the loss, gain, or exchange of an individual essence. There are no contingent haecceities in S5, either.

Since there are no possible worlds in which the minimal God gains, loses or exchanges an essential property, S5 ensures that there are no qualitatively identical worlds that differ only haecceitistically. There are no possible worlds that differ only with respect to, for instance, the particular individuals exemplifying essential omnipotence, omniscience, and moral perfection. Everything is such that, if it exemplifies an essential property \(\Box F_x\), then it necessarily exemplifies \(\Box F_x\)—it exemplifies \(\Box F_x\) in absolutely every possible world. If haecceitism is the view that there are distinct possible worlds that differ only with respect to the individuals exemplifying properties, then S5 rules out haecceitism.\(^{16}\)

\(^{15}\) There is some ambiguity in the notion of exchanging an essential property. Exchanging essential properties does not in general require the loss of any properties.

(1) It is true in w that God and b exchange essential properties \(\Box F_G \& \Box G_b\) in \(w'\) just if it is true in \(w\) that \(\Box F_G \& \Box G_b\) and \(\neg \Box G_G \& \neg \Box F_b\), and there is some (relatively) possible world \(w'\) in which God and \(b\) exist and \(\neg \Box F_G \& \Box G_b\) and \(\Box G_G \& \Box F_b\).

(2) It is true in \(w\) that God and \(b\) weakly exchange essential properties \(\Box F_G \& \Box G_b\) in \(w'\) just if it is true in \(w\) that \(\Box F_G \& \Box G_b\) and \(\neg \Box G_G \& \neg \Box F_b\), and there is some (relatively) possible world \(w'\) in which God and \(b\) exist and it is true that \(\Box F_G \& \Box G_b\) and \(\Box G_G \& \Box F_b\).

\(^{16}\) Haecceitism implies that two worlds differing only with respect to individuals exemplifying properties are qualitatively identical (indiscernible) worlds. My thanks to a referee for Logique et Analyse for the suggested revision. There are a number of formulations of haecceitism, but it is normally assumed that essentialism restricts, but does not rule out
The logic of metaphysical necessity in S4 places fewer restrictions on the essential properties that the minimal God might exemplify. The minimal God exemplifies only essential omnipotence, omniscience, and moral perfection but, despite the weakened restrictions, there are several untenable epistemic consequences. (20) is an S4 theorem.

\( \square F_G \rightarrow \forall S \, \mathcal{P}(\square F_G \mid S) = 1 \)

And (21) follows directly from (20).

\( \exists S \, \mathcal{P}(\neg \square F_G \mid S) > 0 \rightarrow \neg \square F_G \)

According to (21), if there is any evidence at all against the existence of God, then God does not exist. If \( \neg \square F_G \) is true, then \( \neg \square F_G \) is included in our total evidence \( E \) for and against the existence of God. So, if it is true that \( \neg \square F_G \), then \( E \) includes \( \neg \square F_G \), and so \( \square (E \rightarrow \neg \square F_G) \). So we can derive (22).

\( \exists S \, \mathcal{P}(\neg \square F_G \mid S) > 0 \rightarrow P(\neg \square F_G \mid E) = 1 \)

According to (22), if there is any evidence at all against the existence of God, then the non-existence of God is certain on the total evidence \( E \). So no theist could assign any positive probability to the non-existence of God, given any evidence at all.

But it’s worth noting that S4 does not entail that there cannot be states of affairs \( S \) and \( S' \) such that \( S \) constitutes some evidence for \( \neg \square F_G \) and \( S' \) constitutes some evidence for \( \square F_G \). So (23) is possible.

\( \exists S \, \mathcal{P}(\neg \square F_G \mid S) > 0 \& P(\square F_G \mid S') > 0 \)

It is a major epistemic advantage of S4 that (23) is possible. The argument against (23) in S5 showed that it is impossible that there is some evidence for the minimal God and some evidence against the minimal God. But certainly that is possible.

Since it is false in S4 that \( \Diamond \square F_G \rightarrow \square F_G \), we cannot reach the conclusion from (23) that \( P(\square F_G \mid S') = 1 \). So the proposition in (23) is haecceitism. Both claims are false. S5 essentialism does in fact rule out haecceitism and, as we will see, \( K_e \), essentialism does not restrict haecceitism at all. Note that even in non-classical S5 we cannot verify haecceitism as understood here. There are, for instance, no two worlds in S5 such that God exemplifies \( \square F_x \) at \( w \) and \( b \) exemplifies \( \square F_x \) at \( w' \). God will exemplify \( \square F_x \) at \( w' \) as well, even if God is not in \( w' \). Compare Brad Skow, ‘Haecceitism, Anti- Haecceitism and Possible Worlds’, Philosophical Quarterly, Vol. 58, No. 210, (2007) 98-107 and ‘More on Haecceitism and Possible Worlds’, Analytic Philosophy 52 (2011): 267-269 and Sam Cowling, ‘Haecceitism for Modal Realists’, Erkenntnis, 77 (2012) 399-417.
possible. Of course, (21) guarantees that (23) is true only in worlds where it is also true that \( \neg \Box F_G \). In such worlds we can assign some positive epistemic probability to \( \Box F_G \) though we know the proposition is false. But (23) does not entail a contradiction. We might distinguish our knowledge that \( \Box F_G \) is false, for instance, from our certainty that it’s false, and so assign \( \Box F_G \) a probability greater than 0.

There are many other reasonable epistemic positions that are impossible according to S4. It is not possible that there is some evidence against the existence of God and that the probability that God exists on the total evidence is greater (or even much greater) than the probability that God does not exist. (24) and (25) are both impossible.

\[
(24) \exists \text{SP}(\neg \Box F_G \mid S) > 0 \land [P(\Box F_G \mid E) > P(\neg \Box F_G \mid E)]
\]

\[
(25) \exists \text{SP}(\neg \Box F_G \mid S) > 0 \land [P(\Box F_G \mid E) \gg P(\neg \Box F_G \mid E)]
\]

It is also impossible that the probability that God exists on the total evidence is about equal to the probability that God does not exist on the total evidence. From (21) and (22) we know that \( P(\neg \Box F_G \mid E) > 0 \) only if \( P(\neg \Box F_G \mid E) = 1 \), so (26) is impossible.

\[
(26) P(\neg \Box F_G \mid E) = .5 \land P(\Box F_G \mid E) = .5
\]

And in general we cannot assign \( P(\neg \Box F_G \mid E) \) any probability except 0 or 1 on the total evidence \( E \). (27) is also impossible in S4.

\[
(27) P(\neg \Box F_G \mid E) = n \land P(\Box F_G \mid E) = (1 - n), (0 < n < 1)
\]

This proposition follows again from the fact that \( P(\neg \Box F_G \mid E) \) is positive only if \( \Box (E \rightarrow \neg \Box F_G) \).

There are some surprising metaphysical consequences in S4. The formula \( \forall x (\Diamond \Box F_x \rightarrow \Box F_x) \) or, equivalently, \( \forall x (\neg \Box F_x \rightarrow \Box \neg \Box F_x) \) is not theorem in S4. So contingent properties of the minimal God in S4 are not necessarily contingent. The minimal God can survive a contingent property—say, the property of having uttered the name ‘Abraham’ at time \( t \)—becoming an essential property. (28) is possible, so the minimal God can also survive the acquisition of an essential property—say, the property of sempiternality—that it does not currently exemplify.

\[
(28) \Diamond \Box S_G \& \neg \Box S_G
\]

Indeed, S4 allows that the minimal God can become the maximal God, since it is possible that the minimal God acquires maximal greatness. The minimal God might acquire essential omnipresence, aseity, sovereignty, necessary existence, and the remaining attributes of maximal greatness.
If Mx conjoins the properties of maximal greatness—the properties of the maximal God—then it is true in some worlds in S4 that \( (\Box F_G \& \neg \Box M_G) \& \Diamond \Box M_G \). The minimal God is not maximally great, but could have been maximally great.

But God cannot survive the loss of \( \Box M_x \). Since it is a theorem in S4 that \( \forall x (\Diamond \neg \Box F_x \rightarrow \neg \Box F_x) \) or, equivalently, \( \forall x (\Box F_x \rightarrow \Box \Box F_x) \), God cannot survive the loss of any essential properties. So, for instance, God might lack essential maximal greatness in \( w_0 \) and survive the acquisition of essential maximal greatness in \( w_1 \). And, though there are worlds in which God lacks essential maximal greatness, there are simply no worlds in which God survives the loss of essential maximal greatness.\(^{17}\)

According to S4, it is perfectly possible that a contingent property of God should become an essential property of God. Consider the contingent property that God creates some domain of objects. In S4 it is possible that God necessarily creates that domain of objects.\(^{18}\) Supposing, for instance, that we are all contingent beings, S4 ensures that we might all have been necessarily existing beings. And it might have been an essential property of God that the only objects he creates are necessarily existing objects. In S4 it is a matter of contingent fact whether the actual world necessarily obtains and whether everything in it necessarily exists.

\(^{17}\) In every possible world accessible from \( w_1 \) God exemplifies \( \Box O_x \). It is only in the relatively impossible world \( w_0 \) that God fails to exemplify \( \Box O_x \). So God cannot lose an essential property. Here is an S4 model, but it is true in every S4 model that objects cannot lose essential properties.

\[
\begin{array}{ccc}
\neg & \neg & \neg \\
\top & & \\
\Box F_G & & \neg O_G & \Box O_G
\end{array}
\]

\(^{18}\) Consider the S4 model below. Let \( w_1 \) and \( w_0 \) have the same domain and let \( d \) name the objects in the domain of \( w_0 \) and let \( E!x \) be an existence predicate. It is true at \( w_0 \) that the objects in \( d \) contingently exist. It is also true at \( w_0 \) that the objects in \( d \) might necessarily exist.

\[
\begin{array}{ccc}
\neg & \neg & \neg \\
\top & & \\
\neg E!d \\
\top & & \\
\Box \neg E!d & \Diamond \neg E!d
\end{array}
\]

So, if \( w_0 \) is the actual world, then it is true that all (or most) of us contingently exist. But it is also true that all of us might necessarily exist. It is also true that everything that occurs is a contingent event, but that every event—birth, death, coming to be, passing away, etc.—might have been metaphysically necessary.
But it could also have been true that God is essentially incarnate. Since (28) allows that God might acquire a human nature, it might be true that both $\Box F_G \& \Box H_G$. The minimal God might also be human. It is true that God might acquire an essential property, but it is impossible that God might discard an essential property. God cannot discard any nature, human or otherwise, whether or not it is an acquired nature.

But it is also possible in S4 that a property God does not exemplify at all—contingently or essentially—might become an essential property. (29) is also possible.

(29) $\Diamond \Box G_G \& \sim G_G$

In worlds that include no alligators, for instance, (29) ensures that God might acquire the essential properties of an alligator. It could even happen that God acquires the individual essence or haecceity of a particular alligator since there are worlds in which it is true that $\Diamond \Box (A_G \& \forall x(Ax \leftrightarrow x = G))$. In such worlds it is true that, possibly, God is uniquely and essentially a particular alligator.\(^1\)

(30) $\sim A_G \& \Diamond \Box (A_G \& \Box \forall x(Ax \leftrightarrow x = G))$

And there is no guarantee in S4 that no other object B uniquely exemplifies God’s individual essence in some worlds. For instance, it is true in some possible worlds that $\Diamond \Box (F_G \& \Box \forall x(Fx \leftrightarrow x = G))$ and $\Diamond \Box (F_B \& \Box \forall x(Fx \leftrightarrow x = B))$. So, in some worlds G is uniquely the minimal God in every world and in other worlds B is uniquely the minimal God in every world.\(^2\)

\(^1\) Here is an S4 model in which it is true that God does not exemplify the property of being an alligator and acquires the individual essence of an alligator, $\Box A_G \& \Box \forall x(Ax \leftrightarrow x = G)$ where Ax is the property of being an alligator.

\[^{2}\] There are lots of S4 models in which this is true. Here is one example.
Since S4 does not include the theorem $\sim \Box w \rightarrow \Box \sim \Box w$, where $w$ is a maximal set of sentences, S4 ensures that God might contingently create a possible world that itself necessarily obtains. Contrary to standard views on the principle of sufficiently reason, S4 shows that a contingently created world might satisfy even the strongest version of the principle of sufficient reason. Possible worlds might satisfy the principle of sufficient reason as a matter of contingent fact.\(^{21}\)

The epistemological consequences of S4 are implausible, and so are the metaphysical consequences. From the truism that there is evidence for and against the existence of God we can derive the conclusion that our total evidence makes God’s non-existence certain. That consequence is incredible. But it is equally incredible that the only possible values for $P(\sim \Box F_G \mid E)$ and $P(\Box F_G \mid E)$ are 0 and 1. It is possible in S4 that diverse objects might exemplify the same individual essence in different worlds. But it is also possible that $\Diamond \Box F_G \& \sim F_G$, so God might exemplify the essential property $\Box F_x$ in some worlds and not exemplify the contingent property $F_x$ in other worlds\(^{22}\). The minimal God necessarily exists in some worlds in S4 and does not exist at all in others.\(^{23}\)

---

21 It is the standard view that a contingently created world cannot satisfy the strong principle of sufficient reason (PSR). See, for instance, Peter van Inwagen, *An Essay on Free Will*, (Oxford: Clarendon Press, 1983) pp. 202–204; See also his *Metaphysics*, 3rd edition (Boulder: Westview, 2009), p. 150 ff. Jonathan Bennett, *A Study of Spinoza’s Ethics* (Indianapolis: Hackett, 1984), p. 115. William Rowe, *The Cosmological Argument* (New York: Fordham University Press, 1998) p. 94 ff. But here’s a small S4 model in which $w_1$ and $w_2$ both necessarily obtain at themselves, since each of these worlds accesses only itself. But since God in $w_0$ might have actualized either of these worlds, it is a contingent matter whether either of the worlds necessarily obtains. Any world that necessarily obtains satisfies strong PSR, so it is a contingent matter whether $w_1$ or $w_2$ satisfies strong PSR.

22 It is true in some worlds that $\Diamond \Box F_G \& \sim F_G$, but true in no worlds that $\Box F_G \& \Diamond \sim F_G$ or $\Box F_G \& \Diamond \sim F_G$.

23 If we introduce an existence predicate $E!x$ into S4, then we can show that, in some worlds, it is true that $\Diamond \Box E!_G \& \sim E!_G$. In some worlds God does not exist, but possibly, necessarily exists. In general, worlds in which it is true that $\Diamond \Box E!_G \& \sim E!_G$ are impossible relative to worlds in which it is true that $\Box E!_G$, but not vice versa.
4. On the Consequences of $K_{\sigma \rho}$

We can avoid the unwelcome epistemological consequences of S4 and S5 in $K_{\sigma \rho}$. The proposition $P(\square F_G \mid S) > 0 \& P(\neg \square F_G \mid S') > 0$ is possible in $K_{\sigma \rho}$ and so is $P(\square F_G \mid E) > 0 \& P(\neg \square F_G \mid E') > 0$. No contradictions follow from the fact that there is some evidence in favor of the minimal God and some evidence against the minimal God. That’s a good consequence of $K_{\sigma \rho}$. It is possible that $P(\neg \square F_G \mid S) > 0 \& P(\neg \square F_G \mid E) < 1$ and, as we should expect, it is also possible that $P(\neg \square F_G \mid S) > 0 \& P(\square F_G \mid S') \approx 1$. The fact that there is some evidence S for $\neg \square F_G$ is consistent with the probability of $\square F_G$ on $S'$ approximating certainty. The agnostic proposition $P(\square F_G \mid S) = .5 \& P(\neg \square F_G \mid S') = .5$ is also possible. All of these are genuine possibilities in $K_{\sigma \rho}$. Indeed $K_{\sigma \rho}$ has none of the epistemological problems identified in (1) – (30).

But the metaphysical consequences of $K_{\sigma \rho}$ are highly unconventional. Since the characteristic S5 and S4 theorems are invalid in $K_{\sigma \rho}$, contingent properties are not necessarily contingent and essential properties are not necessarily essential. Individuals can gain, lose and exchange essential properties in $K_{\sigma \rho}$ and individuals can gain, lose and exchange haecceities in $K_{\sigma \rho}$.

There are in $K_{\sigma \rho}$ indiscernible worlds $w$ and $w'$ in which one individual exemplifies the properties of the minimal God in $w$ and another individual exemplifies the properties of the minimal God—including the essential properties and haecceities of the minimal God—in $w'$. It is possible in $K_{\sigma \rho}$ that the minimal God exchanges his full profile of properties with another individual. It is even possible that $w$ and $w'$ are indiscernible and every individual in $w$ is replaced with a distinct individual in $w'$. $K_{\sigma \rho}$ does not rule out haecceitism.

It is true in $K_{\sigma \rho}$ that God can survive the acquisition of an essential property and can also survive the loss or exchange of an essential property. Neither conjunct in (31) is true in S5 and the left conjunct is false in S4. But both conjuncts are true in $K_{\sigma \rho}$.

(31) $\Diamond (\Box F_G \& \Diamond \neg \Box F_G) \& \Diamond (\Diamond \Box F_G \& \neg \Box F_G)$

It is even possible in $K_{\sigma \rho}$ for God to exemplify $\square Fx$ and to survive the loss of the property Fx. (32) is possible.

(32) $\Box F_G \& \Diamond \neg \Box F_G \& \Diamond \Diamond F_G$

---

24 Recall that $K_{\sigma \rho}$ is normal, symmetric, and reflexive. It is equivalent to the Brouwer system B.
But (32) can seem impossible. According to (32) God is essentially omnipotent, omniscient, and morally perfect in some worlds and not even contingently omnipotent, omniscient, and morally perfect in others. This is possible because what is essential to objects—including God—in \( K_{\sigma} \) is a purely contingent matter. Unlike S4 and S5, what is essential to God is not necessarily essential.

Could Socrates lose his essential humanity and acquire the essential properties of an alligator? This is certainly possible in \( K_{\sigma} \), since nothing rules out (33) according to which an essentially human Socrates might become contingently human and a contingent human might become essentially an alligator and not even contingently human. Socrates might persist through the exchange of essential properties.

\[
\text{(33)} \quad \Box H_S \land \Diamond \Diamond (\Box A_S \land \neg H_S)
\]

Generalizing on (33), it is true in \( K_{\sigma} \) that Socrates or God or anything else can persist through a complete change in kind. Nothing in \( K_{\sigma} \) precludes the possibility that Socrates becomes essentially an alligator or, for that matter, that Socrates becomes essentially a cat or insect or tree. And the same of course is true for God. Since all essential properties are

25 Here is a \( K_{\sigma} \) model in which it is true at \( w_0 \) that (35) \((\Box F_G \land \Diamond \neg \Box F_G) \land \Diamond \Diamond \neg F_G\).

\[
\begin{align*}
F_G \land \neg \Box F_G & \iff w_2 \iff \neg F_G \\
(\Box F_G \land \Diamond \neg \Box F_G) \land \Diamond \Diamond \Box F_G & \iff \neg F_G \\
\Box F_G & \iff \neg F_G \\
\Box F_G \land \Diamond \Diamond (\Box F_G \land \Diamond \Diamond \Box F_G) & \iff \neg F_G
\end{align*}
\]


Could Socrates have been an alligator? … That depends. We might think of an alligator as a composite typically consisting in a large, powerful body animated by an unimpressive mind with a nasty disposition. If we do, shall we say that any mind-alligator-body composite is an alligator, or must the mind be of a special, relatively dull sort? If the first alternative is correct, then I think Socrates could have been an alligator; for I think he could have had an alligator body.

Actually, in \( K_{\sigma} \) it does not depend at all on what Socrates is essentially or what alligators are essentially. Socrates could lose his essential humanity and acquire the essence of an alligator and survive the change.

27 (36) \( \Box H_S \land \Diamond \Diamond (\Box A_S \land \neg H_S) \) is true at \( w_0 \) in the following \( K_{\sigma} \) model.

\[
\begin{align*}
\Box H_S \land \Diamond \Diamond (\Box A_S \land \neg H_S) & \iff \neg H_S & \iff A_S \land \neg H_S
\end{align*}
\]
contingently essential, there are almost no changes through which an object cannot persist.

According to sortal essentialism, Socrates could not have been an alligator and Aristotle could not have been a centipede. Since Aristotle is essentially a human being and since the property of being human is incompatible with being a centipede, sortal essentialists conclude that Aristotle could not have been a centipede. But both of these claims are false in $K_{σϱ}$. Aristotle can acquire a centipede nature, since it is not a theorem in $K_{σϱ}$ that $∀x(◊□Fx → □Fx)$, and so exemplify both a human nature and a centipede nature. And Aristotle can lose his human nature, since it is not a theorem in $K_{σϱ}$ that $□F_G → □□F_G$, and so exemplify exclusively a centipede nature.

In $K_{σϱ}$ God might also persist through the loss, gain, or exchange of an individual essence or haecceity. There are worlds in which (34) is true, for instance. There are worlds in which God is uniquely and essentially omnipotent, omniscient, and morally perfect. And there are other worlds in which God uniquely exemplifies the essential properties of a tiger.

(34) $□(F_G & □∀x(Fx ↔ x = G)) & ◊◊□(T_G & □∀x(Tx ↔ x = G))$

According to (34) there are worlds in which anything that is identical to God exemplifies the essential properties of a tiger. Nonetheless it might be true in the actual world that anything that is identical to God exemplifies essential omnipotence, omniscience, and moral perfection. So the individual essence that God exemplifies depends on the possible world we are discussing. And of course the same goes for every other object according to $K_{σϱ}$.


29 The sortal argument that Socrates could not acquire the essential properties of an alligator is the same as the sortal argument that Christ could not acquire a human nature and the same as the sortal argument that mental states could not acquire the essential properties of brain states. None of these is impossible in $K_{σϱ}$. There is of course debate concerning auxiliary, non-logical principles such as $□∀x(□Hx → ¬□Ax)$ which rule out the possibility that anything is essentially human and essentially alligator. Compare J.C. Beall, The Contradictory Christ (Oxford: Oxford University Press, 2021).

30 $□(F_G & □∀x(Fx ↔ x = G)) & ◊◊□(T_G & □∀x(Tx ↔ x = G))$ is true in $w_0$ in the following $K_{σϱ}$ model. God is uniquely and essentially omnipotent, omniscient, and morally perfect in $w_0$ but not in $w_2$. In $w_2$ God uniquely exemplifies the essential properties of a tiger.
5. Further Consequences of $K_{\sigma\rho}$ for Philosophical Theology

$K_{\sigma\rho}$ avoids the unwelcome epistemological consequences of S5 and S4. It is not a consequence of $K_{\sigma\rho}$, for instance, that $\neg\Diamond(P(\square F_G \mid S) > 0 \& P(\neg\square F_G \mid S) > 0)$ or $\neg\Diamond(P(\square F_G \mid E) > 0 \& P(\neg\square F_G \mid E) > 0)$. But the metaphysical consequences of $K_{\sigma\rho}$ are extraordinary. $K_{\sigma\rho}$ allows that the minimal God could exemplify all and only the essential properties of an alligator and exemplify none of the traditional essential properties of God. According to $K_{\sigma\rho}$, everything can persist through radical change, including the loss, gain, or exchange of all essential properties. Everything can persist even through the loss, gain, or exchange of haecceities. According to $K_{\sigma\rho}$, it is possible that a contingent property of an object in one world is an essential property of that object in another, and even that a contingent property of an object in one world is an haecceity of that object in another. The metaphysical consequences of $K_{\sigma\rho}$ are extremely unconventional. It is a major advantage of $K_{\sigma\rho}$ that it avoids all of the unwanted consequences in (1) – (30), but it is a highly unconventional consequence of $K_{\sigma\rho}$ that God, and everything else, can survive the loss, acquisition, or exchange of virtually any property.

It is metaphysically important consequence of $K_{\sigma\rho}$ that contingent properties are not necessarily contingent and essential properties are not necessarily essential. $K_{\sigma\rho}$ invalidates both $\forall x(\Diamond \square F_x \rightarrow \square F_x)$ and $\forall x(\square F_x \rightarrow \square \square F_x)$. So, contingent objects—that is, contingently existing objects—are contingently contingent and necessarily existing objects are contingently necessary. Human beings exist contingently, for instance, but might have been both created beings and necessarily existing beings. Contingently existing universes might have been both created universes and necessarily existing universes. And so possible worlds that obtain contingently—our world, for instance—might have obtained necessarily. Our world might have been the only possible world.

The unwelcome consequences of the principle of sufficient reason—the feared loss of contingency—is fully avoidable in $K_{\sigma\rho}$. It is perfectly possible that God creates world $w$, everything in $w$ exists or obtains as a matter of metaphysical necessity and nonetheless everything in $w$ might have been different. In $K_{\sigma\rho}$ it can be true that a created world obtains as a matter contingent necessity, and so true in that world everything exists or obtains as a matter of metaphysical necessity though everything in that world might have failed to exist altogether.

It is also possible in $K_{\sigma\rho}$ that God creates the world $w$ as a matter of chance: God might flip a coin or roll a die. It would then be true in $w$ that everything in $w$ exists or obtains as a matter of metaphysical necessity

---

31 The only relatively possible world. There would still be other absolutely possible worlds—worlds that are possible, but not possible relative to our world.
though everything in w had some chance of not existing or obtaining at all. So, it might be true in w that Smith rides his bike if and only if Smith necessarily rides his bike, while there was a .5 chance that Smith did not ride his bike at all. Nevertheless, since everything in w happens as a matter of metaphysical necessity, everything satisfies the principle of sufficient reason.32

It is a fascinating consequence of $K_{\sigma \rho}$ that we can define both local and global consequence relations over $\rho \sigma$-interpretations and correspondingly we can have both locally valid formulas $\vdash_{K_{\rho \sigma}^*} A$ and globally valid formulas $\vdash_{K_{\rho \sigma}} B$.33

32 It might be true that God contingently actualizes world w and w necessarily obtains. If so, then w and everything in w satisfies the principle of sufficient reason and is fully explained. Of course if w is contingently necessary, then w might not have obtained at all. Here’s an example where the arrows indicate what is possible relative to each world.

<table>
<thead>
<tr>
<th>God chooses a world</th>
<th>$\rightarrow w_1$</th>
<th>God actualizes $w_1$ &amp; $w_1$ necessarily obtains</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rightarrow w_2$</td>
<td>$w_2$ God actualizes $w_2$ &amp; $w_2$ necessarily obtains</td>
</tr>
</tbody>
</table>

But it might also be true that w obtains as a matter of change and w necessarily obtains. Here’s a model in $K_{\rho \rho}$.

<table>
<thead>
<tr>
<th>God flips a coin</th>
<th>$\rightarrow w_1$</th>
<th>Coin falls heads &amp; God actualizes $w_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rightarrow w_2$</td>
<td>Coin falls tails &amp; God actualizes $w_2$</td>
</tr>
</tbody>
</table>

It might be a chancy matter whether $w_1$ or $w_2$ is actualized, but once one of them is actualized, it is necessarily actual. That is, from the point of view of that actualized world, it is impossible that any other world could be actual. From the point of view of that world, there are no other possible worlds. So a world might be necessary even though it is not necessarily created. Suppose there is a .5 chance that the coin falls heads, it does fall heads, and $w_1$ is actualized. In that case everything in $w_1$ occurs as a matter of metaphysical necessity and everything in $w_1$ had some chance of not occurring. Everything that occurs in $w_1$ had, in fact, a .5 chance of not occurring, though everything in w1 happens as a matter of metaphysical necessity.

33 The logic $K_{\rho \rho}$ has the consequence relation $\vdash_{K_{\rho \rho}}$ defined over all $\rho \sigma$-interpretations. An $\rho \sigma$–interpretation for $K_{\rho \rho}$ is a triple $<W, R, \nu>$ where R is reflexive and symmetric. Consider the $\rho \sigma^*$–interpretation for $K_{\rho \sigma} <W, R, R^*, \nu>$ which is just like $<W, R, \nu>$ except that $R^*$ is an equivalence relation defined over a non-empty subset $W^*$ of W. The resulting logic $K_{\rho \sigma}$ has two consequence relations $\vdash_{K_{\rho \sigma}}$ and $\vdash_{K_{\rho \sigma}^*}$ defined over all $\rho \sigma^*$–interpretations. On the unrestricted consequence relation, $\Sigma \vdash_{K_{\rho \sigma}} A$ iff., for all $\rho \sigma^*$–interpretations $(W, R, R^*, \nu)$ and all $w \in W$, if $\nu_w(B) = 1$ for all $B \in \Sigma$, then $\nu_w(A) = 1$. But on the restricted consequence relation $\Sigma \vdash_{K_{\rho \sigma}^*} A$ iff., for all $\rho \sigma^*$–interpretations $(W, R, R^*, \nu)$, and all $w \in W^*$, if $\nu_w(B) = 1$ for all $B \in \Sigma$, then $\nu_w(A) = 1$. The restricted consequence relation is a local consequence relation for regions of worlds $W^*$ governed by S5. $\Sigma \vdash_{K_{\rho \sigma}} A$ only if $\Sigma \vdash_{K_{\rho \sigma}^*} A$, for all $A$, if $A$ is an unrestricted consequence of $\Sigma$ then $A$ is a restricted consequence of $\Sigma$, but the converse is false. In weaker logics like $K_\rho$, the consequence relation $\vdash_{K_\rho}$ defined over all $\rho$–interpretations. An $\rho$–interpretation for $K_\rho$ is a triple $<W, R, \nu>$ where R is simply
The globally valid formulas—formulas true in every interpretation in absolutely every world—will include all of the $K_{\sigma}\varphi$ theorems. But in addition there might be entire regions of possible worlds—regions as large as the entire pluriverse—in which, for instance, the S5 theorems are contingently necessary and locally valid. If $K_{\sigma}\varphi$ is the correct logic of philosophical theology then arguments that are globally invalid might nonetheless be locally valid. Since there could be multiple consequence relations, the validity of ontological arguments, cosmological arguments and arguments from contingency, etc. will depend on the region of metaphysical space you happen to occupy. And the profile of contingent essential properties that God locally exemplifies will of course vary from one region to the next.

Acknowledgements

My thanks to two referees for Logique et Analyse for detailed and insightful comments on an earlier draft of this paper.

References


reflexive. If the logic of philosophical theology is $K_{\sigma}$ then of course there will be $p^*$–interpretations $<W, R, R_{1}, ..., R_{n}, v>$ where $R_{1}, R_{2}$ and so on are each defined over non-empty subsets of $W$, and each defining a local consequence relation for specific regions of metaphysical space. In either case there will be locally valid formulas in some regions that are not valid in others. Every region will have the same globally valid formulas. And sound inferences in some regions of metaphysical space will of course be unsound in other regions.


Michael ALMEIDA
Department of Philosophy and Classics
University of Texas at San Antonio, USA
Email address: michael.almeida@utsa.edu
https://philpeople.org/profiles/mike-almeida