ON THE CONTINGENT NECESSITY OF THE WORLD

1. Introduction

The most widely accepted metaphysics of creation is theistic actualism. Theistic actualism is committed to the view that everything that exists, in the most unrestricted sense, is included in the actual world. According to the metaphysics of theistic actualism, there is a unique, absolutely actual world. And God's creative activity explains why the actual world obtains and why actual beings exist. Since we are the undeserving recipients of our own existence, and our existence is a great good, gratitude to God is both appropriate and morally required. I discuss the account of creation in theistic actualism in section (2).

In section (3) I consider the most serious problem for the actualist account of divine creation. According to Peter van Inwagen's modal collapse argument, ultimate explanation entails that gratitude for one's existence is totally inappropriate. Ultimately, the actual world, and everything in it, is self-explanatory and not a consequence of divine creation.

In section (4) I argue that van Inwagen's argument is unsound. It is consistent with an ultimate explanation of the world that the actual world is *contingently necessary*.\(^1\) If God actualizes the world as a matter of necessity—specifically, contingent necessity—then it is true both that the world has a complete explanation and that gratitude to God for one's existence is perfectly appropriate. It is true that we exist as a matter of necessity,
but that necessary existence is just a contingent fact. There are possible worlds in which we fail to exist altogether.

In section (5) I offer a broader defense of weakening the background logic in philosophical theology to $K_{eq}(B)$ from S5. I offer some concluding remarks in section (6).

2. Creation in Theistic Actualism

According to theistic actualism the totality of creation—everything that exists, obtains, or occurs—is contained in the actual world. There are other possible worlds, other ways things might have been, but even those worlds are contained in the actual world. If there exist impossible worlds—worlds where everything is possible and nothing is necessary—then those too exist in the actual world. There are in any case impossible world-like items existing in the actual world. So all possible and impossible worlds or world-like items, insofar as those worlds exist, are actual objects.

But possible, non-actual, objects—non-actual possibilia—do not exist in the actual world and so do not exist simpliciter. It is true that Wittgenstein might have had a daughter, as the familiar example goes, but there is no actual object that is Wittgenstein's possible daughter. So Wittgenstein's possible daughter does not exist. It is an odd feature of theistic actualism that possible objects exist in possible worlds and possible worlds exist in the actual world, but possible objects do not exist in the actual world. 'Exist in' is oddly non-transitive on theistic actualism.²
The object of divine creation is in fact the actual universe or the actual multiverse. It is of course possible on theistic actualism that a world contains no spatiotemporal regions at all and so contains no universe(s) at all. There are worlds with no universes, and worlds with empty universes. A universe is the largest spatiotemporally connected region in a possible world and includes every spatiotemporally located concrete object in a world. Tigers, humans, water, aluminum, clouds, impure sets, talking donkeys, waves, thoughts and pains are concrete objects existing in a universe. But there are also objects with force but no mass, photons are candidates, and there are objects located but unextended, point-sized objects, for instance, and objects that are unextended in both space and time such as temporal parts. These are all candidates for concrete objects existing in a universe. Not all of them exist in our universe. Pure sets, numbers, geometrical objects, meanings, propositions, properties, and God are all examples of objects not existing in any universe.

According to the metaphysics of creation in theistic actualism, God creates every contingently existing concrete object in the universe. The creation of contingent objects is specifically the instantiation of proxy objects or sets of properties comprising the individual essences of objects. In order to create Smith, for instance, and not merely an indiscernible counterpart of Smith, God must instantiate a set of properties that belong to Smith uniquely. These are Smith's haecceities, the properties that Smith alone exemplifies in every world in which she exists and no other object exemplifies in any possible world. But what then about objects like Mt McKinley? Presumably, if Mt.
McKinley is a genuine object, then God creates Mt. McKinley just if God instantiates the individual essence of Mt. McKinley, and so on. In general, creation is instantiation on this view.

The creation of contingent objects in the universe entails the actualization of contingent state of affairs in the actual world. Creation is not identical to actualization, but entails actualization. In creating Smith atop Mt. McKinley, for instance, God actualizes the state of affairs of Smith's being on top of Mt. McKinley. Apart from the creation of contingent, spatiotemporal objects, everything that God thinks and does also brings about states of affairs in the actual world. God brings about the state of affairs of God's being impressed by a certain set of reasons for creating by being impressed by those reasons. God brings about God's believing that creation is good by believing that it is good and God brings about God's being moved to create the Gobi desert by being moved to create the Gobi.

Possible worlds on this account are not collections of spatiotemporal connected objects. Possible worlds are maximally consistent abstract states of affairs. The largest or most inclusive object requiring explanation is the actual world—the actual world in its totality. The actual world in its totality is the one and only maximal state of affairs that in fact obtains. Every state of affairs S or its negation ~S is included in the actual world, and it could not include even one more state of affairs. In divine creation, God brings about or actualizes the largest consistent collection of states of affairs. God could not actualize
one more state of affairs without actualizing an impossible world. There are further no states of affairs that obtain, but that are not included in the actual world.

Divine creation, according to theistic actualism, involves God choosing one possible world to actualize from infinitely many possible worlds. Since the infinitely many worlds are all possible, God might have actualized any one of them. And since worlds are maximally inclusive, God's actualization of a possible world is the actualization of absolutely everything that contingently obtains. Since God's creative activity contingently actualizes the world, according to theistic actualism, God might have actualized some other possible world.

In section (3) I consider van Inwagen's well-known argument against actualist accounts of divine creation. According to van Inwagen, divine creation provides the ultimate explanation of the world only if there are necessarily no contingent states of affairs. If van Inwagen is right, then the fact of divine creation does not make it appropriate to offer gratitude for our existence. We exist as a matter of metaphysical necessity. But van Inwagen's argument in fact shows that there could be no divine creation at all, if the logic of metaphysical necessity is as strong as S5. The world is self-explanatory.

3. Van Inwagen on Ultimate Explanation

According to Peter van Inwagen, Jonathan Bennett, William Rowe, and several other philosophers, there could be no absolute explanation for the actual world. Absolute explanations are in fact the best sorts of explanations—explanations on which, according
to Richard Swinburne, 'everything really is explained'. The problem with absolute explanations, according to van Inwagen, is that they are incompatible with the contingency of the actual world. If there is an absolute explanation for the actual world, then there are no contingent objects, events, or states of affairs. Here is Swinburne's account of absolute explanation.

... let us delineate a special kind of ultimate explanation, which I shall call absolute explanation. An absolute explanation of E is an ultimate explanation of E in which the existence and operation of each of the factors cited are either self-explanatory or logically necessary. Other explanations cite brute facts that form the starting points of explanations; there are no brute facts in absolute explanations—here everything really is explained.  

But Swinburne also denies that there could be an absolute explanation for the actual world.

I do not believe that there can be any absolute explanations of logically contingent phenomena . . . You cannot deduce anything logically contingent from anything logically necessary . . . These are among the many reasons why it must be held that God is a logically contingent being, although maybe one necessary in other ways.
Absolute explanations are in fact the only explanations that satisfy the principle of sufficient reason. The unfortunate consequence of the absolute explanation of the actual world, according to van Inwagen, is complete modal collapse.

For every state of affairs that obtains, there is a sufficient reason for its obtaining. . . PSR must be rejected, for it has an absurd consequence: the collapse of all modal distinctions. . . In order to see this we must take a brief look at the concept of a sufficient reason. . . First, if x is a sufficient reason for y, then x must entail y. That is, it must be impossible for x to obtain without y's obtaining. For if it were possible for x to obtain and y to fail to obtain, how could the obtaining of x be a sufficient reason for the obtaining of y? Second, no contingent state of affairs may be its own sufficient reason. . . We may now show that PSR leads to the collapse of all modal distinctions. Let P be the conjunction of all contingently true propositions into a single proposition. . . It is evident that P itself is a contingent proposition, for a necessary proposition may not have a single contingent conjunct. Now, according to PSR, there exists a state of affairs S that is a sufficient reason for P. S must be contingent or necessary. But it cannot be either. It cannot be necessary, for, if it were necessary then P (which, by our first principle, is entailed by S) would be necessary. It cannot be contingent, for if it were contingent it would be a conjunct of P; and if it were a conjunct of P . . . it would be P . . . Since S cannot be either
necessary or contingent, it cannot exist and PSR is false. . . Hence if PSR is true, there are no truths but necessary truths: there is no distinction to be made between truth and necessity.7

Let the conjunction of all contingent truths be \( w@ \). According to van Inwagen's argument against contingent creation, (i) a contingent state of affairs \( S \) cannot explain the actual world \( w@ \) since every contingent state of affairs is included in \( w@ \) and (ii) no contingent state of affairs is self-explanatory.

Van Inwagen does not note that the argument against God's contingent creation of the actual world—and against the contingent explanation of a world generally—is valid only if we assume a logic at least as strong as S5. According to S5, every necessary state of affairs is necessarily necessary and every contingent state of affairs is necessarily contingent. Since S5 precludes the possibility of contingently necessary states of affairs, it is impossible—logically impossible, we are to believe—that a contingently necessary state of affairs \( S \) explains why \( w@ \) obtains or is actual. And since it is impossible that a contingently necessary state of affairs \( S \) explains \( w@ \), it is impossible that there are any contingently self-explanatory states of affairs. Assuming S5, then, it is true that no contingent state of affairs could provide an absolute explanation for \( w@ \).

It should be noted that van Inwagen's argument against God's necessary creation of the actual world—and against a necessary explanation of a world generally—is invalid even if we assume a logic as strong as S5. According to van Inwagen's argument against necessary creation, a necessary state of affairs \( S \) cannot explain \( w@ \) since (iii) \( w@ \) is a
maximally consistent, contingent state of affairs and (iv) every necessary explanation S for a contingent state of affairs \( w_@ \) entails that \( w_@ \) is necessary.

Van Inwagen fails to observe that there cannot be a necessary explanation S for the state of affairs \( w_@ \) quite independently of the contingency \( w_@ \) or the necessity of \( w_@ \). It is in general false that S provides any explanation of \( w_@ \) even on the assumption that \( \Box S \) and \( \Box (S \rightarrow w_@) \). It is in general true that \( w_@ \) is self-explanatory on the assumption that \( \Box S \) and \( \Box (S \rightarrow w_@) \). In S5 a state of affairs \( w_@ \) is self-explanatory only if \( w_@ \) is necessarily self-explanatory. So it is impossible that God necessarily creates or necessarily explains \( w_@ \). \( w_@ \) can only explain itself.

So under the assumption of S5 it is impossible that God's creative activity is the absolute explanation for the actual world. Ultimately \( w_@ \) exists necessarily and so is self-explanatory. And since the fact that \( w_@ \) obtains is not explained by God's creative activity, gratitude to God for one's existence is simply inappropriate.

4. On Contingently Necessary Creation.

According to van Inwagen's modal argument against contingent creation, no contingent state of affairs S can explain the actual world \( w_@ \), since every contingent state of affairs is included in \( w_@ \) and no contingent state of affairs is self-explanatory. But van Inwagen's argument depends entirely on the assumption that the logic of modality is at least as strong as S5. And that assumption is almost certainly false.

In the weakened logic \( K_{\alpha} \), (or B) for instance, it is false that no contingent state of affairs are self-explanatory. There are in fact \textit{contingently necessary} states of affairs, and
contingently necessary states of affairs are themselves contingently self-explanatory. Contingently self-explanatory states of affairs are self-explanatory in the way that all necessary states of affairs are self-explanatory. But since they are contingently necessary, there are some possible worlds in which they are self-explanatory and some possible worlds in which they're not.

Contingently necessary states of affairs can provide contingent absolute explanations for possible worlds. God might have actualized a possible world as a matter of contingent necessity, for instance. If the explanation for why a possible world obtains is contingently necessary, then the world itself obtains as a matter of necessity, but might not have obtained as a matter of necessity. There are other possible worlds that could have been necessarily actualized and could have necessarily obtained.

If we let \( P \) be a maximal conjunction of propositions true at the actual world \( w_@ \), then we can describe a simple model permitting contingently necessary worlds. In fact there are any number of more or less complex models permitting contingently necessary worlds in \( K_{\phi} \). In the following model, for instance, \( w_@ \) necessarily obtains and so the principle of sufficient reason is satisfied at \( w_@ \). But \( w_@ \) is contingently necessary, so there are worlds at which \( P \) is false and worlds at which \( \Box P \) is false. Arrows indicate the accessibility relations among worlds.

\[
\begin{align*}
  w_@: & \quad P \& \Box P \\
  w_1: & \quad P \& \sim \Box P \leftrightarrow w_2, P' \& \sim \Box P' \leftrightarrow w_3, \Box P'
\end{align*}
\]

The model shows where the van Inwagen argument goes wrong. It is true at \( w_@ \) that \( \Box P \)—the maximal conjunction \( P \) necessarily obtains—but also true that \( \Diamond \sim \Box P \)—the
maximal conjunction $P$ might not have necessarily obtained. Every state of affairs in $w@$ is either \textit{necessarily} necessary or \textit{contingently} necessary. Since it is true that $\square P$, every state of affairs in $w@$ has an absolute explanation—everything is fully explained—but since it is also true in that $\Diamond \sim \square P$, there are possible worlds in which at least some of the conjuncts of $P$ are false. The truth of $\Diamond \sim \square P$ ensures contingency in $w@$ despite the fact that $\square P$ is true there. Since it is contingently necessary that God actualized $w@$ , another possible world might have been actual.

It is true in some models for $K_{\rho \sigma}$ that not every possible world has direct access to every other possible world. In the small model above, for instance, every world has either direct or indirect access to every other world. $w_1$ is possible relative to $w@$ , and $w_2$ is possibly possible, and so on. But there are some models for S5, too, in which each world has access to itself alone, or to a small isolated cluster of worlds. S5 validates all of the theorems of $K_{\rho \sigma}$, and that's good, but S5 also validates a number of equivalences that conflate important distinctions in philosophical theology.

In the simple model it is true that the only world directly accessible to and from $w@$ is $w_1$, but this feature is not essential. There might be infinitely many possible worlds accessible to and from $w@$. There might be as many possible worlds accessible to and from $w@$ as there are in the entire pluriverse. It is perfectly possible to have vast regions of the pluriverse satisfy S4 or S5 theorems, if you like, or propositions that approximate those theorems.\textsuperscript{11} But the theorems of S4 and S5 will be only contingently necessary in those regions.
In this simple model, P is again contingently necessary at \( w_@ \), and God's creative activity provides an absolute explanation for every state of affairs in \( w_@ \). PSR is again satisfied. In \( w_3 \) and \( w_4 \) P is not necessary at all. Note that in \( w_@ \) and \( w_1 \) all of the S5 theorems are true, including \( \Box P \rightarrow \Box \Box P, \Diamond \Box P \rightarrow \Box P, \Diamond P \rightarrow \Box \Diamond P \) and so on. Those theorems are in fact contingently necessary in \( w_@ \) and \( w_1 \) and regionally govern necessity and possibility in \( w_@ \) and \( w_1 \). For anyone who has doubts about rejecting the S5 theorems, \( K_{\rho_\sigma} \) does not preclude S5 from governing necessity and possibility in your part of the pluriverse. But that would be a matter of contingent fact—contingently necessary fact. \( K_{\rho_\sigma} \) ensures that those theorems are false in other regions of metaphysical space.

God might have actualized \( w_2 \) or \( w_3 \) instead of \( w_@ \), as the theistic actualist says, simply by instantiating the objects existing in those worlds with the properties those objects exemplify in those worlds. God might have actualized a world, for instance, in which none of us exist or only some of us do. So, though it is true that we necessarily exist, it is a contingent fact that we necessarily exist. It is true that \( w_@ \) is necessarily actual, but that too is just a contingent fact about \( w_@ \). It is also true that \( w_@ \) is possibly not necessarily actual. \( w_@ \) is in fact a *contingently necessary world*. 
The small model also shows that God might have necessarily actualized \( w_2 \) instead of \( w_@ \). Had God actualized \( w_2 \), then it would have been true that the principle of sufficient reason was fully satisfied. God's creative activity would have provided an absolute explanation for \( w_2 \) and everything in \( w_2 \) would have been fully explained. The model shows that it's false that no contingent state of affairs is self-explanatory, since \( \Box p \) is both contingent and self-explanatory. The model also shows that it's false that there are no contingent explanations for necessary maximal states of affairs. \( w_@ \) is a necessary maximal state of affairs and \( w_@ \) has a contingent explanation.

5. Theism and \( K_{\Box p} \)

Van Inwagen's argument against contingent and necessary explanation implicitly assumes the logic of S5—as do the similar arguments from Bennett, Rowe, Geirsson, Ross, and others. It is the S5 theorems that guarantee that there are no contingently necessary states of affairs, no contingently necessary explanations, and no contingently self-explanatory states of affairs. But it is the possibility of contingently necessary states of affairs that makes it possible to provide an absolute explanation for a possible world without modal collapse.

S5 generates a number of other problems for philosophical theology. Since S5 is such a strong logic, it conflates a number of philosophically important distinctions. For instance, given the logic of S5, we are unable to distinguish between strongly negative states of affairs and weakly negative states of affairs. We are unable to distinguish between states of affairs that provide *some evidence* against the existence of God and states
of affairs that provide conclusive evidence against the existence of God. We are also unable to distinguish between weakly positive and strongly positive states of affairs. That is, we cannot distinguish between states of affairs that provide some evidence in favor of the existence of God and states of affairs that provide conclusive evidence in favor of the existence of God. In S5, every weakly negative state of affairs is logically equivalent to a strongly negative state of affairs. This is just to say that any state of affairs that provides the slightest evidence against the existence of God also provides conclusive evidence against the existence of God. Weakly positive states of affairs suffer from a perfectly analogous problem.

S5 entails the incredible metaphysical position that either we inhabit a miserable pluriverse or we inhabit an impeccable pluriverse. There are no other alternatives. If we inhabit an impeccable pluriverse, then every possible state of affairs provides conclusive evidence in favor of God's existence. In the impeccable pluriverse every weakly negative or neutral state of affairs is impossible. If we do not inhabit an impeccable pluriverse, then we inhabit a miserable pluriverse. In a miserable pluriverse, every possible state of affairs provides conclusive evidence against God's existence. In a miserable pluriverse all weakly positive and neutral states of affairs are impossible.

S5 does not permit us to distinguish between exemplifying essential properties necessarily and exemplifying essential properties contingently. It is impossible for God or anything else to acquire an essential property it does not already exemplify or to lose an essential property it does exemplify. It is impossible, for instance, that God might
acquire a human nature—as so many theists believe both possible and actual. And it is impossible to lose the essence of humanity, if God does exemplify it. It is impossible, to consider another familiar example, that anything that is essentially water should become essentially wine. It cannot happen as a matter of the logic of essential properties. S5 rules out the possibility of such changes on logical grounds.

On the other hand, $K_{\varphi}$ is consistent with traditional theism and avoids the unwelcome consequences listed above. In $K_{\varphi}$ weakly negative states of affairs do not entail that the traditional God does not exist. Weakly positive states of affairs do not entail that the traditional God does exist. So, the weakly negative fact that Jones endures a serious harm is consistent with the existence of God. And the weakly positive facts that Jones leads a fulfilling life or Smith is reasonably virtuous are consistent with the non-existence of God. And there are simple countermodels in $K_{\varphi}$ to the thesis that we inhabit either a miserable pluriverse or an impeccable pluriverse.

$K_{\varphi}$ allows us to model some features of metaphysical space that are inconsistent in S5. It is possible that the traditional God exists and it is also possible that there obtains widespread intrinsically evil states of affairs. Further, it is possible that the traditional God does not exist and also possible that some worlds are naturally and morally perfect. In the simple four-world model below, the traditional God exists in $w_0$, but the traditional God does not exist in world $w_3$. The arrows again indicate accessibility, every world is assumed to have access to itself. Let $\square F_G$ represent God is essentially omniscient, omnipotent, and morally perfect.
It is true at $w_0$ that, in every possible world, God exemplifies omnipotence, omniscience, and moral perfection. Since it is a matter of philosophical dispute whether the S5 principle $\Box F_G \rightarrow \Box \Box F_G$ is true, $K_{\rho\sigma}$ allows us to model various positions on this issue. The question of whether $\Box F_G \rightarrow \Box \Box F_G$ is true of the essential properties of God depends on how the theological dispute is settled. Different positions are representable in $K_{\rho\sigma}$.

The worlds that are possible at $w_0$ are $w_1$ and $w_2$, but of course there are likely infinitely many worlds—as many worlds as there are in all of metaphysical space—possible at $w_0$. These are enough worlds to represent every possibility. But there are also absolutely possible worlds—$w_3$ for instance—in which the traditional God does not exist and there obtains widespread intrinsic evil. Whether there are infinitely many absolutely possible worlds in which there is intrinsic evil is a metaphysical issue and $K_{\rho\sigma}$ can represent a variety of views on the matter. It is possible to model the position according to which the essential properties of God are contingently non-contingent, and virtually any other plausible view. That issue in $K_{\rho\sigma}$ is philosophical and not logical. It's crucial that $K_{\rho\sigma}$ does not rule out on logical grounds positions that have at least some philosophical merit.

In the simple four-world model, the dispute between traditional theists and traditional atheists is whether the actual world is $w_0$ or $w_3$. The dispute does not involve
either the traditional atheist or the traditional theist holding an absolutely impossible position. Each of the positions is absolutely possible. There is, as we should expect, more or less significant evidence in favor and against theism and atheism throughout the pluriverse. The pluriverse is not a collection of exclusively positive states of affairs or a collection of exclusively negative states of affairs.


Given $K_\varphi$, it is possible that God necessarily actualizes the world $w_\varphi$, every state of affairs in $w_\varphi$ necessarily obtains, every object in $w_\varphi$ necessarily exists, and it is perfectly appropriate to show gratitude to God for one's existence. It is consistent in $K_\varphi$ that $w_\varphi$ has an absolute explanation—God's necessary creation of $w_\varphi$ absolutely explains $w_\varphi$—and that God freely created the world. Since it is possible in $K_\varphi$ that the world is contingently necessary, it is true both that $w_\varphi$ obtains as a matter of necessity and that $w_\varphi$ might not have obtained as a matter of necessity. So God's creation of the world is necessary, but contingently so.

The logic of necessity in $K_\varphi$ invalidates van Inwagen's argument that divine creation ultimately explains the world only if there are no contingent states of affairs. Divine creation ultimately explains $w_\varphi$, the states of affairs in $w_\varphi$ necessarily obtain, but the fact that the states of affairs in $w_\varphi$ necessarily obtain is a contingent fact. Things might have been different—things might have been entirely different. We might all have failed to exist, despite the fact that we necessarily exist. So gratitude is perfectly appropriate.
Finally $K_{QA}$ has all of the advantages described in section (5). The logic provides the basis for making important distinctions in philosophical theology that we cannot make in S5. $K_{QA}$ allows us to maintain the principles of various stronger or weaker logics various regions of the pluriverse, for instance, if there are reasons to believe that those principles are contingently necessary. So it is not as though the S5 and S4 principles are entirely incompatible with the truth of $K_{QA}$. 
Notes

1 It is a distinctive feature of S4-S5 that A is necessarily true only if A is necessarily necessary. In S5, the same goes for contingent truths: if A is contingently true, then it is necessarily contingent. Among the untenable consequences of these strong logics is that we cannot represent the agnostic position—the agnostic position is a logical impossibility. Further, there cannot be evidence S for theism and evidence S' against theism without generating a contradiction, but obviously there is evidence both for and against theism. These consequences are unacceptable. In the weaker logic K_{\omega}\sigma it is possible that propositions are necessary, but only contingently so. Such propositions are in fact necessary, but might not have been necessary. More on this below.

2 This is a crucial fact for actualists who insist that objects included in other possible worlds do not exist, so they welcome the fact that 'exist in' is not transitive.


6 I have in mind the stronger, genuine versions of PSR. All weaker versions of PSR permit at least some brute facts. But genuine versions of PSR are not consistent with the existence of any brute facts.


8 If □S and □(S → w@), then □w@. But if □w@, then w@ is self-explanatory. So, □S necessarily explains w@ only if it doesn't. That is, only if w@ explains itself.

9 There are accounts of divine creation that do not aspire to absolute explanations for w@. In the most developed account of contingent creation, God actualizes our world w@ because God just happens to act on the set of reasons possesses he to actualize w@. In other possible worlds, God just happens to act on the best set of reasons he possesses to actualize those worlds. There is no explanation at all why God chooses to act on the reasons to actualize w@ rather than to act on the reasons to actualize some other world w. It cannot be the case, even, that there is a slightly better set of reasons to actualize w@ rather than some other world. It cannot be the case that there is a slightly better set of reasons to actualize one world rather than another. If there were a slightly better reason to actualize w@ than to actualize any other, then God could not actualize any other world. There are equally good reasons to actualize each possible world in the pluriverse. So, it is just chance that God chooses to act on reasons to actualize w@. See Alexander Pruss, 'The Leibnizian Cosmological Argument' in The Blackwell Companion to Natural Theology, JP Moreland and WL Craig (eds) (Oxford, Wiley-Blackwell Publishers, 2007), chapter (2). 24-100.
10. The logic $K_{qa}$ or B is reflexive and symmetrical but not transitive. $K_{qa}$ is weaker than both $K_{q}$ (S4) and $K_{qta}$ (S5).

11. In this simple model, $w_{@}$, $w_{1}$, and $w_{2}$ differ only with respect to their haecceities. The state of affairs of the world = $w_{@}$ obtains in $w_{@}$ but in no other world, for instance.

12. In particular the S5 theorems $\forall x(◊\Box Fx \rightarrow \Box Fx)$ and $\forall x(\Box Fx \rightarrow \Box \Box Fx)$.

13. It is an S5 theorem that $\forall S \sim S \rightarrow (S \rightarrow \Box F) \leftrightarrow (\Box S \rightarrow \sim \Box F)$, where $S$ ranges over possible states of affairs. The theorem states that, for all states of affairs $S$, $S$ is weakly negative just if $S$ is strongly negative.

14. It is true in S5 that, for all states of affairs $S$, if $S$ is weakly negative, then $◊ S \rightarrow P(\Box F_{G}/S) = 0$.

15. It is a theorem of S5 that every state of affairs in every possible world is strongly positive or every state of affairs in every possible world is strongly negative, $\forall S \Box (S \rightarrow \sim \Box F_{G}) \vee \forall S \Box (S \rightarrow \Box F_{G})$.

16. For instance, we cannot distinguish between not exemplifying $Fx$ essentially and not exemplifying $Fx$ contingently, since it is an S5 theorem that $◊ \sim \Box F_{G} \leftrightarrow ◊ \sim F_{G}$. It ought to be possible to fail to exemplify $Fx$ essentially without it being possible to fail to exemplify $Fx$ contingently.