The Early Arabic Liar:
The Liar Paradox in the Islamic World from the Mid-Ninth to the Mid-Thirteenth Centuries CE*

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Abstract

We describe the earliest occurrences of the Liar Paradox in the Arabic tradition. The early Mutakallimūn claim the Liar Sentence is both true and false; they also associate the Liar with problems concerning plural subjects, which is somewhat puzzling. Abharī (1200-1265) ascribes an unsatisfiable truth condition to the Liar Sentence—as he puts it, its being true is the conjunction of its being true and false—and so concludes that the sentence is not true. Ṭūsī (1201-1274) argues that self-referential sentences, like the Liar, are not truth-apt, and defends this claim by appealing to a correspondence theory of truth. Translations of the texts are provided as an appendix.

Keywords

Liar paradox, truth, self-reference, Arabic logic, Ṭūsī, Abharī

Introduction

The *Liar Paradox* is easy to state but difficult to solve: start with your favorite Liar Sentence—"I am lying", "This sentence is false", "What I say is false", "All that I say is false"; add, if necessary, a few contingent background assumptions; it

follows (depending on exactly how you run the argument) that the sentence is both true and false, or that it is neither true nor false.[1]

The Liar appears in the Ancient Greek, the Medieval Latin, and the Medieval Arabic traditions.[2] We know of no clear and unambiguous instance of the Liar from any Eastern tradition.[3]

The Greek and Latin Liars are well-known. The Arabic Liar is not, presumably because it has gone unnoticed: we have not found a single work by any Western scholar that mentions it.[4]

Paul Spade argues that the Latin Liar does not derive directly from any known Greek sources.[5] Later Latin authors attribute the Liar to Aristotle's *Sophistical Refutations* 25. Spade claims this is a red herring: the passage, he argues, is not about the Liar, and early Latin authors who discuss the passage and say Liar-like things are not actually discussing the Liar.[6] Spade speculates that the development of the Latin Liar was influenced by "other forces": now-

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[1] We use 'The Liar' here to refer to a bundle of closely related paradoxes, including both the Simple Liar ('This sentence is false') and various Contingent Liars (e.g., 'All that I say is false', when that sentence is all that I say).

[2] By 'the Medieval Arabic tradition' we refer to the intellectual tradition within the medieval Islamic world. Work in this tradition was carried out primarily, but not exclusively in the Arabic language.

[3] For an ambiguous instance, see J. E. M. Houben, 'Bhartṛhari's Solution to the Liar and Some Other Paradoxes', *Journal of Indian Philosophy* 23 (1995), 392. Bhartṛhari, a seventh century Sanskrit grammarian, discusses the sentence 'Everything I am saying is false'. Given appropriate assumptions, it is possible to generate a Contingent Liar from this sentence, but Bhartṛhari does not do so. Instead, he points out that the sentence is self-refuting. For similar cases, see Sextus Empircus, *Against the Logicians*, II 55 and Abū Naṣr al-Fārābī, *Kitāb al-taḥlīl*, ed. Rafiq al-ʿAjam (Beirut, 1986), 108. Bhartṛhari's discussion is notable for his focus on the ways in which semantic self-reference generates the problem. For more, see Terence Parsons, 'Bhartṛhari on what cannot be said', *Philosophy: East and West* 51 (2001), 525-534.

[4] But see Ahad Qaramaleki, 'Muʿmmā-ye jezr-e aṣamm nazd-e Motakallemān' ['The Paradox of the Speechless Root in the Opinion of the Mutakallimūn'], *Kheradnameh Sadra* 5,6 (1997), 67-73.

[5] See Paul Vincent Spade, 'The Origins of the Mediaeval Insolubilia-Literature', *Franciscan Studies* 33 (1973), 292-309, reprinted in his *Lies, Language and Logic in the Later Middle Ages* (London, 1988); 'Insolubles', in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta (Stanford, 2005), URL = http://plato.stanford.edu/archives/fall2005/entries/insolubles/.

[6] Paul Vincent Spade, 'Insolubilia', in *The Cambridge History of Later Medieval Philosophy,* eds. Norman Kretzmann, Anthony Kenny, and Jan Pinborg (Cambridge, 1982), 247. Paolo Crivelli has recently argued that *Sophistical Refutations* 25 is about the Liar. See his 'Aristotle on the Liar', *Topoi* 23 (2004), 61-70, and his *Aristotle on Truth* (Cambridge, 2004). Though we remain skeptical, nothing we say here depends on how this question is resolved.

obscure Stoic sources, perhaps; or some now-lost commentary on the *Sophistical Refutations*.[7]

Arabic thought was, of course, influenced by Greek, and Latin by Arabic. So it would be no surprise if the history of the Liar fit this pattern of influence. It is somewhat surprising that Spade does not discuss this possibility.

Our present judgment, however, vindicates Spade on this point. Arabic commentaries on *Sophistical Refutations* 25 do not suggest the Liar. Aristotle's suggestive example—a man who "at the same time says what is both false and true"—is omitted from all but the earliest translations.[8] Neither Avicenna nor Averroes mention it in their commentaries.[9] Both interpret the example they do mention—a man who takes an oath to break an oath—in a very un-Liar-like way: for them, it has to do with whether the man is good or bad, rather than with whether he is or is not an oath-breaker.

In fact, although the Greek, Arabic and Latin traditions are historically connected in countless ways, we have found no hard evidence and little circumstantial evidence that suggests *any* historical connections between the Greek, Arabic, and Latin Liars. Each tradition reads as though it invented the paradox anew.[10]

Systematic historians will despair at the absence of well-bound chains of historical influence. Those who turn to history for new perspectives on familiar problems, however, have reason to rejoice. The history of the Liar presents not one but three extended independent stories, each with its own perspective on the problem. Two of those stories have been told. We here begin to tell the third.

This paper has three parts. In the first, we analyze the Liar as it appears in the early Mutakallimūn. In the second, we analyze Abharī's attempt to solve the Liar by asserting that the Liar Sentence has a special truth condition—its being true, he says, consists in its being both true and false. In the third, we analyze Ṭūsī's solution to the Liar: Ṭūsī argues, on the basis of a correspondence theory of truth, that the predicates 'true' and 'false' do not apply to self-referential sentences.

Ṭūsī's solution is of special interest: we know of no earlier text on the Liar, in any tradition, of comparable sophistication; and we know of no earlier text on the Liar that suggests that, at root, the paradox is a paradox of self-reference. (This observation must be tempered by the observation that the Greek

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[7] Spade, 'Origins', 308-309.

[8] *Manṭiq Arisṭūṭālīs*, ed. ʿAbd al-Raḥmān Badawī (Beirut, 1980), 968. Yaḥyā Ibn ʿAdī (893-974) and Ibn Zurʿah (942-1008) both omit the example.

[9] Avicenna, *al-Safsaṭah*, in *al-Shifāʾ, al-Manṭiq, al-Safsaṭah*, ed. F. al-Ahwānī (Cairo, 1958), 98; Averroes, *Talkhīṣ al-Safsaṭah,* ed. Muḥammad Sālim Salīm (Cairo, 1972), 157.

[10] See Chris Martin, 'Obligations and Liars', in *Sophisms in Medieval Logic and Grammar,* ed. Stephen Read (Dordrecht, 1993), 357-81, for such a reading of the Latin tradition.

tradition survives only in fragments.[11] But we have found no Greek fragments suggesting that the Liar is a paradox of self-reference.)

The work of the early Mutakallimūn also bears study. It is the earliest work on the Liar in the Arabic tradition, and is therefore of significant historical interest both for our understanding of the internal history of the Arabic Liar, and because, if there was Greek influence, this is where it was most likely to be.

Abharī and Ṭūsī together represent the first substantive work on the Liar in the Arabic tradition. They occur early enough that they could have had an impact on the Latin tradition if their work made it West. Abharī's solution, in particular, bears some striking resemblances to the well-known solutions advocated by Bradwardine and Buridan.

There are, then, good reasons to begin the study of the Arabic Liar with a study of these three parts. But we must emphasize that this is only a small part of the full story.

1. The Liar in the Kalām Tradition[12]

The earliest surviving text in the tradition comes from the *Uṣūl al-dīn* (*Principles of Religion*) by ʿAbd al-Qāhir al-Baghdādī (c.961-1037/8), a mathematician and Ashʿarī theologian who lived and worked in Nisapur and Baghdad.[13] But Sayf al-Dīn al-Āmidī (1156/7-1233) reports that the Liar was discussed by Muʿtazilah theologians a generation prior to al-Baghdādī.[14] Relying on al-Āmidī's report, we date the emergence of the Arabic Liar to the late ninth or early tenth century CE.

It arose as part of a debate, within the Muʿtazilah school of *kalām,* over the definition of a declarative sentence (*al-khabar*), where it figured as a counter-example to the proposal that declarative sentences (as opposed to questions and commands) are true or false. Although the debate began within the Muʿtazilah school, the text from Baghdādī, an Ashʿarī, establishes that it was

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[11] Benson Mates, *Stoic Logic* (Berkeley, 1960), 84, provides a characteristically blunt assessment of the situation: "We do not know how any of the competent logicians of antiquity attempted to solve the antinomy". The situation has improved since 1960, but not much.

[12] Qaramaleki, 'The Paradox of the Speechless Root', provides several passages from early *kalām* that, he claims, are on the Liar. But in our judgment, he is wrong about most of those passages. The passages from Baghdādī and Āmidī we discuss here are the only ones we have found.

[13] Baghdādī, *Uṣūl al-dīn* (Beirut, 1981), 13, 217.

[14] al-Āmidī, *al-Iḥkām fī uṣūl al-aḥkām*, v2 (Beirut, ND), 250-252.

later discussed within the competing Ashʿarī school of *kalām*. However, we have found no evidence of the Liar outside *kalām* from this period.

Baghdādī's statement of the Liar is as follows:[15]

> "Declarative sentence" (*khabar*) in the original sense is divided into the true and the false. The true depends on agreement with the subject (that-about-which-something-is-declared), and the false is the opposite of that. There is no declarative sentence that is both true and false together, except one: namely, the declaration by he who has not lied at all, about himself, that he is a liar, and this declarative sentence, from him, is false. And a liar who declares that he is a liar says the truth. And therefore this one declarative sentence is true and false, and it has one subject.

Baghdādī treats the Liar as an exception to the definition of a declarative sentence (*khabar*) in terms of exclusive bivalence.[16] As Baghdādī puts it, declarative sentences are "divided into the true and the false", in the sense that "there is no declarative sentence that is both true and false together, except one", the Liar, a declarative sentence that is *both* true *and* false.

So Baghdādī's solution is to give up exclusive bivalence—at least in this special case.[17] He could have continued to hold that *inclusive* bivalence is the distinguishing mark of a declarative sentence, since prayers and commands are neither true nor false. However, there is no indication that he generalized in this way; he seems instead to take the Liar to be an exception to the definition, rather than grounds for revising the definition.[18]

Writing a bit more than a century later, Āmidī, our second source, was a theologian and philosopher who taught in Baghdad, Cairo, and Damascus. His most important work, *Abkār al-Afkār* (*the Beginning of Ideas*), is both a refutation of philosophy (*falsafah*) and a refutation of the teachings of the Muʿtazilah.

In his *al-Iḥkām fī uṣūl al-aḥkām* (*The Mastery of the Principles of [Islamic] Law*), Āmidī reports on a debate within the Muʿtazilah school, concerning the definition of a declarative sentence. In the context of this debate, the Liar occurs as a threat to the definition:[19]

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[15] *Uṣūl al-dīn*, 13. An almost identical passage occurs at *Uṣūl al-dīn*, 217.

[16] This definition echoes Aristotle, *De Interpretatione* 4, 17a1-6, as well as the Stoic definition of an *axiomata* (e.g., Sextus Empiricus, *Against the Logicians*, II 73).

[17] Graham Priest should be happy to find his favored solution represented here. See his 'The Logic of Paradox', *Journal of Philosophical Logic* 8 (1979), 219–241. Chrysippus wrote a treatise against this sort of solution. See Diogenes Laertius, *Lives of Eminent Philosophers*, VII, 197.

[18] Thanks to Stewart Shapiro for forcing us to clarify this point.

[19] *al-Iḥkām fī uṣūl al-aḥkām*, v2, 250-252.

The Muʿtazilah, Abu ʿAlī al-Jubbāʾī, his son Abū Hāshim, Abī ʿAbd Allāh al-Baṣrī, and others said that a declarative sentence (*khabar*) is a speech (*kalām*) that is pervaded by truth and falsehood. There are four problems brought against this definition.

The four problems that Āmidī reports are, first, that there are counterexamples— declarative sentences that are neither true nor false; second, that the definition is circular; third, that the definition entails that all declarative sentences are both true and false; fourth, that the definition entails that God's declarative sentences can be false. The Liar appears as one of two counterexamples that make up the first of these four problems.[20]

The definition Āmidī mentions is obscure. What thing, exactly, is "pervaded by truth and falsehood", and what does that mean? The third and fourth problems he discusses put pressure on these issues. What we learn from Āmidī's discussion of those problems is this:[21] Contrary to the third problem, the definition does not say that *each* declarative sentence is pervaded by truth and falsehood. Instead, it says of the species, declarative sentence (*khabar*), considered as a whole, that *it* is pervaded by truth and falsehood: its members are all either true or false. (Similar remarks apply to Baghdādī's definition: it is the species *khabar* that is "divided" into true and false.)

So the definition is once again in terms of bivalence: declarative sentences are sentences that are true or false. Our primary interest lies in the first problem Āmidī describes, that there are counterexamples to the definition. The first counterexample Āmidī describes involves a distributive predication with a plural subject:[22]

[This definition] can be undermined by the sentence 'Muḥammad and Musaylimah are truthful in proclaiming the prophecy'. This sentence cannot be true, because [then] Musaylimah's claim would be true, and this sentence cannot be false, because [then] Muḥammad's claim would be false.

To see the force of this counterexample, return to the definitions of truth and falsehood given by Baghdādī: "truth is an agreement with the subject, and falsity is the opposite of that".[23] The problematic sentence has two subjects, and it "agrees" with one of those subjects but not the other. The sentence,

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[20] ibid., 250-1.

[21] ibid., 252.

[22] ibid., 250.

[23] *Uṣūl al-dīn*, 217.

therefore, puts pressure on this definition of truth. To relieve the pressure, one needs to modify one's definition of truth, subject, or declarative sentence.[24]

The second counterexample is the Liar:[25]

> [This definition can also be undermined] by he who said to be false all of his declarative sentences by saying, "All my declarative sentences are false." This declarative sentence cannot be true, for if it were true, then all of his declarative sentences would be false, and this declarative sentence is his only declarative sentence. And it cannot be false, for if it were, then all of his declarative sentences—this being his only declarative sentence—would be false, and therefore he would have spoken truly when he said, "All my declarative sentences are false."

Āmidī's presentation of the paradox differs from Baghdādī's in three ways: he generates the paradox from a universal rather than particular generalization; he presents the paradox as a counterexample to both exclusive and inclusive bivalence rather than as a counterexample to exclusive bivalence alone; he tweaks the background assumptions used to ensure a genuine paradox. We briefly describe these three differences and their significance.

Baghdādī's Liar Sentence is,

L_B I am a liar

which he treats as equivalent to the particular proposition,

L_P Some of my declarative sentences are false.

By contrast, Āmidī's Liar Sentence is a universal proposition,

L_U All of my declarative sentences are false.

In this respect, Āmidī matches his contemporary, Abharī. Perhaps Āmidī is following the conventions of his own period rather than those of the early Mutakallimūn.

Neither sentence is, by itself, paradoxical. L_P can be true without paradox if the speaker says something false other than L_P. L_U can be false without

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[24] Note Baghdādī's cryptic remark, at the end of the passage we quoted above: "this one declarative sentence is true and false, *and it has one subject.*" Baghdādī is emphasizing that, even if sentences with plural subjects can be dealt with in some other way, the Liar, being a sentence with only one subject, cannot.

[25] *al-Iḥkām fī uṣūl al-aḥkām*, 250.

paradox if the speaker says something true other than L_U. To generate a genuine paradox, one must make some further assumptions.

Baghdādī has us assume that the speaker "has not lied at all" prior to saying "I am not a liar." In this way, he blocks the possibility that the speaker said something false other than L_p. Āmidī has us assume that L_U is the speaker's only declarative sentence—that L_U is the only thing he has said. This blocks the possibility that the speaker said something true other than L_U. (It is not clear from what Āmidī says whether the speaker *never* utters another declarative sentence, or whether there is an implicit restriction to what the speaker *now* utters. This is something Abharī makes explicit.)

Baghdādī presents the Liar as a counterexample to exclusive bivalence and "solves" the paradox by allowing an exception to exclusivity. Āmidī presents the Liar as a counterexample to both inclusive and exclusive bivalence. Perhaps he intends to preempt Baghdādī's way out.

For the first counterexample, Āmidī provides a detailed critical account of the different solutions proposed by different Muʿtazilah: according to Abū ʿAlī al-Jubbāʾī (d. 915), a sentence with two subjects is true just in case both subjects agree with the predicate; for Abū Hāshim (d. 933), a sentence with two subjects is really two separate sentences, each separately evaluable for truth or falsehood; for Al-Qādī ʿAbd al-Jabbār (d. 1025), "true" and "false" evaluate the speaker, not what she says. But he does not describe a single Muʿtazilah response to the Liar. His entire discussion of the Liar is contained in the following cryptic remarks:[26]

> With respect to the second part of the first objection, the declarative sentence either corresponds to that about which something is declared or not. If the former, then it is true; if the latter, then it is false, for it is impossible to have together the two contradictories in the case of affirmation or in the case of negation.

Āmidī here makes no attempt to solve the Liar. But he pointedly dismisses Baghdādī's solution: a single sentence, he says, cannot be both true and false, because truth and falsehood are contradictories, and it is impossible to have two contradictories together. Perhaps Baghdādī's solution was *the* dominant solution and widely known, and this explains Āmidī's uncharacteristic brevity.

Āmidī says "it is impossible to have together the two contradictories in the case of affirmation or in the case of negation". This is ambiguous. It may mean that no declarative sentence can be both true and false, whether it is affirmative (the case of affirmation) or negative (the case of negation). Or it

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may mean that, just as no declarative sentence can be both true and false (the case of two contradictories together in an affirmative way), so too no declarative sentence can be neither true nor false (the two contradictories together in a negative way). If the second reading is correct, then Āmidī was casting a wider net: perhaps someone among the early Mutakallimūn defended the view that the Liar Sentence is *neither* true *nor* false. One suspects, however, that Āmidī would have reported on an important disagreement of this sort, given how carefully he reported such disagreements in the other cases he discusses.

Several scholars have argued that the early Mutakallimūn were influenced by Stoic logic and grammar, pointing to parallels between their principles and definitions and those of the Stoics.[27] But there are few striking parallels between what the Mutakallimūn say about the Liar and surviving Stoic fragments on the Liar. We know that the Stoics had a sophisticated take on the Liar: Chrysippus wrote several books arguing against various proposed solutions to the Liar, and proposed a solution of his own.[28] Measured against that apparent sophistication, the work of the early Mutakallimūn is naïve and undeveloped. Hence it seems unlikely that they were working with Stoic sources.

There is one tenuous parallel worth noting. Diogenes lists together three works by Chrysippus:[29]

> Reply to those who solve the Liar Paradox by division;
> Proofs showing that indefinite arguments ought not be solved by division;
> Reply to objections urged against those who condemn the division of indefinite arguments.

What does it mean to solve the Liar "by division"? What does that have to do with "indefinite arguments"?

Suppose that our Liar Sentence is "What I say is false". The subject, "What I say", is, in Stoic terminology, an indefinite term.[30] As an indefinite term, it may refer to any one of several things I say. Hence the following "solution"

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[27] For the details, see S. Horowitz, 'Über den Einfluss des Stoicismus auf die Entwicklung der Philosophie bei den Araben', *Zeitschrift der Deutschen Morgenländischen Gesellschaft* 57 (1903), 177-196; Josef Van Ess, 'The Logical Structure of Islamic Theology', in *Logic in Classical Islamic Culture*, ed. G. E. von Grunebaum (Malibu, 1970), 32; C. H. M. Versteegh, *Greek Elements In Arabic Linguistic Thinking* (Leiden, 1977).

[28] Diogenes, *Lives*, VII 196-198.

[29] ibid., 196-97.

[30] Roughly, definite terms are singular; indefinite terms are general. See A. A. Long and D. N. Sedley, *The Hellenistic Philosophers* (Cambridge, 1987), 207-8.

suggests itself: the Liar sentence is both true and false—true of one of what I say, false of another of what I say. The Liar, then, is solved by division, and, in particular, division of an indefinite term.

Something like this "solution by division" is plausible for the problem of plural subjects: the predicate 'is truthful in proclaiming the prophecy' is true of Muḥammad, but not Musaylamah. Abū Hāshim appears to advocate a solution to the problem of plural subjects on these lines.

So, if we are right about Chrysippus, the Stoics and the Mutakallimūn both associated the Liar with puzzles involving indefinite or plural terms. But it is hard at this point to put much weight on this tenuous parallel.

2. Abharī on the Liar

In the thirteenth century, the Liar was divorced from the problem of giving the definition of a declarative sentence, and treated as a logical fallacy or paradox (*mughālaṭah*) in its own right. In this respect, the role of the Liar in this period resembles its role in the Latin *sophismata* tradition. Several prominent logicians of the time had relatively sophisticated things to say about the Liar:[31] Athīr al-Dīn al-Abharī (1200-1265), Naṣīr al-Dīn al-Ṭūsī (1201-1274),[32] Kātībī al-Quzwinī (d.1276),[33] Saʿad Ibn Manṣūr Ibn Kammūnah (d.1284),[34] and Shams al-Dīn Muḥammad al-Samarqandī (1240-1304)[35] all take a stab at solving the paradox. In this section, we focus on Abharī's solution. In the next, we turn to Ṭūsī.

Our source for both Abharī and Ṭūsī is the chapter '*al-mughālaṭāt*' ('On the logical fallacies') of Ṭūsī's *Taʿdīl al-mīʿyār fī naqd Tanzīl al-afkār* (*The Refinement of Criteria: A Critique of 'Inspired Thoughts'*). Responding to Abharī, Ṭūsī reproduces Abharī's text verbatim.

Abharī was an important thirteenth century mathematician, logician, philosopher, and astronomer. His *Hidyāt al-ḥikmah* (*Guide to Philosophy*), a work on logic, physics, and metaphysics, was widely read and commented

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[31] Some of these logicians—though not their work on the Liar—are discussed in Nicolas Rescher, *The Development of Arabic Logic* (Pittsburgh, 1964), 196-210.

[32] Ṭūsī, *Taʿdīl al-mīʿyār fī naqd tanzīl al-afkār in Manteghe mabāḥeṣ-e alfāẓ* (Tehran, 1974), 235-237.

[33] See Dashtakī, *Risālah fī shubhat al-jadhr al-aṣamm*, ed. Ahad Faramarz Qaramaleki, *Kheradnameh Sadra* 5,6 (1997), 77.

[34] Ibn Kammūnah, *al-Jadīd fī al-ḥikmah* (Baghdad, 1982), 205-206.

[35] See Dashtakī, *Risālah*, 78.

upon within Islamic philosophy. His commentary on Porphyry's *Isagoge* was also quite influential within the tradition; we possess a printed bilingual edition of the text, in Arabic and Latin, from 1625.[36]

Judged against earlier work in the tradition, Abharī shows an increased awareness of the problem the Liar poses, and an increased ambition to address that problem. He describes the Liar as a "difficult fallacy" (*mughālaṭah ṣaʿbah*), and then attempts to pinpoint the fallacious inference that generates the paradox.

Here is his presentation of the paradox:[37]

> One of the difficult fallacies is the conjunction of the two contradictories (*Jamʿ al-naqīḍyan*) when someone says, "All that I say at this moment is false". This sentence (*qawl*) is either true or false. If it is true, then it must be true and false. And if it is not true, then it is necessary that one of his sentences at this moment is true, as long as he utters something. But, he says nothing at this moment other than this sentence. Thus, this sentence is necessarily true and false.

Abharī's presentation of the paradox combines elements found in Baghdādī and Āmidī. His Liar sentence, like Āmidī's, is a universal generalization—'All that I say at this moment is false'. He secures the paradox by assuming that the speaker says only this at this moment, improving on Āmidī's looser characterization of the case. But like Baghdādī, he draws the consequence that the sentence is *both* true and false, not the consequence that it is neither true nor false.

His solution is short and puzzling:[38]

> To solve the paradox we say: we should not concede that if it is false then one of his sentences (*kalām*) is true. For its being true is taken to be the conjunction of its being true and being false. Therefore its being false necessitates the non-conjunction of its being true and being false. And the non-conjunction of its being true and being false does not necessitate its being true.

He generates the paradox by a separation of cases. The Liar sentence is either true or false, and,

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[36] *Isagoge idest, breue introductorium Arabicum, in scientiam logices cum versione Lat.: ac theses sanctæ fidei, fr. Thomæ Nouariensis opera studioq. editæ. Quæ publicè disputabuntur Romæ*, 1625. Available in the Bodleian Library (Bodleian BOD Bookstack MM 32(4) Jur).

[37] *Taʿdīl al-mīʿyār fī naqd tanzīl al-afkār*, 235.

[38] ibid., 236.

(1) If the Liar Sentence is true, then it is both true and false,

while

(2) If the Liar Sentence is false, then it is both true and false.

So either way, it is both true and false.

He locates the fallacy in the reasoning in support of (2). What is that reasoning? When he initially presents the paradox, he defends (2) by saying:[39]

> If it is not true, then it is necessary that one of his sentences at this moment is true, as long as he utters something.

If there is a fallacy, then, it lies in (3):

(3) If the Liar Sentence is not true, then one of the speaker's sentences is true or the speaker says nothing at all.

It is clear that (3) is Abharī's target, for he says, "we should not concede that if it is false then one of his sentences is true".

The Liar Sentence is a universal proposition, and (3) follows from a general principle governing universal propositions:

(4) If a universal proposition is false, then either it has a counter-instance or its subject term is empty.[40]

For example, "All dogs have four legs" is false because there are three-legged dogs; "All unicorns have horns" is false because there are no unicorns. The burden facing Abharī, therefore, is to show that the Liar Sentence is a special case: contrary to (4), it can be false but have neither counter-instance nor empty subject.

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[39] ibid.

[40] The second disjunct is necessary because in Arabic logic, as in Aristotelian logic, universal propositions have existential import. (For a nice discussion of this and related issues in Aristotelian logic, see Terry Parsons, 'The Traditional Square of Opposition', *The Stanford Encyclopedia of Philosophy* (Stanford, 2006), ed. Edward N. Zalta, URL = <http://plato.stanford.edu/archives/win2006/entries/square/>.) This import was especially obvious to Arabic logicians, given how they rendered the Greek copula into Arabic. See Allan Bäck, 'Avicenna on Existence', *Journal of Philosophy* 25 (1987), 353.

Here is what Abharī actually says:[41]

> …We should not concede that if it is false then one of his sentences is true. For its being true is taken to be the conjunction of its being true and being false…

So the Liar is special because it has a special truth condition:

(5) The Liar Sentence's being true is the conjunction of its being true and being false.

What, exactly, does (5) mean? And is Abharī entitled to assert it?
  As a first stab, we might interpret (5) as simple material biconditional:

(5*) The Liar Sentence is true if and only if it is true and false.

One reason (5*) is a plausible interpretation of (5) is that it is easy to reconstruct an argument for (5*) from Abharī's stated commitments. Abharī accepts (1), and (5*) follows from (1) and the trivial observation that if a sentence is both true and false, then it is true.
  From (5), Abharī infers,[42]

> …Therefore its being false necessitates the non-conjunction of its being true and being false…

In other words,

(6) If the Liar Sentence is false, then it is not both true and false.

Assuming that truth and falsehood are contradictories, (6) follows from (5*). It is the next move that is problematic:[43]

> …And the non-conjunction of its being true and being false does not necessitate its being true.

In other words,

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[41] *Taʿdīl al-miʿyār fī naqd tanzīl al-afkār*, 236.
[42] ibid.
[43] ibid.

(7) It is not the case that, if the Liar Sentence is not both true and false, then it is true.

The general principle behind (7) is clear enough: the negation of a conjunction does not entail the negation of a conjunct; so from *not both true and false* you cannot infer *not false* and so *true*. Abharī appears to be saying that the Liar rests on an elementary scope fallacy!

But, of course, Abharī is not entitled to (7). In some cases, the negation of a conjunction *does* entail the negation of a conjunct: "not both P and P" for example, entails "not P". As a general rule, the negation of a conjunction entails the negation of each conjunct whenever the conjuncts are logically equivalent, i.e., whenever the one follows from the other and vice versa. So Abharī is entitled to (7) only if he is entitled to assume that 'The Liar Sentence is true' and 'The Liar Sentence is false' are not logically equivalent. But that assumption would beg the question.

Worse, Abharī has said nothing to undercut the argument for

(2) If the Liar Sentence is false, then it is both true and false,

that he originally presented. The Liar Sentence is a universal proposition, so, if it is (non-vacuously) false, it must have a counter-instance (by (4)); but, assuming that it is the only sentence I utter, its only possible counter-instance is itself; so if it is false, it must be true. There is no step in this line of reasoning that corresponds to the scope fallacy Abharī describes. If this reasoning succeeds, then, given (1), the two conjuncts, "The Liar Sentence is false" and "The Liar Sentence is true" *are* logically equivalent. If they are logically equivalent, then, contrary to (7), the negation of the conjunction does entail the negation of each conjunct.

Abharī's "solution" therefore fails.

We suspect that, if pressed, he would refocus our attention on (5). (5) tells us that the Liar Sentence has a special truth condition: its being true is its being true and false. How might this special truth condition cause trouble for (2)?

Consider an argument for (2) that is a bit different from the one Abharī provides:

(8) The Liar Sentence declares itself to be false.
(9) Assume it is false
(10) So, things are as it declares them to be.
(11) So, the Liar Sentence is true.

The inference from (10) to (11) requires something like:

(P1) A sentence is true if things are as it declares them to be.[44]

Can (5) be used to motivate a denial of (P1)?
   Recall that (1) supports (5):

(1) If the Liar Sentence is true, then it is both true and false,

(1) rests on a line of reasoning that goes something like this: the sentence declares itself to be false; assume it is true; then things are as the sentence declares them to be, so it is false. So (1) depends on the converse of (P1):

(P2) If a sentence is true, then things are as it declares them to be.

If (5) is to undercut (P1), it had better not also undercut (P2), lest it thereby undercut (1), and so undercut (5), and so undercut the basis of Abharī's solution.
   A theory of truth that accepts (P2) but rejects (P1) is a theory that embodies the idea that truth requires *something more* than things being as the sentence says they are. A view of this sort is defended by the 14th century Latin philosopher Thomas Bradwardine.[45]
   Bradwardine held that

(BSP) Every proposition signifies or means contingently or necessarily everything which follows from it contingently or necessarily.[46]

So a proposition like "Zayd is sitting", given (BSP), does not just signify that Zayd is sitting, but also signifies that something is sitting, and—this is the kicker—signifies that "Zayd is sitting" is true.
   Bradwardine also held that

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[44] The precise wording of (P1) and (P2) are both inspired by Ṭūsī. Together, they can be read as glosses on Aristotle's dictum: "To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, and of what is not that it is not, is true" (*Metaphysics*, 1011b25).

[45] Stephen Read, 'The Liar Paradox from John Buridan back to Thomas Bradwardine', *Vivarium* 40 (2002), 189-218.

[46] Read, 'The Liar Paradox', 191.

(BTP) A true proposition is an utterance signifying only as things are.[47]

So the truth of "Zayd is sitting" requires not just that Zayd be sitting, but also that something be sitting, and also—again, the kicker—that "Zayd is sitting" is true.

Everyone agrees that the Liar Sentence signifies that it is false. But, just as "Zayd is sitting" entails "'Zayd is sitting' is true", and so, by (BSP), signifies its own truth, so too the Liar Sentence, by (BSP), signifies its own truth. So the Liar Sentence signifies a contradiction: it signifies both that it is false and that it is true. Like all contradictions, it is false.

That is Bradwardine's solution to the Liar. Returning to Abharī, and to (P1):

(P1) A sentence is true if things are as it declares them to be.

Suppose Abharī held something like (BSP). Distinguish a broad and a narrow sense of things being as the sentence declares them to be—the narrow sense corresponding to what we ordinarily take a sentence to declare; the broad sense corresponding to that along with everything it entails. Given (BTP), in the narrow sense of "declares", (P1) is false; in the broad sense, it is true.

Return to the argument for (2) that relies on (P1):

(8)   The Liar Sentence declares itself to be false.
(9)   Assume it is false.
(10) So, things are as the sentence declares them to be.
(11) So, the Liar Sentence is true.

The argument no longer goes through: the use of "declares" in (10) is equivocal: (10) follows from (8) and (9) only if "declares" is used in the narrow sense; (10) supports (11) only if "declares" is used in the broad sense. So the argument fails.

Bradwardinian principles would also give Abharī grounds for rejecting the principle that a non-vacuously false universal proposition must have a counter-instance: if it has a counter-instance, then things are not as it narrowly declares them to be, so that is one way for the sentence to be false; but also, if it is not true, then things are not as it broadly declares them to be, so that is another way for it to be false, even if it has no counter-instance.

Consider (5) in light of a Bradwardinian analysis. (5) can be read as the claim that for things to be as the Liar Sentence broadly declares them to be is

for the sentence to be both true and false. Hence there is a sense in which the sentence itself is causing the problem, by (broadly) declaring a contradiction. Such a thought seems very much in the spirit of Abharī.

It is therefore tempting to read Abharī as a proto-Bradwardinian. Such a reading is not entirely without support: primarily, it makes sense of Abharī's use of (5). But the reading stretches charity: what Abharī actually says is that one cannot infer the negation of a conjunct from the negation of a conjunction, which, as we have seen, is a mistake.

Proto-Bradwardinian or not, Abharī's work on the Liar occurs early enough that it could have influenced Bradwardine if it made its way West. Since it is with Bradwardine that the Latin tradition picks up steam, one might wonder whether that steam was the result of Arabic forces.[48] Unfortunately, the earliest known Latin translation of any work by Abharī is the bilingual edition of his commentary on the *Isagoge*, from 1625, which is too late, and has nothing to do with the Liar.[49]

It is possible that Abharī's work was known to the West indirectly. Ṭūsī' quotes Abharī, and Ṭūsī"s work in astronomy, if it was available, may have attracted Bradwardine's attention.

3. Ṭūsī on the Liar

Ṭūsī was a thirteenth century polymath who wrote important works in astronomy, mathematics, philosophy, and theology, among other subjects. His most enduring fame stems from his work in astronomy. As founder and director of the famous observatory at Marāgha, his work was central to the reformation of Ptolemaic astronomy. The techniques he and his followers developed—most famously, the so-called "Ṭūsī couple"—were later appropriated by Copernicus.

Just how Copernicus came to know Ṭūsī's work remains a puzzle. There are no surviving Latin translations of any of Ṭūsī's work. Henri Hugonnard-Roche suggests a route through Byzantine sources, reaching Italy in the fifteenth century.[50] Claudia Kren has speculated that "some possibly fragmentary and even garbled version of the al-Ṭūsī device" lies behind a puzzling passage

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[48] In the words of Ralph Strode, Bradwardine was the one "who first came upon something of value concerning insolubles." Spade ('Insolubilia', 249) quotes Strode and concurs: "with him, the *insolubilia*-literature entered its second and most productive phase."

[49] See note 36 above.

[50] 'The Influence of Arabic Astronomy In the Medieval West', in *Encyclopedia of the History of Arabic Science*, V1, ed. Roshdi Rashed, (New York, 1996), 303.

from Nicole Oresme.[51] Y. Tzvi Langermann has suggested that the missing link may be a fourteenth century Hebrew text on geometry, known to a fifteenth century Jewish savant, who in turn was in contact with Christian savants in Northern Italy.[52]

In sum, we have no clear idea how any of Ṭūsī's work came to be known in the West, and, in particular, no reason to suppose that his work on the Liar was ever known in the West.

Ṭūsī frames his discussion of the Liar as a response to Abharī. He quotes Abharī's presentation of the paradox verbatim, mentions that it is a paradox that has been "brought forth by a number of previous thinkers"[53]—a tantalizing but obscure reference—and proceeds to explain how he thinks the paradox "should be arranged"—that is to say, how the paradox should be generated. He then quotes Abharī's solution to the paradox verbatim, briefly criticizes that solution, and begins to describe and develop his own solution, which depends upon a distinction between the negation (*nafī*) of and the nonexistence (ʿ*adam*) of truth and falsity, and upon a restriction of the application of the predicates 'true' and 'false'.

Ṭūsī focuses on (5): when Abharī's says that "its being true is taken to be the conjunction of its being true and false", Ṭūsī says, if he meant that "its being true is the obtaining of the conjunction" then that is something "which we should not concede."[54] Ṭūsī ventures an alternative interpretation: perhaps Abharī means that "its being true is the necessitating of its being true as a result of its being false". Ṭūsī seems to be replacing Abharī's *conjunctive* truth condition ("its being true *and* its being false") with a *conditional* truth condition ("its being true *if* it is false"). He does not say anything to clarify what, exactly, this might mean.

But his critical point does not depend on this. Ṭūsī argues that whatever fancy thing (conjunction, conditional) Abharī wants to identify as the truth condition for the Liar Sentence, it will not matter, because *pace* Abharī, we can generate the paradox without inferring, from the negation of a complex truth condition, the negation of one of its parts. We can argue directly that its being false entails the negation of its being false, and so entails its being true.

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[51] 'The Rolling Device of Naṣīr al-Dīn al-Ṭūsī in the *De spera* of Nicole Oresme?', *Isis* 62 (1971), 497.

[52] 'Medieval Hebrew Texts on the Quadrature of the Lune', *Historia Mathematica* 23 (1996), 34-35.

[53] *Taʿdīl al-miʿyār fī naqd tanzīl al-afkār*, 235.

[54] ibid., 236.

Ṭūsī is making a point we made above: Abharī fails to undercut several obvious and apparently valid lines of argument from *false* to *true*, and therefore fails to show that the paradox rests on the fallacy he describes.

Ṭūsī's preferred solution requires setup. He begins by telling us,[55]

> If a declarative sentence, by its nature, can declare-something-about anything, then it is possible that it itself can declare-something-about another declarative sentence.

In other words, if there are no constraints on the possible subject matter of a declarative sentence, then it is possible to have a meta-linguistic declarative sentence: a sentence that has as its subject another declarative sentence.

He proceeds to describe a case in which one declarative sentence, (D_1), declares another declarative sentence, (D_2), to be false, as in

(D_1) (D_2) is false.
(D_2) Zayd is sitting.

Of such a case, he makes two points:

> The second declarative sentence would become that-about-which-something-is-declared in one respect, and a declarative sentence in another. If the first declarative sentence declares the second declarative sentence to be false, then the first declarative sentence's being true and the second declarative sentence's being false are concomitant.

Ṭūsī is here belaboring two points. First, once we have a sentence declaring something about a sentence, we have a single sentence that plays two grammatical or logical roles: (D_2) is the grammatical subject of (D_1), and it is also a declarative sentence in its own right. The second point concerns the logical relations between sentences like (D_1) and (D_2): if (D_1) is true, then (D_2) is false (and, presumably, vice versa).

Armed with these distinctions, we can consider the case that matters: a sentence that declares something about itself—that is, a sentence that has itself as subject. As Ṭūsī sets it up, we are asked to imagine that our first declarative sentence, (D_1), declares itself to be false, rather than (D_2):

(D_1) (D_1) is false.

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[55] ibid., 235.

Of such a case, Ṭūsī tells us:[56]

> Moreover, if the first declarative sentence declares itself to be false, then [both] its being true, insofar as it is a declarative sentence, and its being false, insofar as it is that-about-which-something-is-declared, are concomitant.

This is just an application of the two points made above. The sentence plays two grammatical roles—subject and sentence—and corresponding to each role, we can consider its truth or falsehood, and the logical relations between them.

Now we are ready to consider the paradox:[57]

> Thus, the following paradox can be generated: The first declarative sentence, which is a declaration (*khabar*) about itself, namely that it is false, is either false or true. If it is true, then it must be false, because it declares itself to be false. If it is false, then it must be true, because if it is said falsely, then it will become true, which is absurd.

It is hard to overemphasize how remarkable this passage is. The contemporary reader will be familiar with the idea that the Liar Paradox is a paradox of self-reference. But Ṭūsī is, as far as we know, the first person to express this idea. This passage has no precedent in any tradition.

Ṭūsī has performed three remarkable feats in short order. First, his Liar Sentence is singular: its subject is itself, and it declares itself to be false. Gone, then, is the choice between universal or particular Liar Sentence, and the associated problem of adding further assumptions to generate a genuine paradox.[58] Second, he has characterized the paradox as one of self-reference. Third, he has identified a key assumption that might be responsible for generating the entire problem: the assumption that a declarative sentence, by its nature, can declare-something-about anything.

Ṭūsī begins to develop his own response in the guise of a recommendation to Abharī:[59]

> It would have been better for the author to have said that it follows that, if it is false, then what it declares about itself, namely that it is false, is true at the same time.

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[56] ibid., 236.

[57] ibid.

[58] It is puzzling that Ṭūsī does not mention this himself, and, as we will see, makes no attempt to apply his solution to non-singular Liar Sentences.

[59] *Taʿdīl al-miʿyār fī naqd tanzīl al-afkār*, 236.

Ṭūsī immediately retracts this advice, in the following difficult passage:[60]

> As for the claim that its being true is necessitated by its being false, the denial of its being false does not necessitate the affirmation of its being true. For the denial of the latter [its being false] will not necessitate the former [its being true].[61] Rather what is necessitated is the denial of its being true. Thus, it is impossible for such a sentence to be true, because its being true necessitates its falsity; and its being false is not impossible, because its being false necessitates the nonexistence (ʿadam) of its being true.

How can he, in the same breath, both accept the validity of the inference from *false* to *true*, and then present an argument against it?

Ṭūsī seems to recognize this. He suggests, in response to the argument just quoted, that one might respond by once again repeating the basic argument from false to true:

> If it is false, then what it declares about itself, namely that it is false, is true, and thus the paradox is regained.

This is, in essence, the same argument that Abharī put forth when he presented the paradox; it is, in essence, the same argument Ṭūsī held up in response to Abharī's solution. And now Ṭūsī seems to have both accepted and rejected it in short order. What is going on?

Ṭūsī knows that the ultimate solution, if it is going to succeed, must explain what is wrong with this argument. He appears to be calling our attention to the fact that Abharī's solution fails to provide such an explanation, and that the confusing argument just vetted—even if it is moving in the right direction—has not done so either.

Ṭūsī responds to this challenge by making a distinction between two different arguments from *false* to *true*:[62]

> The response to this is that its being true, insofar as it is a declarative sentence, and its being false, insofar as it is that-about-which-something-is-declared, are concomitant. However, this is not the issue here. Rather, it is that its being false, insofar as it is a declarative sentence, does not necessitate its being true.

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[60] ibid.

[61] The brackets represent our judgment as to the proper interpretation of 'former' and 'latter'. Note that they cannot refer back to the main clause in the previous sentence ('the denial of its being false does not necessitate the affirmation of its being true'). Hence we interpret them as referring back to the former and the latter parts of the first clause of the previous sentence ('…its being true is necessitated by its being false…').

[62] *Taʿdīl al-miʿyār fī naqd tanzīl al-afkār*, 237.

Ṭūsī wants us to distinguish an argument that trades on the role of the Liar Sentence as subject (as that-about-which-something-is-declared) and an argument that trades on its role as declarative sentence. The former argument, he seems to want to say, succeeds; the latter does not.

The argument that Ṭūsī has now repeated several times,

> If it is false, then what it declares about itself, namely that it is false, is true,

is the argument that Ṭūsī accepts, and so must be the argument that he takes to trade on the role of the Liar Sentence as subject. To bring this out, we abstract the form of the argument:

> If S is P, then what 'S is P' declares about S, namely, that it is P, is true.

The general principle we are relying on here is familiar: if a subject is as a sentence declares it to be, then that sentence is true:

> (P1) A sentence is true if things are as it declares them to be.

Ṭūsī's idea is that, when (P1) is brought to bear in the argument from false to true, the argument begins by focusing on the role of the Liar Sentence as subject—the 'S' role—rather than its role as declarative sentence—the 'S is P' role.

What is the other sort of argument that Ṭūsī has in mind, the argument that begins by focusing on the role of the Liar Sentence as declarative sentence? Picking up where we left off, he says:[63]

> . . . its being false, insofar as it is a declarative sentence, does not necessitate its being true. Instead, *its being false necessitates the denial of its being false, insofar as it is that-about-which-something-is-declared*, and [necessitates] its being false, insofar as it is a declarative sentence. Hence, we should not concede that, in this way, the denial of its being false necessitates its being true.

The key move is in italics. Again, we abstract the form of the argument:

> 'S is P's being false necessitates the denial of S's P-ing, insofar as S is that-about-which-something-is-declared.

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[63] Ibid., our emphasis.

The underlying principle is:

   (P3)   If a sentence is false, then the subject of the sentence is not as the
            sentence declares it to be.

How does (P3) get us an argument from false to true? For simplicity, let our
Liar Sentence be:

   (L)   L is false.

Assume that

   'L is false' is false.

By (P3), we can infer that

   L is not false.

But, Ṭūsī wants to say, without bringing to bear something like (P1)—with-
out, following Ṭūsī's rubric, returning to consider the Liar Sentence insofar as
it is that-about-which-something-is-declared—we cannot get any further; in
particular, we cannot infer, from

   L is not false

to

   L is true.

So Ṭūsī needs to show that there is a way that L can fail to be false without
being true. In his words,

   …We should not concede that in this way the denial of its being false necessitates its
   being true.

To establish this, he appeals to the fact that the Liar Sentence is self-referential:[64]

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[64] ibid.

> If the declarative sentence is the same as that-about-which-it-declares, then it cannot be conceived to be true and false. For agreement cannot be conceived except as between two things, and we cannot conceive them as opposed. For if one thing is affirmed, then nothing will be denied, and if one thing is denied, then nothing can be conceived to be affirmed.

Ṭūsī is arguing that a declarative sentence that "is the same as that-about-which-it-declares"—that is, a sentence that has itself as subject—can be neither true nor false. The argument rests on two principles governing truth and falsehood:

 (T) Truth is an agreement between subject and declarative sentence.
 (F) Falsehood is a disagreement between subject and declarative sentence.

These principles are not new to Ṭūsī. We have already seen them at work within the Arabic tradition with Baghdādī:[65]

> Truth is an agreement with that-about-which-something-is-declared, and falsity is the opposite of that.

However, (T) and (F) are open to interpretation.

On one interpretation, (T) is simply the conjunction of (P1) and (P2),

(P1&2) A sentence is true just in case things are as it declares them to be.

(P1&2) is consistent with a minimalist theory of truth: it does not impose upon us any particular account of the metaphysics of truth.

But the argument makes it clear that Ṭūsī thinks that the agreement and disagreement required by (T) and (F) go beyond this. His central claim is that agreement and disagreement both require distinctness:[66]

> Agreement cannot be conceived except as between two things, and we cannot conceive them as being opposed.

Both agreement and disagreement, he says, require "two [distinct] parts". In the case of a sentence that is its own subject, there are not two distinct

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[65] *Uṣūl al-dīn*, 217.
[66] ibid.

parts—just the same thing playing two different roles—so there cannot be agreement, so there cannot be truth. Likewise, there cannot be disagreement, so there cannot be falsehood. So self-referential sentences are neither true nor false.

In very broad strokes, it appears that Ṭūsī has some sort of correspondence theory of truth (and falsehood) in mind. Truth requires some sort of *real relation* of agreement between subject and sentence, and real relations require distinct relata.

If we grant Ṭūsī this, then he can make a distinction between two ways in which a sentence can fail to be true. First, a sentence can fail to be true because it is false: it is not true because there is no agreement; there is no agreement because there is disagreement. But, second, a sentence can fail to be true because it is self-referential: it is not true because there is no agreement; there is no agreement (or disagreement) because there are not two distinct parts. Ṭūsī expresses this by saying that we can conceive of the nonexistence (ʿ*adam*) of agreement and truth in two ways: the way that "opposes the natural disposition" (the way of disagreement and falsity), and the way that involves the negation (*nafī*) of agreement and truth (the way of neither agreement nor disagreement):[67]

> We can conceive in that case the nonexistence (ʿ*adam*) of an agreement, not in the sense that opposes the natural disposition—i.e., falsity—but rather nonexistence (ʿ*adam*) in the sense of the negation (*nafī*) of an agreement—i.e., the nonexistence (ʿ*adam*) of truth.

Where, then, does this leave matters? Ṭūsī says that the Liar is[68]

> the result of a judgment that applies truth and falsity to something to which they in no way apply, and to apply them in any way is the misuse of a predicate.

In other words, the fallacy is to suppose that the predicates 'true' and 'false' apply to a self-referential sentence like the Liar.

Many of the problems for this sort of solution are well-known and obvious, so we here focus on problems that seem especially pressing given Ṭūsī's concerns.

Many self-referential sentences are non-paradoxically true or false, e.g., "This sentence is six words long" and "This sentence is a sausage". Moreover, it seems right to say that "This sentence is six words long" is true because the

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[67] ibid.
[68] ibid.

subject agrees with the sentence, and that "This sentence is a sausage" is false because the subject disagrees with the sentence. So whatever senses of agreement and disagreement are here in play, it does not seem that the identity or distinctness of subject and sentence matters.

It is puzzling that Ṭūsī's solution does not directly address the Liar Sentences provided by his immediate predecessors, including Abharī. Does the sentence 'All that I say at this moment is false' declare something about itself? There is a clear sense in which it does: the sentence itself is among the things I say at this moment, so it is among the things the sentence declares something about. But if we extend Ṭūsī's solution to cover this sentence in this way, the embarrassments of the previous paragraph become more pronounced: the sentence, "All declarative sentences are sentences", for example, declares something about itself, and so is neither true nor false.[69]

'All that I say at this moment is false' generates a paradox only when we assume that it is the *only* thing I say at this moment. So perhaps Ṭūsī could distinguish sentences that declare something about themselves alone and sentences that declare something about themselves and other things as well. Such a distinction would allow his solution to cover the Liar Sentences found in Abharī and Āmidī without forcing him to deny the truth of every meta-linguistic universal proposition. Perhaps some similar move could be made to handle Baghdādī's version of the paradox, based upon a particular proposition.

Finally, we return to the beginning. Recall that Ṭūsī began with the claim that,

> If a declarative sentence can, by its nature, declare-something-about anything, then…

Does he, or does he not, accept the antecedent of this conditional? Are there, for Ṭūsī, self-referential declarative sentences or not?

Neither Ṭūsī nor Abharī frame the Liar as a counterexample to the definition of "declarative sentence". But it is clear that, for Ṭūsī, no self-referential sentence is truth-apt: they are not the sorts of sentences that are capable of being true or false. If he accepts the definition of the Mutakallimūn, that a declarative sentence is one that obeys bivalence, then he should conclude that a self-referential sentence is not a declarative sentence, and so conclude that

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[69] A consequence Bertrand Russell endorses, but Ṭūsī doesn't seem to notice. See Russell, 'Mathematical Logic as Based on The Theory of Types', *American Journal of Mathematics*, 30 (1908), 224; reprinted in his *Logic and Knowledge* (London, 1956), 63.

it does not declare something about itself, but instead fails to declare anything about anything at all.

In the Arabic tradition, that view does not appear for another century, in the work of Saʿd al-Dīn Taftazānī (d.1390)—but that is for another paper.[70]

4. Conclusion

We have attempted to describe and analyze three important chapters in the history of the Arabic Liar: its earliest appearance in the work of the early Mutakallimūn; Abharī's attempt to solve the paradox by appeal to a special truth condition for the Liar Sentence; and Ṭūsī's remarkable, if inadequate, attempt to appeal to a correspondence theory of truth in support of the claim that self-referential sentences, like the Liar Sentence, are neither true nor false.

Much remains to be done. The Arabic Liar continues for at least another four centuries after Ṭūsī. Broader historical questions of inter-traditional influence remain unanswered; or, rather, the only answer we can give at the moment is a negative epistemic: no inter-traditional lines of historical influence have come to light.

Appendix

Translation from Al Āmidī, 'On the Truth of the Khabar *and its Divisions'*[71]

The Muʿtazilah, such as Abu ʿAlī al-Jubbāʾī, his son Abū Hāshim, Abī ʿAbd Allāh al-Baṣrī and al-Qāḍī ʿAbd al-Jabbār and others said that the declarative sentence (*khabar*) is a speech (*kalām*) that is pervaded by truth and falsehood. There are four problems brought against this definition.

First, [this definition] can be undermined by the sentence "Muḥammad and Musaylimah are truthful in proclaiming the prophecy". This sentence cannot be true, because [then] Musaylimah's claim would be true, and this sentence cannot be false, because [then] Muḥammad's claim would be false. And furthermore, by he who said to be false all of his declarative sentences by saying, "All my declarative sentences are false". This declarative sentence cannot be true, for if it were true, then all of his declarative sentences would be false, and this declarative sentence is his only declarative sentence. And it

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[70] Taftazānī, *Sharḥ al-Maqāṣid*, ed. ʿAbd al-Raḥmān ʿUmayrah (Beirut, 1989), 286-287.

[71] *al-Iḥkām fī uṣūl al-aḥkām*, v2, 250-252.

cannot be false, for if it were, then all of his declarative sentences—this being his only declarative sentence—would be false. And therefore he would have spoken truly when he said, "All my declarative sentences are false".

Second, the definition of the declarative sentence—that which is either true or false—is a circular definition. For the determination of the true and the false depends on knowing the declarative sentence, since something is true if it agrees with that-about-which-something-is-declared, and it is false if it does not, and that is impossible.

Third, truth and falsehood are opposite to each other, and it is inconceivable that they be together in one declarative sentence. And that necessitates that either it is impossible for the declarative sentence to exist in an absolute sense, which is absurd, or the declarative sentence exists together with the impossibility of the possibility of truth and falsehood pervading through it, and thus the defined thing would be understood without it saying anything about the thing which is defined by it, and that too is absurd.

Fourth, God's [word] must be a declarative sentence, but God's word cannot be false.

With respect to the first objection, Abu ʿAlī al-Jubbāʾī replies to the one who utters 'Muḥammad and Musaylimah are truthful', that this sentence (*kalām*) signifies the truth of one of them provided the truth of the other, that is, one of them says the truth provided that the other says the truth […]

Abū Hāshim replies that this declarative sentence acts as two declarative sentences, one declares the truth of the prophet and the other declares the truth of Musaylimah […]

Al-Qādī ʿAbd al-Jabbār replies that what we mean by "A declarative sentence is either true or false" is that language does not prevent us from judging a speaker by saying, "what you said is true or false" […]

Abū ʿAbd Allāh al-Baṣrī replies that it is false because in such a sentence truth is applied to both […]

With respect to the second part of the first objection, the declarative sentence either corresponds to that-about-which-something-is-declared or not. If the former then it is true; if the latter, then it is false, for it is impossible to have together the two contradictories in the case of affirmation or in the case of negation.

With respect to the second objection, al-Qādī ʿAbd al-Jabbār replies […]

With respect to the third objection, it has been said in reply that the defined thing is the species of the declarative sentence, and it admits the pervading of truth and falsehood through it in same way that black and white are conjoined in the species of color. This is not correct because even if the

definition is of the species of the declarative sentence, the definition must exist in every individual declarative sentence, since otherwise an individual declarative sentence would exist without its definition, and that is impossible. And it is clear that in each individual declarative sentence there is no conjunction of truth and falsehood, because even though it appears to be an absolute conjunction, what is intended by the conjunction 'and' is the repetition of the two parts in a figurative sense.

With respect to the fourth objection, it has been said that the reply to it is the same as the reply to the previous objection, which we have already explained.

Translation of Ṭūsī, 'On the logical fallacies', from his The Refinement of Criteria: A Critique of 'Inspired Thoughts'[72]

He [Abharī] says: One of the difficult fallacies is the conjunction of the two contradictories (*jamʿ al-naqīdyan*) when someone says, "All that I say at this moment is false". This sentence (*qawl*) is either true or false. If it is true, then it must be true and false. And if it is not true, then it is necessary that one of his sentences at this moment is true, as long as he utters something. But, he says nothing at this moment other than this sentence. Thus, this sentence is necessarily true and false.

I say: This fallacy is brought forth by a number of previous thinkers. Let us [first] state how it should be arranged:

If a declarative sentence, by its nature, can declare-something-about anything, then it is possible that it itself can declare-something-about another declarative sentence. Thus, the second declarative sentence would become that-about-which-something-is-declared in one respect, and a declarative sentence in another. If the first declarative sentence declares the second declarative sentence to be false, then the first declarative sentence's being true and the second declarative sentence's being false are concomitant. Moreover, if the first declarative sentence declares itself to be false, then [both] its being true, insofar as it is a declarative sentence, and its being false, insofar as it is that-about-which-something-is-declared, are concomitant.

Thus, the following paradox can be generated: The first declarative sentence, which is a declaration (*khabar*) about itself, namely that it is false, is either false or true. If it is true, then it must be false, because it declares itself to be false. If it is false, then it must be true, because if it is said falsely, then it will become true, which is absurd.

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[72] *Taʿdīl al-mīʿyār fi naqd tanzīl al-afkār*, 235-237.

He says: To solve the paradox we say: we should not concede that if it is false then one of his sentences (*kalām*) is true. For its being true is taken to be the conjunction of its being true and being false. Therefore its being false necessitates the non-conjunction of its being true and being false. And the non-conjunction of its being true and being false does not necessitate its being true.

I say: When he said, "its being true is taken to be the conjunction of its being true and being false", he either meant that its being true is the obtaining of the conjunction, which we should not concede, or that its being true is the necessitating of its being true as a result of its being false. [But if that is the case] then we should not concede that its being false necessitates the non-existence (*ʿadam*) of that necessitating. Rather, it necessitates the negation (*nafī*) of the falsity of the sentence (*kalām*). Hence, the sentence (*kalām*) is not false, and what we mean by 'being true' is exactly that. Hence, it appears that its being false necessitates its being true. Thus, the paradox can be regained.

It would have been better for the author to have said that it follows that, if it is false, then what it declares about itself, namely that it is false, is true at the same time. What I mean by this is that being true and being false are contradictory, so that to deny one is to affirm the other. As for the claim that its being true is necessitated by its being false, the denial of its being false does not necessitate the affirmation of its being true. For the denial of the latter [its being false] will not necessitate the former [its being true].[73] Rather what is necessitated is the denial of its being true. Thus, it is impossible for such a sentence to be true, because its being true necessitates its falsity; and its being false is not impossible, because its being false necessitates the nonexistence (*ʿadam*) of its being true.

One may respond by saying: Just as its being true necessitates its being false, concomitantly, its being false necessitates is being true. Because, if it is false, then what it declares about itself, namely that it is false, is true, and thus the paradox is regained.

The response to this is that its being true, insofar as it is a declarative sentence, and its being false, insofar as it is that-about-which-something-is-declared, are concomitant. However, this is not the issue here. Rather, it is that its being false, insofar as it is a declarative sentence, does not necessitate its being true. Instead, its being false necessitates the denial of its being false, insofar as it is that-about-which-something-is-declared, and [necessitates] its

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[73] See note 61 above.

being false, insofar as it is a declarative sentence. Hence, we should not concede that, in this way, the denial of its being false necessitates its being true.

Upon investigation, truth and falsity apply to every declarative sentence that is distinct from that-about-which-it-declares,[74] so that one can conceive a declarative sentence and a declarative sentence opposite to it standing on contrary sides, so that, if one of truth or falsity is assigned to that-about-which-it-declares, the other should not be assigned to it, so that one of them will be true and the other of them will be false. Furthermore, the denial of one of them will result in the affirmation of the other, and the affirmation of one of them will result in the denial of the other.

However, if the declarative sentence is the same as that-about-which-it-declares, then it cannot be conceived to be true and false. For agreement cannot be conceived except as between two things, and we cannot conceive them as opposed. For if one thing is affirmed, then nothing will be denied, and if one thing is denied, then nothing can be conceived to be affirmed.

Moreover, if this declarative sentence declares itself to be false, then we cannot conceive in that case an agreement between its two parts such that it is true. But we can conceive in that case the nonexistence (ʿ*adam*) of an agreement, not in the sense that opposes the natural disposition—i.e., falsity—but rather nonexistence (ʿ*adam*) in the sense of the negation of an agreement—i.e., the nonexistence (ʿ*adam*) of truth.

Therefore, it is not true that this declarative sentence is either true or false, and judging it to be either results in an impossibility. Rather, it is true that this sentence is neither true nor false. And the affirmation of the one does not result from the negation of the other, and so no impossibility follows.

Now, from what we have discussed, it is clear that this fallacy arises as the result of a judgment that applies truth and falsity to something to which they in no way apply, and to apply them in any way is the misuse of a predicate.

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[74] Literally, "its that-about-which-something-is-declared."