

On the fundamental role of massless form of matter in physics

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Abstract

In the article, with the help of various models, the thesis on the fundamental nature of the field form of matter in physics is considered. In the first chapter a model of special relativity is constructed, on the basis of which the priority of the massless form of matter is revealed. In the second chapter, a field model of inert and heavy mass is constructed and on this basis the mechanism of inertia and gravity of weighty bodies is revealed. In the third chapter, the example of geons shows the fundamental nature of a massless form of matter on the Planck scale. The three-dimensionality of the observable space is substantiated. In the fourth chapter, we consider a variant of solving the problem of singularities in general relativity using the example of multidimensional spaces. The last chapter examines the author's approach to quantum gravity. The conclusions do not contradict the main thesis of the article on the fundamental nature of the massless form of matter.

Introduction

At the turn of the 20th and 21st centuries, when the number of open particles approached several hundred, the nonlinear field concept of Einstein became popular again: particles are clusters of a certain material field, i.e. Formations localized in a small spatial region. In favor of this point of view, in particular, the fact of mutual transformation of particles in collisions. Various well-known physicists have considered different variants of the nonlinear field theory of particles: Louis de Broglie, Werner Heisenberg, Dmitri Ivanenko, Tony Skirm, Jacob Terletsky, Ludwig Faddeev, and others.

In the article, with the help of various models, the thesis on the fundamental nature of the field form of matter in physics is considered. In the first chapter a model of special relativity is constructed, on the basis of which the priority of the massless form of matter is revealed. In the second chapter, a field model of inert and heavy mass is constructed and on this basis the mechanism of inertia and gravity of weighty bodies is revealed. In the third chapter, the example of geons shows the fundamental nature of a massless form of matter on the Planck scale. The three-dimensionality of the observable space is substantiated. In the fourth chapter, we consider a variant of solving the problem of singularities in general relativity using the example of multidimensional spaces. The last chapter examines the author's approach to quantum gravity. The conclusions do not contradict the main thesis of the article on the fundamental nature of the massless form of matter.

1 To special relativity

1.1 Model of special relativity

Create different kinds of models plays an important role in scientific knowledge . Therefore, the construction of a visual model of the special relativity is of great importance for the explanation of the phenomena (length contraction, time dilation processes) inaccessible to direct perception of human senses.

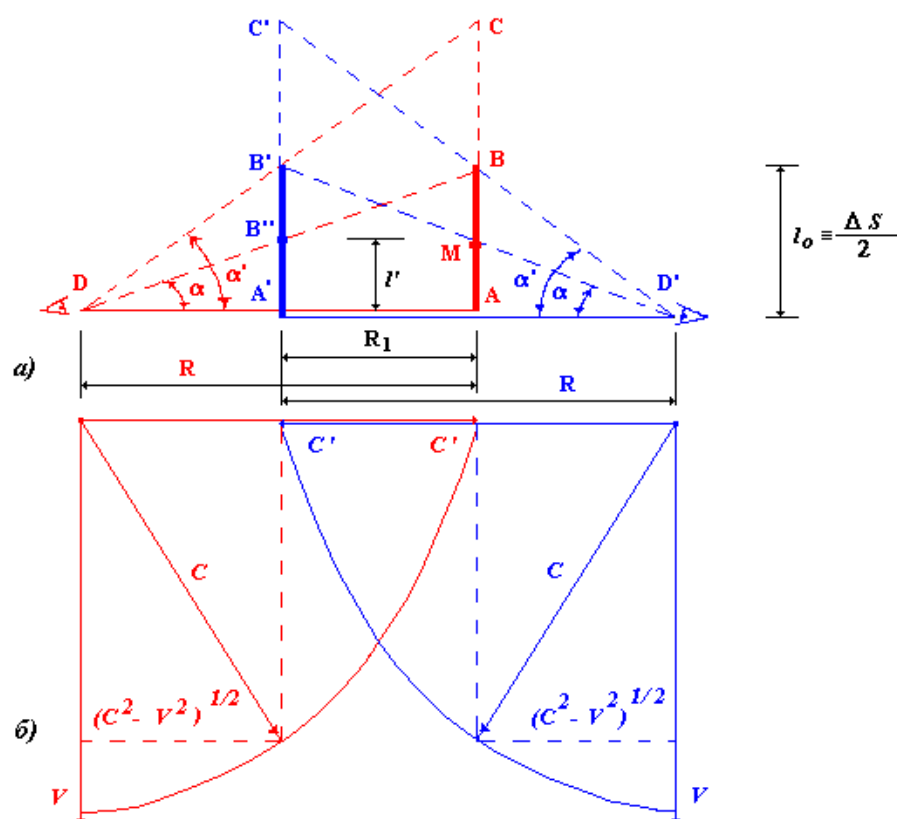


Figure 1: Model of special relativity

Model of special relativity is a system of two observers and two rods (Fig.1a). Here AB and $A'B'$ are rods with a length l_0 . At points D and D' are observers. R is a constant distance, R_1 is a variable distance. Thus, each observer associated with a respective rod (own reference system indicated in red or blue).

From Fig.1a is easy to obtain equations that are valid with respect to both observers

$$l' = l_0 \left(1 - \frac{R_1}{R} \right) \quad (1.1)$$

$$\tan \alpha' = \frac{\tan \alpha}{1 - R_1/R} \quad (1.2)$$

$$R \tan \alpha = \tan \alpha' (R - R_1) = \textit{invariant} \quad (1.3)$$

The equation (1.1) characterizes the apparent decrease in the length of one rod with respect to the other rod as a function of the distance R_1 . The equations (1.2) and (1.3) characterize the invariance of the lengths of both rods as the distance R_1 changes, that is, the length l_0 is an invariant of the transformations. We note that in (1.1), the decrease in the length l' is not the result of the action of some internal molecular forces in the rods. This is analogous to special relativity (SR) where, according to Einstein, the "compression" of rods is an inevitable consequence of kinematics, and not the result of a change in the balance of forces between solid-state molecules during motion, according to Lorentz and Poincare.

If we consider the motion of the light signal from the point A to the point B and back to the point A in the indicated model, then it is not difficult to show that for the light signal of the formula (1.1), (1.2), (1.3) take the following form

$$l' = l_0 \sqrt{1 - \frac{v^2}{c^2}} \quad (1.4)$$

$$\Delta t' = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (1.5)$$

$$c \Delta t_0 = c' \Delta t' = (c^2 - v^2)^{1/2} \Delta t' = (c^2 \Delta t'^2 - \Delta x'^2)^{1/2} = \Delta S = \textit{inv} \quad (1.6)$$

Here l' is the distance that the light signal travels during the time $\Delta t_0/2$ with respect to the rod $A'B'$ and is the *projection* of the light beam on this system; $\Delta t_0 = 2 \tan \alpha (R/c)$ and $\Delta t' = 2 \tan \alpha' (R/c)$ are the total times of the light signal's movement back and forth with respect to its own and other frames of reference; c is the speed of light; v is a quantity with the speed dimension (the so-called spatial component of the speed of light, (see Fig.1b); $c' = \sqrt{c^2 - v^2}$ is the so-called "transverse", time component of the speed of light; $\Delta S = 2l_0$ is an invariant quantity characterizing the unchanged length of the rods and expressed in terms of the spacetime characteristics of the light signal, which runs through the length l_0 of the rod twice, from A to B and back to A .

Since the formulas (1.4), (1.5), (1.6) are completely analogous to the formulas in SR, then all the conclusions of special relativity are visually and easily displayed in the constructed model of SR.

In Fig.1, we can explicitly show the value of the velocity v . Since $c' = \sqrt{c^2 - v^2}$ or $c^2 = c'^2 + v^2$, which is the equation of the circle, we get Fig.1b. Figure 1b shows that for $v \ll c$ we obtain $l'/l_0 \approx 1$ and $\Delta t'/\Delta t_0 \approx 1$, which is a transition from Lorentz transformations to Galileo transformations. For $v > c$, the SR-model becomes meaningless. The constancy of the speed of light c in the model is shown by the constancy of the radius of circles in Fig.1b, regardless of the value of the velocity v (that is, regardless of the mutual arrangement of the two frames of reference). The form for the velocity $c' = \sqrt{c^2 - v^2}$ is due to the fact that the signal transmission rate has a limit and the speed of light in vacuum is the highest signal transmission rate. The existence of a limit

for the velocity v and, consequently, the very form of the equation $c^2 = c'^2 + v^2$, is physically explained in the *Field model of the inert and heavy mass* (see chapter 2).

In the SR-model, you can define the so-called "event space". It is a half-plane over the line DD' , where each point can be characterized by time and place. Let us consider how the problem of simultaneity of two events is displayed in the model. Let light signals be emitted from the point M lying in the middle between the points A and B to the points A and B . An observer at point D will find out that these signals will come to points A and B simultaneously. However, at the points A' and B' from the point of view of the observer in D these signals will not come simultaneously. Thus, the concept of simultaneity becomes relative, depending on which frame of reference this process is being examined. An observer will arrive at similar conclusions at the point D' .

From the SR-model it is also seen that the "shortening" of the length of the rod l' is closely related to the concept of the simultaneous arrival of light signals to the ends of the rods. Indeed, if we send light signals from the point M (Fig.1) to the points A and B , then the observer in D will find out that these signals will come to the points A and B simultaneously. In relation to the rod $A'B'$, the light signals will come simultaneously to the points A' and B'' . But the distance $A'B''$ is the "reduced" length l' . Thus, with respect to the rod $A'B'$, the SR-model adequately reflects the "contraction" of the original length. And, as in SR, in the SR-model (Fig.1a), this "reduction" is also connected with the concept of simultaneity.

Note that the length of the rod can also be determined in such a way that the positions of the ends of the rod $A'B'$ that are simultaneous in the improper reference system are measured. That is, here the light signals must be sent from the middle of the rod $A'B'$ to points A' and B' . In this case, the Lorentz transformations will be followed not by a "reduction" but by an "increase" in the length of the rod. In the SR-model in Fig.1, this is reflected in the fact that, relative to the rod AB from the point of view of the observer in D , the light signals will come simultaneously to the points A and C , and the initial length of the rod $A'B'$ will appear "increased" and equal to AC . In this case, instead of the previous relation, we would have the following equation

$$l' = \frac{l_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (1.7)$$

However, relativistic physics prescribes, when measuring the length, to make a simultaneous reading in the system in which the measurement is made, and thus eliminates the ambiguity of the results. The considered example of the length relativity clearly indicates that the length of the object is not some absolute property associated with the very existence of the object, but, on the contrary, the numerical value to be compared with the length depends on the conditions of the measurement.

As noted by Pauli, Lorentz reduction is not a property of one scale, but represents a fundamentally observed mutual property of two scales that move relative to each other. It is satisfactory to regard the relative motion as the cause of the Lorentz contraction, since this latter is not a property of one scale, but a ratio between two scales.[1] The above remark of W.Pauli is reflected in our model in Fig.1a by the presence of two rods AB and $A'B'$.

What does it mean to reduce the length of the ruler? First of all, it is clear that no compression of the ruler can occur. This follows from the basic principle underlying

the SR, the principle of the equality of all inertial reference frames (IRF). In all IRF, the physical state of the ruler is the same. Therefore, there can be no question of the occurrence of any stresses leading to deformation of the ruler. The "shortening" of the ruler occurs solely due to various methods of measuring the length in two frames of reference. On the other hand, the detectable relativity of the length of the ruler is not an illusion of the observer. This result is obtained for any reasonable method of measuring the length of a moving body. Moreover, considering the physical phenomena in a given frame of reference, it is necessary to take the length l' for the length of the body, and not the length l_0 . [2]

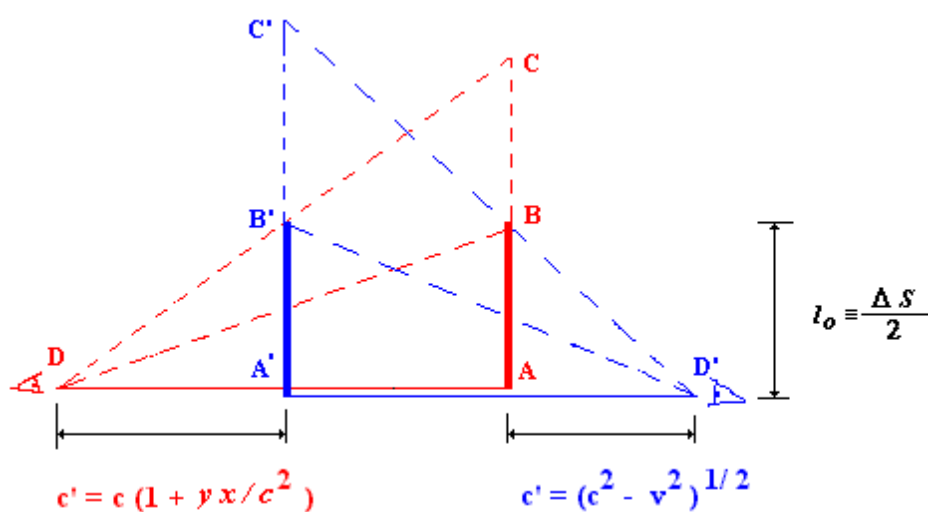


Figure 2: Model of special relativity

In SR, the speed of light is determined from the expression $\Delta S = 2l_0 = 0$. How this situation is displayed in the SR-model. In this case, for an observer in D , the length of the rod AB is 0, i.e. own reference system no longer exists. There remains only a light signal and there is nothing to correlate with. That is, the light signal can not be a reference system. For light there is no own frame of reference. If light signals are taken as clocks, then these clocks do not go, they cost. The counting of the time process (movement of the light beam) can occur only with respect to the rods AB or $A'B'$, but not with respect to itself.

It is seen from the model that the invariant interval ΔS maps the immutable extent of the moving body l_0 , which the light signal passes twice (back and forth). This interval is expressed in terms of the characteristics of the light signal r and $c\Delta t'$.

In the model, one can display the situation when one of the reference frames moves uniformly-accelerated (Fig.2).

In this case, the quantity c' (in Fig.2 on the left) will have the form

$$c' = c(1 + \gamma x/c^2) \quad (1.8)$$

where γ is the uniform acceleration. On the right as before, c' has the form $c' = \sqrt{c^2 - v^2}$. As can be seen from Fig.2, the symmetry of two frames of reference (their equality) is already lost. The change in speed c' under the influence of acceleration naturally changes the speed of the remaining time processes in the accelerated frame of reference. Such a change in the velocity c' in the accelerated frame of reference and solves the so-called "twin paradox".

Thus, the SR-model constructed adequately reflects the spacetime relations in the SR and, by studying the model, we can better understand the essence of the special of relativity.

1.2 On the philosophy of special relativity

In support of all of the above, we point out an article by the well-known physicist and close friend of Einstein, Nobel laureate Max Born, "Physical reality" [3] in which he emphasizes that the essence of the special theory of relativity lies in the logical distinction between the fact that often the measured quantity is not a property of the object, but the property of his attitude to other subjects and shows examples of this. As an example, he shows a figure from a cardboard circle and shadows, which he throws from a remote lamp on a flat wall. Rotating this cardboard shape, you can get any value of the length of the axis of elliptical shadows from zero to maximum. This is an exact analogy with the behavior of length in the theory of relativity, which in any state of motion can have any value between zero and maximum. A similar example Max Born also shows with respect to mass behavior in the theory of relativity.

Max Born shows that the majority of dimensions in physics do not refer to the things that interest us, but to some kind of their *projections* in the broadest sense of the word. *Projection* is defined relative to the reference frame. In the general case, there are many equivalent frames of reference. In any physical theory, a rule is given that links the *projection* of one and the same object to different frames of reference. This rule is called the law of transformations (in the special of relativity it is Lorentz transformations); all these transformations have the property that they form a group, that is, the result of the two subsequent transformations is a transformation of the same kind. Invariants are quantities that have the same meaning for any frame of reference and are therefore independent of the transformations. And the main progress in the structure of concepts in physics is to discover that a certain quantity that was considered as a property of an object is in reality only a property of *projection*. And it turns out that in the relativistic theory the maximum length (length l_0) and the minimum mass (rest mass m) are relativistic invariants. The idea of invariants is the key to the rational concept of reality.

Some physicists consider (Max Born attributed to them the well-known physicist Paul Dirac) that there is no need to be interested in the question of whether there is anything behind the projections. Max Born states that behind projections is physical reality, which is displayed through *projections*. We observe only *projections*, which are variable and dependent on devices (frames of reference). But their combination makes it possible to find the properties of reality itself, no longer dependent on instruments. These ways of transition from *projections* to the reality itself are developed by the theory

of invariants. At the same time, *projections* can not be denied in reality only because they are not invariant. *Projection* is the result of the actual interaction of an object with a reference system. Example: the thermal effect of the solar disk (the projection of the Sun) on the observer depends on the distance between the observer and the Sun. But the physical reality is only the Sun itself, and not its *projection*.

We quote a quote from another author confirming this point of view. Spatial-temporal relations and properties of bodies do not depend on the frame of reference, but only differ in different systems. In general, physical quantities that depend on the reference frame and in this sense are relative, are a kind of *projection* of more general quantities that do not depend on the reference frame. In accordance with this, Minkowski gave a four-dimensional formulation of the laws of relativistic mechanics and electrodynamics. Nevertheless, Minkowski's view of the theory of relativity was not accepted by physicists in all its depths. The point of view of relativity, taking any phenomenon in relation to one or another frame of reference, was more familiar, firstly, because this is the real position of the experimenter, the observer, and secondly, because the theorist also considers phenomena using this or that coordinate system. But there was also a third point - positivist philosophy, which essentially attached importance to reality only to what was given in direct observation; all the rest that is contained in the theories of physics is interpreted by it not as an image of reality, but as a construction that only links the observational data. From this point of view, Minkowski's four-dimensional world is nothing more than a scheme that does not reflect any reality beyond what is already expressed in the original exposition of the theory of relativity. Thus, two different approaches to the theory of relativity were defined. The first is Minkowski's approach, which is based on the idea of spacetime as a real absolute form of the existence of the material world. The second is a purely relativistic approach; the main thing in it is one or another frame of reference. It is clear that the first approach is of a materialistic nature and corresponds to the natural logic of the object: "its form determines its relative manifestations." The second approach turns out to be positivistic, denying that the relative is only a facet of the manifestation of the absolute. [4]

In conclusion, we note that the SR-model describes the spatial-temporal characteristics of the light signal. This may indicate that massless quanta of energy are the basis of spatio-temporal processes and material formations. This hypothesis is developed in the construction of the *Field model of inert and heavy mass* (see chapter 2). In the field mass model, the reason for the impossibility of motion of weighty bodies with a velocity exceeding the speed of light is substantiated.

2 About the mass

2.1 Field model of inert and heavy mass

Consider a thought experiment.

Let the reference frame K' contain a weightless cylinder of height h (Fig.3). We denote the upper cylinder cover by S_2 , the lower one by S_1 . Let this frame of reference K' , together with a weightless cylinder rigidly fixed to it, move in the direction of positive values of z with the acceleration γ . Suppose that a photon with energy E_0 is emitted

from S_2 in S_1 and we consider this process in some system K^0 , which does not have acceleration. Suppose that at the moment when the radiation energy is transferred from S_2 to S_1 , the system K' has a speed equal to zero relative to the system K^0 . The light quantum will reach S_1 after the time h/c (in the first approximation), where c is the speed of light. At this moment, S_1 has a velocity $\gamma h/c = v$ relative to the system K^0 . Therefore, according to the special of relativity, the radiation reaching S_1 does not have the energy E_0 , but has a large energy E_1 , which is in first approximation related to E_0 by the relation [5]

$$E_1 \approx E_0 (1 + v/c) = E_0 (1 + \gamma h/c^2) \quad (2.1)$$

The momentum transmitted by the radiation to the wall S_1 is equal to

$$P_1 = E_1/c = E_0 (1 + \gamma h/c^2)/c \quad (2.2)$$

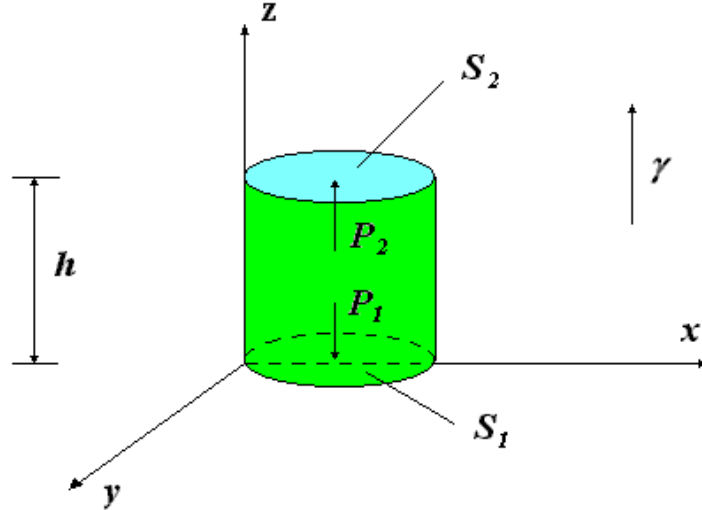


Figure 3: Model of mass

Let a light quantum with the same energy E_0 be emitted from S_1 toward S_2 . Then the radiation energy reaching the wall S_2 and the transmitted momentum will have the following form

$$E_2 \approx E_0 (1 - v/c) = E_0 (1 - \gamma h/c^2) \quad (2.3)$$

$$P_2 = E_2/c = E_0 (1 - \gamma h/c^2)/c \quad (2.4)$$

If in the system K' we simultaneously emit two quanta of light of the same energy - one toward S_1 and the other towards S_2 , the recoil momentums are compensated, and

the momentums (2.2) and (2.4) will play the main role. We get

$$\Delta P = P_1 - P_2 = (2E_0/c^2)(\gamma h/c) = 2m \Delta v \quad (2.5)$$

where $2m = 2E_0/c^2$ is inert mass; coefficient 2 corresponds to two photons.

The equation for the force F acting for time Δt will have the form

$$F = \Delta P/\Delta t = 2m \Delta v/\Delta t = 2m \gamma \quad (2.6)$$

in accordance with Newton's second law.

Thus, a weightless cylinder in which radiation is located has an inertial mass $2m$. The momentum $\Delta \vec{P}$ of this inertial mass, as is easily seen from Fig.3, is directed in the direction opposite to the acceleration vector $\vec{\gamma}$. Thus, a weightless cylinder with a cloud of photons inside the cylinder exerts resistance to its accelerating force, which is one of the characteristic manifestations of that physical property called "mass." The inertial mass characterizes the resistance exerted by the body to any change in its velocity both in magnitude and in direction. In modern physics, the phenomenon of inertia of massive material bodies is still an unsolvable riddle. The above mechanism removes the aura of mystery from the phenomenon of inertia of massive bodies.

So, a massive particle can be represented as a weightless vessel with mirror walls, in which there is a cloud of massless particles. Momentums of particles on average are uniformly transmitted to mirror walls in all directions and the vessel will be in a state of relative rest (i.e. *dynamic equilibrium*). When the vessel is accelerated, the total transferred to the vessel walls by massless particles becomes nonzero in the direction opposite to the acceleration vector. This manifests itself in the form of inertness of the vessel. When the force applied to the vessel disappears, the *dynamic equilibrium* of the vessel is restored, but the vessel acquires velocity v relative to the primary frame of reference. The inertial mass model clearly shows that the inertia of material bodies is their internal property and the Mach principle to tangible material bodies is inapplicable. Such a cylinder will have inertia in the absence of a horizon of distant stars.

In accordance with the principle of Mach, it is assumed that the gravitational field is the source of inertia of bodies in the universe. However, although the Mach principle played a definite heuristic role in the construction of the theory of relativity, there is reason to believe that in this formulation it is unacceptable. As was shown above, the bodies do not cease to possess inertial properties, even in an empty, mass-free space.

It is also clear why the huge energy reserves are "hidden" in the mass. This field energy is only partially released in nuclear fission processes in nuclear reactors, as well as in nuclear fusion processes spontaneously flowing in the Sun and other stars.

Further. Let the weightless cylinder (Fig.3) do not accelerate, but is located on a support and is in the weak gravitational field of the Earth. Suppose that in S_1 the field potential is equated to zero, and at the height h it is equal to φ . Taking into account the equivalence principle, we can write $\gamma h = \varphi$. Now let a light quantum with energy E_0 be emitted from S_2 in S_1 . Then the energy and momentum of the photon change according to the relations

$$E_1 \approx E_0 (1 + \varphi/c^2) \quad (2.7)$$

$$P_1 = E_1/c = E_0 (1 + \varphi/c^2)/c \quad (2.8)$$

On the other hand, emitting a photon of energy E_0 from S_1 to S_2 we obtain

$$E_2 \approx E_0 (1 - \varphi/c^2) \quad (2.9)$$

$$P_2 = E_2/c = E_0 (1 - \varphi/c^2)/c \quad (2.10)$$

As a result, the difference of P_1 and P_2 is equal to

$$\Delta P = P_1 - P_2 = (2E_0/c^2)(\Delta\varphi/c) = 2m(\Delta\varphi/c) \quad (2.11)$$

and directed towards the center of the Earth. Here $2m = 2E_0/c^2$ is a heavy mass. Therefore, the force acting on S_1 , is

$$F_z = \Delta P/\Delta t = -2m(\Delta\varphi/c\Delta t) \quad (2.12)$$

For light in the field of the Earth vertically $c\Delta t = \Delta z$, then $F_z = -2m(\Delta\varphi/\Delta z)$ or, more generally

$$F(\vec{r}) = -2m \text{grad} \varphi(\vec{r}) \quad (2.13)$$

where $\varphi(\vec{r}) = -GM/r$; G is a gravitational constant; M is the mass of the Earth.

We have received expression for the force of gravity acting on the cylinder, it follows from Newton's theory of gravitation.

It follows from the mass model that the free motion of a material structure in a gravitational field (dip or motion along a geodesic) is associated with a constant redistribution of the momentums of massless energy quanta with respect to the structure of the body. Removing the stand, we force the cylinder to move under the action of the difference of the photon momentums ΔP , as a result of which it appears in the region of the field with a larger potential difference $\Delta\varphi'$ than at the previous time. This generates a large difference in the momenta $\Delta P'$ and the process repeats. This is how the cylinder accelerates in the gravitational field.

Thus, the field model of mass adequately reflects the inert and heavy properties of massive bodies.

2.2 Philosophy of Mass

The above approach interprets the general theory of relativity as a method of describing the gravitational field with the help of the concept of *curved spacetime*. Indeed, according to the modern approach, spacetime is not a thing that can be "warped". This is an abstract mathematical model of what we call ordinal and metric relations: we can measure distances and time intervals, events occur in a certain sequence, and so on. The very relationships are determined by the distribution of matter, its motion and interaction. The model of all this is *spacetime*. The results of our measurements are in better agreement with the curved spacetime than with the flat one.

Light, possessing energy, is thus subject to the influence of the gravitational field. This circumstance affects the law of the propagation of light, and hence, on the general laws of establishing spacetime relations. In other words, the presence of a gravitational field should exert a certain influence on the properties of spacetime, which actually takes place. The gravitational field, through the influence of the light propagating through its influence, determines the geometry of space. *The movement of the rays of light determines*

the spatio-temporal structure of the world. On the other hand, the physical basis for introducing the concept of "curved spacetime" for massive bodies and their free motion along geodesic trajectories is the field model of the inert and heavy mass described above, formed by massless energy quanta. It becomes understandable the universal nature of the effect of the gravitational field on all material bodies in nature (massless and massive). It consists in the effect of the gravitational field on the character of the propagation of massless energy quanta in the free and bound states. Thus, *the basis of the observed world is the massless form of matter.*

It is obvious that a cylinder with weightless walls can be replaced by a system of two (or more) interacting massless particles. Massless particles oscillate within certain limits, repeatedly reflected from imaginary "walls", which are nothing but *a potential barrier*. A bound system of such particles will have the same inert and heavy properties as the above-discussed weightless cylinder. Here, Einstein's assumption is justified, that *matter is the concentration of field energy in a small space.*

Modern physics has come to the conclusion that in the Planck scale all matter particles (both real and virtual) do not yet have masses (that is, they are all like a photon) and, therefore, the above mechanism of mass formation applies to them.

The constructed model of the inert and heavy mass is a universal model, since inertia and gravity in matter can not arise in various ways.

An interesting question is whether our weightless cylinder can move at a speed exceeding the speed of light? Since our cylinder is analogous to the aggregate of massless quanta of energy, connected by some kind of interaction, the question of whether it reaches a speed greater than the speed of light amounts to the following: will light be able to overtake the light? The answer is obvious - can not. The system of two coupled photons, as we have shown above, possesses inertial properties and therefore will move with a velocity less than the speed of light or will be at rest. It is clear that the speed of light for such a system will be the limiting speed. A body made up of light can not move faster than light.

From the foregoing it follows that the massless form of matter is primary, fundamental, and the massive form of matter is a secondary, derivative form. We arrived at the same conclusion when constructing a model of special relativity.

Indeed, under conditions (at least) of the Great Unification, the processes of electromagnetic, weak and strong interactions are indistinguishable. This state has such characteristic features: 1) Particles of matter (both real and virtual) "for the time being" do not have masses (that is, they are like a photon). 2) The potential energy of particle interaction is "regulated" by special scalar Higgs fields.

The primary nature of the field can also be traced in quantum mechanics. As a result of the quantization of the field, the concept of a particle arises as a characteristic of the excitation of an electromagnetic wave with a certain length. The idea of perceiving particles as quantum states of oscillators of a certain field turned out to be fruitful. It permeates all modern theoretical physics. The field is the primary concept. Elementary particles arise as a result of its quantization.

2.3 Dirac equation and the correspondence principle

The Dirac equation can be proved with the help of the correspondence principle. The energy and momentum of a particle can be expressed by the equation

$$E^2 = p_1^2 c^2 + p_2^2 c^2 + p_3^2 c^2 + m^2 c^4 \quad (2.14)$$

This equation can be divided into E on both sides. We obtain

$$E = \frac{v_1}{c} p_1 c + \frac{v_2}{c} p_2 c + \frac{v_3}{c} p_3 c + \frac{v_0}{c} m c^2 \quad (2.15)$$

where $v_0 = \sqrt{c^2 - v^2}$ and $v^2 = v_1^2 + v_2^2 + v_3^2$

Really

$$\frac{p_1 c}{E} = \frac{m v_1 c (c/v_0)}{m c^2 (c/v_0)} = \frac{v_1}{c}$$

and so on, and

$$\frac{m c^2}{E} = \frac{m c^2}{m c^2 (c/v_0)} = \frac{v_0}{c}$$

The Dirac equation has the form

$$i\hbar \frac{\partial \psi}{\partial t} = (\alpha_1 \hat{p}_1 c + \alpha_2 \hat{p}_2 c + \alpha_3 \hat{p}_3 c + \alpha_0 m c^2) \psi \quad (2.16)$$

where α_j is matrix ($j = 0, 1, 2, 3, \dots$). We obtain from (2.15) and (2.16): $v_j/c \rightarrow \alpha_j$.

In fact, in quantum mechanics it shows that the relativistic velocity operator $v_\nu = dx_\nu/dt$; ($\nu = 1, 2, 3$) has the form $v_\nu = c\alpha_\nu$, ie is a matrix operator (see [8] p.340-342).

Really

$$\frac{dx_\nu}{dt} = \frac{\partial x_\nu}{\partial t} + [H, x_\nu] \quad (2.17)$$

where

$$H = \alpha_\nu p_\nu c + \alpha_0 m c^2 \quad (2.18)$$

Since the operator x_ν does not depend on time, it will be $dx_\nu/dt = [H, x_\nu]$. We get

$$\frac{dx_\nu}{dt} = [(\alpha_\mu p_\mu c + \alpha_0 m c^2), x_\nu] \quad (2.19)$$

The matrix α_μ commutes with x_ν , so that the matrix α_μ can be factored out. Finally we have

$$dx_\nu/dt = c\alpha_\mu [p_\mu, x_\nu] = c\alpha_\mu \delta_{\mu\nu} = c\alpha_\nu \quad (2.20)$$

The eigenvalues of the matrix of the velocity operator equal to $\pm c$, but as the speed of the operator does not commute with the Hamiltonian operator, then the experience is always measured by the average value of the relativistic velocity operator, and it is less than c .

Thus, the correspondence between equations (2.15) and (2.16) is confirmed.

The Dirac equation predicts a fast oscillating motion of an elementary particle - "trembling motion". To understand the effect of electron jitter, it suffices to adopt an inert and heavy mass model. According to this model, a massive particle is a hollow weightless cylinder with internal mirror walls and a cloud of photons inside. Suppose

that the electron is a cylinder with one photon. Reflecting from one wall, the photon transmits its impulse to the cylinder. The cylinder moves to the right at the speed of light c . Having reached the opposite wall and having reflected already from it, the photon also transmits its impulse to it and the cylinder shifts to the left at the speed of light c . And the process is repeated. As a result, a jitter of a weightless cylinder simulating a massive electron occurs. From a distance, it seems that the cylinder is at rest. The length of the cylinder is $l = 10^{-13}m$, the speed of light is $c = 3 * 10^8 m/sec$, so the jitter frequency is $\nu \sim c/l \approx 10^{21} Hz$. [9]

In the next chapter, following the accepted logic, we will consider geons - massive particles made up of massless energy quanta.

3 Geons

Introduction

In the paper [7], p.525, the geons are defined as follows. This metastable union of the energy of electromagnetic or gravitational waves, held together by its own gravitational attraction.

The following considerations are used in constructing geons. The gravitational acceleration required to keep radiation in a circular orbit of radius r , in order of magnitude, is c^2/r . The acceleration due to gravitational attraction in a cluster of radiant energy with mass M is of the order of magnitude GM/r^2 , where G is the Newtonian gravitational constant. Both these accelerations coincide in order of magnitude when the radius $r \sim GM/c^2$. Under these conditions, it is possible to obtain a radiation bundle that keeps itself by its own gravitational field. [10] p.64–66.

In this case, the geons are an unquantized classical mass that has nothing to do with the physics of elementary particles.

In this paper we will consider a system consisting of two gravitationally interacting photons. It will be shown that at a certain energy the classical geon turns into the Planck geon, i.e. into a particle with size $\ell_P = 10^{-33}$ cm, mass $m_P = 10^{-5}$ g and a complex internal structure. The Planck geon is a micro black hole in spacetime. Probably, such objects could appear in the first fractions of a second of the "Big Bang". Therefore, a theoretical analysis of the occurrence of Planck geons is of some interest. Planck geons can act as the fundamental basis of massive elementary particles and be an alternative to string theory. Within the framework of the model of geons, the three-dimensionality of the observed space is substantiated.

3.1 Qualitative quantum-theoretical analysis of the formation of geons

From general relativity it is known that any form of energy, including the energy of massless quanta, is capable of generating a gravitational field. It follows that two single photons can interact with one another gravitationally and thus form a connected system - the geon.

In the classical physics of Newton, the potential energy E_{pot} generated by the gravi-

tational fields of the masses M and m has the form

$$E_{pot} = -\frac{GMm}{r} \quad (3.1)$$

where G is Newton's gravitational constant, M and m are gravitating masses, r is the distance between masses.

We use the relation (3.1) as applied to a system of two gravitationally interacting photons of the same energy. It is possible to show [6], p.25, that for photons instead of masses M and m it is necessary to substitute the values of the momenta of photons divided by the speed of light, i.e. P/c . Then (3.1) can be rewritten as follows

$$E_{pot} = -\frac{GP^2}{c^2r} \quad (3.2)$$

With qualitative analysis, the analogy of photons with massive particles is acceptable. Therefore, the problem of the motion of two photons interacting only with one another can be reduced to the problem of the motion of a single photon. The "reduced" momentum of a system of two identical photons is $P' = P/2$, where P' is the "reduced" momentum, P is the momentum of each of the photons.

Then the total energy of the geon (in the first approximation) takes the following form

$$E = E_{kin} + E_{pot} = P'c - \frac{GP^2}{c^2r} = \frac{Pc}{2} - \frac{GP^2}{c^2r} \quad (3.3)$$

where $E_{kin} = P'c$ is the kinetic energy of a system of two photons; c in the first term - the relative speed of photons, equal to the speed of light.

Equation (3.3) can be rewritten as follows

$$E = \frac{Pc}{2} \left(1 - \frac{2GP}{c^3r}\right) = \frac{Pc}{2} \left(1 - \frac{r_g}{r}\right) \quad (3.4)$$

where $r_g = 2GP/c^3$ is the so-called gravitational radius of the geon, which, as is easy to see, is similar in form to the gravitational radius of ordinary particles, which has the form $r_g = 2GM/c^2$. But in the geon M must be replaced by P/c .

We note that equations (3.3) and (3.4) are valid not only for massless particles, but also for massive ultrarelativistic microobjects. In this paper, we focus only on the properties of massless particles, as a more fundamental (from the point of view of the author) form of matter.

Equation (3.3) is analogous to the equation for the total energy of a hydrogen atom. It is known from quantum mechanics that it is possible to estimate the energy of the ground state of a hydrogen atom using the Heisenberg uncertainty relation. We will do the same in this case. In order to use equation (3.3) in quantum theory (in the qualitative approximation), we consider the quantities P and r entering into equation (3.3) as the uncertainties of the momentum and the coordinate. We note that r in (3.3) characterizes the size of the region occupied by the geon. On the other hand, r can be treated as the radius of curvature of the trajectory of photons.

According to the uncertainty relation, the quantities P and r are related to each other. Let $Pr = \hbar$, where \hbar is the reduced Planck constant. Using this relation, we eliminate r from (3.3). We get

$$E(P) = \frac{Pc}{2} - \frac{GP^3}{\hbar c^2} = \frac{Pc}{2} \left(1 - \frac{2P^2}{P_P^2}\right) \quad (3.5)$$

where $P_P = \sqrt{\hbar c^3/G}$ is the Planck's momentum.

The function $E(P)$ has a maximum at some value $P = P_1$. We denote it by E_1 . The value of E_1 can be considered as an estimate of the energy of the ground state of the geon, and $r_1 = \hbar/P_1$ as an estimate of the linear dimensions of the geon. Equating the derivative dE/dP to zero, we find that

$$P_1 = \sqrt{\hbar c^3/6G}; \quad r_1 = \hbar/P_1 = \sqrt{6\hbar G/c^3} \approx 10^{-33} \text{cm}; \quad E_1 = \sqrt{\hbar c^5/54G} \approx 10^{19} \text{Gev} \quad (3.6)$$

As can be seen from the estimates obtained, a geon consisting of two gravitationally interacting photons has Planck length and Planck mass.

Using the uncertainty relation, we find from (3.3) the function $E(r)$. We have

$$E(r) = \frac{\hbar c}{2r} - \frac{2\hbar G}{c^2 r^3} = \frac{\hbar c}{2r} \left(1 - \frac{2\ell_P^2}{r^2} \right) \quad (3.7)$$

where ℓ_P is the fundamental Planck length.

The graph of the function $E(r)$ has the form (see Fig.4).

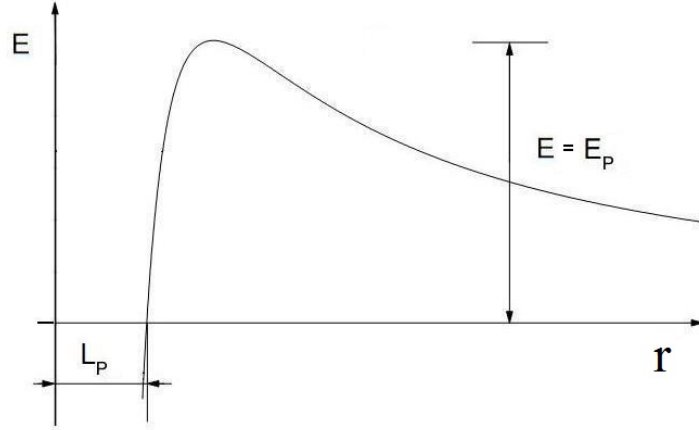


Figure 4: The graph of the function $E(r)$

It can be seen from Fig.4 that on energy level 10^{19} GeV, the massless energy quanta interact with each other, turning into microscopic black holes with a size of 10^{-33} cm. The point of intersection of the graph of the function $E(r)$ with the r axis in Fig.4 corresponds to the event horizon, separated from the singular state of the geon $r = 0$ at a distance of $\sqrt{2}\ell_P$.

A complete solution of the problem of the motion of a particle in a central field can be obtained by starting not only from the laws of conservation of energy, but also from the moment. It is clear that for a more complete analysis it is necessary to turn to the general theory of relativity, which describes strong gravitational fields.

3.2 Geons in general relativity

Let us consider the motion of a "reduced" photon in a centrally symmetric gravitational field (a system of two gravitationally interacting photons). As in any central field, the

motion will occur in one plane passing through the origin; we choose this plane as the plane $\Theta = \pi/2$. We use the Hamilton-Jacobi equation, taking into account the fact that the mass of the particle is zero. [11], § 101

$$g_{ik} \frac{\partial S}{\partial x_i} \frac{\partial S}{\partial x_k} = 0 \quad (3.8)$$

where S is the action.

We take the coefficients g_{ik} from the Schwarzschild solution. Then we obtain the equation of motion of the "reduced" photon in a centrally symmetric gravitational field

$$e^{-v} E^2 - e^v P'^2 c^2 - \frac{N^2 c^2}{r^2} = 0 \quad (3.9)$$

where $e^v = 1 - r_g/r$ and $r_g = 2GP_r/c^3$ is the gravitational radius of the geon; $P' = P_r/2$; P_r is the momentum of each of the photons; N is the orbital angular momentum of the "reduced" photon; Nc/r is the centrifugal energy of the "reduced" photon.

We rewrite (3.9) as follows

$$E^2 = \left(1 - \frac{2GP_r}{c^3 r}\right)^2 \frac{P_r^2 c^2}{4} + \left(1 - \frac{2GP_r}{c^3 r}\right) \frac{N^2 c^2}{r^2} \quad (3.10)$$

Equation (3.10) is the basic equation for the total energy of the geon. Its exact solution will be of great importance for Planck physics. If $N = 0$, then we return to equation (3.4). We discuss the equation (3.10) we found using simple qualitative arguments. Our conclusions will not claim rigor and completeness, and they can be regarded more as an exploration of the ways of further research than as clearly formulated results. Instead of an exact solution, we confine ourselves to qualitative consideration.

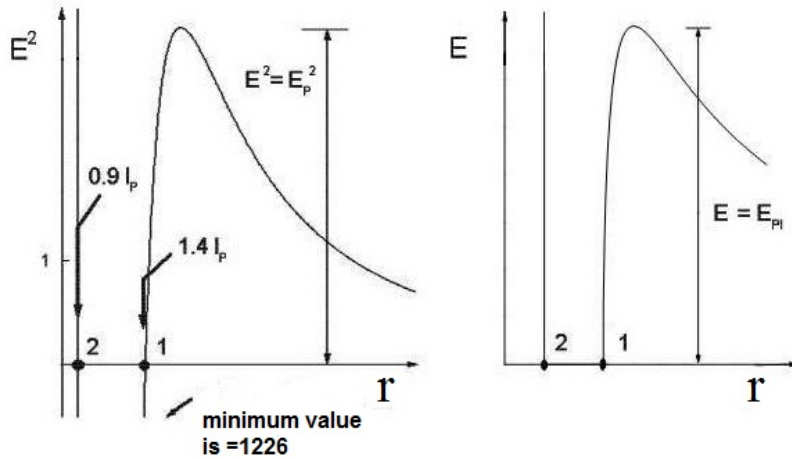


Figure 5: A graph of the function $E(r)$ with allowance for the angular momentum of the geon

Let $2\pi r$ be the length of the n -th Bohr orbit. A "reduced" photon with a de Broglie wavelength $\lambda_r = 2\pi\hbar/P_r$ moves along the orbit. On the length of the orbit, the wavelength of the "reduced" photon λ_r should fit n times. Hence $2\pi r = n\lambda_r$

Hence we obtain the Bohr condition for quantizing the orbits $P_r r = n\hbar$ or $P_r = n\hbar/r$, where n is the principal quantum number.

Further. The observed value of the square of the angular momentum of a micro object is expressed by the formula

$$N^2 = \hbar^2 l(l+1) \quad (3.11)$$

where l is an integer.

If we take into account that the total angular momentum of the N "reduced" photon consists of two terms: the orbital angular momentum l and the spin moment s , which are added in vector form, then (3.11) is rewritten as follow

$$N^2 = \hbar^2 j(j+1) \quad (3.12)$$

where j is the quantum number of the total angular momentum. We substitute $n\hbar/r$ instead of P_r , and $\hbar^2 j(j+1)$ instead of N^2 , then we get

$$E^2 = \frac{n^2 \hbar^2 c^2}{4r^2} \left(1 - \frac{2\ell_P^2 n}{r^2}\right)^2 + \frac{\hbar^2 c^2}{r^2} \left(1 - \frac{2\ell_P^2 n}{r^2}\right) [j(j+1)] \quad (3.13)$$

where ℓ_P is the fundamental Planck length, which here appears automatically due to the above expression for the gravitational radius of the geon.

The total angular momentum significantly affects the graph of the function $E(r)$ of the total energy of the geon near the singular point $r = 0$. Indeed, let $n = 1, N = 1$.

Then the graph of the function $E(r)$ will have the form (see Fig.5).

From Fig.5 that the presence of a centrifugal energy Nc/r changes the behavior of the geon near the singular point $r = 0$. Geon in this case has two event horizons - external and internal (points 1 and 2), separated by an interval of $0.9\ell_P$. The singular state is reached by the geon at $r = 0$. However, from Fig.5 that when the geon approaches the singular state its total energy increases, which corresponds to repulsion from the singularity. Thus, the region r , less than $0.5\ell_P$, corresponds to antigravity. The growth of the total energy of the geon and, accordingly, the repulsion from the singular point is due to the centrifugal energy of the geon.

In the case when massless energy quanta also have charges, the expression for e^v from the relation (3.9), according to the Riesner-Nodstrom solution, must be written as follows

$$e^v = 1 - \frac{r_g}{r} + \frac{GQ^2}{c^4 r^2} \quad (3.14)$$

where Q is the total charge of the geon.

Then equation (3.10) for the total energy of a charged geon takes the form

$$E^2 = \left(1 - \frac{r_g}{r} + \frac{GQ^2}{c^4 r^2}\right)^2 P_r^2 c^2 + \left(1 - \frac{r_g}{r} + \frac{GQ^2}{c^4 r^2}\right) P_\varphi^2 c^2 \quad (3.15)$$

where $P_\varphi = N/r$.

Recall now that the particle charge can be expressed in terms of the fine-structure constant α : $Q^2 = \alpha \hbar c$.

Note that the fine structure constant α for $v = c$ in Planck scales should be equal to 1. Here gravitation becomes a strong interaction. Since a charged massless quantum moves at the speed of light, it must have: $Q^2 = \hbar c$. Then

$$\frac{GQ^2}{c^4 r^2} = \frac{G\hbar}{c^3 r^2} = \frac{\ell_P^2}{r^2} \quad (3.16)$$

For qualitative analysis substitute $n^2 \hbar^2 / r^2$ in place of P_r^2 in (3.15); instead of P_φ^2 the value $\hbar^2 j(j+1) / r^2$; instead of the charge Q^2 the value $\hbar c$. Then we obtain the following equation

$$E^2 = \frac{n^2 \hbar^2 c^2}{4r^2} \left(1 - \frac{n\ell_P^2}{r^2}\right)^2 + \frac{\hbar^2 c^2}{r^2} \left(1 - \frac{n\ell_P^2}{r^2}\right) [j(j+1)] \quad (3.17)$$

This equation is analogous to equation (3.13).

We emphasize that in this section, when we are talking about a geon, we do not mean simply the union of two photons into a system such as a hydrogen atom. Such a system, as seen from Fig.4, at the maximum point is unstable and can not exist for a long time. When we talk about the geon, then by this word we mean primarily two photons that have collapsed into the black hole state. It is exactly such an object, having Planck dimensions and mass, we call the Planck's geon and it interests us first of all.

We also note that now we no longer have the right to write an expression for the invariant interval dS without metric coefficients g_{ik} , since in Planck scales they can not be equated to unity even in inertial frames of reference (see chapter 5).

Indeed, in the case of the Schwarzschild solution, the interval has the following form

$$dS^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \quad (3.18)$$

where $r_g = 2kM/c^2$

This solution describes the geometric properties of spacetime, caused by the point mass M , located at the beginning of the spherical coordinate system. At large distances, when r_g/r can be neglected in comparison with 1, the expression for the interval (3.18) goes over into the interval of the special of relativity, written in a spherical coordinate system, namely

$$dS^2 = c^2 dt^2 - dr^2 - r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \quad (3.19)$$

The Schwarzschild metric (3.18) describes, in good agreement with the experiment, three known effects: the displacement of the perihelion of the planet's orbit, the deviation of the sun's rays and the gravitational red shift of the spectral lines. However, now we see that in the small (Planck) scales the interval (3.19) must inevitably have the form

$$dS^2 = \left(1 - \frac{2\ell_P^2}{r^2}\right) c^2 dt^2 - \left(1 - \frac{2\ell_P^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \quad (3.20)$$

It is clear from (3.20) that even in the micro world, up to distances of 10^{-30} cm, we can still use the interval (3.19) with great accuracy, i.e. up to these distances, special relativity is valid. See also equations (5.13) and (5.14).

Equations (3.10) and (3.15) for the total energy of the uncharged and charged geons can be investigated not only qualitatively, but also subjected to rigorous mathematical

analysis. Author does not consider this task and offers it for those readers who are interested in this problem. Nevertheless, the above qualitative consideration of the interaction of photons in the Planck scales makes it possible to understand many significant aspects of the behavior of matter at the deepest level of physical reality.

In this regard, we recall that, according to Bohr, the physical picture of the phenomenon and its mathematical description are additional. The creation of a physical picture requires the neglect of details and leads away from mathematical accuracy, and an attempt to accurately describe the phenomenon of the phenomenon makes its clear understanding difficult. The physical picture of the phenomenon is described above.

3.3 Why the space is three-dimensiona

Let us show that within the framework of the geon model, one can answer the question: "Why is the observable space having exactly three dimensions?" We will use the results obtained in due time by P. Ehrenfest. [13]

Ehrenfest considers "physics" in the n -dimensional space U^n . In this case, he derives the law of interaction with the point center (in analogy with the three-dimensional case) from the differential Poisson equation in U^n for the potential that determines this interaction. Fundamental physical laws of interactions are given in a variational form. The Lagrangian for the simplest case of a scalar massless field $\varphi(t, x_1, x_2, \dots, x_n)$ has the form

$$L = \left(\frac{\partial \varphi}{\partial t} \right)^2 - \sum_{k=1}^3 \left(\frac{\partial \varphi}{\partial x^k} \right)^2 \quad (3.21)$$

This Lagrangian leads to the Poisson equation and, consequently, to the point center field $\varphi \sim r^{n-2}$ ($\varphi \sim \ln r$ for $n = 2$). The dimension of space is taken into account in (3.21) only in the form of a condition on the set of values that the index k can take. B $(3 + 1)$ -dimensional case $k = 1, 2, 3$. Thus, (3.21) allows us to obtain the corresponding part of physics in a space of any dimension. The Poisson equation is mathematically equivalent to the indicated Lagrangian (with a natural generalization to other fields).

In the spherically symmetric case in U^n from the Poisson equation or from the Gauss law for the field strength, the expressions for the potential energy have the form

$$E_{pot} = -\frac{k M m}{(n-2)r^{n-2}}; \quad n \geq 3 \quad (3.22)$$

$$E_{pot} = k M m \ln r; \quad n = 2 \quad (3.23)$$

$$E_{pot} = k M m r; \quad n = 1 \quad (3.24)$$

where M, m are the masses of bodies, k is the interaction constant in an n -dimensional space. With the usual Newton constant G , it is connected with the help of the potential matching for the 3-dimensional space and the corresponding n -dimensional space. Then for the gravitationally interacting photons (for example, in the geon or in the Planck black hole), the expressions (3.22), (3.23), (3.24) take the following form (taking into account that instead of the masses M and m we substitute P/c and $P \approx \hbar/r$)

$$E_{pot} = -\frac{kP^2}{c^2(n-2)r^{n-2}} = -\frac{k\hbar^2}{c^2} \frac{1}{(n-2)r^n}; \quad n \geq 3 \quad (3.25)$$

$$E_{pot} = \frac{k}{c^2} P^2 \ln r = \frac{k\hbar^2}{c^2} \frac{\ln r}{r^2}; \quad n = 2 \quad (3.26)$$

$$E_{pot} = \frac{k}{c^2} P^2 r = \frac{k\hbar^2}{c^2} \frac{1}{r}; \quad n = 1 \quad (3.27)$$

The total potential energy of the system includes the centrifugal energy of the geon $P_\phi c = Nc/r$, whose shape, however, does not depend on the dimensionality of space, just as the form for the kinetic energy "of the reduced" of the photon $E_{kin} = P'c$, where $P' = P/2$. On the other hand, the centrifugal energy plays a role only in the third approximation; therefore, in the expressions for the total energy of the geon in the spaces U^n , we will not take it into account (in order to simplify the graphs). Then the equations for the total energy of the geon $E = E_{kin} + E_{pot}$ in the spaces U^n will have the form (provided that $k = c = \hbar = 1$).

$$E(r) = E_{kin} + E_{pot} = \frac{Pc}{2} - \frac{kP^2}{c^2(n-2)r^{n-2}} = \left(1 - \frac{2}{(n-2)r^{n-1}}\right) \frac{1}{2r}; \quad n \geq 3 \quad (3.28)$$

$$E(r) = E_{kin} + E_{pot} = \frac{Pc}{2} + \frac{k}{c^2} P^2 \ln r = \left(1 + \frac{2 \ln r}{r}\right) \frac{1}{2r}; \quad n = 2 \quad (3.29)$$

$$E(r) = E_{kin} + E_{pot} = \frac{Pc}{2} + \frac{k}{c^2} P^2 r = \frac{1,5}{r}; \quad n = 1 \quad (3.30)$$

We construct graphs of the functions of the total energy of the geon $E(r)$ in spaces with dimensions $1, 2, 3, 4, 5, \dots, n$ in accordance with relations (3.28), (3.29), (3.30) (Fig.6).

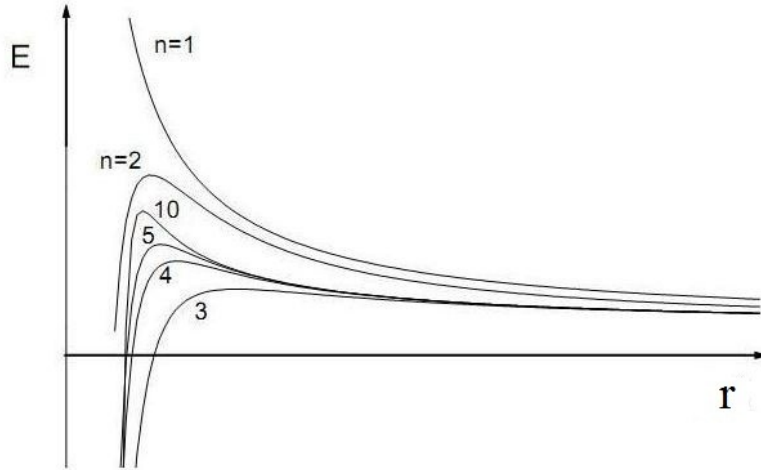


Figure 6: Graphs of functions $E(r)$ in n -dimensional spaces

Fig.6 shows that the maxima of the curves $E(r)$ in the spaces $U_1, U_2, U_4, U_5, \dots, U_n$ lie above the maximum of the curve $E(r)$ in U_3 . This means that the formation of

Planck black holes, from the energy point of view, is most advantageous in U_3 . It is seen from Fig.6 that Planck black holes can be formed in spaces of other dimensions (except U_1), but the minimum photon energy necessary for the formation of Planck black holes is inherent in the 3-dimensional space.

Every system tends to come into a state with a minimum of energy, allocating an excess of its available energy. A system that has a store of energy (an excited system) always has a "desire" to get rid of it, to come to the lowest energy state. Therefore it is quite obvious that, thanks to the mechanism of formation of Planck black holes in n -dimensional spaces, the choice of three-dimensional space from all other possibilities in the formation of the observed Metagalaxy was predetermined.

Indeed, the basis of the "tissue" of the universe is the vacuum at the Planck level. It consists of virtual Planck black holes formed by massless quanta of energy and the generation of which, as was shown above, is energetically most advantageous in a space of dimension three. The property of a space to be n -dimensional at a point p is topologically invariant. Therefore, as a source of the dimension of real spacetime, we must adopt a four-dimensional character (space plus time) of an elementary physical event in the micro world. But at the Planck level, there are no other events except the generation of virtual black holes. Thus, the three-dimensionality of the observed space (or four-dimensionality of spacetime) is due to the "boiling" of Planck's vacuum. In Planck scales, empty space is not at all empty - it is a repository of violent physical processes. And these processes are nothing more than a gravitational collapse, which is continuously and everywhere accomplished. At the same time, a reverse process is also taking place. The collapse under the Planck scale of lengths occurs everywhere and continuously in the form of a quantum fluctuation in the geometry and topology of space. And since the formation of Planck black holes is energetically most profitable in 3-dimensional space, this also determines the three-dimensionality of the observed space.

3.4 Summary

In the chapter 3 the model of the Planck geon is considered, that is, an object with a linear dimension of $\ell_P \approx 10^{-33}$ cm and a mass of $m_P \approx 10^{-5}$. Here we come very close to the region, where the laws of Planck physics work. What conclusions can be drawn?

The fundamental Planck length $\ell_P \approx 10^{-33}$ cm and Planck mass $m_P \approx 10^{-5}$ g, apparently, can appear only in the geon's model. It is here that the constants \hbar , c , G unite naturally. In contrast to the phenomenological concepts of theories, where Planck values are forcibly introduced into the 4-dimensional continuum, within the framework of the geon's model ℓ_P and m_P appear automatically as a consequence of the gravitational interaction of massless energy quanta.

Planck geons can claim the role of "truly elementary particles." In this case, as it appears from the article, "truly elementary particles" ultimately turn out to be microscopic black holes, which, most likely, solves the problem of ultraviolet divergences in quantum field theory. Indeed, as noted in [14], p.469, numerous attempts to introduce a fundamental length in the framework of the special theory of relativity in order to construct a theory that is free from divergences inevitably leads to a violation of the causality principle. However, in the same place [14], p.479, it was stated that within the framework of the general theory of relativity, the length ℓ_P would deprive the concept of space inside the Schwarzschild sphere of its physical meaning, and r_g would separate this region from the

real world of physical phenomena, preserving in it the causal connections in their original form.

It was also noted that one of the most perceptible troubles in the dynamic (based on the Hamiltonian formalism) quantum field theory is the emergence of divergent integrals in the solution of quantum field problems. Elimination of divergences by means of renormalization of masses and charges is some successful half-measure that always caused physicists have a certain feeling of dissatisfaction. The emergence of divergences is in all probability due to the use in modern field theory of the metric of the special theory of relativity. This is due to the neglect of gravitational effects in the field theory. The latter circumstance, apparently, leads to an essential defect in the theory: to its inapplicability for very small regions of space and to large-momentum divergences. It is entirely possible that quantum field theory should be built on the basis of the general theory of relativity, that is, on the basis of a general covariant formalism, it is necessary to solve the problem of quantizing non-linear field equations, which, as we know, represents a certain mathematical difficulty. It can be hoped that in this way it will be possible to construct a quantum field theory applicable to arbitrarily small regions of space and devoid of such defects as divergence. In such a theory it will be possible to find the relations between the "seed" and the experimental masses and charges. Having determined the "seed" masses and charges from these relations, we get rid of the need to perform a procedure for renormalizing masses and charges.[15], p.6. Indeed, from the expression for the invariant interval (3.20) it follows that at the Planck level $r = \ell_P$ the invariant interval dS is bounded from below by the Planck length; on this scale, division by zero occurs, which means the formation of real and virtual Planck black holes.

Ultra-small distances can be "probed" with the help of high-energy massless energy quanta (photons, etc.). But since massless quanta transform into microscopic black holes (collapsing) with the Planck energy 10^{19} GeV, then in this case there is no longer a tool for investigating distances less than $\ell_P \approx 10^{-33}$ cm. Consequently, the idea of distances less than $\ell_P \approx 10^{-33}$ cm, that is, beyond their possible physical verification, is pointless. This would contradict the principle of observability, according to which in principle one can not introduce in principle unobservable quantities, in this case distances smaller than the Planck length.

Perhaps real Planck geons with a mass of $m_P \approx 10^{-5}$ g do not "evaporate", but are stable formations. The fact is that the entire mass of the black hole can "evaporate," except for that part of it that is related to the energy of zero, quantum oscillations of the substance of the black hole. Such oscillations do not raise the temperature of the object and their energy can not radiate. The residual mass is $m_P \approx 10^{-5}$ g, regardless of what was the initial mass of the black hole [16]. In this case Planck geons can serve as "seed" nuclei of other elementary particles. The observed masses of elementary particles, which are much smaller than the Planck mass, can be a consequence of a mass defect in the formation of a bound system of several geons.[14]

4 To the singularity problem

One of the difficulties of the general theory of relativity is the problem of singularities, which actually originated with the receipt of the non-stationary Friedman cosmological solutions of the equations of general relativity, and even more aggravated due to the

problem of relativistic gravitational collapse. Singularity refers to a state of infinite density of matter, which indicates the failure of the general theory of relativity. These problems are solved in a multidimensional space.

4.1 How to place the Universe at the point.

Consider the obvious example. Take an ordinary book, 3-dimensional object. The amount of information in the form of letters in a book occupies a volume V . Let this same amount of information must be placed in the two-dimensional space, i.e. in the plane. In the form of lines of information will occupy an area of a square with the side $a(2)$. It is clear that $a(2) > a(3)$, where $a(3)$ is the side three-dimensional cube depicting book.

The same amount of information is located in a one dimensional space in the form of a line with length $a(1)$, and

$$a(1) > a(2) > a(3)$$

Intuitively, it is clear that if we increase the number of dimensions of space to accommodate the same amount of information (in the form of letters), we construct an n -dimensional cube with a smaller side $a(n)$, that is

$$a(1) > a(2) > \dots > a(k) > \dots > a(n)$$

It is not difficult to show that $a(n)$ and $a(k)$ are related as follows

$$a(n) = a(k)^{k/n} \tag{4.1}$$

Indeed, (4.1) is a consequence of an equal amount of information (or atoms) in one or other n -dimensional space

$$V(1) = V(2) = V(k) = \dots = V(n)$$

where $V(n)$ is the «volume» n -dimensional spaces, which have an equal number of units of information (or atoms) which are located in nodes n -dimensional cubic lattices with a pitch d in that or another n -dimensional space (see Fig.7)

So how

$$V(1) = a(1)^1; V(2) = a(2)^2; \dots; V(k) = a(k)^k; \dots; V(n) = a(n)^n;$$

Then we obtain (4.1). Here, for example, $a(1) = d \cdot t$, where t is the number of steps of the lattice.

If the space is three-dimensional, we obtain from (4.1)

$$a(n) = a(3)^{3/n} \tag{4.2}$$

From equation (4.2) should be an interesting conclusion. Suppose that we need to place the observable universe, together with the substance in the elementary n -dimensional "cube" and the side of the cube is equal to $10 \ell_P$. Here $\ell_P = 10^{-33}$ cm is the Planck length. How many dimensions of space is needed?

The size of the observable Universe is 10^{28} cm., or in units of Planck length $10^{61} \ell_P$. From (4.2) we have

$$10^1 \ell_P = (10^{61} \ell_P)^{3/n} \tag{4.3}$$

Hence, $n = 183$. Thus the observed Universe can be placed in 183-dimensional "cube". Rib "cube" is $10\ell_P$.

The density of matter in a "183-cube" is equal to the density of a substance in 3-dimensional space of the observable Universe. Indeed, the density of the matter in the n -dimensional space is defined as follows: $\rho(n) = M/V(n)$, where M is the mass of the substance of the observable Universe; $V(n)$ is the volume of n -dimensional space; $\rho(n)$ is the density of material in an n -dimensional space. And since, by hypothesis, $V(3) = V(183)$, then $\rho(3) = \rho(183)$.

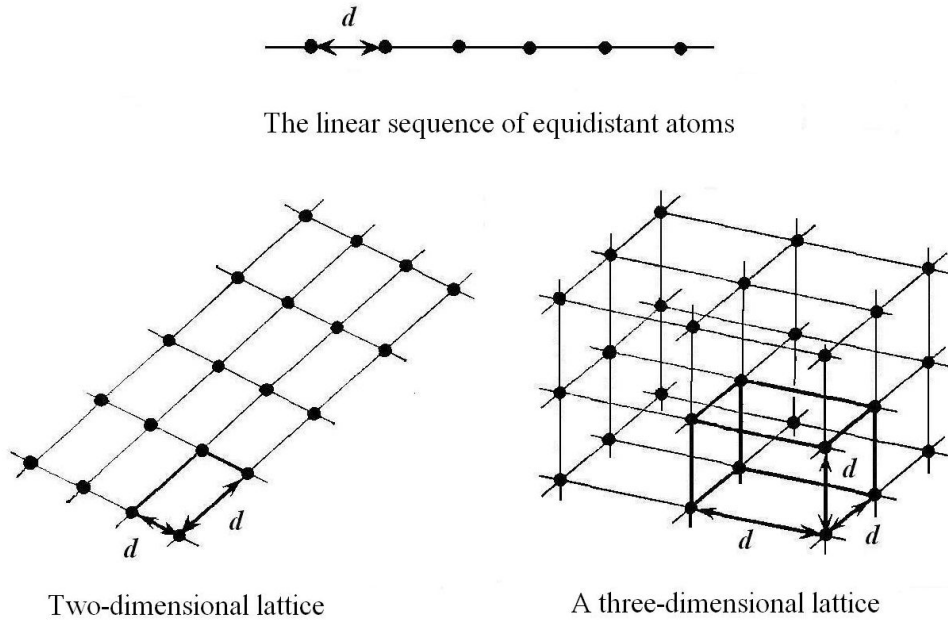


Figure 7: Multi-dimensional lattice

A good example: folding a one-dimensional string with length r_1 into a flat two-dimensional "rug" in the form of a spiral with a diameter of r_2 or in a three-dimensional ball of diameter r_3 . It is clear that $r_1 > r_2 > r_3$, that is, the compactness of the placement of the string grows with the increase in the dimensionality of the space, but the density of the string substance remains the same (the atoms of the string matter will still be located at a distance d from each other in the direction of each n -th coordinate axis, (see Fig.7).

It is obvious that in an infinite-dimensional "point" one can place a finite-dimensional space of any volume.

Proceeding from the above, we can assume that the singular "point" (that is, a very small domain of space), from which, according to the general of relativity, our Universe arose, was multidimensional.

It can also be assumed that, in the collapse of black holes, matter in the center of

a black hole (in a singularity) is squeezed out into other dimensions of space that are compactified into rings of diameter about the Planck length.

Thus, the three-dimensionality of the outer observable space is due to the energy advantage in the formation of virtual Planck black holes (quantum foam), and the multidimensional nature of the singularities of black holes hidden under the event horizon solves the problem of the infinite density of collapsing matter.

5 To the quantum gravity

In the final chapter, the author's own approach to quantum gravity is considered. The conclusions do not contradict the main thesis of the article on the fundamental nature of massless energy quanta.

5.1 Integration of Einstein's equations

Einstein's equation is

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (5.1)$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{R}{2}g_{\mu\nu}$ is the Einstein tensor, which combines the Ricci tensor, the scalar curvature and the metric tensor, Λ is the cosmological constant, $T_{\mu\nu}$ energy-momentum tensor of matter, π is the number, c is the speed of light, G is Newton's gravitational.[11]

This equation can be written as

$$\frac{1}{4\pi}(G_{\mu\nu} + \Lambda g_{\mu\nu}) = 2 \left(\frac{G}{c^3} \right) \left(\frac{1}{c} T_{\mu\nu} \right) \quad (5.2)$$

where $\left(\frac{1}{c} T_{\mu\nu} \right)$ is the density of the energy-momentum of matter.

In the derivation of his equations, Einstein suggested that physical spacetime is Riemannian, ie curved. A small domain of it is approximately flat spacetime.

For any tensor field $N_{\mu\nu\dots}$ value $N_{\mu\nu\dots}\sqrt{-g}$ we may call a tensor density, where g is the determinant of the metric tensor $g_{\mu\nu}$. The integral $\int N_{\mu\nu\dots}\sqrt{-g}d^4x$ is a tensor if the domain of integration is small. It is not a tensor if the domain of integration is not small, because it then consists of a sum of tensors located at different points and it does not transform in any simple way under a transformation of coordinates.[17] Here we consider only small domains. This is also true for the integration over the three-dimensional hypersurface S^ν .

Thus, Einstein's equations (5.2) for small spacetime domain can be integrated by the three-dimensional hypersurface S^ν . Have

$$\frac{1}{4\pi} \int (G_{\mu\nu} + \Lambda g_{\mu\nu}) \sqrt{-g} dS^\nu = \frac{2G}{c^3} \int \left(\frac{1}{c} T_{\mu\nu} \right) \sqrt{-g} dS^\nu \quad (5.3)$$

Since integrable spacetime domain is small, we obtain the tensor equation

$$\boxed{R_\mu = \frac{2G}{c^3} P_\mu} \quad (5.4)$$

where $P_\mu = \frac{1}{c} \int T_{\mu\nu} \sqrt{-g} dS^\nu$ is the component of the 4-momentum of matter, $R_\mu = \frac{1}{4\pi} \int (G_{\mu\nu} + \Lambda g_{\mu\nu}) \sqrt{-g} dS^\nu$ is the component of the radius of curvature small domain.

Since $P_\mu = McU_\mu$ then

$$R_\mu = \frac{2G}{c^3} McU_\mu = r_s U_\mu \quad (5.5)$$

where r_s is the Schwarzschild radius, U_μ is the four-velocity, M is the gravitational mass. This entry reveals the physical meaning of the R_μ values as a component of the gravitational radius r_s . Here $R_\mu R^\mu = r_s^2$ (compare $dx_\mu dx^\mu = dS^2$).

There is a similarity between the obtained tensor equation and the expression for the gravitational radius of the body (the Schwarzschild radius). Indeed, for static spherically symmetric field and static distribution of matter have $U_0 = 1, U_i = 0$ ($i = 1, 2, 3$). We obtain

$$R_0 = \frac{2G}{c^3} McU_0 = \frac{2G M}{c^2} = r_s \quad (5.6)$$

We draw attention to the external similarity of equation (3.4) for the gravitational radius of the geon $r_g = (2G/c^3)P$ and equations (5.4) $R_\mu = (2G/c^3)P_\mu$ and (5.6) $R_0 = (2G/c^3)McU_0$. This indicates their mutual connection and supports the thesis about the primary character of the massless form of matter.

In a small area of spacetime is almost flat and this equation can be written in the operator form

$$\hat{R}_\mu = \frac{2G}{c^3} \hat{P}_\mu = \frac{2G}{c^3} (-i\hbar) \frac{\partial}{\partial x^\mu} = -2i \ell_P^2 \frac{\partial}{\partial x^\mu} \quad (5.7)$$

where \hbar is the Dirac constant, ℓ_P is the Planck length. Then commutator operators \hat{R}_μ and \hat{x}_μ is

$$[\hat{R}_\mu, \hat{x}_\mu] = -2i\ell_P^2 \quad (5.8)$$

From here follow uncertainty relations

$$\boxed{\Delta R_\mu \Delta x_\mu \geq \ell_P^2} \quad (5.9)$$

Substituting the values of $R_\mu = \frac{2G}{c^3} McU_\mu$ and $\ell_P^2 = \frac{\hbar G}{c^3}$ and cutting right and left of the same symbols, we obtain the Heisenberg uncertainty principle

$$\Delta P_\mu \Delta x_\mu = \Delta(McU_\mu) \Delta x_\mu \geq \frac{\hbar}{2} \quad (5.10)$$

Since $R_\mu = (2G/c^3) P_\mu$ and $P_\mu = \hbar k_\mu$, then $R_\mu = 2\ell_P^2 k_\mu$, where k_μ is the wave 4-vector. That is, R_μ (Schwarzschild radius) is quantized, but the quantization step is extremely small.

It can be seen that in quantum gravity, spacetime is not quantized, but the radius of curvature of spacetime is quantized.

In the particular case of a static spherically symmetric field and static distribution of matter $U_0 = 1, U_i = 0$ ($i = 1, 2, 3$) and have remained

$$\Delta R_0 \Delta x_0 = \Delta r_s \Delta r \geq \ell_P^2 \quad (5.11)$$

where r_s is the Schwarzschild radius, r is radial coordinate. Here $R_0 = r_s$ and $x_0 = ct = r$, since the matter moves with velocity of light in the Planck scale.

5.2 Estimation of the equations of general relativity

Last uncertainty relation (5.11) allows make us some estimates of the equations of general relativity at the Planck scale. For example, the equation for the invariant interval dS in the Schwarzschild solution has the form

$$dS^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - r_s/r} - r^2(d\Omega^2 + \sin^2 \Omega d\varphi^2) \quad (5.12)$$

Substitute according to the uncertainty relations $r_s \approx \ell_P^2/r$. We obtain

$$dS^2 \approx \left(1 - \frac{\ell_P^2}{r^2}\right) c^2 dt^2 - \frac{dr^2}{1 - \ell_P^2/r^2} - r^2(d\Omega^2 + \sin^2 \Omega d\varphi^2) \quad (5.13)$$

It is seen that at the Planck scale $r = \ell_P$ spacetime metric is bounded below by the Planck length, and on this scale, there are real and virtual Planck black holes.[18] Note that equation (5.13) coincides with equation (3.20).

Similar estimates can be made for other equations of general relativity. In macroscopic physics, when encountering a heavy body, it is first of all necessary to estimate the ratio of the gravitational radius to the distance to the center of attraction $\zeta = r_g/r$ and we will already know much about the magnitude of the effects associated with total mass. For example, in the macrocosm by the parameter ζ , the scale of the change in the clock is determined. For the Sun, the parameter ζ is approximately $4 \cdot 10^{-6}$ or 1.76 ang.sec., that is, a ray of light passing near the edge of the solar disk will deviate by about $4 \cdot 10^{-6}$ radians. For Mercury, this parameter will be 10^{-7} , which for one hundred years of the Earth gives for displacement of the perihelion of Mercury 43 ang.sec. The parameter ζ is included in all other estimates. But, as we explained above, the parameter $\zeta = r_g/r$ at the Planck level has the form $\sim \ell_P^2/r^2$, so in order to estimate any ratio obtained within the framework of the general of relativity as applied to the Planck scale, it is necessary to replace the ratio r_g/r by the expression $\zeta \sim \ell_P^2/r^2$.

The parameter ζ determines on the Planck level the fluctuations in the spacetime metric. The spacetime metric $g_{00} \approx 1 - \Delta g = 1 - \ell_P^2/(\Delta r)^2$ fluctuates generating the so-called spacetime quantum foam consisting of virtual Planck black holes and wormholes.[19] But these fluctuations $\Delta g \sim \ell_P^2/(\Delta r)^2$ in the macroworld and in the world of atoms are very small in comparison with 1 and become noticeable (about 1) only on the Planck scale. Fluctuations of Δg must be taken into account when using the special relativity metric $(+1, -1, -1, -1)$ in very small regions of space and at large momenta. That is, the expression for the invariant interval dS in spherical coordinates must have the form

$$dS^2 \approx \left(1 - \frac{\ell_P^2}{(\Delta r)^2}\right) c^2 dt^2 - \frac{dr^2}{1 - \ell_P^2/(\Delta r)^2} - r^2(d\Omega^2 + \sin^2 \Omega d\varphi^2) \quad (5.14)$$

However, in view of the smallness of $\ell_P^2/(\Delta r)^2$, the expression for the invariant interval in special relativity is always written in Galilean form, which actually does not correspond to reality. The correct expression must take into account the fluctuations in the spacetime metric and the presence of virtual black holes at Planck scale distances.

For the vacuum at the Planck level, the uncertainty relation (5.11) will be characteristic, since the state of motion or the velocity vector cannot be ascribed to the vacuum. In Minkowski space, due to its high symmetry for all inertial reference systems, the vacuum

is one and the same state. In any reference system, the vacuum will look at rest (static). Therefore, the Planck vacuum, according to the specified uncertainty relation, will generate wormholes and tiny virtual black holes (quantum foam). Thus, the very fabric of space-time is a seething foam of wormholes and tiny black holes a hundred billion billion times smaller than a proton. It can be seen that Lorentz invariance is violated on the Planck scale.

For example, it is known that the speed of light in some place with the gravitational potential $\varphi = -Gm/r$ is equal to $c' = c(1 + 2\varphi/c^2) = c(1 - r_g/r)$. Then, on the Planck scale, because of the quantum potential fluctuations, the expression for the speed of light will have the form $c' = c(1 - \ell_p^2/(\Delta\lambda)^2)$. Here λ is the wavelength of the light emitted by the source. The greater the distance from the light source, and the shorter its wavelength, the stronger the dispersion of the rays due to accumulated distortion. In this case, the inhomogeneities of the photon velocity $\Delta c = c\ell_p^2/(\Delta\lambda)^2$ are determined not by the Planck length, but by its square, so that these inhomogeneities are immeasurably small (of the order $10^{-56}c$ for $\lambda = 10^{-7}$ m) and images of remote sources will be sharp even at metagalactic distances.[20], [21]

As noted in [22], for the spacetime region with dimensions l , the uncertainty of the Christoffel symbols should be of order ℓ_p^2/l^3 , and the uncertainty of the metric tensor is of order ℓ_p^2/l^2 . If l is a macroscopic length, the quantum constraints are fantastically small and can be neglected even on atomic scales. If l is comparable to ℓ_p , then the maintenance of the former (conventional) concept of space becomes more and more difficult and the influence of micro curvature becomes obvious.

The expression for the metric fluctuations agrees with the Bohr-Rosenfeld uncertainty relation $\Delta g (\Delta L)^2 \geq 2\ell_p^2$. [23]

As underlined in [24], these small-scale fluctuations indicate that everywhere in space there is always something like a gravitational collapse that the gravitational collapse is essentially constantly taking place, but the reverse process is constantly happening, that in addition to the gravitational collapse of the Universe and stars must also be considered the third and most important level of gravitational collapse at Planck scale distances.

Virtual Planck black holes are important for the theory of elementary particles. When performing calculations in modern quantum theory and, in particular, in calculating the self-energy of particles, the contribution of intermediate states with arbitrarily high energy is usually taken into account, which leads to the appearance of known divergences. Taking into account the gravitational interaction of the corresponding virtual particles and the possibility of virtual (short-lived) black holes appearing in the intermediate state should lead to the elimination of these divergences.[19]

It is possible that Planck's black hole (maximon) is the final product of the evolution of ordinary black holes, stable and no longer subject to Hawking radiation. The Planck black holes characterize an extremely small cross section for the interaction, of the order of $10^{-66}cm^2$. The smallness of the cross section of the interaction of neutral maximons with matter leads to the fact that a significant (or even basic) part of the matter in the Universe at the present time could consist of maximons, without leading to a contradiction with observations. In particular, maximons could play the role of an invisible substance (dark matter), whose existence is now recognized in cosmology.[19]

Finally, an analysis of the Hamilton-Jacobi equation in spaces of different dimensionality with respect to the Planck scale (see subchapter 3.3) showed that the appearance of virtual Planck black holes (quantum foam, the basis of the "tissue" of the universe) is

energetically most profitable in three-dimensional space, which most likely predetermined the three-dimensionality of the observed space.

6 Conclusion

The aim of the author was to show that the field form of matter lies at the base of a weighty form of matter. This hypothesis makes it possible to explain such properties of matter as 1. its inertness and weight; 2. inability of a weighty form of matter to reach a speed greater than the speed of light; 3. the universal nature of the effect of the gravitational field on all physical matter; 4. The ability of mutual transformations of elementary particles into each other. On the basis of the model of the geon, it is shown that Planck black holes that arise from the massless form of matter in the first fractions of a second of the Big Bang can act as "bricks" of elementary particles. Planck black holes (maximons) have a number of quantum numbers, which explains the diversity of elementary particles. Within the framework of the geon model, the nature of the Planckian vacuum consisting of virtual black holes is shown. A substantiation of the three-dimensionality of the observable space is given and a hypothesis about the multidimensional character of the singularities arising in black holes is stated. This solves the problem of the infinite density of matter in singular regions. In the last chapter, the Einstein gravitational equations are integrated, which allowed us to establish new uncertainty relations and, with their help, to estimate the gravitational equations with respect to the Planck scale.

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