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Gravitational decoherence: A thematic overview

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ABSTRACT

Gravitational decoherence (GD) refers to the effects of gravity in actuating the classical appearance of a quantum system. Because the underlying processes involve issues in general relativity (GR), quantum field theory (QFT), and quantum information, GD has fundamental theoretical significance. There is a great variety of GD models, many of them involving physics that diverge from GR and/or QFT. This overview has two specific goals along with one central theme: (i) present theories of GD based on GR and QFT and explore their experimental predictions; (ii) place other theories of GD under the scrutiny of GR and QFT, and point out their theoretical differences. We also describe how GD experiments in space in the coming decades can provide evidence at two levels: (a) discriminate alternative quantum theories and non-GR theories; (b) discern whether gravity is a fundamental or an effective theory.

I. INTRODUCTION

Gravitational decoherence (GD) is a fundamental issue of theoretical physics because gravitation is a universal interaction and decoherence is an essential factor in how a quantum system assumes some classical behavior. GD encompasses the issues of both the quantum-to-classical and the microscopic-to-macroscopic transitions. The quantum-to-classical transition enables the description of a quantum system in terms of classical physics. The microscopic-to-macroscopic transition seeks effective descriptions of systems with 10^{23} degrees of freedom in terms of a few observables, as in thermodynamics.

By their very nature, GD and gravitational entanglement (GE) also involve basic quantum information issues, starting with the effects of gravity on the quantum-to-classical transition,[1–3] and continuing to the construction and probing of gravitational cat states.[4] The possibility of direct experimental feedback about a nontrivial interplay between gravity and quantum theory is what makes these phenomena so interesting from a theoretical viewpoint.

Note that one does not need to appeal to theories of quantum gravity at the Planck scale to see the contradictions between the quantum and gravitation, to ask questions about the quantum nature of gravity, and how the quantum-informational features show up. Their manifestations are already present in low energy physics, namely, in the nonrelativistic (NR) and weak field (WF) limits of general relativity (GR) and quantum field theory (QFT). This is the regime we live in where laboratory experiments on Earth and in space can offer invaluable observational data to cross-examine the above-mentioned theoretical issues of fundamental importance.

A. Quantum decoherence

For a long time since the advent of quantum mechanics, the issues of the quantum-to-classical transition and of the emergence of the classical world have been a subject of curiosity for debates among the philosophers of science. Physicists reclaimed the subject in the 1980s with full rigor in three lines of development.

1. Consistent/decoherent histories (DH)

This program was developed by Griffiths,[5] Omnés,[6] and Gell-Mann and Hartle.[7] The key idea is that quantum theory must be expressed in terms of histories, i.e., sequences of properties of a system at different moments of time, rather than in terms of evolving single-time quantum states. The decoherent histories program places great emphasis on the logical structure of propositions about histories, which leads to a rigorous implementation of the crucial notion of coarse-graining. As a result, the DH program offers the most sophisticated account of the quantum-to-classical transition in a unified way with the microscopic-to-macroscopic transition. However, DH guarantees neither the uniqueness nor the stability of the classical world of our experience,[8] while it offers only minor improvements over Copenhagen quantum mechanics on the measurement problem.[9]

2. Environment-induced decoherence (EID)

This program originates from Zurek,[10] and Joos and Zeh,[11] following an earlier work by Zeh.[12] The key idea is that quantum systems classicalize as a result of their interaction with the environment. This program went hand in hand with the development of the theory of open quantum systems. Most prominent are the master equations built on the work since the 1960s, from Schwinger[13] and Feynman and Vernon[14] on the ubiquitous quantum Brownian motion, to the 1980s, in the Caldeira–Leggett Markovian master equation[15] valid for Ohmic environments at high temperatures, and to the 1990s in the Hu–Paz–Zhang non-Markovian master equation[16] valid for a general environment at all temperatures. The applicability of the program is limited to setups that admit a natural system-environment split and, despite its many successes in matching with experimental outcomes, it does not provide, by itself, a fully satisfactory solution to the quantum measurement problem.[17]

3. Wavefunction collapse models

This program introduces a scale that separates the microscopic from the macroscopic, and postulates that quantum mechanics as we know it which applies to a plethora of natural phenomena from subatomic scales up, no longer holds for macroscopic phenomena. There are two major directions. The first direction originates from Ghirardi *et al.*[18] and Pearle[19] (hence, GRWP), and it culminates in the formulation of dynamical reduction or continuous state localization (CSL) models.[20] The second direction, exemplified by Diosi[2] and Penrose[3] (DP model), focuses on wavefunction collapse due to gravity, and we will discuss it separately.

Unlike the previous two, this approach postulates a fundamental modification of quantum theory, and for this reason, we will refer to the theories in this class as alternative quantum theories (AQT). CSL models start from the pragmatic aim of introducing a scale where macroscopic phenomena overtake the microscopic quantum mechanical behavior through a mathematically consistent modification of quantum dynamics. This mostly works in nonrelativistic physics. Still, the whole procedure is *ad hoc*, with no fundamental explanation, and one encounters grave problems when attempting to formulate a relativistic version of CSL models appropriate for QFT. Despite significant progress in recent years,[21] a CSL theory that is fully consistent with special relativity has yet to appear.[22]

B. Gravitational decoherence

The main body of work on gravitational decoherence began a decade later. We shall mention the main lines of development in the next section but give an overall perspective here.

One special feature in this research subfield is a widened scope of possibilities, invoking novel features from gravity as well as quantum theory. We observe that both the DH and the EID programs do not call on alteration of GR or QFT. DH allows for generalizations of quantum theory for systems that are difficult to treat with usual methods, or for the definition of more elaborate observables, but the physical predictions in the domain of current theories are the same as the standard ones. EID highlights the role played by the environment in determining the classical outcomes of a quantum system. As such, it requires tools and concepts in open quantum systems and nonequilibrium statistical mechanics, but it does not need to alter existing theories at the fundamental level.

With gravitational decoherence, we see proposals that call for the alteration of quantum mechanics, such as in the GRWP and DP theories. The motivation ranges from pragmatic considerations as in CSL to the "gravitization" of quantum mechanics, proposed by Penrose.[24] Now that gravity enters the picture many proponents find it convenient to use GR, even altering it, to explain the quantum-to-classical transition. This is done by invoking fluctuations either in the manifold structure or in the spacetime metric, under the labels of "intrinsic," "fundamental," or "quantum gravity" decoherence. We shall point out how such models infringe on GR in subtle ways.

C. Features of this overview

An excellent review is available on this subject[23] which covers a broad range of topics with prominent representation of CSL and AQTs. For this reason, we shall narrow our goals in this overview to two: (i) Present theories of gravitational decoherence based on GR and QFT and their experimental predictions. (ii) Place other theories of GD under the scrutiny of GR and QFT, and point out the theoretical differences. In fact, we would urge all proponents of new and old alternative theories to present, alongside with what they consider as their proposals' attractive features, also a self-scrutiny against GR and QFT as benchmarks. The reason is simply that, these two theories are the pillars of modern physics, and they serve as the yardstick by which to distinguish between proposals based on "known physics" or conservative extensions thereof and proposals based on "new physics" which violate either GR or QFT or both, in ways big or small.

In Sec. II, we present the main theories of gravitational decoherence classified according to the physical origins of decoherence. Decoherence may originate from viewing gravity as intrinsically classical, from quantum gravity processes, or from spacetime fluctuations. In Sec. III, we present the Anastopoulos–Blencowe–Hu (ABH) theory of gravitational decoherence[25,26] that is rooted completely in GR and QFT. In Sec. IV, we briefly review estimates for gravitational decoherence effects, mainly comparing the predictions of the DP and the ABH model. In the Appendix, we answer a few basic questions which may arise from a novice reader with an acute mind.

II. THEORIES OF GRAVITATIONAL DECOHERENCE

A. Decoherence versus dephasing

The term "gravitational decoherence" is sometimes used in the literature when the authors actually refer to dephasing due to gravity.[27] Decoherence is an irreversible process, while dephasing may also occur without irreversibility. A classic case of mere dephasing without decoherence is provided by the spin-echo experiments.[28] In these experiments, phase information is apparently lost, but a simple operation on the system will recover this information. In the absence of dissipation (an irreversible process), the recovery of information is complete.[29] In decoherence, the phase information is lost forever, or at least for a very long time (of the order of the Poincaré recurrence time of the environment). Another analogy in terms of energy is the following.

Mere dephasing is in the nature of Landau "damping" a la the Vlasov equation which describes unitary time evolution, is time-reversal invariant of the Hamiltonian type; decoherence is in the nature of the nonunitary Boltzmann equation with real energy dissipation.

Here, we will only discuss models with actual irreversible decoherence.

B. Decoherence from classical gravity

This is the oldest approach and most influential approach to gravitational decoherence. It originates with Karolyhazy[1] in the 1960s. An underlying idea is that gravity is intrinsically a classical channel of interaction. This implies that gravity can act as an agent of decoherence in a quantum system.

1. The Diosi–Penrose theory

According to Penrose,[3] decoherence is a plausible consequence of the fundamental incompatibility between GR and quantum theory, especially in their respective treatments of time. Time in quantum theory is essentially classical and forms a background to all quantum phenomena, while in GR time is fundamentally dynamical as it is determined by the spacetime metric that is dynamical observable.

The contradiction between the role of time in GR and in quantum theory is most emphatically manifested when considering the superposition of macroscopically distinct states for the matter. By Einstein's equations, each component of the superposition generates a different spacetime. Since there is no canonical way of relating time parameters in different spacetime manifolds, there is a fundamental ambiguity in the time parameter of the evolved quantum state. According to Penrose, this ambiguity is manifested even at low energies, when the gravitational interaction can be effectively described by the Newtonian theory. It may be expressed as a mechanism of gravity-induced decoherence for superpositions of states with different mass densities $\mu_1(x)$ and $\mu_2(x)$. Penrose proposed that the timescale of decoherence is of the order of the gravitational self-energy of the difference in the two mass densities.

Diosi[2] constructed a master equation in which he postulated a collapse term, with noise correlator proportional to gravitational potential. The master equation is

$$\frac{\partial \hat{\rho}}{\partial t} = -i\left[\hat{H}, \hat{\rho}\right] - \frac{G}{2}\int d\boldsymbol{r} d\boldsymbol{r}' \frac{\left[\hat{\mu}(\boldsymbol{r}), \left[\hat{\mu}(\boldsymbol{r}'), \hat{\rho}\right]\right]}{|\boldsymbol{r}-\boldsymbol{r}'|}, \tag{1}$$

where $\hat{\mu}(\boldsymbol{r})$ is the mass density operator. Key value in this master equation is that predictions do not involve any free parameters, except for a high-frequency cutoff Λ that is necessary for the definition of the mass density operator. The natural scale of Λ for nonrelativistic particles is the nuclear scale, in the sense that Λ^{-1} corresponds to a nucleon's radius, nuclei being the simplest particles most affected by this decoherence mechanics. The decoherence rate is typically of the order of $1/\Delta E$ where ΔE is a regularized gravitational self-energy difference associated with a macroscopic superposition of mass densities.

Penrose's arguments for gravitational decoherence are not model-specific, but his proposal for the decoherence timescale is the same as the one obtained from Diosi's model, hence the name Diosi–Penrose for this theory.

Diosi's model predicts a small violation of energy conservation, which can be used to test the theory. A recent experiment rules out the most natural version of the model where Λ is the nuclear scale.[30] There is no obvious physical justification for the smaller value of the cutoff.

There are also some theoretical problems in the D–P theory. A recent analysis[31] of the quantization of weak gravity interacting with the matter has shown that the ambiguity pointed out by Penrose is not related to the gravitational self-energy. The latter appears the same in all gauges/reference frames. The frame-dependent terms on the Hamiltonian are different, and they have no obvious physical interpretation.

Moreover, the Diosi master equation implies a coupling of quantum matter to gravity that differs from that of GR, in effect, through a nonunitary channel. This nonunitary interaction will likely be manifested at the classical macroscopic level of GR. The reason is that the Diosi master equation does not preserve energy. Even tiny violations of energy conservation would accumulate when taken over a sufficiently large region filled with matter, and one would expect that they would invalidate the "conservation equation" for $T_{\mu\nu}$, namely, $\nabla_\mu T^{\mu\nu} = 0$. But this equation is fundamentally enforced by Einstein's equation, and in particular, by the diffeomorphism symmetry of the action. Hence, the classical gravity theory compatible with the Diosi master equation cannot be GR. Most probably, such a theory would contain a non-Hamiltonian channel.

Even if the avowed aim of Penrose is to gravitize quantum mechanics, this specific approach apparently requires strong modifications in our current theory of gravity, and not only in quantum mechanics.

2. The Newton–Schrödinger equation

An early candidate for the coupling of classical gravity to quantum matter is the so-called Moller–Rosenfeld (MR) theory,[32,33] according to which the source in Einstein equations in the expectation value of the stress-energy tensor: $G_{\mu\nu} = 8\pi G\langle \hat{T}_{\mu\nu}\rangle$ is taken as fundamental in this theory, rather than an approximation to a quantum gravity theory. (See Ref. 35 for how the MR theory, at least the way it is interpreted by recent followers, differs from semiclassical gravity (SCG) theory proper,[47] based solely on GR + QFT.) It is far from obvious that the MR theory is consistent, or even that it makes mathematical sense, but at least in the weak field, nonrelativistic regimes, one can subject it to tests against theories based on GR + QFT. In these regimes, adherents of the M–R theory like to work with the Newton–Schrödinger equation (NSE),[34] which is a nonlinear equation for the wavefunction,

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi + m^2 V_N[\psi]\psi, \tag{2}$$

where $V_N(\mathbf{r})$ is the (normalized) gravitational (Newtonian) potential given by

$$V_N(\mathbf{r}, t) = -G\int d\mathbf{r}' \frac{|\psi(\mathbf{r}', t)|^2}{|\mathbf{r}-\mathbf{r}'|}. \tag{3}$$

Note, as we have emphasized in Ref. 37, such a single ($N=1$) particle NS equation is not derivable from GR+ QFT. To see this, we have to recall that standard quantum theory cannot lead to any nonlinear evolution for the quantum state (either in open or in closed systems). The most general evolution law is given by completely positive maps, which are linear with respect to the density matrix. Equation (2) does not lead to a linear equation for the density matrix.

The specific procedure leading one from SCG to an NS equation in the description above is the treatment of $m|\psi(\mathbf{r})|^2$ as a mass density for a single particle described by the wavefunction $\psi(\mathbf{r})$. The problem with this procedure is that the mass density is in fact an observable (rather than a part of the wave function), and it corresponds to an operator $\hat{\mu}(\mathbf{r})$ defined on the space of the associated QFT. Equation (2) is obtained by substituting the mass density operator with its expected value in the Poisson equation for the gravitational potential. The substitution of an operator with its mean value is a good approximation only if the system is presupposed to behave classically. In QFT such an approximation is meaningful only at the mean-field description of an N-particle system at the limit $N \to \infty$. Indeed, in this regime, the NS equation is a special case of Hartree's equation, but $\psi(\mathbf{r})$ is a collective variable, not the wave function of an individual particle. This is the crucial difference between the correct semiclassical gravity theory[47] rooted in GR + QFT and the N–S equation.

When considering a single particle, the NS equation has no justification in terms of GR and QFT. Its derivation requires the postulate that the semiclassical Einstein equations are valid even in regimes where QFT would assert they are not a meaningful approximation, because the fluctuations of the stress-energy tensor $\hat{T}_{\mu\nu}$ are too large.

C. Decoherence from quantum gravity processes

This involves proposals about decoherence from Planck scale processes that cannot be identified by low energy physics. Early models attempted to find a connection with nonunitarity suggested by the black hole information paradox.[39] Now, we are well aware that there is no direct relation: the presumed nonunitarity in black hole evaporation is accompanied by the loss of Cauchy surfaces, hence, with the loss of the global notion of a state. Therefore, it is incompatible also with the nonunitarity of Markovian master equations that describe the evolution of a globally defined state.[40]

Gravitational decoherence of this type is presumed to originate from specific features of quantum gravity theories, for example, string-theory couplings to massive-string states[41] or emergent nonlocality from spacetime foam.[42] The key critique of this type of models is presented in Ref. 25.

Any entity of the quantum gravity realm, such as spacetime foam, exists at the Planck scale, before spacetime with a Lorentz structure emerges. One needs a strong case to show that some Planck-scale properties can escape the coarse-graining and scaling which subsumes their effect to that of the average as the large scale manifold structure of spacetime emerges, and emerge at low energies. The most reasonable assumption is that their average behavior is contained in the effective description that survives at the low energy limit, namely, GR; we can think of no counter-example in all fields of physics. Even if some Planck-scale effects survive at low energies, they must be much smaller than the effects that originate from the gravitational dynamics of GR. Hence, decoherence from quantum gravity effects will be subdominant compared to all decoherence effects that originate from GR, or are postulated on the basis of GR.

D. Decoherence from spacetime fluctuations

In these models, decoherence originates from some fundamental imprecision in the measuring devices (starting with clocks and rulers)[43] or uncertainties in the dynamics,[44] or treating time as a statistical variable.[45] Usually, the fluctuations are assumed to originate from Planck-scale physics.

These approaches make very strong assumptions about the physics of the Planck scale. There is absolutely no reason to expect that such uncertainties can be modeled by stochastic processes which are intrinsically classical, as is commonly the case in this approach. The modeling of uncertainties by classical noise is a valid assumption for randomness at the macroscopic scale. In contrast, quantum uncertainties are different in nature as they involve nonlocalities and correlations with no analogs in the classical theory of stochastic processes.

To explain this point, we note that the limitations posed by the Planck length are not *a priori* different from those placed by the scale $\sqrt{\hbar/c^3}q/m$ in quantum electrodynamics: At this scale quantum field effects are strong, and the fluctuations from these effects are fully quantum. Any effect they cause at low energy is also inherently quantum. One needs to specify the conditions (e.g., Gaussian systems) or the regime for the quantum field, and justify the means by which they could be treated like classical fluctuations described by a stochastic process. In particular, the effects of the fluctuations of the electromagnetic (EM) field at low energies ($E \ll mc^2$) have been well studied. It has been shown that the "noise" induced by these fluctuations is non-Markovian and does not cause significant decoherence effects in the microscopic regime.[46] In other words, the coherence of the EM vacuum does not allow for the *a priori* generation of classical (i.e., decohering) fluctuations in the quantum motion of the particle. The assumption that the gravitational field exhibits a different behavior is completely *ad hoc*, with no justification unless one postulates that gravity is intrinsically classical.

Note that the theory of semiclassical stochastic gravity (SCSI)[47] can introduce stochastically associated solely with the quantum fluctuations of matter fields. The backreaction of the mean and the fluctuations of the stress-energy tensor of a quantum matter field is taken into account by way of the Einstein–Lanolin equation[48] governing the dynamics of the induced metric fluctuations. Simple dimensional analysis shows that these induced metric fluctuations are dominant only at the Planck scale and thus are on the same footing as spacetime foams, albeit Wheeler wanted his spacetime foams to admit also topology changes. In SCSI the noise is fundamentally quantum, they are not put in by hand, and the theory is entirely based on GR+QFT. The drop-off behavior of metric fluctuations in Minkowski spacetime has been derived in Ref. 49. Indeed, the strength of quantum noise in ScStG provides a useful measure of the validity of semiclassical gravity.[50] In contrast, the noise employed by the models discussed above is defined by a purely classical stochastic process, often catered to the particular wishes of the designers.

The second problem in these approaches is that they often contradict the symmetries of GR. Fluctuations in the time or space coordinates of an event are indistinguishable mathematically from time and space reparameterizations of the system, only such reparameterizations are viewed as stochastic. However, time and space reparameterizations are pure gauge variables in classical relativity (even in the linearized approximations); they do not have any dynamical content. The invariance of the theory under space and time reparameterizations follows from the diffeomorphism invariance of the classical action, a fundamental symmetry of general relativity.

The assignment of dynamical contents to such fluctuations implies that they are not treated as gauge variables. Doing so violates the fundamental symmetry of classical GR. Any theory with this property would have far-reaching implications which go beyond the gravitationally induced decoherence effects. The diffeomorphism symmetry affects both the dynamics and the kinematics of general relativity, and its abandonment ought to be manifested in other gravitational phenomena.

We also note that time and space reparameterizations decouple from the terms describing Newtonian interaction, already at the classical level. Hence, there is no reason for Newton's constant in GR to modulate the strength of decoherence effects.

We may also place in this category the so-called event operator formalism of Ralph *et al.*[51,52] This model originates from a modification of standard QFT that allows for unitary evolution in the presence of closed time-like curves. This model does not lead to decoherence of individual particles, but it predicts rather a strong decorrelation of entangled photon pairs.

The presumed physics in this model is rather implausible from the perspective of gravity theory. The event operator formalism relies on the presence of closed-time-like curves[53] without accompanying quantum gravity effects. This strongly contradicts the well-motivated chronology protection conjecture[54] which asserts that the laws of physics, including quantum phenomena, do not allow for the appearance of closed time-like curves. Chronology protection is valid also for semiclassical gravity.[55] All this strongly suggests that closed time-like curves can emerge only as Planck scale quantum gravity effects (like Wheeler's spacetime foam[56]) if at all.

III. ABH THEORY: ROOTED IN GR AND QFT

A. Key ideas and results

In the theory of gravitational decoherence of Anastopoulos and Hu[25] and Blencowe[26] (ABH) the source of decoherence comes as noise (fluctuations) from gravitational waves (classical perturbations) or gravitons (quantized linear perturbations). The transverse-traceless tensor perturbations are the dynamical (propagating) degrees of freedom satisfying the linearized Einstein's equations. The gravitational noise source may be cosmological[26] (gravitons produced in the early universe near the big bang or from inflation pre or post), astrophysical[57] or structural of a thermo-hydrodynamical nature, if gravity is an emergent theory (see Ref. 58 for an explanation of the differences between quantum gravity and emergent gravity). Note also the later work, Ref. 59, which derives a related theory of gravitational decoherence for matter and light.

There is one parameter in the master equation of the ABH theory which is the noise temperature Θ. It coincides with the graviton temperature if the origin of perturbations is cosmological. Early works in this direction include the study of decoherence due to the graviton vacuum,[60] and of open system dynamics of particles in graviton baths.[61,62] The Power–Percival[63] collapse model also falls in the same category, but the perturbations there are restricted to conformal waves.

It is important to distinguish the ABH theory from models of gravitational decoherence due to metric perturbations that do not require the perturbations to satisfy the linearized Einstein equations. Such models have been developed by Kok and Yurtsever,[64] Breuer *et al.*,[65] and Asprea *et al.*[66] These models describe the perturbations of the metric by a stochastic process, but they do not distinguish between true and pure gauge degrees of freedom in the perturbations. Treating variables that are pure gauge in GR as stochastic is not compatible with GR, because they do not implement the diffeomorphism symmetry. (Unless one assumes a stochastic matter source of unknown origin and physics, to which the pure gauge stochastic perturbations are slaved.) They are intermediate between the ABH model and the models of Sec. II D, in that they postulate stochastic behavior of pure gauge quantities, but they also include a contribution from true degrees of freedom.

The ABH analysis applies to any type of quantum matter fields in addition to scalar, any number of particles, in the ultra-relativistic as well as the nonrelativistic regimes. The specific methodology is the following.

The system under consideration is a quantum field (a massive scalar field in the simplest case), interacting with a gravitational field as its environment. Gravity is described by classical general relativity. In the weak-field limit, we describe gravitational perturbations in the linearized approximation. Hence, one starts from a linearization of the Einstein–Hilbert action around Minkowski spacetime and constructs the associated Hamiltonian through a $3+1$ decomposition. The constraints of the system are solved classically, thereby allowing us to express the Hamiltonian in terms of the true physical degrees of freedom of the theory, namely, the transverse-traceless perturbations for gravity and the scalar field. We then quantize the scalar field and the gravitational perturbations and trace out the contribution of the latter.

A key input in this stage is the specification of an initial state for the gravitational perturbations. We consider an initial condition that interpolates between the regime of negligible (vacuum) perturbations and strong classicalized perturbations. The initial state is defined in terms of a free parameter Θ that can be loosely interpreted as the noise temperature of the perturbations. Θ conveys coarse-grained information reflective of the micro-structures of spacetime, similar to temperature with regard to molecular motion, or the spectral density function of the environment in Brownian motion. It is in this sense that we think gravitational decoherence may reveal the underlying "textures" of spacetime beneath that described by classical general relativity.

Following the standard methodology of open quantum systems, a second order (perturbative) master equation for the matter field is obtained. This master equation applies to configurations with any number of particles. We project the master equation to the single-particle subspace and we derive a master equation for a single particle. The latter simplifies significantly in the nonrelativistic regime, leading to the ABH master equation,

$$\frac{\partial \hat{\rho}}{\partial t} = -i\left[\hat{H}, \hat{\rho}\right] - \frac{\tau}{16m^2}\left(\delta^{ij}\delta^{kl} + \delta^{ik}\delta^{jl}\right)\left[\hat{p}_i\hat{p}_j, \left[\hat{p}_k\hat{p}_l, \hat{\rho}\right]\right], \tag{4}$$

where τ is a constant of dimension time and $\hat{H} = \hat{p}^2/2m$. Crucially, the master equation (4) preserves energy.

For motion in one dimension, the ABH master equation simplifies to

$$\frac{\partial \hat{\rho}}{\partial t} = -i\left[\hat{H}, \hat{\rho}\right] - \frac{\tau}{2}\left[\hat{H}, \left[\hat{H}, \hat{\rho}\right]\right], \tag{5}$$

where τ is a constant of dimension time and $\hat{H} = \hat{p}^2/2m$.

In the ABH model

$$\tau = \frac{32\pi G\Theta}{9} = \frac{32\pi}{9}\tau_P(\Theta/T_P), \tag{6}$$

where $T_P = 1.4 \times 10^{32}$ K is the Planck temperature.

The master equation (5) appears in models by Milburn,[44] Adler,[68] Diosi,[69] and Breuer *et al.*[65] where τ is obtained from postulated stochastic fluctuations of time, discreteness of time, stochastic fluctuations of the metric, or even stochastic fluctuations of $\hbar$. In these models, the value of τ is not fixed, but the natural candidate is the Planck-time $\tau_P = 10^{-43}$ s. In Ref. 25, it is argued that τ need not be restricted to τ_P but can be a free parameter depending on the underlying structures of spacetime at different scales which may vary in different theories.

The ABH model can be generalized to photons, where we can obtain a master equation for a general photon state.[70] For a single photon,

$$\frac{\partial\hat{\rho}}{\partial t} = -i\left[\hat{H},\hat{\rho}\right] - \frac{\tau_{ph}}{2}\left(\delta^{in}\delta^{jm} - \frac{1}{3}\delta^{ij}\delta^{nm}\right)\left[\frac{\hat{p}_i\hat{p}_j}{\hat{p}_0},\left[\frac{\hat{p}_n\hat{p}_m}{\hat{p}_0},\hat{\rho}\right]\right], \tag{7}$$

where $\hat{H} = |\hat{\mathbf{p}}|$ and $\tau_{ph} = 4G\Theta$.

B. Observational constraints to the ABH

If we regard the parameter Θ in the ABH theory as a noise temperature—originating from some emergent gravity theory—, then Θ need not be related to the Planck temperature and even $\Theta \gg T_P$ is perfectly acceptable from a theoretical point of view. Θ is only a measure of the power P carried by the noise, $P \sim \Theta\Delta\omega$, where $\Delta\omega$ is the band-width of the noise.

Some bounds to Θ can be estimated from the nonrelativistic analysis of Ref. 60. This paper employs the Feynman–Vernon influence functional method, which has the benefit of providing a simple stochastic equation for the semiclassical evolution of a particle interacting with a heat bath. The results that we will present are new, but they follow from the direct use of the "thermal" noise of the ABH model to the analysis of Ref. 60.

The effective semiclassical equation for a nonrelativistic particle in the presence of classical gravitational perturbations of noise temperature Θ turns out to be

$$\ddot{x} + \frac{2G}{15}\ddot{x}^2\dot{x} = 2\ddot{x}\xi, \tag{8}$$

where $\xi(t)$ is Gaussian noise with $\langle\xi(t)\xi(t')\rangle = \eta(t-t')$, where η is known as the noise kernel. The dissipative term is relatively weak as it corresponds to energy loss due to gravitational wave radiation, so we can ignore it.

The noise kernel for the ABH model is

$$\eta(s) = \frac{G}{2}\int_0^{\Lambda} dkk\cos(ks)\coth\frac{k}{2\Theta}, \tag{9}$$

where Λ is a cutoff and Θ the noise temperature. The physically relevant regime corresponds to $\Theta \gg \Lambda$, whence,

$$\eta(s) = \pi G\Theta\delta(s). \tag{10}$$

By Eq. (8), the noise behaves like stochastic fluctuations of the particle's inertial mass of order

$$\frac{\delta m}{m} \sim \sqrt{\xi^2(t)}. \tag{11}$$

With the noise kernel (10), we find

$$\frac{\delta m}{m} \sim \sqrt{\frac{\Theta}{T_P}}, \tag{12}$$

where T_P is the Planck length. We can use Eq. (12) to establish bounds to Θ from cosmological and solar-system measurements.

Cosmology does not lead to strong constraints in Θ. If we assume that the Lambda-cold dark matter model holds we can identify the maximum value of $\delta m/m$ with the relative error in the determination of baryon mass density, hence $\delta m/m \sim 10^{-2}$, which implies that $\Theta < 10^{-4}T_P$. However, if we take into account the changes in the values of the baryon mass density in different dark energy models, a bound $\delta m/m \sim 10^{-1}$ is more plausible, but even this may be too restrictive. We need a model that includes intrinsic stochastic gravitational perturbations in the evolution of the Universe, in order to estimate a proper bound to the size of these fluctuations.

Solar-system measurements provide a better constraint to Θ, if we assume that Eq. (12) also applies to large astronomical bodies like planets. This assumption is by no means evident, because the derivation of Eq. (8) treats particles as point-like, or at least, much smaller than the typical wavelength of gravitational perturbations. Equation (12) overestimates the effects of the gravitational noise. In an extended system, part of the noise would be expended on the moment of inertia and on higher moments of the mass density, leading to weaker effects on the center of mass motion.

In any case, if we apply Eq. (12) to planets, we can use the relative accuracy in the measurement of Earth's mass to place an upper bound $\delta m/m < 10^{-4}$, which implies that $\Theta < 10^{-8}T_P$. This estimate is probably too stringent, because the most accurate measurements of mass in astronomical bodies come from the measurement of the gravitational acceleration on its surface, not from its orbit in the solar system, as would be required for comparison with Eq. (8). An exact bound on Θ will require an analysis of the motion of planet-sized bodies under ABH-type noise. However, our simple analysis shows that under pessimal assumptions, sufficiently large values of Θ that are compatible with observable decoherence effects cannot be ruled out on the basis of existing measurements.

IV. TYPES OF EXPERIMENTS

In this section, we provide order-of-magnitude estimates for ABH-type decoherence, in comparison with the D–P model.

A. Optomechanical experiments

Consider a body brought into a superposition of a zero momentum and a finite momentum state, corresponding to an energy difference ΔE. For the ABH model, the decoherence rate for the center of mass is then

$$\Gamma_{ABH} = \frac{(\Delta E)^2\tau}{\hbar^2}, \tag{13}$$

where τ is the free parameter in the master equation (4). A value for Γ_{ABH} of the order of 10^{-3} s may be observable in optomechanical systems, as it is competitive with current environment-induced-decoherence timescales. Hence, to exclude values of $\tau > \tau_P$, we must prepare a quantum state with $\Delta E \sim 10^{-14}$ J.

In the DP model, the decoherence rate for a sphere of mass M of radius R in a quantum superposition of states with different center of mass positions (though the predicted decoherence rate is largely independent of the details of the prepared state) is of the order of

$$\Gamma_{DP} = \frac{GM^2}{\hbar\sqrt{R^2 + \ell^2}}, \tag{14}$$

where ℓ is a cutoff length, originally postulated to be of the order of the size of the nucleus, but recently constrained to $\ell > 0.5 \times 10^{-10}$m (see Ref. 30). Alternative models postulate ℓ up to a scale of 10^{-7} m. For an optomechanical nanosphere with $M \sim 10^{10}$ amu and $R \sim 100$ nm, $\Gamma_{DP} \sim 10^{-3}\ \mathrm{s}^{-1}$, a value that is in principle measurable in optomechanical experiments.

B. Matter-wave interferometry

In far field interferometry, the ABH model [but not the 1D master equation (5)] leads to loss of phase coherence of the order of $(\Delta\Phi)^2 = m^2 v^3 \tau L/\hbar^2$, where L is the propagation distance inside the interferometer. While the exact derivation of $(\Delta\Phi)^2$ requires a dynamical analysis, it is of the order of $\Gamma_{ABH} t_{int}$, where $t_{int} = L/v$ is the average time of the particle in the interferometer. Setting an upper limit of $L = 100$ km, and $v = 10^4$ m/s, decoherence due to cosmological gravitons requires particles with masses of the order of 10^{16} amu. If Θ is a free parameter, experiments with particles at 10^{10} amu will test up to $\Theta \sim 10^{-5} T_P$. For comparison, the heaviest molecules used to date in quantum mechanical interference experiments are oligoporphyrines with mass of "only" 2.6×10^4 amu.[67]

The Diosi–Penrose model and other models that lead to decoherence in the position basis can also be tested by near-field[71] and far-field[72] matter-wave interferometry. A rough estimation for the loss of phase coherence is $(\Delta\Phi)^2 \simeq \Gamma_{DP} L/v = Gm^2 L/\hbar R v$, where R is the radius of the particles. In contrast to the ABH model, this loss of coherence is enhanced at low velocities. Assuming $L = 100$ km, $v = 10$ m/s, and $R = 100$ nm, an experiment would require a mass $M \sim 10^9 - 10^{10}$ amu to observe decoherence according to the DP model.

C. Wave-packet spread

The intrinsic spreading of a matter wave-packet in free space is a hallmark of Schrödinger evolution. ABH-type models predict negligible deviations in the wave-packet spread from that of unitary evolution. The DP model and all other models that involve decoherence in the position basis predict a wave-packet spread of the form,

$$(\Delta x)^2(t) = (\Delta x)_S^2(t) + \frac{\Lambda}{2m^2} t^3, \tag{15}$$

where $(\Delta x)_S^2(t)$ is the usual Schrödinger spreading, and Λ depends on the model. The changes from free Schrödinger evolution become significant at later times. An exact estimation of this effect depends on properties of the initially prepared state, and is rather involved. The MAQRO proposal estimates that for a free-propagation time equal to 100 s (accessible in their setup) it is possible to constrain CSL-type models and some models of quantum gravity decoherence, but not decoherence of the D–P type.

In contrast, the Newton–Schrödinger equation predicts a retraction of the wave-packet spread for masses around 10^{10} amu.[73] An osmium nanosphere of radius $R \simeq 100$ nm would require a couple of hours of free propagation in order to observe significant deviation from Schrödinger spreading.[74]

D. Decoherence of photons

Only the ABH model has been generalized for photons.[59,70] For interferometer experiments with arm length L, the model predicts loss of visibility of order $(\Delta\Phi)^2 = 8G\Theta E^2 L/\hbar^2 c^6$. For $L = 10^5$ km, $\Theta \sim T_P$ and photon energies E of the order of 1 eV, this implies a loss of coherence of the order of $\Delta\Phi = 10^{-8}$. In principle, this would be discernible with EM-field coherent states with mean photon number $\bar{N} > 10^{16}$, though it would be very challenging to suppress all other systematic errors to this degree.

The linear dependence of $\Delta\Phi$ on energy implies that decoherence is significantly stronger at high frequencies. For interferometry in the extreme UV, $\Delta\Phi$ may increase by two orders of magnitude or more. Alternative setups, such as the formation of effective Fabry–Pérot "cavities" with mirrors could increase the effective propagation length by many orders of magnitude, and hence, lead to stronger signatures of ABH-predicted, photon gravitational decoherence.

V. CONCLUSION

Quantum gravity, the quest for theories of the microscopic constituents of spacetime and to achieve the fusion of quantum and gravity ($Q \times G$) at the Planck scale (10^{-35} m), has occupied the attention of a significant number of theoretical physicists for the past seven decades. Yet the lack of observable experimental data has prevented any of the resulting theories to claim success. Instead of chasing this lofty yet unattainable goal, with little chance of finding directly verifiable evidence at today's energy, we should set our targets at the union of quantum and gravity ($Q + G$) because the contradictions and inconsistencies between quantum theory and general relativity (GR) already show up acutely at today's accessible low energy scales and there are earthbound and space experiments which can probe into issues at the joining of these two fundamental theories.

In particular, taking advantage of long baselines and small environmental influence, deep space experiments[71,72,75,76] can provide significant novel information about the coexistence of quantum and gravity—no matter how precarious it is—and separate them from the alternative theories. In this context, gravitational decoherence experiments have far-reaching theoretical significance in at least two respects: (i) discriminating alternative quantum or gravity theories, such as those mentioned earlier, based on their predictions, against that from theories based on general relativity and quantum field theory, such as the ABH theory described above; (ii) the possibility of discerning the nature of gravity, whether it is a fundamental theory or an effective/emergent theory.

Concerning the first aspect, it is easy to make the demarcation because most proposals for gravitational decoherence involve a violation of quantum theory, or of GR, or (usually) of both, whereas the ABH theory respects both QFT and GR. Precision experimental data can quickly discern these two categories of theories. Concerning the second aspect—why it is important and how one can make such a distinction—we offer some background perspectives in the following.

GR is commonly accepted as the best theory for the description of macroscopic spacetime, but whether quantizing GR will yield the true theory of the microscopic structure of spacetime at the Planck scale remains an open question. GR could well be an effective theory emergent from some fundamental theory of quantum gravity, valid only at the macroscopic scale we are familiar with. The ABH model distinguishes these two alternatives. In the fundamental theory view, Minkowski spacetime is the ground state of a quantum gravity theory. In the emergent theory view, Minkowski spacetime is a low energy collective state or macrostate of quantum gravity, whereby one could associate a thermodynamic description. The key difference between a ground state of a fundamental theory and a macrostate of an effective theory lies not only in the energy scale where each operates, which could have a big discrepancy but also in the strength of fluctuations. Assuming that we are blind to the origin of the Minkowski spacetime we live in (fundamental or emergent), the magnitude of its fluctuations nevertheless reveals: thermodynamic fluctuations are more powerful than quantum fluctuations in spacetime, and they can cause significantly stronger decoherence.

Therefore, if we can see evidence of gravitational decoherence, then, using a theory based on GR + QFT such as the ABH theory, even crude orders of magnitude differences in the observation data could provide a useful discriminant separating gravity as a fundamental theory from an effective one.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts of interest to disclose.

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

APPENDIX: SOME QUESTIONS ON THE BASICS OF GR + QFT AND NSE

1. Q: If one wants to combine GR + QFT, one must make assumptions about the way they are coupled. Which ones?

A: A quantum field is coupled to a curved spacetime via the Laplace–Beltrami operator containing the metric of the curved spacetime, and the wave/field equation in curved spacetime contains a term proportional to the scalar curvature of the background spacetime. The coupling constant distinguishes different types of coupling, minimal, conformal, etc. See, e.g., Ref. 38.

2. Q: GR is fundamentally nonstochastic, how does it contribute to a master equation which is stochastic?

A: Indeed so. Einstein equation describing the dynamics of the gravitational field is deterministic. But then almost all discussions of decoherence invoke some noise source in some stochastic equation like a Langevin equation or a master equation. The noise in gravitational decoherence is due to fluctuations in the gravitational waves, or in gravitons. This is because there is an intermediate step involved: The noise in these stochastic equations can be shown to arise from an environment described by deterministic dynamical variables. Examples abound, the most common perhaps is Brownian motion, where the environment can be an n-oscillator heat bath or a scalar field, and a spectral density (for the oscillator bath) or a Gaussian functional identity (a la Feynman–Vernon) can be used to describe these environments in terms of noises with different properties (e.g., colored, multiplicative). Through these procedures, noise can be rigorously (at least for Gaussian systems) defined. Neither GR nor QFT need to be a stochastic theory—they are not—for one to investigate decoherence based on these theories.

3. Q: You call the semiclassical gravity (SCG) theory[47] “proper” to distinguish it from the Moller–Rosenfeld (MR) theory which is followed by many authors. Don’t they both invoke GR + QFT? What is the crucial distinction?

A: SCG proper is the large N limit of quantum gravity,[36] like the Hartree approximation in atomic physics. Large N is important. One cannot deduce the one-particle or a few-particle-NSE from GR + QFT. The crucial difference lies in whether one treats the mass density as an operator which is the correct way, or takes its mean value, a c-number, as the source. This renders the NSE nonlinear. The equations evolving quantum states are linear in quantum theory, including QFT. Nonlinear evolution equations have many pathologies.

REFERENCES

[1] F. Karolyhazy, Nuovo Cimento A **42**, 390 (1966).

[2] L. Diosi, Phys. Lett. A **120**, 377 (1987); Phys. Rev. A **40**, 1165 (1989).

[3] R. Penrose, “Gravity and state vector reduction,” in *Quantum Concepts in Space and Time*, edited by R. Penrose and C. J. Isham (Clarendon, Oxford, 1986); “Gravity and state vector reduction,” in Gen. Relativ. Gravitation **28**, 581 (1996).

[4] C. Anastopoulos and B. L. Hu, Classical Quantum Gravity **32**, 165022 (2015).

[5] R. B. Griffiths, J. Stat. Phys. **36**, 219 (1984); *Consistent Quantum Theory* (Cambridge University, Cambridge, 2002).

[6] R. Omnés, J. Stat. Phys. **53**, 893 (1988); Rev. Mod. Phys. **64**, 339 (1992); *The Interpretation of Quantum Mechanics* (Princeton University, Princeton, 1994).

[7] M. Gell-Mann and J. B. Hartle, “Quantum mechanics in the light of quantum cosmology,” in *Complexity, Entropy, and the Physics of Information*, edited by W. Zurek (Addison-Wesley, Reading, 1990); , “Quantum mechanics in the light of quantum cosmology,” in Phys. Rev. D **47**, 3345 (1993).

[8] F. Dowker and A. Kent, J. Stat. Phys. **82**, 1575 (1996).

[9] E. Okon and D. Sudarsky, Stud. His. Philos. Mod. Phys. **48**, 7 (2014); R. B. Griffiths, arXiv:1501.04813 (2015); E. Okon and D. Sudarsky, Stud. Hist. Philos. Mod. Phys. **52**, 217 (2015).

[10] W. H. Zurek, Phys. Rev. D **26**, 1862 (1982).

[11] E. Joos and H. D. Zeh, Z. Phys. B **59**, 223 (1985).

[12] H. D. Zeh, Found. Phys. **1**, 69 (1970).

[13] J. S. Schwinger, J. Math. Phys. **2**, 407 (1961).

[14] R. P. Feynman and F. L. Vernon, Ann. Phys. **24**, 118 (1963).

[15] A. O. Caldeira and A. J. Leggett, Physica A **121**, 587 (1983).

[16] B. L. Hu, J. P. Paz, and Y. Zhang, Phys. Rev. D **45**, 2843 (1992).

[17] C. Anastopoulos, Int. J. Theor. Phys. **41**, 1573 (2002); M. Schlosshauer, Rev. Mod. Phys. **76**, 1267 (2005); Phys. Rep. **831**, 57 (2019).

[18] G. C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986).

[19] P. Pearle, Phys. Rev. D **13**, 857 (1976); Int. J. Theor. Phys. **18**, 489 (1979); Phys. Rev. A **39**, 2277 (1989).

[20] G. C. Ghirardi, P. Pearle, and A. Rimini, Phys. Rev. A **42**, 78 (1990); G. C. Ghirardi and A. Bassi, Phys. Rep. **379**, 257 (2003).

[21] R. Tumulka, J. Stat. Phys. **125**, 821 (2006); Proc. R. Soc. **462**, 1897 (2006); D. J. Bedingham, Found. Phys. **41**, 686 (2011); D. Bedingham, D. Dürr, G. C. Ghirardi, S. Goldstein, R. Tumulka, and N. Zangh, J. Stat. Phys. **154**, 623 (2014); P. Pearle, Phys. Rev. **91**, 105012 (2015).

[22] C. Jones, G. Gasbarri, and A. Bassi, J. Phys. A **54**, 295306 (2021).

[23] A. Bassi, A. Groardt, and H. Ulbricht, Classical Quantum Gravity **34**, 193002 (2017).

[24] R. Penrose, Found. Phys. **44**, 557 (2014).

[25] C. Anastopoulos and B. L. Hu, Classical Quantum Gravity **30**, 165007 (2013).

[26] M. Blencowe, Phys. Rev. Lett. **111**, 021302 (2013).

[27] I. Pikovski, M. Zych, F. Costa, and C. Brukner, Nat. Phys. **11**, 668 (2015); New J. Phys. **19**, 025011 (2017).

[28] E. L. Hahn, Phys. Rev. **80**, 580 (1950).

[29] C. Anastopoulos and N. Savvidou, Phys. Rev. E **83**, 021118 (2011).

[30] S. Donadi, K. Piscicchia, C. Curceanu *et al.*, Nat. Phys. **17**, 74 (2021).

[31] C. Anastopoulos, M. Lagouvardos, and K. Savvidou, Classical Quantum Gravity **38**, 155012 (2021).

[32] C. Moller, "Les theories relativistes de la gravitation," in *Colloques Internationaux CNRX 91*, edited by A. Lichnerowicz and M.-A. Tonnelat (CNRS, Paris, 1962).

[33] L. Rosenfeld, Nucl. Phys. **40**, 353 (1963).

[34] M. Bahrami, A. Groardt, S. Donadi, and A. Bassi, New J. Phys. **16**, 115007 (2014).

[35] B. L. Hu, J. Phys. **504**, 012021 (2014).

[36] J. B. Hartle and G. T. Horowitz, Phys. Rev. D **24**, 257 (1981).

[37] C. Anastopoulos and B. L. Hu, New J. Phys. **16**, 085007 (2014).

[38] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University, Cambridge, 1982).

[39] J. Ellis, J.S.Hagelin, D. V. Nanopoulos, and M. Srednicki, Nucl. Phys. B **241**, 3 (1984).

[40] W. G. Unruh and R. M. Wald, Rep. Prog. Phys. **80**, 092002 (2017).

[41] J. Ellis, N. E. Mavromatos, and D. V. Nanopoulos, Phys. Lett. **293**, 37 (1992).

[42] L. J. Garay, Phys. Rev. Lett. **80**, 2508 (1998).

[43] R. Gambini, R. Porto, and J. Pullin, Phys. Rev. Lett. **93**, 240401 (2004); New J. Phys. **6**, 45 (2004).

[44] G. J. Milburn, New J. Phys. **8**, 96 (2006); Phys. Rev. **44**, 5401 (1991).

[45] R. Bonifacio, Nuovo Cimento B **114**, 473 (1999).

[46] P. M. V. B. Barone and A. O. Caldeira, Phys. Rev. A **43**, 57 (1991); C. Anastopoulos and A. Zoupas, Phys. Rev. D **58**, 105006 (1998); P. R. Johnson and B. L. Hu, *ibid.* **65**, 065015 (2002).

[47] B. L. Hu and E. Verdaguer, *Semiclassical and Stochastic Gravity–Quantum Field Effects on Curved Spacetimes* (Cambridge University, Cambridge, 2020).

[48] E. A. Calzetta and B. L. Hu, Phys. Rev. D **49**, 6636 (1994); B. L. Hu and A. Matacz, *ibid.* **51**, 1577 (1995); E. A. Calzetta, A. Campos, and E. Verdaguer, *ibid.* **56**, 2163 (1997); F. C. Lombardo and F. D. Mazzitelli, *ibid.* **55**, 3889 (1997).

[49] R. Martín and E. Verdaguer, Phys. Lett. B **465**, 113 (1999); Phys. Rev. D **61**, 124024 (2000); **61**, 26 (2000).

[50] B. L. Hu, A. Roura, and E. Verdaguer, Phys. Rev. D **70**, 044002 (2004).

[51] T. C. Ralph, G. J. Milburn, and T. Downes, Phys. Rev. **79**, 022121 (2009).

[52] T. J. Ralph and J. Pienar, New J. Phys. **16**, 085008 (2014).

[53] D. Deutsch, Phys. Rev. D **44**, 3197 (1991).

[54] S. W. Hawking, Phys. Rev. D **46**, 603 (1992).

[55] B. S. Kay, M. J. Radzikowski, and R. M. Wald, Commun. Math. Phys. **183**, 533 (1997).

[56] J. A. Wheeler, Phys. Rev. **97**, 511 (1955).

[57] S. Reynaud, B. Lamine, A. Lambrecht, P. Maia Neto, and M. T. Jaekel, Gen. Relativ. Gravitation **36**, 2271 (2004).

[58] B. L. Hu, J. Phys. **174**, 012015 (2009).

[59] T. Oniga and C. H.-T. Wang, Phys. Rev. D **93**, 044027 (2016).

[60] C. Anastopoulos, Phys. Rev. D **54**, 1600 (1996).

[61] Z. Haba, Mod. Phys. Lett. A **15**, 1519 (2000); Z. Haba and H. Kleinert, Int. J. Mod. Phys. A **17**, 3729 (2002).

[62] S. Reynaud, B. Lamine, A. Lambrecht, P. M. Neto, and M. T. Jaekel, Int. J. Mod. Phys. A **17**, 1003 (2002).

[63] W. L. Power and I. C. Percival, Proc. R. Soc. London, Ser. A **456**, 955 (2000).

[64] P. Kok and U. Yurtsever, Phys. Rev. D **68**, 085006 (2003).

[65] H. P. Breuer, E. Göklü, and C. Lämmerzahl, Classical Quantum Gravity **26**, 105012 (2009).

[66] L. Asprea, G. Gasbarri, H. Ulbricht, and A. Bassi, Phys. Rev. Lett. **126**, 200403 (2021).

[67] Y. Y. Fein *et al.*, Nat. Phys. **15**, 1242 (2019).

[68] S. Adler, *Quantum Theory as an Emergent Phenomenon* (Cambridge University, Cambridge, 2004).

[69] L. Diosi, Braz. J. Phys. **35**, 260 (2005).

[70] M. Lagouvardos and C. Anastopoulos, Classical Quantum Gravity **38**, 115012 (2021).

[71] R. Kaltenbaek *et al.*, EPJ Quantum Technol. **3**, 5 (2016).

[72] R. Kaltenbaek *et al.*, Exp. Astron. **34**, 123 (2012).

[73] D. Giulini and A. Groardt, Classical Quantum Gravity **28**, 195026 (2011).

[74] A. Groardt, Phys. Rev. A **94**, 022101 (2016).

[75] L. Mazzarella *et al.*, Proc. SPIE **11835**, 118350J (2021); M. Mohageg *et al.*, arXiv:2111.15591 (2021).

[76] A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht, Rev. Mod. Phys. **85**, 471 (2013); G. Gasbarri, A. Belenchia, M. Carlesso *et al.*, Commun. Phys. **4**, 155 (2021); S. Kanno, J. Soda, and J. Tokuda, Phys. Rev. **104**, 083516 (2021).