Countering Justification Holism in the Epistemology of Logic: The Argument from Pre-Theoretic Universality

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Forthcoming in the *Australasian Journal of Logic*.

Abstract

A key question in the philosophy of logic is how we have epistemic justification for claims about logical entailment (assuming we have such justification at all). Justification holism asserts that claims of logical entailment can only be justified in the context of an entire logical theory, e.g., classical, intuitionistic, paraconsistent, paracomplete etc. According to holism, claims of logical entailment cannot be atomistically justified as isolated statements, independently of theory choice. At present there is a developing interest in—and endorsement of—justification holism due to the revival of an abductivist approach to the epistemology of logic. This paper presents an argument against holism by establishing a foundational entailment-sentence of deduction which is justified independently of theory choice and outside the context of a whole logical theory.

Keywords

Deduction; Semantics; Bootstrapping; Logical Theories; Epistemic Justification; Logical Abductivism; Anti-Exceptionalism about Logic

1 Introduction

1.1 Abductivism, Justification Holism, and Logical Theories

Recently there has been a renewed interest in an abductivist approach (to be defined) in the epistemology of logic.[1] Some of the contemporary abductivists are motivated by *anti-exceptionalism about logic*, which, roughly speaking, says that logic doesn't differ from (empirical) science in any interesting way.[2] This view, and the general abductive approach, has historically been associated with Quine (1953, 1986),[3] [4] who argued that logic is neither necessary, analytic nor *a priori*.[5] Modern varieties of anti-exceptionalism, however, come in less radical forms, e.g., by denial of logic's a priori-status (Hjortland, 2017) and/or analyticity (Williamson, 2007) without full-blown Quinean commitments.

According to Gillian Russell, *abductivists* endorse two central claims:

> The heart of the abductivist approach consists in two claims. The first is holism about the justification of logic: it is entire logics—rather than isolated claims of consequence—that are justified (or not). The second is that what justifies a theory is adequacy to the data, and the possession of virtues and absence of vices. (Russell, 2019, p. 550)

For abductivists the object of justification is logical theories *en bloc* rather than in-

[1] See (Williamson, 2007, 2017b, 2020, 202X; Priest, 2008, 2014, 2021; Russell, 2014, 2015, 2019; Beall, 2017; Hjortland, 2017, 2019, 2022; Martin, 2021c,a,b, 2022; Zanetti, 2021; Martin and Hjortland, 2021, 2022; Rossberg and Shapiro, 2021; Sagi, 2021; Becker Arenhart, 2022a,b; Carlson, 2022; Tajer, 2022; Sforza et al., 2023; Martin and Hjortland, 202X).

[2] In a recent paper Martin and Hjortland (2022) distinguish between different kinds of anti-exceptionalism about logic. Usually anti-exceptionalism is taken to be a stronger claim than abductivism, e.g., *methodological* anti-exceptionalism proposes a similarity between the methodology in logic and science which is not necessary for abductivism.

[3] One should not simply identify modern versions of abductivism with Quine's ditto. See for instance (Martin, 2021b) for some important differences.

[4] Note also the seminal work on the abductive approach by Nelson Goodman (1983).

[5] Bear in mind the internal tension in (the development of) Quine's philosophy. On the one hand, *Quine the holist* (1953) takes logic to be revisable, it's just that our beliefs concerning such matters are closer to the center of our web of beliefs, and hence hard to revise, whereas beliefs about "more synthetic" statements are closer to the periphery of the web, and thus easier to revise. On the other hand, *Quine the conservative* (1986) thinks that classical first-order logic is "the realm of the obvious" and that any attempt of non-classical revision amounts to changing the subject.

dividual claims of logical entailment.[6] [7] Abductivists endorse justification holism claiming that whatever justification we have for holding particular claims of logical entailment must be in virtue of the logical theory to which they belong. It's not that one is not able to have justification with respect to individual sentences about entailment, the point is rather that such justification is dependent on a choice of logical theory, say, classical, intuitionistic, paraconsistent, paracomplete etc.[8] Further, abductivists hold that the grounds for justification of a logical theory is how well it fits with relevant data (frequently taken to be our intuitive judgments about logical inferences) plus its theoretical virtues and lack of vices, e.g., its strength in terms of ratified consequences (in logic and wider scientific context), how aesthetically elegant and simple it is, and how ontologically parsimonious. Abductivism is succinctly summarized by Ben Martin:

> According to this account of logical epistemology, logical propositions are not directly justified by intuitions or definitions, but rather logical theories are justified by their ability to best accommodate relevant data. In other words, logical theories are justified by abductive means. (Martin, 2021b, p. 9070)

To be sure, the term 'logical theory' must at minimum be understood as a set of sentences logically closed under a given entailment-relation (modeling the concept of validity). Indeed, according to Ole Hjortland there is something like a consensus that the main function of a logical theory is to tell us which inferences are valid (Hjortland, 2019, p. 252). However, some authors add to this deflationary understanding a demand that theories should account for features like provability, truth-preservation, formality, and consistency, as well (Priest, 2008; Hjortland, 2017).[9]

[6] We'll use the terms 'proposition', 'sentence', and 'claim' interchangeably throughout this paper.

[7] It's unclear in the contemporary literature on abductivism whether we should distinguish between a *logic* and a *logical theory*. Consult (Mortensen, 2013) for an example of someone who draws a clear distinction between the two.

[8] Further details about the position *justification holism* can be found in §2.3 below.

[9] As an anonymous reviewer points out, the minimal characterization of a logical theory stated above can be thought to miss a potential distinction between a logical *system* and a logical *theory*; where the former is taken to be a formal apparatus with a vocabulary, a proof-theory, a semantics etc., while the latter is an applied system that models particular target-phenomena. According to some, logical theories should not only tell us which inferences are valid, but ideally also tell us *why* these inferences are valid (and other inferences invalid). Theories shouldn't merely give us a set of sentences logically closed under a given entailment relation or supply a list of inferences

One should also bear in mind that, in some cases, e.g., Carnap (1937), Dummett (1991), and Shapiro (2014), logical theories are claimed to be solely about language, i.e., metalinguistic, but often they are taken to be non-metalinguistic (Russell, 1918; Sider, 2013; Maddy, 2014; Williamson, 2013, 2017b). Williamson, for instance, takes logical theories to consist of unrestricted generalizations about the world, not just language.[10]

1.2 Justification Atomism

For the present purposes it's crucial to note that abductivism is incompatible with *justification atomism*:

> One view that is incompatible with abductivism is a view on which individual claims about entailment are justified atomistically, rather than in the context of a whole theory. (Russell, 2019, p. 552)

The justification atomist opposes the holist part of the abductivist methodology by insisting that: *individual claims about entailment can be justified point-wise rather than in the context of a whole logical theory*.

Importantly, justification *holism* is not claiming that one cannot have justification for an individual claim that, say, 'double negation elimination is valid'.[11]

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or laws that are valid, they should also provide an account of why these inferences or laws are valid. Thus—according to some—logical theories are *about* a particular (extra-systematic) subject matter, and for a theory to be *correct* it should get the subject matter right. For example, the modal logics **S5**, **S4**, etc., can be characterized as logical systems of sets of sentences, given by some system of proofs or models. But to adopt one of these as a *theory* is to, in addition, adopt this or that system as part of an explanation of what follows from what (and what doesn't follow). You wouldn't adopt both **S4** and **S5** as logical theories of the same phenomenon (say, some given notion of necessity), when they give different accounts of the truth of modal statements that can differ in truth value. Note that while this distinction between logical system and logical theory is a plausible one, it won't change the main result of the present paper whether we commit to it or not. See for instance §2.3.4 for an explanation how the main argument of §2 is compatible with different logical analyses.

[10] One might frame anti-exceptionalism about the content or subject matter of logical theories as *metaphysical* anti-exceptionalism about logic. An illustrative example of such position is Bertrand Russell's universalism: "*Logic is concerned with the real world just as truly as zoology, though with its more abstract and general features*." (Russell, 1919, p. 169). Metaphysical anti-exceptionalism is importantly distinct from *epistemological* anti-exceptionalism, and as noted by Martin and Hjortland (2022), one can be an anti-exceptionalist about one without being an anti-exceptionalist about the other.

[11] Let 'φ' denote a meta-variable and let the symbol '$\neg$' denote negation. Then double negation elimination is the entailment from φ to $\neg\neg\varphi$.

For one could easily obtain such individual justification via a proof *within* some logical theory. The key point here is that, according to the holist, any such justification presupposes the context of an entire logical theory, and depends on a choice of such theory, e.g., choosing a classical theory rather than an intuitionistic one.

The *atomistic* view is incompatible with holism because the atomist holds that there can be cases of individual entailment-sentences such that these are justified outside the context of a whole logical theory, viz., counterexamples to holism.

Of course, some holists may be more sensitive to counterexamples than others. Tim Williamson's work on the problem of overfitting in epistemology (2007; 2017a; 2020) suggests that he would be reluctant to give up holism due to a single counterexample, for instance; while Gillian Russell's work on logical nihilism (2017; 2018a; 2018b) indicates that she has a great respect for the normative force of individual counterexamples. Accordingly, the announced argument against justification holism (cf. §2) will have the greatest impact on those who are ill-disposed to counterexamples.

It's also worth stressing that the contemporary abductivists are not always explicit about what kind of epistemic justification they are interested in, and whether this is a kind that only logical experts can possess. *Prima facie*, the kind of justification one can expect agents to have with respect to logical propositions and theories varies with their logical background knowledge. Contrast, for example, the kinds of justification we would expect a novice and a logical expert to have, respectively. The expert may have firm convictions regarding logical theories and principles, while it's unlikely that the novice would even fathom what a logical theory is. However, it seems that the distinction between *fundamental* and *non-fundamental* sources of justification could dissolve this issue. Deductive proofs may be seen as a fundamental source of justification, while testimony could be considered a non-fundamental source enabling transmission of justification only. Insofar as we are interested in fundamental justification alone, it is straightforward to suppose that the justification of entailment-sentences is an esoteric business of logical experts, and that is what we will assume here.

Further, we'll suppose that the abductivists are interested in *propositional* rather than *doxastic* justification, i.e., the justification of logical propositions rather than belief-tokens about such propositions. Doxastic justification is a property that a belief has when one believes a proposition for which one has propositional justification, and this belief is based on that which propositionally justifies it. We will focus on propositional justification since—assuming we can give a good account of propositional justification and that this account can be exploited as the basis for

the relevant beliefs—we can have doxastic justification as well.[12]

1.3 E-Sentences and E-Literals

Before getting down to business it will be helpful to introduce some technical terminology concerning logical entailment. *E-sentences* are atomic sentences in which the main predicate is given by the symbol '$\models$' (or its natural language equivalents) (Russell, 2019).[13] Examples are:

- $[\varphi \vee \psi, \neg\psi \models \varphi]$
- $[\models \neg(\varphi \wedge \neg\varphi)]$
- $[\varphi \wedge \neg\varphi \models \psi]$[14]

These sentences are *atomic* in the sense that they are the simplest kind of sentences of a given meta-language. To see this, we observe that symbols like '$\vee$','$\neg$', '$\wedge$' are not used but merely mentioned in E-sentences, whereas '$\models$' is a metalinguistic symbol placed between terms referring to schemas (or sentences) of an object-language.[15]

[12] We'll leave it as an open question whether the distinction between justification *internalism* and *externalism* is of great importance to the holist. Note, however, that basing your beliefs about logical propositions on proofs in deductive logic could be seen as a kind of (evidential) proper basing of propositional justification, which would amount to doxastic justification on standard internalist accounts. Similarly, forming your beliefs about logical propositions via proofs in deductive logic could be counted as a reliable (or safe) method of belief-formation on standard externalist accounts of doxastic justification. For details on internalism in the form of evidentialism, consult e.g., (Feldman and Conee, 1985; Conee and Feldman, 2004). For accounts of the epistemic basing relation, see e.g., (McCain, 2012, 2014; Carter and Bondy, 2019; Neta, 2019; Korcz, 2021). For details regarding externalism in the form of process reliabilism, consult e.g., (Goldman, 1979, 1986). For externalist accounts involving modal properties like safety and sensitivity, see e.g., (Dretske, 1971; Nozick, 1983; Williamson, 2000; Pritchard, 2005).

[13] 'E-sentence' is shorthand for 'entailment-sentence'. As indicated by (the standard use of) the double turnstile-symbol '$\models$', E-sentences and E-literals should be thought of in semantic terms, not proof-theoretic ones (more on our exclusive semantic focus in footnote 20). We use square brackets around entire E-sentences and E-literals rather than corner-quotes around schemas to ease readability.

[14] Let lowercase Greek letters be meta-variables. Let the symbols '$\vee$', '$\neg$', '$\wedge$', and '$\rightarrow$', denote disjunction, negation, conjunction, and material implication, respectively.

[15] Note that my use of the object-language/meta-language distinction presupposes that there is a hierarchy of languages in logic. A number of logicians reject this. Notoriously, they think (i) it's implausible that there be meta-languages for English or any other natural language, and (ii) one does not even need a hierarchy of languages for the purposes of a theory of truth. Examples

An *E-literal* is either an E-sentence or its negation. Thus, all E-literals are E-sentences, but not *vice versa*. Examples of E-literals are:

- $[\varphi \rightarrow \psi, \varphi \nvDash \psi]$
- $[\vDash \varphi]$
- $[\varphi \wedge \neg\varphi \nvDash \psi]$

E-literals are central to the epistemology of logic as their truth-value tells us what follows from what, and what doesn't follow. On the common view that logic is the study of (valid) inferences, the importance of E-literals is given, but in virtue of what are our E-literals justified, and is it possible for individual E-literals to be propositionally justified outside the context of a whole logical theory? Those are the central questions of this paper.[16] Justification holism gives one possible all-encompassing answer, but as we shall see now, there are good reasons to think that holism is false.

2 A Foundational E-Sentence of Deduction

This paragraph aims to show that the E-literal $[\forall xPx, \Gamma \vDash Pa]$, where '$a$' refers to an element of domain D of some model $\mathfrak{M}$, and 'Γ' denotes a (possibly empty) set of side-conditions, is true under any acceptable deductive entailment-relation, and denying its truth would mean giving up on deduction altogether.[17] In other words, the aim is to establish that a liberal version of the E-literal about *universal instantiation* is a foundational E-literal for which we have propositional justification independently of theory choice and outside the context of an entire logical

are dialetheists, such as Graham Priest (2006) and Jc Beall (2011), as well as proponents of paracomplete logics like Saul Kripke (1976). These logicians endorse non-hierarchical truth theories and semantics. It's well beyond the scope of this paper to go deeper into these issues, so we'll have to make do with the following observation. Look in any logic textbook and you shall find a formal object-language plus a logical entailment-relation for that object-language defined in a meta-language, which is usually English (perhaps with bits of mathematical notation). In this sense, the notion of logical entailment is clearly meta-linguistic.

[16] Note that this is a separate question from the question of what makes an agent entitled in her *disposition* to reason in accordance with some rule (Boghossian and Peacocke, 2000).

[17] A (Tarskian) model $\mathfrak{M}$ in first-order logic is an ordered pair $\mathfrak{M} = \langle D, I \rangle$, where D is a domain of objects and I is an interpretation function specifying referents for constant symbols, predicate symbols, and function symbols. We say that $\mathfrak{M}$ is a model of a well-formed formula φ if φ is true in $\mathfrak{M}$. A countermodel $\mathfrak{M}^*$ to φ is a model of $\neg\varphi$.

theory; thus constituting a counterexample to the holistic doctrine.[18]

The plan for the rest of the paragraph is as follows. In §2.1 universal instantiation is defined and some crucial notions, viz., *Universality* and *Universality Booting*, are introduced and motivated.[19] In §2.2 the main argument against justification holism is put forward. If successful, it shows that justification holism is false. As this result will strike many readers as being too bold, §2.3 aims to address some objections to it. In particular, the straightforward objection from *free logic* will be discussed in §2.3.

2.1 Terminology and Lemmas

Some preliminary remarks.

First, universal instantiation ('UI') is a well-known syntactic inference rule. Under one plausible semantic interpretation it says: any instance of '*Everything is P*' entails '*t is P*', where '*t*' refers to an individual term. When the rule is stated formally in standard notation, it looks like this:

$$\frac{\forall v P v}{P t}$$

When this schema is interpreted in the standard way, we take the quantifier denoted by '$\forall$' as ranging over a domain of objects, the predicate denoted by 'P' as referring to a property, and the term denoted by 't' as replacing all occurrences of the variable given by 'v'. Accordingly, we can state an E-literal about UI as follows: '$[\forall x P x, \Gamma \vDash Pa]$', where '$a$' refers to an element of domain D of some model $\mathfrak{M}$, and 'Γ' denotes a set of side-conditions based on one's favored logical analysis. Since Γ is usually left empty, we'll simply write '$[\forall x P x \vDash Pa]$' by default in order to ease readability. We'll discuss a special case where Γ is non-empty in §2.3.

Second, we'll assume that, in semantics, *Universality* is a necessary property of every acceptable deductive entailment-relation. That is to say, any acceptable deductive entailment-relation—modeling the concept of validity—must involve universal quantification over cases, be it in the form of possible worlds, constructions, situations, truth-makers etc. One could, for instance, say:

[18] From this point on we'll frequently use the adjective 'foundational' about a particular E-literal and simply take this to mean *an entailment claim for which we have propositional justification independently of theory choice and outside the context of an entire logical theory*.

[19] 'Universality Booting' is shorthand for 'Universality Bootstrapping'.

> A valid inference is one whose conclusion is true in every case in which all its premises are true. (Jeffrey and Burgess, 2006, p. 1)

Or

> . . . [D]eductive validity can be adequately accounted for by means of quantification over possible worlds: an argument is deductively valid (or equivalently, the relation of consequence holds between its premises and conclusion) if and only if in all possible worlds in which the premises are true/holds, so is/does the conclusion. (Dutilh Novaes, 2020, pp. 14-15)

In these and similar ways universal quantification is standardly thought to be embedded in the semantic characterization of deductive entailment. And furthermore, Universality is widely thought to be exactly what gives deduction necessary force, i.e., demarcating it from induction and abduction (Beall and Restall, 2000, 2006; Cohnitz and Estrada-González, 2019; Dutilh Novaes, 2020; Douven, 2021). Thus, Universality is an extremely well-motivated property of acceptable deductive entailment.[20]

Third, let's make the crucial observation that the E-literal about universal instantiation, i.e., $[\forall xPx \vDash Pa]$, is a universal sentence about *true* universal sentences. For the main predicate of $[\forall xPx \vDash Pa]$ is given by the entailment-symbol, which is exactly a universal claim (by Universality). This is crucial because, in our modelings of the concept validity, we'll have that any model $\mathfrak{M}$ which makes $[\forall xPx \vDash Pa]$ true must itself be a fact of universal quantification over cases; and note that this fact will need to be a *pre-theoretic* counterpart of UI. That is to say, any $\mathfrak{M}$ making the E-literal $[\forall xPx \vDash Pa]$ true must itself be a fact of universal quantification which lies outside the bounds of logical theorizing; since any acceptable deductive entailment-relation—modeling the concept of validity—must adhere to brute universal quantification over cases. Or, in yet other words, the E-literal about UI is *doubly* universal in containing both a universal statement and in stating a fact

[20] A natural constraint on the main result below is imposed by our exclusive focus on semantic accounts of deduction. Proof-theorists need not adhere to universal quantification over cases in their modelings of validity as their definitions of the concept presuppose the particular *there's a proof* rather than the universal *in all cases*. Structurally, however, a similar foundational point could be made with respect to the particular quantifier, but we'll leave proof-theoretic specifications of validity out of the picture here, as they are strictly speaking irrelevant to the aim of this paper.

of entailment, which is itself a brute fact of universal quantification.[21] Let's name this special feature of $[\forall xPx \vDash Pa]$ '*Universality Booting*'.

Here's an intuitive elaboration. Consider the following E-literals:

1. $[\forall xPx \vDash Pa]$
2. $[\varphi \wedge \psi \vDash \varphi]$

Now, (1) induces Universality Booting, whereas (2) doesn't bring about anything like "Conjunctive Booting". For (1) is a universal sentence about true universal sentences, while (2) is a universal sentence about true conjunction-sentences. Hence, while any $\mathfrak{M}$ making (1) true must itself be a pre-theoretic fact of universal quantification over cases, it would be false to suggest that any $\mathfrak{M}$ making (2) true must itself be a pre-theoretic fact of conjunction elimination. And consequently, the E-literal $[\forall xPx \vDash Pa]$ has a pre-theoretic booting-property which other E-literals like $[\varphi \wedge \psi \vDash \varphi]$, $[\varphi \vDash \varphi \vee \psi]$, $[\neg\neg\varphi \vDash \varphi]$ etc. don't have.

In slogan-form: *Whatever logical theory you prefer, it will be booting in a state of universality!*

2.2 Countering Justification Holism

Based on the preliminaries from §2.1, we are now equipped to show that $[\forall xPx \vDash Pa]$ is a foundational E-literal of deduction.

> *The Argument from Pre-Theoretic Universality*
>
> Assume that Universality is a necessary property of any acceptable deductive entailment-relation, and let '$\vDash$' denote any such relation. Suppose further that $[\forall xPx \vDash Pa]$ is false. Then there exists a counter-model $\mathfrak{M}^*$ to the E-literal $[\forall xPx \vDash Pa]$, i.e., a model such that $[\forall xPx \nvDash Pa]$ and $a \in D$. By Universality Booting, any $\mathfrak{M}$ making $[\forall xPx \vDash Pa]$ true is itself a pre-theoretic fact of universal quantification over cases. Yet, by assumption $[\forall xPx \vDash Pa]$ is false, so there can be no such

[21] The brute fact of universal quantification referred to above is perhaps easiest to register when thinking in terms of counterexamples. If you've got a model of the premises of an argument which is not a model of the conclusion. Then you are making a transition from an *instance* to the falsity of a *universal* claim. This is an *implicit* appeal to UI. Thanks to an anonymous reviewer for their very detailed comments on this paragraph.

> pre-theoretic fact. But then, by Universality, $\models$ cannot be an acceptable deductive entailment-relation. For there exists a counterexample to universal quantification over cases, viz., $\mathfrak{M}^*$. Therefore, either $[\forall xPx \models Pa]$ has no counter-model, or Universality is not a necessary property of acceptable deductive entailment. By assumption, Universality is a necessary property of acceptable deductive entailment. Ergo: $[\forall xPx \models Pa]$ is true under any acceptable deductive entailment-relation.

Cut your theoretical cake anyway you please, some E-literals—like $[\forall xPx \models Pa]$ as demonstrated—are propositionally justified independently of theory choice and outside the context of an entire logical theory. And importantly, the upshot is not just that all acceptable logical theories should include $[\forall xPx \models Pa]$, perhaps for different reasons, rather the argument shows that $[\forall xPx \models Pa]$ is foundational in such a way that it leaves any theoretical specifications—within the bounds of deduction—redundant with respect to its justificational status. If one were to deny the truth of $[\forall xPx \models Pa]$, this would amount to giving up on deduction altogether (by denial of Universality). So, to carve out the point: $[\forall xPx \models Pa]$ is a foundational E-literal of deductive entailment, and hence justification holism must strictly speaking be false.[22]

Now, finally, before taking on some pressing objections to the Argument from Pre-Theoretic Universality, two quick clarifying comments are called for.

First, the argument above doesn't fall prey to a conflation of the distinction between quantification in *object-language* and quantification in *meta-language*. The argument appeals to the brute fact that any acceptable deductive entailment-relation—semantically understood—will be booting up in a state of universality with respect to its cases, be it in the form of possible worlds, constructions, situations, truth-makers etc. As this fact must be taken for granted by any logical theory, it will need to be presupposed in whatever semantic entailment-relation one can come up with, and no matter the meta-language one might fancy.

Second, neither does the argument conflate *first-order* and *higher-order* quantification. It uses no quantification over properties at all (or anything in that vicinity).

[22]It's worth flagging that the argument relies on inferential strategies such as *reductio ad absurdum* ('reductio'), which is unacceptable to some non-classical logicians, e.g., dialetheists like Graham Priest (2006). However, even for dialetheists who reject reductio as a general strategy, it's still safe to use it in consistent contexts. For Priest, reductio is "quasi-valid", i.e., valid if the premises are consistent. So, while reductio is used in the argument above, it's fair to suppose that this is in a consistent context, and thus, that even Priest would be fine with this particular use.

2.3 Objections

2.3.1 Charity to Holists

One potential worry about the above argument concerns how one should interpret the position referred to by the label 'justification holism' and whether the result in §2.2 really poses a problem for the holist under a charitable interpretation.[23] In this paper, the holist position was introduced as follows:

> (a) Holism about the justification of logic: it is entire logics—rather than isolated claims of consequence—that are justified (or not). (cf. §1.1)

But when countering this claim, it was established that:

> (b) Some E-literals—like $[\forall xPx \vDash Pa]$—are propositionally justified independently of theory choice and outside the context of an entire logical theory. (cf. §2.2)

Now, would the truth of (b) be problematic for the holist position as it is expressed in (a)? One may suspect that the central argument resulting in (b) is off the mark because a charitable interpretation of the holist position seems able to take on board the whole story of §2.2. After all, the upshot of the argument is that assuming some very general features of deductive entailment, the E-literal $[\forall xPx \vDash Pa]$ will be true under all acceptable entailment-relations, which perhaps doesn't amount to showing that $[\forall xPx \vDash Pa]$ *is justified outside the context of an entire logical theory*, but rather that the E-literal is justified *independently of theory choice* in the sense that no matter what theory you consider it in the context of, it will be justified. Compare, for instance, to a contextualist position about knowledge attributions: the proposition expressed by the claim that '*Subject*, *S*, *knows that S exists*' is not true independently of context in a sense that refutes contextualism, but in the sense that it is true in every context. Thus, on a charitable reading, what the holist claims is that a particular E-literal, like $[\forall xPx \vDash Pa]$, cannot be justified outside the context of a logical theory because a logical theory is what specifies "the bounds of deduction". And so, the holist could accept all the central claims made in §2.2 as part of a broad holistic justification-enterprise.

While this objection completely misses the central point about the booting-property of $[\forall xPx \vDash Pa]$ and how this special feature of the E-literal about universal instantiation gives rise to pre-theoretic justification, let's just assume for the

[23] Thanks to an anonymous reviewer for raising this issue.

sake of argument that the E-literal $[\forall xPx \vDash Pa]$ doesn't provide us with a direct counterexample to justification holism under a charitable reading of the position. This notwithstanding, the Argument from Pre-Theoretic Universality would pose an indirect challenge to the holistic claim that entire logical theories, not individual E-literals, are the primary bearers of justification in the epistemology of logic, i.e., that whatever justification we may have for our individual claims of entailment must be due to the justifiedness of logical theories *en bloc*. Since the propositional justification of foundational E-literals like $[\forall xPx \vDash Pa]$ is orthogonal on the issue of theory choice—illustrated by the argument in §2.2—we could just as well have the opposite order of dependence: whatever justification we have for our logical theories must be due to the basic justifiedness of certain foundational E-literals. It's plainly arbitrary to say that logical theories rather than foundational E-literals are primary without further argument at this point. In fact, at least one of the abductivist virtues, viz., simplicity, seems to support the primacy of a very limited set of foundational E-literals.

This reply can even be strengthened if we notice that not everything hinges on the success of the argument in §2.2 as there are plausible candidates of foundational E-literals other than $[\forall xPx \vDash Pa]$. Consider for instance the E-literal about the inference rule *uniform substitution* instead of universal instantiation. In the end—on the pain of nihilism about deductive entailment—certain entailments need to go through no matter our theoretical differences because giving up on them would mean giving up on deduction as such. Foundational E-literals, like the ones suggested in the present paper, should come across as a very suitable basis of justification in the epistemology of logic, or at least they should be on par with entire logical theories in this respect.[24]

2.3.2 Circularity

Another objection to the Argument from Pre-Theoretic Universality is that while the proclaimed aim of the argument was to establish $[\forall xPx \vDash Pa]$ as a foundational E-literal of deduction, it ended up merely presupposing the truth of $[\forall xPx \vDash Pa]$.

To unpack this objection a bit, consider the following pattern of reasoning. Suppose that a deductive entailment is valid when all cases where all its premises are true also make its conclusion true. If so, entailment—semantically under-

[24]Further, it has been argued that abduction cannot serve as a neutral arbiter in foundational disputes about logic since in order to use abduction one must first point out the relevant data to assess, and which data is found relevant is not independent of one's foundational views regarding many of the disputes one may hope to solve via abduction (Hlobil, 2021).

stood—is essentially tied up with universal quantification over cases. And thus, if the E-literal $[\forall xPx \vDash Pa]$ is true, we get that from '*In all cases where all premises of a valid entailment are true, its conclusion is true*' it follows that '*If this particular model, $\mathfrak{M}$, makes all the premises of a valid entailment true, $\mathfrak{M}$ also makes its conclusion true*'. But how can the fact that this latter claim follows justify the E-literal for UI itself? Or, in other words, how does this fact "ground" the truth of the E-literal $[\forall xPx \vDash Pa]$ in a non-holistic way rather than simply presupposing it?

In response to this, one should simply bite the bullet and observe that while there was undeniably some circularity involved in establishing the foundational truth of $[\forall xPx \vDash Pa]$, this was both expected and unproblematic from an atomistic perspective. Indeed, the relevant kind of circularity was already highlighted in §2.1 under the label 'Universality-Booting' as a special fact about $[\forall xPx \vDash Pa]$. What makes $[\forall xPx \vDash Pa]$, and perhaps a few other E-literals, stand out from the rest as a foundation of deduction is at least partly their bootstrapping nature, so the relevant kind of circularity is a distinguishing feature of foundational E-literals rather than a bug in the main argument.[25]

2.3.3 Truth-Aptness

Yet another objection to the result from §2.2 is that if UI is definitional with respect to the universal quantifier, then UI is not truth-apt, i.e., the Argument from Pre-Theoretic Universality involves a certain category mistake.

In response, one should notice, yet again, that the argument concerns the E-literal *about* UI, i.e., $[\forall xPx \vDash Pa]$, not the rule UI. In other words, it concerns the claim *that* [UI is valid], or *that* [$\forall xPx$ entails Pa]. As $[\forall xPx \vDash Pa]$ is truth-apt, the argument clearly doesn't fall prey to the suggested category mistake.

2.3.4 Free Logic

A final obvious worry is based on the fact that UI fails in standard theories of free logic (Williamson, 1999; Sider, 2010; Nolt, 2021). From this it can be argued that something must be wrong with the argument in §2.2 since $[\forall xPx \vDash Pa]$ cannot be

[25]Note also the literature on the more or less related topics, e.g., *Adoption Problem* (Carrol, 1895; Kripke, 1974; Berger, 2011; Padro, 2015; Besson, 2019; Cohnitz and Estrada-González, 2019; Finn, 2019; Williamson, 202X); the *Background Logic Problem* (Martin, 2021a,b), and *Hinge Propositions* (Wittgenstein, 1969; Wright, 2004a,b; Coliva and Moyal-Sharrock, 2016; Ranalli, 2020).

a foundational E-literal of deductive entailment if it fails in logical theories like the standard ones of free logic. Let's spell out the details of this objection.

On standard semantic accounts, the proponent of a free logic has two alternatives. On the one hand, a model of free entailment might allow for two disjoint domains D and D^*, where D is an "inner" domain, which on the standard interpretation consists of existing objects and is the domain of quantification, while D^* is an "outer" domain, usually thought to consist of non-existing objects like, say, Big Foot, Pegasus, the golden mountain etc. While either domain can be empty, their union must be non-empty (by definition). In such models, it's possible for $D \cup D^*$ to be larger than the domain of quantification, and thus $[\forall xPx \vDash Pa]$ could be false. Suppose, for instance, that model $\mathfrak{M}$ is specified such that $D = \{x : x \, is \, human\}$ and the symbol 'P' refers to the property of being human. Here, the proposition expressed by the sentence '$\forall xPx$' is true in $\mathfrak{M}$. But suppose then that $D^* = \{Pegasus\}$. This would make $[\forall xPx \vDash Pa]$ false in $\mathfrak{M}$ since the name 'a' could denote Pegasus, who is not human. On the other hand, the proponent of free logic could make do with models that only include the usual domain D (of existing objects), while at the same time allowing for D to be empty and with the interpretation function being partial (leaving the interpretation of some names undefined).

To get our reply going, let's first make the following observation. Free logicians reject UI as we have understood it above and replace it with their own UI-principle based on their preferred logical analysis. In some cases, their analysis would involve an extra clause stating that 'object a exists' (perhaps using an existence predicate denoted '$E!$'). So, as a statement of UI, instead of having $[\forall xPx, \Gamma \vDash Pa]$ with Γ empty, they may have something like $[\forall xPx, E!a \vDash Pa]$. These are two completely general, not relativized, rival principles of universal instantiation, which makes the tension between them a genuine case of *logical disagreement* (Williamson, 1988; Hattiangadi, 2018; Andersen, 2020; Hjortland, 2022; Rossi, 2023). Some free logicians may accept that $[\forall xPx \vDash Pa]$ in case 'a' is not an empty name, but reject that this is the (correct) principle of universal instantiation, and endorse $[\forall xPx, E!a \vDash Pa]$ instead. We can make an analogy to the famous case of double negation elimination ('DNE'). It may be that the intuitionist accepts DNE for a limited number of cases that one can specify as an extra clause added to the original DNE-principle, but that doesn't mean they accept DNE; they still reject it.[26]

Nonetheless we don't need to launch anything like a campaign against the le-

[26]Thanks to an anonymous reviewer for pressing this point.

gitimacy of theories of free logic *tout court* in order to steer clear of the objection. Even the free logician would accept that, in semantics, Universality is a necessary property of every acceptable deductive entailment-relation, i.e., any modeling of the concept validity must involve universal quantification over cases; and this is all the agreement needed to get the Argument from Pre-Theoretic Universality off the ground. A friend of free logic can thus run the whole story from §2.2 with a version of UI they accept (based on their favored logical analysis). This will not change the brute fact that their preferred logical theory—whatever it may be—is booting in a state of universality.[27]

Acknowledgments

I would like to thank the audiences at various Arché seminar meetings in 2019/2020, University of St Andrews, for constructive feedback on earlier versions of this paper. From memory these included Aaron Cotnoir, Catarina Dutilh Novaes, Jace Snodgrass, Justin Snedegar, Katharina Bernhard, Katherine Hawley, Lara Jost, Matteo Nizzardo, Paolo Savino, Roy Sorensen, and Sandy Goldberg. Further, I would like to thank the audiences at the conferences Contemporary Philosophical Issues, January 2021, University of Rijeka; APA Pacific, April 2021; and 15th Cambridge Graduate Conference on the Philosophy of Mathematics and Logic, January 2022, University of Cambridge, for helpful Q&A-sessions. Finally, I want to express special gratitude to James Skinner and Gillian Russell as well as my supervisor team, i.e., Greg Restall, Franz Berto, and Jessica Brown, for written comments on my drafts.

Funding

My research is supported by the Niels Bohr Foundation (Royal Danish Academy of Sciences) and the Royal Institute of Philosophy (London).

[27] A similar reply goes against other theories of logic in which UI fails, e.g., certain theories of quantified modal logic. Such theories are notoriously controversial, however, and it is way beyond the scope of the present paper to dive into this intricate debate.

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