Induction: Shadows and Light

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**1.0 Introduction**

The purposes of this essay are to identify the assumptions that underlie the Uniformity Principle, and to examine the process of exclusion that supports induction. The hope is that this essay will carry forward the project of finding a justification of induction.

Section 2.0 examines the inductive syllogism. This section first identifies the problem of confirming inductive conclusions: no matter how many successful observations occur, nothing follows, for there is no reason why the next observation must conform to earlier ones. Next the essay describes the structure of the inductive syllogism: the Uniformity Principle as major premise, and an observed event as minor premise.

Sections 2.1 and 2.2 examine the derivation of the Uniformity Principle. These sections identify the assumptions that permit the trial and error process to exclude data based on nonuniformity, and so to estimate the outer boundaries of the event that will more likely reach a reliable prediction. There are three such axioms: Existence, Identity, and Continuity. Together they are necessary and sufficient to permit an estimate that a given observed event will recur; they enable falsifiability.

Section 2.3 discusses the process of elimination and the central role of the contrapositive form, which reshapes the hypothesis and enables the process of exclusion to work. Section 2.4 applies the essay’s analysis to David Hume’s example of a game of billiards and to Carl Gustav Hempel’s Paradox of the Ravens.

**2.0 Induction as syllogism**

**2.0.1 The problem of circularity**

Induction is a “Generalization from Experience” (Mill 1882, p. 223, §1). But Hume identifies a serious problem with such generalizations. As summarized by Marc Lange, “[W]e are not entitled to *any degree of confidence whatever*, no matter how slight, in any predictions regarding what we have not observed” (p. 43, emphasis in original). Colin Howson gives the question the deserving label, Hume’s Problem: “there is no justification for regarding what has been observed to happen in the past as any sort of reliable guide to the future” (Howson, p. 1). Howson describes the central problem, circularity:

Nor can such an inference [from the observed to the as-yet unobserved] be justified, without circularity, as a ‘probable argument’, based on but not entailed by observational data…. Since ‘the course of nature may change’, indeed proceed from here in a virtually uncountable number of different ways, the inference that the future will proceed or even probably proceed in any one of them must beg the question. Since also, as we shall see later, each of these extrapolates the data under some true description of those data, and therefore according to that description ‘resembles’ the past in that way, we can conclude with Hume— his *circularity thesis*— that

all inferences from experience suppose, as their foundation, that the future will resemble the past[.]

(Howson, p. 10, emphasis in original, quoting *An Enquiry into Human Understanding*). Since Hume set forth his argument in *A Treatise of Human Nature* (1739) and later in *An Enquiry into Human Understanding* (1748), philosophers have made numerous attempts at finding an adequate response (Howson, Lange).

To show that the future may not be predicted from the past, Hume applies this test: whether, given a set of observations and expectations, a differing result is nonetheless “conceivable” (Hume 1902, §25; *THN*, I, 3, 3). Given the latitude of this assumption, that from specific experiences any result remains conceivable, Hume cannot predict anything. He is truly frozen in the moment. What is past is settled. The future cannot be known. Hume does not know what he will say next. He does not know that his pen might wilt in his hand. He does not know that his lungs will accept oxygen. Each of these events, however arbitrary they might appear, is still conceivable.

Seen in this way, Hume’s Problem seems related to Zeno’s Arrow Paradox (Mazur, p. 4-5). As Zeno’s arrow may never move forward from moment to moment, because in any given moment it is stationary in the air, so Hume’s knowledge can never reach beyond the present. In Hume’s world, each bit of data is but a description of a moment. Zeno concludes that no movement is possible; Hume concludes that any movement is possible. Hume’s Problem and Zeno’s Arrow Paradox probably have the same solution.

This essay describes a nonarbitrary world. It must be conceded that the world posited by Hume, where “the course of nature may change” from moment to moment, precludes the possibility of rational prediction (Hume, 1902, §§30, 32). Until the day that controversy is satisfactorily resolved, induction must rest on the principle of uniformity and its underlying axioms.

**2.0.2 The inductive syllogism**

The general form is straightforward. “[E]very induction may be thrown into the form of a syllogism” (Mill 1882, p. 224). The two premises are, first, the Uniformity Principle, which is the major premise (Mill 1882, p. 224-25); and second, a statement that at least one event relevant to the major premise has in fact occurred, which statement becomes the minor premise (Mill 1882, p. 452).

The inductive syllogism outlines the process of falsification of hypotheses and elimination of nonuniform results. The major premise is not in the form “P therefore Q”, and the minor premise is not the affirmative statement “P has occurred”. Rather, the major premise is the contrapositive statement, “Not-Q therefore Not-P”. The minor premise is the observation “Not-Q has occurred”. The conclusion is Not-P; in other words, whatever the content of P might be, it does not include the event Not-P that was found by observation.

**2.1 The Uniformity Principle; the major premise**

The “ultimate major premise of all inductions” is the “uniformity of the course of nature” (Mill 1882, p. 224). If a given occurrence has caused a certain result, then future similar occurrences will cause similar results (Hume 1902, ¶29).

What is the source, if any, of the Uniformity Principle? Many philosophers conclude that there is none; rather, the principle is a subjective belief supported by common observations and expectations about nature, but nothing more. Hume concludes that the Uniformity Principle assumes what it sets out to prove (1902, ¶30) and ascribes it to “general habit, by which we always transfer the known to the unknown…” (1902, ¶84, fn. 19). Hume considers such expectations part of humanity’s “natural instincts” (1748: 46–7). John Stuart Mill is satisfied that when “we consult the actual course of nature, we find that the assumption is warranted” (1882, p. 223). Carl Gustav Hempel believes that the Principle is “a rule psychologically guided and stimulated by antecedent knowledge of specific facts, [but] its results are not logically determined by them” (1945, p. 4).

This essay defines the Uniformity Principle as follows: The relationship between two objects remains constant until interrupted by a third object. Or: If two objects have a given relationship R, and no third object is present, then, given two similar objects, relationship R will recur. Finally, in exclusion form: If relationship R has not recurred, then either the two objects did not have relationship R, or a third object was in fact present.

The Uniformity Principle usually refers to cause and effect relationships. Here, the principle applies to any relation between two things, regardless of whether one appears to cause the other. For example, take two factors: the color blue and the weight one kilogram. There is no reason why something weighing one kilogram should have the color blue; such a relationship is random. However, even a random relationship can be uniform over time, and finding some correlation between color and weight would be notable. In that example, the absence of any cause-and-effect relation is the uniformity, and the later appearance of such a relationship is the dissimilarity.

In this essay the term “object” is used broadly, and so includes separate qualities within something that is ordinarily considered a single entity. One example is a tree. Each tree is a single object, while the shape of its leaves and the shape of its canopy are separate qualities. In this essay, the leaves and canopy would be termed objects.

**2.2 Deriving the Uniformity Principle from fundamental axioms**

**2.2.1 The purpose of the axioms**

Deduction reaches its conclusion by defining a target area and determining whether an observation fits inside it. If there is a fit, then the conclusion is valid. So: All emeralds are green; this gemstone is an emerald; thus this gemstone is green.

Induction approaches its conclusion from the opposite direction. Induction defines an area and determines whether an observation falls outside it. If so, then the definition of the target area is refined by excluding the new data. A series of such exclusions allows an estimate of the outer boundaries of the target area. So: All non-green things are not emeralds; this gemstone is non-green; thus this gemstone is not an emerald. Although the process never tells the observer affirmatively what is inside the definition of “emerald”, that definition is adjusted to exclude the new data.

The collection of Not-P data forms the outer boundary of that Object P which is most likely to avoid Result Not-Q. This exclusionary process never permits the affirmative prediction “P thus Q”.

**2.2.2 The axioms of uniformity**

How can an observer know when a result must be excluded from the content of the minor premise? The axioms that underlie the Uniformity Principle are those necessary and sufficient to conclude that the world is uniform, and so enable the observer to identify examples of nonuniformity.

*A. Existence*

1. Object P exists independently of the observer.
2. Object Q exists independently of the observer.
3. The relation between Objects P and Q exists independently of the observer.

The existential assumption is part of the structure that allows the exclusion of nonuniform results. Without the assumption that Objects P and Q, and their relationship, exist independently, the observer cannot compare the predicted result against an observed result. Both exclusion and simple enumeration become impossible. The real result might be as predicted, but the observer cannot know that; a successful observation might be only a reflection of the observer themselves. If the actual result depends on the observer’s preconceived beliefs about the world, or if the observer believes that they have the power to alter reality, the result might simply be the event that the observer wanted.

Immediately the problem grows more complex. Without the existential assumption, the work of two different observers cannot be compared. Finally, there is the question of whether the result is independent of only some observers, but not all. The existential assumption is one of the fundamental axioms of uniformity.

The existential assumption requires readjusting beliefs to conform to known data, rather than ignoring or changing the data. One example is the discovery of the planet Neptune. In the 1820s astronomical tables described the orbit of the planet Uranus, at the time the farthest known planet; i.e., given this data, then that orbit is predicted. But it became clear that in the planet’s actual orbit, Uranus deviated from predictions; i.e., the predicted orbit is false. Thus the original tables were false. Astronomers of the day set to work calculating the position of some planetary object that would explain the deviation. The reaction was not to maintain that original astronomical tables were correct and then adjust the data to justify the nonexistent orbit. Neptune was found in 1846, almost precisely where observers had placed it in the sky (Pannekoek, p. 359-62).

When an object is in fact found to be unsupported or contradicted by observation, inquiry stops. Two examples here are phlogiston and luminiferous aether. First posited in the late 1600s, phlogiston was said to exist in combustible material and was released during combustion. But by the late 1700s, the data made such a belief untenable (Krebs, p. 336-37, 514). Aether was said to fill the universe as the substance that carried electromagnetic waves, such as radio waves and light. The theory arose in the 1600s. Again, the data showed that there could be no such substance, and the belief dissipated by the late 1800s (Krebs, p. 335, 387-88; Motta).

*B. Identity*

1. Object P is what it is.
2. Object Q is what it is.
3. The relation between P and Q is what it is.

If Object P is also Not-P at the same time, then nonconforming test results cannot be excluded. Any test result, regardless of its outcome, is adequately explained by the fact that both P and Not-P are true simultaneously. Again, both exclusion and simple enumeration become impossible. The identity assumption here is the same as that in the Laws of Thought (Horn, sec. 1), and extends to the relationship between two objects, not only to the objects themselves.

Apparent violation of the identity assumption can prompt further inquiry. “In formal logic, a contradiction is the signal of a defeat: but in the evolution of real knowledge it marks the first step in progress towards a victory” (Whitehead, p. 267). Alfred North Whitehead’s observation follows his example where two experiments had arrived at two different weights for nitrogen (p. 266-67). The experimenters obtained nitrogen by different methods, “each equally effective for that purpose”. But the two results found different weights for nitrogen. So the results showed that nitrogen weighed x and not-x.

Whitehead’s point is that the experimenters did not hide the results nor declare the end of chemistry (p. 266). Instead, further research discovered argon mixed in the nitrogen, which explained the differing weights (p. 267).

What if P or Q is changeable? Nelson Goodman offered “The New Riddle of Induction”, which discussed the problems of confirming a hypothesis (Goodman, p. 73-75). Assume first that, before a future time *t*, each emerald examined is green. But then introduce a new category, *grue*. *Grue* is the color of things examined before time *t* just in case they are green, but also to other things, not examined before time *t,* just in case they are blue. If, before time *t*, a set of emeralds is examined and each is green, then this series confirms the hypothesis that all emeralds are green. However, the same observations, obtained before time *t*, also confirm that all emeralds are grue. Thus, before time *t*, the two hypotheses, "All emeralds are green" and "All emeralds are grue" are both confirmed by the same set of observations.

The category grue does not violate the Law of Identity. Goodman’s examination focuses on time *t* and its impact on color. The assumption does not say that an emerald is at the same time green and not-green; rather, before time *t*, tests using simple enumeration will confirm the quality of the emerald as both green and grue. At and after time *t*, the emerald might be blue. But at any given time, the emerald remains what it is.

The Law of Identity does not resolve Goodman’s paradox. The only point here is that the paradox does not suggest that the Law of Identity has an exception.

*C. Continuity*

1. Object P will not change without a sufficient reason.
2. Object Q will not change without a sufficient reason.
3. The relation between P and Q will not change without a sufficient reason.

The relation between P and Q might be that of cause and effect, or there might be no relation at all beyond randomness. But whatever that relation might be, it will remain constant until acted upon by a third object.

The continuity of individual objects is essential to the exclusionary process. If the individual Objects P and Q may change without a reason, then projection into the future becomes impossible. If, as time passes, Objects P and Q can become Not-P or Not-Q for no reason, then any test result must be expected and none can be excluded.

Suppose research shows that three A's, when combined, produce B. But later research cannot reproduce this result. Instead, new research shows that no combination of A's can ever produce B. Thus the original conclusion appears wrong. But if relationships are nonuniform anyway, or if the presence of uniformity is unknown, then both conclusions are meaningless. The second research result nominally falsifies the first, but in fact there is no such effect. In the absence of some assumption that present and future events will be similar in similar circumstances, the two conclusions cannot be compared. Rather, the conclusions are simply two unrelated statements.

The continuity assumption is the Principle of Sufficient Reason applied to induction. “For every fact *F*, there must be an explanation why *F* is the case” (Melamed and Lin 2016, §1). The Principle of Sufficient Reason is most closely associated with Gottfried Wilhelm Leibniz (Melamed and Lin 2016, §3), although forms of the principle first appeared in antiquity. Leibniz joined the principles of noncontradiction and sufficient reason:

31. Our reasonings are grounded upon two great principles, that of contradiction, in virtue of which we judge false that which involves a contradiction, and true that which is opposed or contradictory to the false;

32. And that of sufficient reason, in virtue of which we hold that there can be no fact real or existing, no statement true, unless there be a sufficient reason, why it should be so and not otherwise, although these reasons usually cannot be known by us.(Leibniz, cites omitted; Melamed and Lin 2016, §3)

In the case of the Uniformity Principle, the relation “P thus Q” is so, and not otherwise, because there is no intervening object making it otherwise.

The continuity assumption is the center of the matter; namely, that the observed event and its result will be repeated by a future similar event and its result. Two questions arise.

First, why is this one axiom not sufficient by itself to support the Uniformity Principle? Why are the existential and identity assumptions necessary if continuity is available to carry the relation “P thus Q” into the future?

The answer is that the first two assumptions, existence and identity, describe only the present. The identity assumption pertains to the first moment when P or Q exists, but makes no statement about the future. In practical terms, the identity assumption describes the first observation of Object P relative to Object Q. The continuity assumption pertains to moments after the first observation, to events not yet observed. Similarly, the existential assumption pertains to the first moment of Objects P and Q. The continuity assumption deals with the future; it says that the status of these Objects will not change without a sufficient reason for that change.

The second question also involves the sufficiency of the axioms. Once the Principle of Sufficient Reason is taken as a given, why does that assumption not settle the whole question of the justification of induction? Why is it necessary to use the contrapositive form? Is it now possible to simply prove “P thus Q” affirmatively?

The continuity axiom does not validate the process of simple enumeration. The accumulation of successful observations never leads to a valid conclusion larger than the total of such observations. The problem is that the syllogism used in enumeration has an undistributed middle term; the Principle of Sufficient Reason does not cure this problem. This issue is addressed in Section 2.3.1.2, below.

Hume denies the validity of the Principle of Sufficient Reason. In *A Treatise of Human Nature*, he considers several arguments in its support, including those from Thomas Hobbes and John Locke (Hume, *THN*, I, 3, 3). Although this principle is said to be “impossible for men in their hearts really to doubt”, Hume finds “no mark of any such intuitive certainty” (Hume, *THN*, I, 3, 3).

[A]s all distinct ideas are separable from each other, and as the ideas of cause and effect are evidently distinct, it will be easy for us to conceive any object to be non-existent this moment, and existent the next, without conjoining to it the distinct idea of a cause or productive principle. (Hume, *THN*, I, 3, 3)

This statement is a clear expression of why the denial of cause-and effect negates the possibility of an inductive conclusion. The assumption that an object can exist one moment and non-existent the next, without any reason, destroys any possibility of continuity, so long as a discontinuous result may be conceived.

**2.2.3 Falsifiability**

When a statement complies with all three axioms, the statement becomes falsifiable: the relationship of two independently existing objects will continue until some third object intervenes. The inductive process of finding events that render the statement false can proceed.

The affirmative statement of the Uniformity Principle is “P thus Q”, which is equivalent to “Not-P or Q”. The falsification of “Not-P or Q” is “P and Not-Q”. The purpose of the axioms is to create a statement that has sufficient precision to allow the event “P and Not-Q” to be identified. The process begins as Object P is tested. When Object Not-Q occurs, the event “P and Not-Q” has been observed and “P thus Q” has been falsified. The relation of two independently existing objects did not continue, although some third object did not intervene. Because this result violated one or more of the axioms of uniformity, something went wrong with the hypothesis.

**2.3 Assembling the minor premise; the process of exclusion**

The Uniformity Principle states the generalization, “if P then Q”, but it does not identify the content of that Object P which always produces Object Q. Simple enumeration of a series of successful events “P thus Q” does not identify that content; however, the exclusion of unsuccessful events “P and Not-Q” allows an accurate estimate of the outer boundary of Object P.

Mill takes the major premise, the Uniformity Principle, as a given. He sees the minor premise as the challenge:

The universe, so far known to us, is so constituted, that whatever is true in any one case, is true in all cases of a certain description; the only difficulty is, to find what description (Mill 1882, p. 223).

Hume denies that simple enumeration of individual observations could support to a general conclusion (1902, ¶30). Francis Bacon arrives at the same conclusion (1902, Bk I, §CV), but he and Mill see exclusion of observations as an effective way to isolate a cause (Bacon 1902, Bk I, §CV; Mill 1882, p. 281).

To estimate the content of that description, induction uses the contrapositive of the hypothesis that is under examination. Estimates of content are not made from within, so to speak; they are not made by identifying the likely content of an Object P and then looking for Object Q. Rather, such estimates are made from the outside; they are made by identifying occurrences which produced Object Not-Q, and then, in future tests, avoiding the occurrences which did so. As this process eliminates occurrences that produce unsuccessful results, it approaches the outer boundary Object P. By eliminating the shadow, the process finds the source of the light.

Both Bacon and Mill notice this elimination process, without connecting it to the contrapositive. Bacon sees the problem with the simple enumeration of observations: “The induction which proceeds by simple enumeration… leads to uncertain conclusions, and is exposed to danger from one contradictory instance…” (Bacon 1902, Bk I, §CV; Broad 1926). But he also sees the solution:

[A] really useful induction for the discovery and demonstration of the arts and sciences, should separate nature by proper rejections and exclusions, and then conclude for the affirmative, after collecting a sufficient number of negatives. (Bk I, §CV).

Bacon sees exclusion as the “first work of legitimate induction” (Bk. II, XVI, 147). His process divides observations into sets of positive results (Bk. II, XI, 121), negative results (Bk. II, XII, 123), and results that are positive or negative “to a certain degree” (Bk. II, XIII, 123). “After an exclusion correctly effected, an affirmative form will remain as the residuum, solid, true, and well defined, while all volatile opinions go off in smoke” (1902, Bk. II, XVI, 147).

Mill systematizes the process through his Canons of Induction (1882, pp. 278-91; Copi and Cohen 2009, pp. 519-58). Two of the canons “are methods of *elimination*” (Mill 1882, p. 281; emphasis in original): the Method of Agreement and the Method of Difference (p. 280-81). These canons look for similarities and differences among observations, in order to build what Mill calls

…the foundation of experimental inquiry, namely: the successive exclusion of the various circumstances which are found to accompany a phenomenon in a given instance, in order to ascertain what are those among them which can be absent consistently with the existence of the phenomenon (p. 281).

The statement “P thus Q” has a logically equivalent contrapositive, “Not-Q thus Not-P”. Assume that an experiment tests the statement “P thus Q”, and finds its falsification “P and Not-Q”. Not-Q is the premise of the contrapositive; Not-P is the conclusion. The observer now knows, at least for this one data point, what Object P is not; when tested, Object P resulted in Not-Q. Whatever the content of Object P may be, the content must be different in some manner from that of Not-P.

This process yields no new information about the internal content of Object P, but it allows the external limits of P to be estimated. Through repeated exclusions, the accumulation of results continually approaches the “residuum” identified by Bacon (Bk. II, XVI, 147).

Karl Popper accepts Hume’s critique of induction but denies that science uses induction at all (Thornton, sec. 3). Still, with his focus on falsifiability, Popper arrives at a similar conclusion about the process of exclusion:

…Popper holds that experience cannot *determine* theory (i.e., we do not argue or infer from observation to theory), it rather *delimits* it: it shows which theories are false, not which theories are true. (Thornton, sec. 4(d), emphasis in original)

This exclusionary process avoids simple enumeration; the process tests the contrapositive of the hypothesis.

**2.3.1 The failure of simple enumeration**

To support a successful prediction, the content of future Object P must match the content of the Object P that is defined in the major premise. But the repeated observation of Object P, always accompanied by Object Q, proves nothing. There are two reasons for this failure of simple enumeration. One is the use of the contrapositive, ”Not-Q thus Not-P”. The observation of Object Q only denies the antecedent, which yields no conclusion. The second reason is that the specific syllogism used in enumeration fails to distribute its middle term, so no valid conclusion follows from the premises.

**2.3.1.1 Contraposition; denying the antecedent**

Assume the testing goal is identification of that Object P which confirms “if P then Q”. After a large number of tries, the result has always been exactly the predicted event: Object P followed by Object Q.

But that consistent outcome does not mean that Object P has been found and “if P then Q” confirmed. The process of exclusion uses the contrapositive statement, “if Not-Q then Not-P”. A finding of Object Q says only that “Not-Not-Q” is true. Thus, in relation to the contrapositive, Object Q denies the antecedent. Nothing follows from denying the antecedent. This absence of a valid conclusion prevents predictions based on the accumulation of individual successful tries, as a large number of tries produces merely a large number of invalid conclusions.

**2.3.1.2 The undistributed middle term**

The four categorical statements are: A (All P are Q); E (No P are Q); I (Some P are Q); and O (Some P are not Q). Each syllogism has three terms: subject, predicate, and middle. These statements and terms can be assembled into 256 syllogisms (Barker, p. 35, 62-63). The syllogism that describes enumeration is invalid.

The following syllogism is that used in simple enumeration. The formal name for this syllogism is AAA in the second figure:

All emeralds are green.

This particular emerald is green.

Therefore: All emeralds are green.

The premises are both A statements; in an A statement, the subject is distributed but the predicate is not.

A term *[P]* occurring as the subject of a categorical sentence is said to be distributed in that sentence if and only if the sentence, in virtue of its form, says something about *every kind of [P]*. A term *[Q]* occurring as the predicate of a categorical sentence is said to be distributed in that sentence if and only if the sentence, in virtue of its form, says something about *every kind of [Q]*. (Barker, p. 43, emphasis in original).

In order for a syllogism to be valid, the middle term must be distributed in the premises at least once (Barker, p. 69). In the syllogism AAA in the second figure, the middle term (green) is undistributed in both premises. The conclusion is invalid. A revised statement, “All green things are emeralds”, distributes the term “green”, but that is not the statement under examination.

Without a distributed middle term, nothing links the two premises and supports a conclusion that is larger than both premises alone. No matter how many times such observations occur, they will never support a valid conclusion.

**2.4** T**he process of exclusion applied**

Hume’s example of billiards illustrates the exclusionary process. In a series of rhetorical questions, Hume submits that prediction is impossible because any assumption of cause and effect is arbitrary (Hume 1902, §25). This section of this essay applies the Uniformity Principle and the process of exclusion to Hume’s example.

Hume’s scenario assumes an observer who, in advance of an observation, knows almost nothing about the world, even about the effects of gravity.

When I see, for instance, a Billiard-ball moving in a straight line towards another; even suppose motion in the second ball should by accident be suggested to me, as the result of their contact or impulse; may I not conceive, that a hundred different events might as well follow from that cause? May not both these balls remain at absolute rest? May not the first ball return in a straight line, or leap off from the second in any line or direction? All these suppositions are consistent and conceivable. Why then should we give the preference to one, which is no more consistent or conceivable than the rest? (Hume 1902, §25)

Regarding the preference for any one prediction, the answer lies partially in Hume’s assumptions. In a complete vacuum, free of information, any prediction is truly random in relation to a future event, because there is no basis for any particular conclusion, regardless of its content.

But given the axioms underlying the Uniformity Principle, things change immediately. Once the cue ball strikes the object ball, there is one observation of actual behavior. The first observation circumscribes the range of future predictions. Whatever the second shot might be, predictions about it cannot contradict the observed behavior of the first. As the number of observations increases, the range of useful predictions narrows, as known failures tell the player which strategies to avoid.

Assume that David Hume, champion of billiard parlors across the Scottish Enlightenment, wants to strike the cue ball so that the object ball travels at a 30-degree angle to the path of the cue ball. To accomplish that result, the cue ball must strike the object ball at a specific angle. Call that position Angle L; finding the technique that will produce that angle is the goal.

Call Object P the event when the cue ball has struck the object ball at the correct angle. Call Object Q the event when the object ball moves at the desired angle.

Here is the deductive form: If the cue ball strikes the object ball at Angle L, then the object ball moves at a 30° angle in relation to the motion of the cue ball. Here is the contrapositive: If the object ball does not move at a 30° angle in relation to the movement of the cue ball, then the cue ball did not strike the object ball at Angle L.

Player Hume begins. The first shot sends the object ball at a 90° angle. Now Hume knows that the first technique, whatever it was, produced Object Not-Q. The technique used was thus an instance of Object Not-P. Our player excludes this technique from future tries.

The next shots send the object ball at angles of 45° and 25°. Those techniques are eliminated. Tests continue. Object Not-Q recurs, but the resulting angles approach 30°. The outline of Object P, the event that will produce Angle L, gradually appears in focus.

There are obvious practical restrictions, notably human mortality. To permit a continual approach to Angle L, the process assumes that attempts will continue indefinitely. But the number of attempts by a single player must be finite. Hume’s final test result must accept not a single angle, but a range of angles. The likelihood that this range includes Angle L is expressed as some fraction less than one.

**2.4.1 The paradox of the ravens**

Suppose player Hume tries again. On his next attempt he ignores the cue ball. Instead, he grabs an apple and throws it out the window. Is that eccentric event truly part of a search for Angle L? The short answer is that, although marginally useful, Hume’s action is indeed an attempt at finding Angle L, and becomes one more observation of an unsuccessful strategy that may be excluded.

Here the analysis encounters the Paradox of the Ravens. Hempel raised The paradox when discussing the confirmation of hypotheses (1945, p. 11-21). Assume the observer wants to prove that all ravens are black. If something is a raven, then it is black: “if R then B”. To that end, the observer begins watching ravens. With each observation of a thing that is both a raven and black, the hypothesis appears to be correct.

The contrapositive is the logical equivalent and has the same truth value. If something is not black, then it is not a raven: “if Not-B then Not-R”. The paradox begins. Apparently irrelevant information now supports the original hypothesis, “if R then B”.

The observer begins to watch the rest of the universe. The observer records white shoes and blue planets. None are black and none are ravens. Still, because of the logical equivalence of the original hypothesis and its contrapositive, such apparently irrelevant observations support the original statement that all ravens are black (1945, p. 11).

Hempel resolves the paradox by finding a process of exclusion. He observed that when that when drawing conclusions, people tacitly use information already known to them (1945, pp. 19-20). Hempel describes testing the statement, “All sodium salts burn yellow.” The contrapositive is, “If a substance does not burn yellow, then it is not sodium salt.” Water ice does not burn yellow or any other color; thus, holding ice over a flame would confirm the statement about a different substance: sodium salts. However, to perform such a test in real life, people use their existing knowledge about ice; they know that water ice neither burns nor contains sodium salt. This knowledge eliminates a large set of tests and results from the examples that would confirm the hypothesis. In effect, Hempel concludes that people use their existing knowledge as part of an unspoken process of elimination; “the paradoxes vanish” (1945, p. 20).

This essay argues that the resolution of the paradox lies in the requirements of the exclusionary process. Attempts at verification must continue indefinitely. Proving the statement, “if Not-black then Not-raven”, requires observation of the entire universe outside the group of black ravens. Each unit of information eliminates one point in an area that gradually circumscribes the outer boundary of the group of things that are both ravens and black.

When player Hume throws the apple out the window, that event is indeed an attempt at locating Angle L. This try produces an unsuccessful result (Not-Q) and thus reveals an event (Not-P) to be avoided. Hume now knows to disregard that particular strategy. In actual practice, where reality permits only a finite number of attempts, the mortal Hume abandons his assumption that he knows nothing of billiards; instead, he concedes to Hempel, uses what he knows, and so concentrates on focusing his aim.

**3.0 Conclusion**

At ceremonies marking the 300th anniversary of the death of Francis Bacon, C.D Broad offered this aspiration for the future:

May we venture to hope that when Bacon's next centenary is celebrated the great work which he set going will be completed; and that Inductive Reasoning, which has long been the glory of Science, will have ceased to be the scandal of Philosophy? (Broad 1926)

With the approach of the quatercentenary, glory and scandal remain. But perhaps the reason for both is more clear.

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