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The System S5 in Modal Logic

Modal logic, a subfield of logic beyond introductory propositional logic, in which the knowledge of modal operators, kripke models, assorted topics in set theory, etc. is added to the logic skill set, involves the usage of modal operators to determine the necessity and possibility of a sentence or set of sentences, among other things. In the beginning of one's journey to learn modal logic, one will likely encounter the term "S5" in reference to a specific system of modal logic. So, what exactly is this illusive "S5," so heavily used by logicians and philosophers alike? This short article seeks to answer the above question, with some acknowledgment of various other details of the system S5, such as history, use, etc.

S5, one of five original systems of modal logic, which include S1, S2…, was originally introduced by pragmatist Clarence Irving Lewis and logician Cooper Harold Langford in their 1932 work entitled "Symbolic Logic." The modal logics were originally developed in a form of proof used by Lewis and Langford. Four other systems, solely for proof-based use, were classified by Lewis and Langford, in which methods of proving the "truth" (validity) of something modally were to be formalized. Ideas about this new system flourished, and, since the publishing of "Symbolic Logic" by Lewis and Langford in 1932, a variety of systems has been created, each with their individual proponents.

Modal logic, namely the system S5, differs from ordinary propositional logic in that, as opposed to merely breaking down the validity of sentences and proving validity (or lack of it), as propositional logic does, modal logic (S5) considers modal expressions in relation to said sentences ("necessarily," "possibly," "possibly necessarily," etc.) in order to determine the validity of those sentences with the modal expressions applied. The system S5 is primarily built upon the typical syntax of propositional logic (Not, if then, if and only if, biconditionality, disjunction, conjunction, etc.) with the addition of the modal operators of possibility (◊) and necessity (□). In the system S5, the modal operators behave similarly to the symbols of quantification in ordinary quantificational logic (∀ and ∃, which denote the quantity of x assigned to the operators (∃x means 'for some x', ∀x means 'for all x', ∀x(Hx) means that for all x, x is an H, and so on). S5 is typically considered to be a foundational system, as it is often too weak to prove certain axioms, such as the following: □A→A. The above sentence, which states that if a sentence A is necessary then A, is difficult to prove using only S5. □A→A is logically valid, but can't be shown to be valid by S5 alone.

The modal logic system S5, which entails the use of the operators of necessity and possibility in order to judge the validity of various sentences, is a relatively new, as far as logic goes, system of logic. The system is often somewhat insufficient for more complicated sentences involving necessity. S5, though insubstantial, forms the very beginning of one's path towards learning and using the other systems, which, when combined with S5, provide a sufficient logic for classifying and quantifying most sentences of necessity.