Two Pre-Theoretic Counterexamples to Justification Holism in the Epistemology of Logic

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This article has been accepted for publication in *Proceedings of the Aristotelian Society*.

Published by Oxford University Press

Recently an abductivist approach to the epistemology of logic has gained traction. A necessary component of logical abductivism is justification holism, asserting that claims of logical entailment can only be justified in the context of an entire logical theory, e.g., classical, intuitionistic, etc. One view that is incompatible with abductivism is an atomistic view on which individual entailment-claims can be justified point-wise rather than in the context of a whole theory. This paper provides two atomistic counterexamples to justification holism in the epistemology of logic. Both examples appeal to pre-theoretic commitments of deductive validity. The main aim is to show that there are some foundational entailment-claims for which we can have propositional justification independently of theory choice and outside the context of a whole logical theory. If one were to give up on these foundational claims, all semantic and syntactic accounts of deductive validity would be non-starters.

I

Preliminaries. Recently there has been a great interest in an abductivist approach to the epistemology of logic (Arenhart 2024; Beall 2019; Erickson 2024; Hjortland 2017; Russell 2014, 2015, 2019; Priest 2008, 2014, 2021; Williamson 2007, 2017, 2020). Some of the contemporary logical abductivists are motivated by *Anti-Exceptionalism about Logic* ('AEL'), which, roughly put, asserts that logic doesn't differ from (empirical) science in any significant way.

AEL contrasts with the common perception of logic as an *exceptional* discipline in the sense of it being normatively, epistemically, methodologically, and metaphysically different from (empirical) science (Ferrari et al., 2023). Through various historical periods logical laws have been seen as special, and logic has commonly been viewed as a foundational field of study underlying all other fields. Whereas the laws of physics apply only to physical systems, those of logic have typically been conceived as exceptionally general, i.e., applying to all domains and entities. It has been the exceptionalist conception that principles of logic are necessary and analytic—i.e., not responsive to evidence from the empirical realm—as well as *a priori*, leading to traditional views like Rationalism (BonJour 1998) and Semanticism (Ayer 1952). According to such exceptionalist views, it follows that logical evidence, justification, and knowledge must either stem from direct intuitions about the realm of logic or epistemic analyticity.[1]

AEL in its many forms challenges all the above mentioned exceptionalist aspects of logic. Historically, AEL has been associated with Quine (1951; 1986) who argued that logic is neither necessary, analytic, nor *a priori*. Modern varieties of AEL, however, come in less

[1]A sentence is *metaphysically* analytic if and only if it is true in virtue of meaning. A sentence is *epistemically* analytic exactly when anyone who understands it is justified in taking it to be true.

extreme versions. Much attention in recent debate has been paid to the question of whether logic is *epistemically* exceptional. Contrary to the traditional views, it has become increasingly popular to suggest that epistemic justification[2] in the context of logic is a matter of showing that a given logical theory better accommodates the relevant data than its rivals (along with arguing for its possession of theoretical virtues and lack of vices). This view is now known as *Logical Abductivism* ('LA') and is summarized by Gillian Russell as follows:

> The heart of the abductivist approach consists in two claims. The first is holism about the justification of logic: it is entire logics—rather than isolated claims of consequence—that are justified (or not). The second is that what justifies a theory is adequacy to the data, and the possession of virtues and absence of vices. (Russell 2019, p. 550)

For LA the target of justification is entire logical theories rather than individual claims of logical entailment. Abductivists endorse justification holism asserting that whatever justification one may have for particular entailment-claims must be in virtue of the logical theory to which they belong. Of course, it's not that one is not able to have justification with respect to individual entailment-claims, the point is rather that any such justification is dependent on a choice of logical theory, say, classical, paraconsistent, paracomplete, etc.

Further, Russell underscores that LA is incompatible with justification atomism (Russell 2019, p. 552). The justification atomist opposes the holist part of LA by insisting that: *some individual entailment-claims can be justified point-wise rather than in the context of a whole logical theory*. We'll return to the theme of LA and its commitment to holism below in sections II and III, where we'll argue that the holistic doctrine is false via counterexamples.

Before we get to the main event, however, we'll introduce some technical terminology regarding logical entailment. *E-sentences* are atomic sentences in which the main predicate is given by one of the turnstile-symbols '⊨' or '⊢', or its natural language equivalents (Russell 2019). Examples: [$A \rightarrow B, A \vdash B$]; [$A \vee \neg A \vDash B$]; [$\vDash A$]. Surely these sentences don't look *atomic* in the ordinary syntactic sense, but they are atoms in the sense of them being the simplest kind of sentences of a given metalanguage—notice that symbols such as '∨','¬', '→' are not used but merely mentioned in E-sentences, whereas '⊢' and '⊨' are metalinguistic symbols.[3]

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[2] We'll take 'epistemic justification' to refer to propositional justification rather than doxastic justification, i.e., the justification of propositions rather than belief-tokens about propositions.

[3] In this paper we stay neutral regarding the debate between metalinguistic and non-metalinguistic perspectives on the subject matter of logic. Importantly, for Williamsonians, statements about *validity* will be about language since they consider these metatheoretical statements about object-language conditionals (Williamson 2013). But Williamsonians will also assert that logic is *not* primarily about validity:

> In logic's auxiliary role of drawing out deductive consequences of hypotheses and theories, what matters is the relation of logical consequence. But logic also has another role, in codifying very general structural truths about the world; for that purpose what matters are the true universal generalizations corresponding to logical truths. Their primary significance is in what they tell us about the world, not in what they tell us about logical truth or validity; our interest in them is no more primarily metalinguistic than is our interest in the true sentences of the language of physics. (Williamson 20XX, p. 8)

Of course, as should be noted, it is not just Williamsonians who take logic to deal primarily with mostly non-metalinguistic stuff, see for instance (Maddy 2014).

From this definition of E-sentences we can define *E-literals*, where an E-literal is either an E-sentence or its negation. Thus, all E-sentences are E-literals, but not *vice versa*. Examples: [A → B, A ⊭ B]; [⊢A]; [A∨ ¬A ⊭ B].

II

First Counterexample (Semantic Entailment). In this section we'll provide an argument (a counterexample) against justification holism. The argument was first proposed in (Andersen 2023), but with the important caveat of targeting semantic accounts of deductive validity only. In section III we'll break new ground by showing how the style of argument can be extended to target syntactic accounts of validity as well.

More specifically, the present section aims to show that the E-literal [∀xPx, Γ ⊨ Pa], where 'a' refers to an element of domain D of some model M, and 'Γ' picks out a (possibly empty) set of side-conditions, is true under any acceptable deductive entailment-relation (semantically understood), and denying its truth would mean giving up on deduction altogether.[4] In other words, we aim to establish that a liberal version of the E-literal about universal instantiation is a foundational E-literal for which we have propositional justification independently of theory choice and outside the context of a whole logical theory.

In what immediately follows, universal instantiation will be defined and some crucial components of our argument—Universality and Universality Booting—will be introduced. On the heels of this, the first counterexample to justification holism will be put forward. We'll also briefly address an objection from free logic, i.e., a case where the set of side-conditions Γ in [∀xPx, Γ ⊨ Pa] is non-empty, before proceeding to section III.

Universal instantiation ('UI') is a syntactic inference rule. Under a plausible semantic interpretation, it says: any instance of 'Everything is P' entails 't is P', where 't' refers to an individual term. In standard notation:

$$\frac{\forall v P v}{P t}$$

The quantifier denoted by '∀' ranges over a domain of objects, the predicate given by 'P' refers to a property, and the term given by 't' replaces all occurrences of the variable given by 'v'. We can state an E-literal about UI: [∀xPx, Γ ⊨ Pa]. Since Γ is usually left empty, we'll just write '[∀xPx ⊨ Pa]' by default.

In our argument against justification holism we'll assume that—in formal semantics—Universality is a necessary property of every acceptable deductive entailment-relation. That is to say, any acceptable deductive entailment-relation must involve universal quantification over cases, e.g., possible worlds, constructions, truth-makers, situations, etc.

[4] A model M in first-order logic is an ordered pair M = ⟨D, I⟩ such that *D* is a domain of objects and *I* is an interpretation function specifying referents for constant symbols, predicate symbols, and function symbols. M is a model of a formula A if A is true in M. M* is a countermodel to A when it is a model of ¬A.

Also, we'll need to appreciate that the E-literal about universal instantiation is a universal sentence about true universal sentences. For the main predicate of [∀xPx ⊨ Pa] is given by the semantic entailment-symbol, which is indeed a universal claim (assuming Universality). This is important to our argument because it means that any model M making [∀xPx ⊨Pa] true must itself be a fact of universal quantification over cases; a pre-theoretic counterpart of UI.[5] Or, in other words, the E-literal about UI is *doubly* universal in containing both a universal statement and in stating a fact of entailment, which is itself a basic fact of universal quantification.[6] Let's call this special feature of [∀xPx ⊨ Pa] 'Universality Booting'. In slogan-form: *Whatever logical theory you prefer, it will be booting in a state of universality!*

We are now in a position to state the *Argument from Pre-Theoretic Universality*:

> Assume that Universality is a necessary property of any acceptable deductive entailment-relation, and let '⊨' denote any such relation. Suppose that [∀xPx ⊨ Pa] is false. Then there exists a countermodel M* to the E-literal [∀xPx ⊨ Pa], i.e., a model such that [∀xPx ⊭ Pa] and *a* ∈ D. By Universality Booting, any M making [∀xPx ⊨ Pa] true is itself a pre-theoretic fact of universal quantification over cases. Yet, by assumption [∀xPx ⊨ Pa] is false, so there can be no such pre-theoretic fact. But then, by Universality, ⊨ cannot be an acceptable deductive entailment-relation. For there exists a counterexample to universal quantification over cases, viz., M*. Therefore, either [∀xPx ⊨ Pa] has no countermodel, or Universality is not a necessary property of acceptable deductive entailment. By assumption, Universality is a necessary property of acceptable deductive entailment. Hence [∀xPx ⊨ Pa] is true under any acceptable deductive entailment-relation. ⊣

Upshot: some E-literals are propositionally justified independently of theory choice and outside the context of an entire logical theory. And crucially, the result is not just that all acceptable logical theories should include [∀xPx ⊨ Pa], perhaps for completely different reasons, rather the argument shows that [∀xPx ⊨ Pa] is foundational in such a way that it leaves any theoretical specifications redundant with respect to its justificational status.[7]

Note finally that an important worry about the argument is that UI fails in standard theories of free logic (Williamson 1999; Sider 2010; Nolt 2021). Free logicians reject UI as it is understood above and replace it with their own version of universal instantiation. In some cases, their analysis would involve an extra condition stating that 'object *a* exists' (using an

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[5] The operational sense of 'pre-theoretic': *playing an epistemic, justificatory role prior to theory choice.* Notice that in the development of any kind of general theory there is going to have to be—as a matter of when you are developing the theory—a relationship between the general claim and instances of it.

[6] This basic fact of universal quantification is easiest to notice when thinking in terms of counterexamples. If we have a model of the premises of an argument which is not also a model of the conclusion, then we are making a transition from an *instance* to the falsity of a *universal* claim; an implicit appeal to UI.

[7] The argumentation strategy used to establish this pre-theoretic counterexample looks like *classical* reductio: It proceeds from not-not-valid to valid. Crucially, however, *valid* is understood as not having a counterexample (a negative notion). So, we get a version of reductio that even the constructivist accepts, i.e., we aim to prove not-*p* ("not being invalid"), thus we assume not-not-*p* and derive a contradiction. That is triple negation elimination, which is acceptable even by intuitionistic standards.

existence predicate 'E!'). So, instead of having [∀xPx, Γ ⊨ Pa] with Γ empty, they may have [∀xPx, E!a ⊨ Pa]. Yet, even the free logician would accept that, in formal semantics, Universality is a necessary property of every acceptable deductive entailment-relation, and this is all the agreement needed to get the argument going.

III

Second Counterexample (Syntactic Entailment). This section aims to extend the style of argument from II such that it covers syntactic accounts of deductive validity too. More specifically, the aim of the present section is to establish that the E-literal about existential introduction is a foundational truth under any acceptable deductive entailment-relation (syntactically understood), and that giving up on it would amount to giving up on deduction altogether.

Existential introduction ('EI') is a syntactic inference rule, which informally states that if a specific object has property P, then there exists an object with P. Formally:

$$\frac{Pt}{\exists v Pv}$$

Here 't' is substitution free for 'v' in 'Pv', and 'Pv' is the result of replacing all instances of 't' with 'v'. We can state an E-literal about EI: [Pa ⊢ ∃xPx].

Now, proof-theorists need not adhere to universal quantification over cases in their accounts of deductive validity as their views presuppose the particular *there is a derivation* rather than the universal *in all cases*. Structurally, however, a similar foundational point can be made with respect to the existential quantifier. In the argument below, we'll assume that Existentiality is a necessary property of every acceptable entailment-relation, syntactically conceived. That is, any acceptable deductive entailment-relation must involve existential quantification over derivations, e.g., in the form of axiomatics, natural deductions, sequent calculations, etc.

Further, let's make the crucial observation that the E-literal about EI is an existential sentence about true existential sentences. This is important because any proof Π making [Pa ⊢ ∃xPx] true must itself be a fact of existential quantification over derivations; a pre-theoretic counterpart of EI.[8] In other words, the E-literal about EI is *doubly* existential in containing both an existential statement and in stating a fact of entailment, which is itself a brute fact of existential quantification. Let's call this special feature of [Pa ⊢ ∃xPx] 'Existential Booting'. In slogan-form: *Whatever logical theory you prefer, it will be booting in a state of existentiality!*

The *Argument from Pre-Theoretic Existentiality*:

> Assume that Existentiality is a necessary property of any acceptable deductive entailment-relation, and let '⊢' denote any such relation. Suppose that [Pa ⊢ ∃xPx] is

[8] A proof Π of a formula A from a set of formulas Γ in first-order logic is a finite sequence $\langle \alpha_0, \ldots, \alpha_n \rangle$ of formulas such that α_n is A and for each k ≤ n, either (a) α_k is in Γ ∪ Λ, where Λ is a (possibly empty) set of axioms; or (b) α_k is obtained from earlier formulas in the sequence (via deductive inference rules). A counterproof Π* is a proof of ¬A.

false. Then there exists a counterproof Π* to the E-literal [Pa ⊢ ∃xPx], i.e., a proof such that [Pa ⊬ ∃xPx]. By Existential Booting, any Π making [Pa ⊢ ∃xPx] true is itself a pre-theoretic fact of existential quantification over derivations. Yet, by assumption [Pa ⊢ ∃xPx] is false, so there can be no such pre-theoretic fact. But then, by Existentiality, ⊢ cannot be an acceptable deductive entailment-relation. For there exists a counterexample to existential quantification over derivations, viz., Π*. Therefore, either [Pa ⊢ ∃xPx] has no counterproof, or Existentiality is not a necessary property of acceptable deductive entailment. By assumption, Existentiality is a necessary property of acceptable deductive entailment. Hence [Pa ⊢ ∃xPx] is true under any acceptable deductive entailment-relation. ⊣

IV

Conclusion. We have an argument against justification holism, which is taken to be a necessary component of LA. Justification holism asserts that entailment-claims can only be justified in the context of an entire logical theory. Yet the counterexamples above have shown that the E-literals about UI and EI are of special epistemic status. These E-literals are foundational for deductive validity, meaning that their propositional justification is independent of our theory choice in logic, and that their justifiedness emerges from outside the context of an entire logical theory.

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