Aristotle’s Definition of Scientific Knowledge

(APo 71b 9–12)

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Abstract

In Posterior Analytics 71b9–12, we find Aristotle’s definition of scientific knowledge. The definiens is taken to have only two informative parts: scientific knowledge must be knowledge of the cause and its object must be necessary. However, there is also a contrast between the definiendum and a sophistic way of knowing, which is marked by the expression “kata sumbebekos”. Not much attention has been paid to this contrast. In this paper, I discuss Aristotle’s definition paying due attention to this contrast and to the way it interacts with the two conditions presented in the definiens. I claim that the “necessity” condition ammounts to explanatory appropriateness of the cause.

Introduction

In Posterior Analytics 71b9–12, we find a definition of a higher-level kind of knowledge which can be labelled scientific knowledge. At first glance, there are only two important parts in the definiens: scientific knowledge is defined in terms of knowledge of the cause and in terms of the necessity of its object. However, there is also a contrast between the definiendum and a sophistic way of knowing, which is marked by the mysterious expression “kata sumbebekos”. This makes the passage harder to decipher, since it is unclear how important this contrast is and how it is related to the two defining conditions. The text runs as follows:

T1: We think we have knowledge of something simpliciter [i] (and not in the sophistical way, incidentally), [ii] when we think we know of the cause because of which the explanandum holds that it is its cause, [iii] and also that it is not possible for it to be otherwise. (71b9–12, Barnes’s translation modified)

My aim in this paper is to give a full discussion of this definition and to show that it is consistent with other features of Aristotle’s theory of scientific knowledge in the Posterior Analytics. In the remainder of this introduction, I will outline my main claims. Some of those claims are controversial and unorthodox, and they are all closely interrelated. I hope that a brief announcement of how they are tied together will make them seem more plausible to the reader. Then, in the next sections, I will discuss each of them in detail.

First of all, I take T1 to be giving a definition of scientific knowledge and not of knowledge in general. Whatever knowledge in general might be, scientific knowledge is a higher-level kind of knowledge for which more demanding requirements are in play.¹

¹ See Fine 2010a, 326f. I disagree with Barnes 2014, 81–94, who takes 71b9–12 to be offering an account of knowledge, not of scientific knowledge.
Given that Aristotle’s definiendum is such a specific sort of knowledge, mention of a general notion of “knowing” in the definiens is not circular. When Aristotle uses the expressions “we think we get to know” in step [ii] of T1, he is relying on a generic notion of “knowing” in order to add the peculiar features that make a piece of knowledge specifically scientific. There has been some debate about the merits of translating the definiendum in T1 as “understanding”. I have nothing against this translation, but I prefer “scientific knowledge”.

As for the definiens, it might seem that only steps [ii] and [iii] put forward conditions required for knowledge to be scientific; but I will argue that step [i], even if it should not be taken as presenting a further condition, presents a foil very useful for understanding what Aristotle had in mind.

There are three claims that will be substantiated in my discussion below:

Claim (1): in step [ii] of T1, pragma must be understood as an explanandum with predicative structure. In its more general core meaning, “pragma” means “what one is concerned with”. Now, when it comes to scientific knowledge, “what one is concerned with” is explanation, and what admits of explanation is a predicative relation in which some attribute is present in some subject.

Claim (2): in step [iii] of T1, the pronoun “touto” (71b12) refers to the relation between pragma and its cause; in grammatical terms, the referent of “touto” is not “pragma”, but the sentence “hoti ekeinou aitia esti” started in the previous line. What cannot be otherwise, if a piece of knowledge is to be scientific, is the relation between explanans and explanandum. Only one cause is the primary cause that makes the explanandum what it is. This cause can be called necessary in the sense of being “the required one” for the most appropriate explanation. Thus, what seems to be a modal terminology referring either to bare objects and their mode of being or to propositions and their truth value turns out to be an expression concerned with explanatory relevance. Aristotle is not concerned at this step with ontological claims about the “necessary being” of the objects of scientific knowledge – for instance, he is not suggesting that mathematical entities qualify as objects of scientific knowledge inasmuch as they are unchangeable, in contrast with natural bodies, which would not qualify as objects of scientific knowledge inasmuch as they are changeable. Nor is Aristotle concerned at this step with claims about the necessary truth of the predications that constitute a body of scientific knowledge – he is not insisting that your conclusion and your premises must be necessarily true sentences. Aristotle’s point is that scientific knowledge must satisfy the following requirement: the explanation of the explanandum at stake must proceed from the cause that is “the required one” for the fully appropriate explanation.

Claim (3): in step [i], the expression “kata sumbebekos” refers to the way in which a middle term introduces a cause that explains why the pragma obtains. In order to understand the

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2 See Barnes 1993, 90; McKirahan 1992, 276, fn. 11.
4 To my knowledge, Lennox 2001, 7–10, was the only one to pay due attention to step [i]. For a different view, see Gifford 2000, 172–178.
contrast between scientific knowledge and this sort of incidental knowledge, the implied expression “epistasthai kata sumbebekos” must be supplied with “hekaston” (71b9) and “pragma” (71b11), which keeps the parallel needed for the contrast with the expression “epistathai haplôs”. More importantly, the structure “kata + accusative” in the expression “kata sumbebekos” must be taken seriously instead of being flattened into an adverbial locution. This expression points to the factor – characterised as a sumbebekos – that is doing the explanatory work for a given explanandum formulated (or presupposed) in predicative form. The term “sumbebekos” does not mean contingent predicate in this context. It refers to a middle term that is only called a sumbebekos in reference to the explanation at stake: it is a sumbebekos in the sense that it just “comes together” with the terms of the explanandum (C and A) without delivering the most appropriate explanation of it. “Coming together with the terms of the explanandum” means that the middle term might be an attribute that is necessarily and even essentially true of C (and so will “accompany” C when C has the attribute A), but is not the most appropriate explanatory factor needed to explain why C has the attribute to be explained, A. Accordingly, the sophistic way of knowing that serves as a foil for unqualified knowledge (the definiendum in T1) is an attempted explanation that uses a sumbebekos as middle term and is called sophistic because its purpose is to produce a false semblance of appropriate explanation.

1. Pragma as explanandum with predicative structure

The word “pragma” is used in many ways: sometimes it seems to pick out the subject-matter of a given science (e.g., 46a25), sometimes it introduces a definiendum (96a35), sometimes it refers to an attribute to be explained (98b30). It can also refer to the subject of a predication. It is not my purpose here to give an exhaustive survey of Aristotle’s uses of this word in APo. Rather, I want to argue that in T1 and in several other passages, “pragma” is best understood as introducing the notion of an explanandum with predicative form. In these passages, “pragma” refers to the fact that some subject C has attribute A, not to the bare subject C.

There is no definitive philological evidence for taking “pragma” in this way in T1. Aristotle’s use of the word is very flexible and context-sensitive. But his discussion of the objects liable to scientific investigation in APo II.1–2, as well as the contexts in which he connects “pragma” with the notion of cause or explanatory factor, lend support to my interpretation.

First, when Aristotle talks about the connection between cause and its pragma in APo 98b28–31, the word is clearly used to pick out the attribute to be explained, not the subject. To take one of Aristotle’s examples, “pragma” picks out being eclipsed but not the Moon. My contention here is not that the passage 98b28–31 lends support to my claim that “pragma” can refer to an explanandum with predicative structure. Rather, my claim is

5 Henceforth, I follow Aristotle’s practice of employing schematic letters in the first-figure syllogistic framework to express explanatory relations: the explanandum is the attribute A (the major term) predicated of C (the minor term), whereas the explanans is the middle term B (cf. 78a31–21ff.; 81b11–12; 89b16–17; 93a30–31; 94a28ff., etc.).
that the passage clearly tells against the assumption that “pragma” should refer (always or preferentially) to the subject of predication within a scientific field.

Secondly, when Aristotle discusses the objects of scientific investigation in APo II.1–2, he makes it clear that all of them involve a predicative structure. It is uncontroversial that “knowing that” and “knowing why” have predicative propositions as their objects. But Aristotle argues that a predicative structure and, together with it, the notion of a middle term introducing the explanatory factor, are also involved in “knowing whether there is” and “knowing what it is”. His discussion, which has many intricacies, has received a good deal of attention in the recent literature. My contention is that – to use one of Aristotle’s own examples – “knowing whether there is thunder” amounts to knowing whether thunder (or a specific sort of noise) is attributed to the clouds or, more precisely, to knowing that there is a middle term that causes thunder to be attributed to clouds. In the same way, “knowing what thunder is” also involves a predicative structure: it involves the predication in which thunder is the definiendum and its definiens is the predicate, but also a more basic predication which is packed inside the definiens account, namely, the attribution of noise of a certain sort to the clouds. This is enough to show that interpreting “pragma” in T1 as a mere subject (instead of as an explanandum with predicative structure) is not very promising.

Returning to APo I.2, there is another point in favour of taking “pragma” as an explanandum with predicative structure. When Aristotle comes to the requirements for the starting-points of scientific demonstration just a few lines below, he says that scientific demonstration must proceed from items which are “cause of the conclusion” (71b22). Now, the term “conclusion” in Aristotle normally refers to predicative sentences. If scientific knowledge is defined in terms of knowing the cause of the pragma and then Aristotle fleshes out his theory by saying that the starting-points must be “the cause of the conclusion”, it is reasonable to infer that there is a match between “pragma” and “conclusion”: we might reasonably say that a conclusion of a scientific demonstration states exactly the pragma of which we have scientific knowledge when we have understood it through its appropriate cause. The conclusion as well as the pragma should be understood in terms of the attributive relation C-A, for which B is the middle term in the syllogistic framework.

Before arguing for my Claim (2), let me clarify how I deal with the issue of the syllogistic form of scientific knowledge, for this will allow me to clarify some of my assumptions.

Some scholars argue that Aristotle’s notion of scientific knowledge is built around the concept of a proof concerned with establishing or certifying that something is true. In other words, it is assumed that the main target of scientific knowledge is one of the following options: (i) to determine (in the first place) whether a given proposition is true; (ii) to certify on solid grounds that a given proposition is true. Within the first option, the proposition to be proved has a problematic truth-value in the sense that one cannot ascertain whether it is true or not without proof; within the second option, the proposition to be proved is already taken to be true but on insufficient grounds to assure its truth. Within the first picture, in order to determine that a proposition is true, one must find a proof based on principles already known to be true, and, ultimately, the chains of proofs must be based on undeniable axioms. The only relevant difference within the second picture is that the conclusion is not taken as problematic but as unsecured or unwarranted until its truth is

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backed up by the certified truth of the premises (and, ultimately, of the axioms). Now, since Aristotle defines scientific knowledge in terms of knowing the cause and by appealing to some notion of necessity (71b9–12), these interpretations need to be fleshed out. One might then add that the main target of scientific knowledge is to prove, about propositions that are either problematic or already known to be true, that they are necessarily true, and that such a proof will require appeal to causes explaining why the conclusions are necessarily true. Thus, let us assume that a proposition of the form “every C is A” is already known to be true before one gets scientific knowledge of it. On the view I am depicting, scientific knowledge aims at proving that it is necessarily true on the ground of some axiomatic premises; in other words, scientific knowledge aims at finding out the more basic propositions, whose truth, being self-evident and necessary, allows us to produce a proof that not only establishes that our conclusion is necessarily true, but also explains why it is necessarily true. Within this picture, one can say that his conclusion is necessarily true because it follows from principles that are necessarily true, and these necessarily true principles constitute the premises of a sound deduction explaining why the conclusion cannot be false.

I disagree with this picture of Aristotle’s notion of scientific knowledge. In APo I.9 (75b36–40, 76a28–30) Aristotle explicitly says that to prove a given proposition from principles that are true, indemonstrable and immediate is not enough for having scientific knowledge of it. Some scholars complain about Aristotle’s remarks at 75b36–40 and suggest that they are inconsistent with Aristotle’s theory in APo. Against this interpretation, I argue that Aristotle’s remarks at APo I.9 are entirely in tune with his definition of scientific knowledge at T1. My argument will become clearer with my discussion of Claim (2): Aristotle’s definition of scientific knowledge does not rest on the requirement that a proposition must be necessarily true; it rather rests on the requirement that a proposition must be explained by its appropriate cause. Before exploring Claim (2), let me stress that, in my picture, the syllogistic framework is not out of place in Aristotle’s theory of scientific demonstration. The issue at stake is the role played by the syllogistic in Aristotle’s account. Now, syllogisms would not be interesting tools for scientific demonstrations if scientific knowledge were just a matter of proving that p is necessarily true from a set of axiomatic propositions (or just a matter of “explaining” that p is necessarily true because it follows from necessarily true axioms). But syllogisms are adequate tools if scientific knowledge is concerned with displaying the causal relations between explananda and their appropriate explanantia, since causal relations for Aristotle must be formulated as triadic

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7 For approaches along these lines, see Scholz 1975, 50; Barnes 1975, 65; Barnes 2007, 360; Mignucci 2007, 152; Corcoran 2009, 1; Smith 2009. Perhaps also Malink 2013, 217.
8 The references in the previous footnote apply here too. For a more nuanced view, see Ferejohn 2013, 66–72.
10 The main outlines of the rival interpretation I reject are also rejected by Kosman 1973; Burnyeat 1981, 108–115; Matthen 1981, 4–10; Taylor 1990, 116f.; McKirahan 1992, 26–32; Lesher 2001, 46; Ferejohn 2013, 66–97; Fine, 2010; Goldin 2013; Bronstein 2016. (For discussion, see Tierney 2001). My view is nearer to theirs, but to my knowledge none of them connect the notion of appropriate explanation with the “necessity requirement” in the way I do.
relations, in which explananda must be phrased as predications (“why a given predicate is attributed to a given subject”), and explanatory factors are introduced as middle terms. Aristotle is not interested in importing from syllogistic a (supposed) axiomatised procedure, nor is he primarily interested in the truth-preservation feature of a syllogistic deduction. Of course, he takes syllogisms to be truth-preserving. But his reason for thinking that syllogisms are suited to express scientific demonstrations is that syllogisms are adequate tools for displaying appropriate explanations.

2. The Necessity Requirement: scientific knowledge grasps something that “cannot be otherwise”.

According to standard interpretations, in section [iii] of T1 the predicate “cannot be otherwise” attaches to “pragma” (which is taken to be the referent of the pronoun “touto” at 71b12) and should be taken to indicate Aristotle’s concern with necessary truth: the object of scientific knowledge – apart from its relation to its cause – must be something that is necessarily what it is. Accordingly, the objects of scientific knowledge should be basic sentences that are necessarily true. Since the pragma is expressed as a syllogistic conclusion, the Necessity Requirement stresses, according to the standard interpretation, that the conclusion of a demonstration must be a necessarily true sentence in itself, namely, apart from its relation to the premises. I wish to challenge this interpretation. I do not need to challenge the assumption that conclusions of scientific demonstrations are “normally” necessarily true predications. What I am rejecting is the exegetical claim that the point of Aristotle’s Necessity Requirement in T1 is to stress the need for necessarily true predications.

In my view, the pronoun “touto” in 71b12 refers back to the previous sentence, “that this is the cause of it” (“hoti ekeinou aitia esti”, 71b11–12) and so focuses on the causal relation between aitia and pragma: what cannot be otherwise is that this cause is the (primary) cause of the explanandum at stake. It might seem that I am smuggling the adjective “primary” into T1, but I hope that my ensuing discussion will show that I am justified in suggesting that Aristotle defines scientific knowledge by appeal to primary causes from the very beginning of APo. Thus, a scientific knower knows that the cause selected as the explanatory factor is the (primary) cause of the explanandum and knows that “this” (namely, this cause’s being the primary cause of the explanandum) cannot be otherwise. A scientific knower knows that, among several causes available for her explanatory story, the one she has chosen is uniquely able to deliver the most appropriate explanation of the explanandum. Now, there is an important question about what “the most

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13 The necessity of the premises will then be included in the picture in APo I.4 and I.6 according to controversial moves of modal reasoning. See Barnes 1993, 110f.; Mignucci 2007, 162f., 170f.
14 My “normally” is just a reminder that most sciences are constituted of sentences that are true not necessarily but for the most part. Ferejohn 2013, 82, suggests that Aristotle’s use of “anankaion” is not meant to exclude what is true for the most part but not necessarily.
15 To my knowledge Lloyd 1981, 157, fn. 2, is the only one who entertains something similar to what I am proposing.
appropriate explanation” means. I will flesh out this notion below. For the time being, let me stress that, if my interpretation is right, Aristotle is implying that the cause that is able to deliver scientific knowledge of its explanandum is unique. By being the only one that delivers the fully appropriate explanation, it can be called (if not a “necessary cause” ipsis litteris) a “necessary principle” (see 74b5–6).

One might suggest that the cause that delivers scientific explanation of the explanandum should be called “a necessary cause” in the sense that, once it obtains, its pragma also obtains (see 98b29–30). Such a cause will be a “necessitating cause”. But I am not suggesting that this is Aristotle’s point in T1. The meaning of the expression “cannot be otherwise” in T1 can only be captured if we start with the explanandum and ask for its most appropriate explanation, but not if we start with the cause and asks whether the pragma will follow it or not. Aristotle is concerned with explanatory appropriateness (or primacy) from the very beginning of APo. In order to have scientific knowledge of the explanandum C-A, one must know that C-A obtains because of its cause B and one must know that this relation of appropriate explanation cannot be otherwise. The cause B can thus be characterised as the “necessary principle” (cf. 74b5–6) in the sense of being the required one for the most appropriate explanation of C-A. Another cause might be good enough for yielding some explanation, but scientific knowledge demands the cause that will uniquely deliver the most appropriate explanation.16

Now, let me flesh out the notion of appropriate cause (or appropriate principle, cf. 71b23). First of all, since causes are expressed as middle terms in syllogistic demonstrations, a given cause is an appropriate cause for a given explanandum if it performs its explanatory work in the most appropriate way as a middle term. Given the priority attributed to Barbara syllogisms in Aristotle’s theory of demonstration, I will focus on them (henceforth, my use of “syllogism” should be taken to mean “syllogism in Barbara” unless I remark otherwise). As a middle term of this sort, an appropriate cause has three basic features, which can be put in a gradual series.

An appropriate cause is:

1) a sufficient condition for its explanandum to obtain;
2) a necessary condition for its explanandum to obtain;
3) what makes the explanandum what it is.

2.1. Appropriate cause as sufficient condition for its explanandum to obtain

An appropriate cause is expressed as a middle term of a sound syllogism. The truth of its attribution to the minor term C entails (together with the truth of the major premise) the truth of the conclusion, in which A is attributed to C. Thus, an appropriate cause (expressed as a middle term B) is a sufficient condition for its explanandum to obtain.

It goes without further argument that such a feature, however important, is only a sine qua non condition for a cause to be an appropriate one. A given conclusion can be soundly deduced from a myriad of different middle terms. If demonstration were just a matter of

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16 As I will explore at the end of this paper, my interpretation allows us to understand what Aristotle says at 74b15–18 without attributing him the false view that, if conclusion C is proved and the premises are necessarily true, C is demonstrated. See Barnes, 1993, 126, and Mignucci 2007, 171.
certifying the truth of the conclusion from true premises, one scientist will end up with a swarm of demonstrations of the same explanandum. This swarm might sound attractive to some philosophical temperaments, but not to Aristotle’s.\(^{17}\) It will not help to introduce necessary truth into this story. A given conclusion that is necessarily true can be soundly deduced from a myriad of premise-pairs in which each premise is necessarily true. If demonstration were just a matter of “explaining” that the conclusion is necessarily true because it follows from necessarily true premises, a scientist will end up again with a swarm of demonstrations of the same explanandum.\(^{18}\)

2.2. Appropriate cause as a sine qua non condition for its explanandum to obtain and the coextensiveness requirement

An appropriate cause is also a sine qua non condition for its explanandum to obtain. The truth of the attribution of an appropriate cause to the minor term \(C\) not only entails (together with the truth of the major premise) the truth of the conclusion, but is also a necessary condition for it – which implies, as will be explored below, that \(A\) and \(B\) are coextensive terms. Thus, if the minor premise “every \(C\) is \(B\)” were false, the conclusion “every \(C\) is \(A\)” would also be false. Special attention to terminology is needed here: the reader might wonder whether condition (2.2) for an appropriate cause is my motivation for calling it a “necessary cause”, thus making it the bearer of the necessity in \(T_1\). But this is not so. My reason for taking the cause as the focus of the Necessity Requirement in \(T_1\) rests on the next condition, which concerns the notion of explanatory appropriateness. And Aristotle’s reason for suggesting that an appropriate cause is a “necessary principle” is not that it satisfies condition (2.2), but rather that it satisfies condition (2.3). For these reasons, I have employed the expression “sine qua non” instead of “necessary” as the title for my condition (2.2).

It is important to stress that the conjunction of conditions (2.1) and (2.2) yields the result that an appropriate cause \(B\) is coextensive with the predicate to be explained, \(A\). If, for any \(C\), being \(B\) is not only a sufficient but also a necessary condition for being \(A\), then \(B\) and \(A\) are coextensive terms. Coextensiveness between terms in demonstrative syllogisms is one of the most important claims Aristotle deploys in his theory in \(APo\). The pivotal notion of a katholou predicate (as developed in 73b26-39) is surely not exhausted by mere extensional features like coextensiveness, but nevertheless coextensiveness is in play.\(^{19}\) Aristotle explicitly recognises that a cause (as a middle term \(B\)) and its explanandum (as a major term \(A\)) might be coextensive with each other in the first part of \(APo\) I.13: “being near the Earth” and “not-twinkling” are coextensive predicates (at least in the domain of celestial bodies, which is Aristotle’s concern in the passage). One might object that

\(^{17}\) It is possible to prove that “every human is mortal” from many sets of premises that are necessarily true and involve different middle terms (e. g., “animal”, “biped animal”, “mammal”, “animal having lungs” etc.). How should we decide which of these proofs is a scientific explanation? For sensitivity to this issue, see Hankinson 1998, 161f.

\(^{18}\) The same examples from the previous footnote apply here.

\(^{19}\) The importance of coextensiveness (as a necessary but not sufficient condition for scientific demonstration) has been stressed by Lennox 2001b, 46f., and Ferejohn 2013, 81–95. See also Hasper 2006, 274–284, who focuses on \(APo\) I.5. I have left \(APo\) I.5 out of my picture because the intricacies of that chapter cannot be dealt with in this paper.
Aristotle only mentions that the major and the middle terms can be coextensive in some cases without making coextensiveness a requirement for a causal relation to obtain. Of course, coextensiveness is not a requirement for every causal relation to obtain. However, it is a requirement for an appropriate causal relation to obtain. When Aristotle talks about priority in *Categories*, he makes it clear that the specific sort of *causal* priority that a cause has over its *pragma* does not depend on any purely “logical” asymmetry: there can be a mutual implication between cause and *pragma*, and this symmetrical relation does not affect the asymmetrical explanatory relation (14b10–23).\(^{20}\)

Furthermore, Aristotle also makes it clear that appropriate explananda are such that coextensiveness between the middle term \(B\) and the attribute \(A\) is involved:

\textbf{T2}: 

[i] Hence, if the cause holds, it is necessary for its *pragma* to hold, but, if the *pragma* holds, it is not necessary for every cause to hold, but it is only necessary that some cause holds (not every).  
[ii] Or, if the problem is always universal, not only the cause is a whole, but also that of which it is the cause is universal too? [iii] E.g. shedding leaves is commensurate to a given whole, even if there are species of it, and it is universal in relation to them (be it plants or plants of a given kind); hence, the middle term and that of which it is the cause must be equal in these cases, and convertible. (98b29–35; my translation)

Step [i] of \textbf{T2} seems to make room for different causes explaining the same *pragma* in different circumstances (or for different subjects), but Aristotle is only setting up the problem in order to announce his position in steps [ii]–[iii]: given that an appropriate explanandum must be a *katholou* predication, as he has insisted in *APo* I.4–5, the middle term must be coextensive with the major term too.\(^{21}\)

Moreover, Aristotle does indeed argue for the coextensiveness condition in *APo* I.13, 78b13–31: a (primary) cause must be coextensive with the attribute to be explained. He makes his point with a syllogism that does not capture the *cause* (I insist: the *appropriate cause*) of the conclusion. The passage is made much more difficult by his compressed language and, in particular, by his use of negative statements and two different words that can usually be appropriately translated “cause”. In my interpretation, the text reads as follows:

\textbf{T3}: 

[i] why do walls not breathe? Because they are not animals. [ii] Now, if this were the [primary] cause of their not breathing, then being an animal would have to be the cause of their breathing, [iii] just as, if the negation is the cause of [the explanandum’s] not holding, the affirmation is the cause of its holding [...] and similarly, if the affirmation is the cause of its holding, the negation is the cause of its not holding. [iv] Now, for the cases such as the one mentioned above, the condition I have just stated is not satisfied: for not every animal breathes. [v] The syllogism for explanations of this kind comes about in the second figure. Let \(A\) be animal, \(B\) breathing, \(C\) wall. (78b15–25, Barnes’s translation modified)

Aristotle has in mind an attempted demonstration with this *Camestres* syllogism:


\(^{21}\) The first three sentences in step [ii] are very compressed but deliver a clear argument: if the problem (i.e., the attribution of the major to the minor) is universal (i.e., minor and major are coextensive with each other), then the middle must be coextensive with both. Aristotle is stating this consequence in two steps: “then, the middle is a universal (attributed to the minor in the minor premise), and the major is a universal too (namely, attributed to the middle in the major premise)”. Coextensiveness between major and middle is then explicitly stated in step [iii].
“Everything which breathes is an animal”;  
“No wall is an animal”;  
“therefore, no wall breathes”.

Now, a wall’s not being an animal is at least a partial explanation of the fact that walls do not breathe. In addition, there is no doubt that the deduction is sound. However, Aristotle is not satisfied with this sort of explanation and asks for more than a mere sound deduction of the conclusion. For this reason, it is clear that this passage is describing a requirement for being a cause of a superior sort. I take it to be a requirement for being a primary or appropriate cause. Indeed, the notion of a primary cause is in play in the context, from 78a26 on (cf. 78b4). Thus, we have a counterfactual at step [ii], 78b16–17: if the cause stated in this Camestres syllogism (“not being an animal”) were the appropriate cause of the wall’s not breathing (rather than a mere cause delivering a sound deduction and a partial explanation), its negation (“being an animal”\textsuperscript{22}) would be the cause of breathing. What we have in step [iii] (after “hoion”, 78b17) is a general point: Aristotle is spelling out a requirement every appropriate cause must meet, but the requirement is not restricted to the Camestres syllogism formulated above. Thus, if $B$ is an appropriate cause of its explanandum $C$-$A$, “its negation” (namely, the minor premise in which $B$ is denied of $C$) will be a sufficient condition for $A$’s not being attributed to $C$, as we have in the Camestres above; in addition, “its affirmation” (namely, a minor premise in which $B$ is affirmed of $C$) would be a sufficient condition for $A$’s being attributed to $C$, as we would have in a syllogism in Barbara with the original major premise converted (and this is what we fail to get in the Camestres syllogism above). Aristotle’s peculiar terminology makes the passage difficult to decipher, but he is saying precisely that, for any appropriate cause $B$ (as expressed in the syllogistic framework), the truth of “$C$ is $B$” is sufficient for the truth of “$C$ is $A$”, just as the truth of “$C$ is not $B$” would be sufficient for the truth of “$C$ is not $A$” (which is tantamount to saying that the truth of “$C$ is $B$” is also a necessary condition for the truth of “$C$ is $A$”). Thus, the first point spells out condition (2.1), whereas the second spells out condition (2.2). For saying that the truth of “$C$ is not $B$” is sufficient for the truth of “$C$ is not $A$” is tantamount to saying that being $B$ is a necessary ($\textit{sine qua non}$) condition for being $A$. What is wrong in the foil-case presented by our Camestres is that its major premise is not convertible and, therefore, “being an animal” works only as a necessary but not sufficient condition for being a breathing thing. As Aristotle has stressed in the counterfactual mode, if “not being an animal” were the primary or appropriate cause that explains why walls do not breathe, being an animal should also be sufficient for a wall to be a breathing thing. But this is not the case, since (for Aristotle) not every animal breathes. Therefore, the Camestres syllogism above is given as a counterfactual example in which condition (2.2) is satisfied but condition (2.1) is not (see 78b21–22). The whole passage, in sum, is arguing for the coextensiveness requirement for appropriate causes.

\textsuperscript{22} This is the result of “not” being applied to “not being an animal”.

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2.3. Explanatory appropriateness: making the explanandum what it is.

Aristotle does not reduce the notion of an appropriate cause to the conjunction of the two previous features (2.1) and (2.2), which can be treated from a merely extensional standpoint. Explanatory appropriateness is also (and mainly) a matter of making the explanandum what it is. In order to flesh out this notion of explanatory appropriateness, I will rely, in the next section, on the contrast between scientific knowledge and incidental knowledge. But some key points should be mentioned now.

First, condition (2.3) implies the previous two but not the other way around. It is possible for a given cause to satisfy (2.1) without satisfying (2.2) and (2.3). Any middle term not coextensive with the major in a sound *Barbara* will do as an example: for instance, *mammal* as a middle term explaining why humans are mortals. Again, it is possible for a given cause to satisfy (2.2) without satisfying (2.1) and (2.3), which is the situation Aristotle has illustrated with the *Camestres* syllogism about walls: being an animal is just a necessary but not sufficient condition for something to be a breathing thing. Furthermore, and more importantly, it is possible for a given cause to satisfy both (2.1) and (2.2) without satisfying (2.3). Aristotle stresses this point in the first half of *APo* I.13: a middle term in an attempted demonstration might well be coextensive with the major term but not capture the cause that appropriately explains the fact expressed in the conclusion. This is what happens when one soundly deduces that every planet is near the Earth through “not-twinkling” as the middle term. This attempted demonstration is a sound deduction of its conclusion, but only establishes that it is the case without explaining why it is the case (78a28–38). However, if a given cause satisfies condition (2.3) and thereby counts as an appropriate cause, it also satisfies both conditions (2.1) and (2.2).

A second point about appropriate causes is that they seem to bring to an end a series of why-questions: if an appropriate cause is attained, no one could sensibly keep asking why. This feature follows from the idea that an appropriate cause is primary in the sense that it is not because of something else (or not as being something else) that it appropriately holds of its explanandum: there is no further cause that makes an appropriate cause what it is.

A third point is that Aristotle does not frequently use the expression “*oikeion aition*” (or something equivalent), nor does he define the notion. As for the absence of a definition, this does not tell against the existence or the importance of the notion. The notion of form is one of the most important in Aristotle’s metaphysics and philosophy of nature, but he never explicitly defines it either. In *APo* I.13, where a definition of the notion of appropriate cause would be in its proper place, Aristotle seems to rely on the intuitiveness of his examples: even if there is mutual entailment between the planet’s not-twinkling and the planet’s being near the Earth, it is clear enough that their nearness is the cause of not-twinkling but not the other way around (78a28–b4). Nor does the scarcity of occurrences of the expression “*oikeion aition*” poses any serious problem for my interpretation. Aristotle’s terminology


24 Aristotle is characterising this feature of causes in 85b27–86a3.

25 See *Meteorologica* 346a30 for “*oikeiota tê aitia*”. In 80b18, 21, “*oikeion meson*” does not have the meaning of an appropriate middle term in the sense of explanatorily appropriate.
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is flexible and context-sensitive. In my view, the notion of appropriate cause as I am describing it here is what Aristotle has in mind when he uses expressions such as “appropriate principles” (“archai oikeiai”, 71b23; 72a6), “appropriate” (“oikeion”, 74b26), “primary cause” (“proton aition”, 78a25, 78b4), “necessary principles” (“anankai archai”, 74b5–6) and “of the same family” (“suggenes”, 76a30), as well as certain absolute occurrences of the adjective “necessary” (“anankai archai”, 74b7, 74b12, 31) and the noun “cause” (either “aitia”, as in 71b10–11, or “aition”, as in 71b30–31, 75a35, 78b15, 98a35ff.). As I will try to show in my next section, Aristotle also has in mind this notion of an appropriate cause when he uses the expression “the principle of the thing [i.e., the explanandum] as such” (76a5–6).

Finally, a fourth point about appropriate causes will pave the way for my Claim (3) about Aristotle’s definition of scientific knowledge in T1. An appropriate cause captures what makes its explanandum what it is. Since the factor that makes X what X is turns out to be (at least) an element in the definiens account of X, it follows that the explanatory appropriateness of a cause is closely related to the notion of definitional priority. As Aristotle says:

T5: And this [sc. the middle term B] is the account of the one extreme, i.e., in this case of A; for an eclipse is a screening by the Earth. (93b6–7, Barnes’s translation)

A few lines below he stresses the same point:

T6: and B is indeed an account of A, the first extreme (93b12, Barnes’s translation)

And there is still a third explicit statement:

T7: and the middle term is an account of the first extreme (99a21–22, Barnes’s translation)

Thus, an appropriate cause for an explanandum X is either the whole definiens account of X or at least an important factor contained in the definiens account. Since X is interpreted as being the major term, the first option will imply that the major premise must be an inverted definition, in which the definiens will be the subject and the definiendum will be the predicate. Since this is highly controversial, the other option seems preferable: the middle term will express the most important factor in the definiens account of the major term. Aristotle’s cautious use of “logos” instead of “horos” or “horismos” at T5–7 suggests this option: although “logos” can be used in the sense of a full definition, it can also be taken in a different way, as pointing to the most important explanatory factor. Aristotle clarifies his point in APo II.16:

T8: It is clear that the eclipse is not the cause of the Earth’s being in the middle but rather the latter is the cause of the eclipse; for the Earth’s being in the middle is present in the account of the eclipse. (98b21–23, Barnes’s translation modified)

Since “being present in the account” is different from “being the whole account”, it is fair to infer that the appropriate cause for an explanandum is one of the elements through which the explanandum must be defined. Now, since a definition is the expression of the essence of its definiendum, the explanatory appropriateness of a cause is closely related to the idea that explanations must be formulated in terms of essences. Actually, the notion of an appropriate cause is one of the ways to understand Aristotle’s claim that “to know what
3. Incidental knowledge

Let me first clarify some points about the grammar of step [i] of T1. The expression “kata sumbebekos”, which is a gloss on the “sophistical way of knowing”, should be taken ultimately to modify “epistasthai” and to provide a contrast with “epistasthai haplôs”, which is Aristotle’s definiendum. Now, since “epistasthai haplôs” has “hekaston” (71b9) and “pragma” (71b11) as its objects, it is reasonable to take its foil to have the same objects too. This amounts to taking “kata sumbebekos epistasthai” as a way of knowing an object that has predicative structure (cf. 76a1–7). Now, since “epistasthai haplôs” is defined by the knowing the cause and, as I have argued, by knowing that the cause is the required one for the most appropriate explanation, it is reasonable to infer that its foil will be characterised by the failure of at least one of these conditions. Given that not knowing any cause whatsoever entails not knowing the most appropriate cause, there are only two options: knowledge will be incidental either when one does not attain the knowledge of any cause or when one knows some cause, but not the most appropriate one.

One might be attracted by the first option: scientific knowledge is knowledge of the most appropriate cause, whereas incidental knowledge is a flat knowledge of a given predication without involving any explanatory claim. Scientific knowledge would be expressed in sentences such as “I know that the planets are not-twinkling because they are near the Earth”, whereas incidental knowledge would be expressed in sentences such as “I know that planets are not-twinkling” full stop. Now, this would be a very weak foil, and even if someone argues that its weakness is suited to an introductory remark or a sketchy outline of the definiendum, one has to admit that a much more powerful foil will be attained if

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26 There is no room to discuss the controversial issues surrounding these passages in this paper, but two things are worth mentioning. First, the explanandum is treated as the major term A, which seems to tell against my contention that “pragma” in T1 must be taken as an explanandum with predicative structure. However, in my view, Aristotle’s treatment of A does not raise an objection. The explanandum is the predicative relation between an attribute A and its proper subject C. But the predicate A is much more important than the subject C for making the explanandum the specific explanandum it is. If you want to explain (e.g.) the longevity of birds, it is clear that you are interested in some fact about birds (C), but the attribute longevity (A) is a much more important factor to make that explanandum the explanandum it is. This priority of the predicate over the subject might have led Aristotle to use terms standing for the attribute to point out to the whole predicative explanandum as such. Second, the major term itself is interpreted in different ways: e.g., in T5, A is interpreted as “eclipse” (which is the pragma itself), whereas in T6, where the example has switched to thunder, A is interpreted as “noise” (presumably “noise of a given kind”, cf. 93a22–3). Aristotle shifts with no further notice (nor embarrassment) from one to another interpretation of the explanandum. There are several subtleties which I cannot address here, but I do not think my story is jeopardised by them. Thunder is the proper explanandum and can also be treated as a major term A. However, its explanation presupposes a previous step in which its elements are articulated (“thunder is a noise of a given kind in the clouds”), so that “noise” becomes the major term.

27 I have developed this point in my 2014. For Book II of APo, see Charles 2000; Charles 2010; Goldin 1996; Bronstein 2016. Aristotle’s remark at 90a31–4 suggests that the picture is at least applicable to Book I as well, since 2 R is one of the central examples in Book I.
incidental knowledge has the same formal structure as scientific knowledge, namely, “I know that every \( C \) is \( A \) because every \( C \) is \( B \) etc.”. The contrast between them will hinge on the explanatory appropriateness of the cause selected on each side: scientific knowledge is the knowledge of the most appropriate cause, whereas incidental knowledge will be the knowledge of any cause that is not the most appropriate one.\(^{28}\) Furthermore, an interpretation along these lines fits well with what Aristotle says about pseudo-demonstrations (such as Bryson’s squaring of the circle) in \( APo \) I.9: those pseudo-demonstrations, which convey only incidental knowledge, are faulty because their attempted explanations appeal to something “common” and not suited to the explanandum at stake (cf. 76a1–3).

One might object that this way of construing the contrast will not work, because scientific knowledge will be too restrictive, whereas incidental knowledge will cover too many cases. For a given explanandum, only one explanation will count as scientific knowledge, whereas all other attempted explanations will count as incidental knowledge. Call this the Statistical Imbalance between scientific knowledge and its foil. One might try to argue that the contrast between scientific knowledge and incidental knowledge in terms of different sorts of explanation does not depend on the Statistical Imbalance. However, in my view such an imbalanced picture is exactly what Aristotle offers us:

\( T9: \) It is difficult to tell whether you have [scientific] knowledge of something or not, for it is difficult to tell whether or not our knowledge of something proceeds from its principles – and this is what it is to know something. We think we have scientific knowledge of something if we possess a syllogism from some true and primary items, but this is not so. (76a26–29, Barnes’s translation modified)\(^{29}\)

Moreover, the way in which Aristotle appeals to incidental knowledge as a foil at an important juncture provides strong evidence for my view that it has an explanatory structure:

\( T10: \) We understand something non-incidentally when we know it in virtue of that in virtue of which it holds and from the principles of that thing as such. (\( APo \) I.9, 76a4–6, Barnes’s translation, modified)

There is a good deal of controversy about the referents of the pronouns in this passage, but my present point does not depend on this. What I want to stress is that the grammatical structure “\( kata + \) accusative” is related to the way in which an explanatory factor is introduced to explain a given explanandum.\(^{30}\) The expression “in virtue of that in virtue of which” translates “\( kat’ ekeino […] kath’ ho \)”, and “incidentally” translates “\( kata sumbebekos \)”. Aristotle is saying that, when one captures the appropriate explanatory factor (the one that will be called “the principle of the explanandum as such”), her knowledge is not \( kata sumbebekos \). By contrast, when one’s knowledge is \( kata sumbebekos \), its explanatory factor is not the appropriate one. The habit of flattening the expression “\( kata sumbebekos \)” into a non-elucidative adverbial expression, such as “incidentally” or “accidentally”, has

\(^{28}\) See my 2013 for a similar approach to the distinction between (scientific) knowledge and (explanatory) opinion in \( APo \) I.33. For a different view, see Fine 2010b, 146–148, who resists that opinion and incidental knowledge can be identified.

\(^{29}\) See Hasper 2013, 320, for an interpretation of this passage in a similar direction.

\(^{30}\) For a similar approach, see McKirahan 1992, 175, and Hasper 2006, 279–284 (on “\( kata touto \)” in 74b2), and Hasper 2013, 318–320.
blunted our sensitivity to what the Greek expression means in each context. The undue connection of “sumbebekos” with the notion of a contingent predicate has also strongly contributed to this insensitivity. Now, the core-meaning of the term “sumbebekos” is just “what comes together with X” with an additional and important feature: despite its coming together with X, the sumbebekos is not relevant or is not the most important factor for the “target” under which X is taken. But the “target” is differently conceived of in different contexts, and the notion of a contingent predicate is just one of the common ways in which Aristotle employs the expression “kata sumbebekos” or the term “sumbebekos”. There is no room in this paper for an exhaustive examination of the ways in which the expression “kata sumbebekos” is employed in Aristotle’s texts. Nevertheless, I will briefly discuss some of these ways in order to flesh out my claim that, in T1 as well as in T10, the expression “kata sumbebekos” might be paraphrased as: “according to a middle term which is not the most appropriate one for the intended explanation (even if this middle term is a necessary/essential predicate of its subject in the minor premise)”.

I will rely on the reasonable assumption that the expression “kata sumbebekos” is sometimes used in opposition to the expression “kath’ hauto” (or equivalent expressions). The opposed expressions are used in many ways by Aristotle and I will not discuss them exhaustively. It is enough for my purposes to highlight in general outlines two uses of the opposition in order to prepare the way for a third use, which is, I will argue, the one in play in T1 and in T10.

3.1. First use of the opposition “kata sumbebekos” / “kath’ hauto”

Three points are relevant for my purposes. The first is that, within the first use of this opposition, the expressions “kata sumbebekos / kath’ hauto” qualify predicative relations. The second point is that these expressions focus on the subject of a predication. (They can also focus on predicates, but I will skip this complication for strategical reasons; it is more important to stress that, if the expression is taken as a qualifier of a sentence without paying attention to the structure of the sentence, much of its meaning is lost). The third point is that the expressions say in what way that subject must be taken for the sentence to make sense. In other words, the expressions inform us about the way in which the thing picked out by the subject-term must be considered in order to be a subject that can admit of that kind of predicate. The truth of the sentence is not at stake: the question is whether the subject of the sentence is the kind of thing of which it makes straightforward sense to predicate the attribute of the sentence. The subject can be taken (i) either according to its essence (ii) or according to some further predicative connection which obtains and is assumed in the context, but is not asserted in the sentence. The expression “kath’ hauto” applies in the former case, whereas the expression “kata sumbebekos” applies in the latter. Thus, saying that “the surface is [kath’ hauten] white” (Met. 1022a30–31) is tantamount to saying that being musical does not pick out the relevant feature that makes the thing a subject of the appropriate kind: it is not
by being musical, but by being something else (namely, a human), that the thing qualifies as a subject of the appropriate kind; being musical just happens to come together with being a human without introducing the human at stake under the feature that is relevant to understand his ability to be white.32

3.2. Second use of the opposition “kata sumbebekos” / “kath’ hauto”

Again, three points are relevant for my purposes (the first two are the same as in the previous case). Firstly, the expressions “kata sumbebekos/kath’ hauto” qualify predicative relations. Secondly, they focus on the subject of a predication. Thirdly, they say how that subject must be taken to guarantee the truth of the sentence. The subject might be taken (i) either according to its essence, (ii) or as being something else besides what is implied or involved in its essence. The expression “kath’ hauto” applies in the former case, while the expression “kata sumbebekos” applies in the latter. Thus, saying that “the triangle is [kath’ hauto] a figure” is tantamount to saying that the essence of the triangle guarantees that the predication is true, whereas saying that “the triangle is [kata sumbebekos] a brazen object” is tantamount to saying that the essence of the triangle on its own does not guarantee that the predicate holds of triangles.

It is within the second use of the expression “kata sumbebekos” that the notion of a contingent predicate is found. However, there are plenty of uses of “kata sumbebekos” in which the inference from “S is kata sumbebekos P” to “P is a contingent attribute of S” does not work. Aristotle says at Met. 981a18–20 that the physician who heals Socrates heals a human “kata sumbebekos”, and his explanation for this terminology is that “being a human is attributed to Socrates” (“[Socratei] sumbebeke anthropon einai”). He is surely relying on the natural connection between the expression “kata sumbebekos” and the verb “sumbebeke”. Now, being a human, described as something which “happens to” or “comes together with” Socrates, is not a contingent attribute of Socrates. But being a human is an attribute that, even being essentially true of Socrates, is not the most important for the target of the sentence, which is to present Socrates as appropriately liable to the physician’s activity. The relevant property that makes Socrates liable to the physician’s activity is his being sick. Now, being a human might be taken as a necessary condition for being sick in the relevant sense, since a physician heals human beings, not beasts. Even so, the most important property for making Socrates liable to the physician’s healing activity is not the plain property of being a human, and this fact is enough to call “being a human” a “sumbebekos” of Socrates in this context.33

I have explored this example to stress the claim that the term “sumbebekos” is far from being equivalent to the notion of a contingent predicate. Being a human can be described as a sumbebekos of Socrates, just as being white can be described as a sumbebekos of snow (cf. 120b21–22), and sciences might be said to study the sumbebekos of their subject-matter (cf. 1003a25, 193b32–33). Being a human is an essential predicate of Socrates, but this does not prevent it from being called a “sumbebekos” in contexts where the property

32 See also the elucidation of uses of “kata sumbebekos” in Physics 211a19–23, as well as the explanation of coming-to-be in Physics 1.8; see Clarke, 2015, 140. My sketchy outline does not cover many subtleties and complications, but is enough for my purposes.

33 Similarly, “being a sphere” is a sumbebekos of a brazen sphere (1033a29–31).
of being a human, although essential to Socrates, is not the most important for the target at stake.\footnote{A similar move can be found in Hasper 2006, 269–273, concerning “hos en merei holon” in 74a9. What makes triangle a “distributive (or compartmentalized) whole” (“hos en merei holon”) is not that one of its species is being taken as subject to which 2 R is attributed (since triangle as a whole is taken as the subject), but the way in which the proof is conducted.}

3.3. Third use of the opposition “kata sumbebekos” / “kath’ hauto”

Although this use still depends on predicative relations, it qualifies exactly the relation between explanandum and explanans, which is a triadic relation in which the explanandum has predicative form and the explanans is the middle term. Thus, we have a shift from a dyadic perspective within the first two uses to a triadic perspective within this third use; I say “dyadic” because a predicative sentence has only two elements, the subject and the predicate; I use “triadic”, by contrast, to point out that an explanatory context involves a triplet of terms and three predications ordered according to an explanatory relation.

Within the third use of their opposition, these expressions focus on middle terms as middle terms. What I mean by that will become clearer in my next step and from the discussion of examples, but the main idea is that the expressions do not focus on terms in the context of isolated predications (as if the whole concern were to see how the middle term is related as a predicate to the minor, then to see how the middle term is related as a subject to the major, and then mechanically to sum up the result).

The most important point is that these expressions say how the middle term is doing the work proper to middle terms, namely, in what way the middle term is introducing the explanation for a given explanandum:

(i) the expression “kath’ hauto” (or some equivalent expression, like “hei auto” or “hei ekeino”) applies when the middle term introduces an explanation by taking the explanandum as what it is and by presenting the explanatory factor that makes it what it is.\footnote{For “kath’ hauto”: 85a23–24; 74b6-7; “hei auto”: 85b7; “hei ekeino”: 75b38, 76a1, 6; some of these passages will be discussed below.}

(ii) the expression “kata sumbebekos” applies when the middle term introduces an explanation by taking some explanatory factor which is not the primary (or the most appropriate) one, i.e., is not the factor that makes the explanandum what it is – even if this factor delivers (in the minor premise) a predication that is a necessary truth, or an essential predication, or even the full definition of its subject.

The most important thing here is the following: in order to say something about the explanatory work a middle term is doing in relation to a given explanandum, it is not enough to pay attention to each predication in which it appears; one should rather ask whether those predications taken together are appropriately explanatory of the explanandum, but this explanatory appropriateness cannot be reduced to a mere sum of predicative relations of a given sort. As I will explore below in my discussion of specific examples, in order to ascertain whether a given syllogism is a scientific demonstration, it is not enough to check that each premise is a predication of a (supposedly) convenient kind – it is not enough to proceed in the following way: “the major premise is an essential predication; so is the minor premise; then, since both my premises are essential predications, I have got...
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a demonstration”, or “the major premise is a per se predication; so is the minor premise; then, since both my premises are per se predications, I have got a demonstration”. There are clear counterexamples to such attempted demonstrations (as I will explore below). These counterexamples show that the explanatory work to be done by a demonstrative middle term cannot be reduced to any sum of merely dyadic predicative relations.

Now, in my view, in the contexts where we find the third use of the opposition between “kata sumbebekos” and “kath’ hauto” (or some equivalent expression, like “hei auto” or “hei ekeino”), the pronoun “hauto” (or “auto”, or “ekeino”) refers to the explanandum as such.36 Consequently, the expressions “kath’ hauto”, “hei auto” and “hei ekeino” point out to the way in which the explanandum must be considered in order to make the explanation a strict scientific demonstration: the attempted explanation will be a scientific demonstration if the middle term captures the primary explanatory factor that makes the explanandum exactly what it is.37

Aristotle’s discussion about the superiority of what he calls “universal demonstrations” in APo I.24 provides support for my interpretation. I disagree with standard readings of the first part of the chapter. Barnes, for instance, takes several of the occurrences of “hoti” as “that” (not as “because”) with the result that, on his view, concerns with explanatory factors are not relevant in Aristotle’s “paragone”: Aristotle’s worry is whether or not the conclusion presents a minor term coextensive with the major.38 In my view, the concern with coextensiveness between the terms in the conclusion is not the most important point. The most important point focuses on the explanatory factors selected in each sort of demonstration involved in the comparison.39

The key passage for the introduction of the problem runs as follows:

TII: [i] if we have knowledge of something better when we know it per se (“kath’ hauto”) than when we know it in virtue of something else [“kat’ allo”] […] and [ii] if the universal demonstration proves something because it is something else [“hoti allo”], not because it is itself [“ouk hoti auto”] (iii) e.g., that the isosceles [sc. has 2 R] not because it is isosceles [“hoti isoskeles’”] but because it is triangle [“hoti trigonon”]; [iv] if the particular demonstration proves something because it is itself [“hoti auto”], […] then particular demonstrations will be better. (85a23–31, my translation)

Aristotle is still settling the question, not giving his ultimate solutions. Step [i] introduces the strong premise that guides the comparison, but in steps [ii] and [iii] Aristotle plays devil’s advocate: step [ii] presents what an adversary might argue against the primacy of

36 As I have said, Aristotle normally takes the explanandum to be the whole predicative relation expressed in the conclusion of a syllogism, but sometimes he refers to the major term itself as being the explanandum. But in all those contexts the major term is being introduced qua attributed to the minor, so that reference to the major term as the explanandum actually collapses into reference to the pragma with predicative form as expressed in the conclusion.

37 Ferejohn 2013, 86, 89, makes a good point when he takes “qua itself” as an extensional requirement to be applied only when previous intensional conditions are met. My disagreement is that “auto” refers back to the attribute to be explained, not to the subject.

38 See Barnes 1993, 184; Mignucci 2007, 71, 227f. For different views, see McKirahan 1992 175f.; Ross 1949, 588.

39 A similar point has been made by Hasper 2006 about APo I.5 (see especially 278 and his discussion about the last paragraph of APo I.5 in 279–284): coextensiveness is only a necessary but not sufficient feature of scientific demonstrations, for demonstrations are concerned with an intensional feature, the conceptual ground for the conclusion.
universal demonstrations, and step [iii] accordingly fleshes out the adversary’s point with an example. At 85b4–7 Aristotle will present his own solution, which involves a rejection of the arguments presented in steps [ii] and [iii] above. Now, my main contention is that the uses of “kata + accusative” in step [i] are pointing out to the way in which an explanatory factor is presented in each demonstration, and that several occurrences of “hoti” in the subsequent steps replace that expression and, consequently, have an explanatory meaning.

The example given in step [iii] has “isosceles” as minor term and presumably the attribute “2R” as major term (cf. 85b5–6). Now, this is not an example of what Aristotle would take to be a strict universal demonstration, since the minor term is not coextensive with the major (cf. 74a1–3). However, a mere extensional failure is not Aristotle’s main point here. Aristotle’s point is exactly the appropriateness of the explanatory factor chosen in each demonstration. Step [iii] of T11 compares, on the one side, a universal demonstration (or a demonstration that is nearer to the universal one) in which the attribute 2R will be explained for isosceles triangles by the fact that they are triangles, and, on the other side, a particular demonstration in which the attribute 2R will be explained for isosceles triangles by the fact that they are isosceles triangles. Coextensiveness is important to the story: but the main issue is coextensiveness in one of the premises (either between the middle and the major term or between the middle and the minor term), not coextensiveness between minor and major terms in the conclusion. What makes the universal demonstration be called “universal” in this context is rather the fact that its middle term is coextensive with the major, while what makes the particular demonstration be called “particular” in this context is the fact that its middle term is coextensive with the minor but not with the major. Aristotle’s adversary argues that the universal demonstration is inferior because it introduces a mismatch in the minor premise: in order to explain why isosceles have 2R, isosceles is taken “not as itself” (namely, not as defined by its proper, specific feature), but as “something else”, namely, as triangle.

What is Aristotle’s answer to that? He responds with the same argument introduced at step [ii] of T11, but he corrects its interpretation: “Or, in the first place, one of those arguments do not apply more to the universal than to the particular demonstration?” (85b4–5). “One of those arguments” picks up precisely what we have in step [ii] of T11: “to prove something because it is something else [‘hoti allo’], not because it is itself [‘ouk hoti autο’]”. Aristotle agrees that the demonstration satisfying this description will be inferior. But he disagrees with his adversary’s interpretation of how each demonstration, the universal and the particular, falls or fails to fall under that description. As I will explore, Aristotle’s point can be described as a correction of the reference of the pronouns

40 See a similar view in McKirahan 1992, 175. Mignucci 2007, 71, 227, seems to take “kat’ allo” with some explanatory force (“in base ad altro”) but surprisingly does not extend the same story neither to “kath’ hauto” nor to “hoti allo”, which gives him a strange argument with no parallelism in the comparison (no wonder that on his view Aristotle has not given a precise answer to this adversary).

41 There is a “shift” in the use of “hoti” in the same line 85a27, but I do not see any reason why we should be worried about that. Indeed, the first “hoti” at 85a27 means “because” (pace Ross, Barnes and Mignucci), after which there is a parenthetical remark in 85a27–28 with three occurrences of “hoti”: the second and the third occurrences mean “because” too, whereas the first means “that” (see Ross 1949, 590). This is a perfectly natural reading. It is not difficult to find a string of sentences in English in which “that” will similarly shift from one meaning to another.

42 For similar terminology, see APr. 48a35.
“auto” and “allo” in the argument advanced in T11, as if he says: “you are right: to prove something because it is something else [hoti allo], not because it is itself [ouk hoti auto], is the mark of an inferior demonstration; however, the most important element in the explanandum as such is the attribute, not the subject; you have taken ‘auto’ to refer to the minor term, which is the subject within the explanandum, but it should be taken to refer to the major term, which introduces the attribute; with this correction, you will see that it is rather the particular demonstration that proves something because it is something else [hoti allo], not because it is itself [ouk hoti auto]”.

The passage in which Aristotle presents his answer runs as follows:

T12: For if having two right angles holds of something not as isosceles but as triangle, then if you know [sc. that the isosceles has 2 R] because it is isosceles, you will know it less as such [hei auto] than if you know [sc. that the isosceles – or the triangle – has 2 R] because it is triangle.

(85b5–7, my translation)

One difficulty is that Aristotle’s text leaves it open as to which term is the minor in the second demonstration alluded to in T12 (the universal one): it might be either “triangle” or “isosceles triangle”. Now, the question of determining which term is the minor is important for other issues, but it will not affect my point because in either case (either with “triangle” or with “isosceles triangle” as the minor term) Aristotle’s main target is the relation between the middle and major terms in the particular demonstration. Thus, according to my interpretation, T12 together with T11 sketch two syllogisms that can be outlined in the following way:

(Warning about “notation”: the item on the left of “a” is the subject, the item on the right is the predicate, so that “2R” is the major term in both syllogisms; “a” stand for the universal affirmative)

Syllogism 1:
Definition of isosceles triangle a 2 R
Isosceles triangle a Definition of isosceles triangle

Syllogism 2:
Definition of triangle a 2 R
[Isosceles] triangle a Definition of triangle

[Isosceles] triangle a 2 R

– Definition of isosceles triangle: “rectilinear plane figure enclosed by three sides with two angles (and two sides) equal to each other”;
– Definition of triangle: “rectilinear plane figure enclosed by three sides”;

My formulation of the minor premise in Syllogism 1 is meant to capture what Aristotle means by “to know that isosceles has 2 R because it is isosceles” (85b6, cf. 85a27–28), whereas the minor premise in Syllogism 2 (either with “triangle” or with “isosceles triangle” as the minor term) captures what Aristotle means by “to know that isosceles [or triangle] has 2 R because it is triangle” (85b7, cf. 85a28). The important point on which the comparison hinges is the middle term selected as explanatory factor. The minor premise in Syllogism 1 introduces a definition: the middle term is the definiens of its subject. Even so, it does not deliver the required explanation. In the expression “hei auto” at 85b7, the
pronoun “*auto*” refers back to the explanandum as such, namely, 2 R (as an attribute to be ascribed to its proper subject). The problem with Syllogism 1 is that its minor premise smuggles into the explanation an explanatory factor that is not relevant for making 2 R what it is. The attribute 2 R “*qua* itself” is not something that depends on two angles of a triangle being equal to each other. The fact that the middle term is the definiens of its subject does not add to its explanatory appropriateness: the specific feature that defines isosceles triangles *qua* isosceles (namely, having two angles, and two sides, equal to each other) is not even a *sine qua non* condition for 2 R to obtain, let alone the most important factor for making 2 R what it is. Thus, the middle term in Syllogism 1, even though it is the definiens of its subject, satisfies only condition (2.1) but not conditions (2.2) and (2.3) for being an appropriate cause. Therefore, it does not deliver what Aristotle defines at 71b9–12, i.e., scientific knowledge.

As for Syllogism 2, (either with “triangle” or with “isosceles triangle” as the minor term), the most important point for Aristotle’s comparison is the appropriateness of the middle term in relation to the attribute to be explained, 2 R. The middle term is the definition of triangle, but what is more important for its explanatory appropriateness is rather the fact that it captures exactly the factor that primarily makes 2 R what it is. If the minor term is “triangle”, Syllogism 2 will probably count as a genuine demonstration in the strictest sense, and to say that Syllogism 1 delivers “less knowledge of the explanandum as such than Syllogism 2 does” is compatible with saying that “Syllogism 2 itself expresses knowledge of the explanandum as such”. But, if the minor term is “isosceles triangle”, Syllogism 2 does not count as a genuine demonstration in the strictest sense (because it fails to present a universal predication in the conclusion, cf. 74a1–3), but the appropriateness of its middle term would entitle Aristotle to say that it delivers “more knowledge of the explanandum as such than Syllogism 1 does” in the sense that it is nearer to what will strictly count as a genuine demonstration expressing knowledge of the explanandum as such. Therefore, Syllogism 2, either as an example of universal demonstration or as a demonstration that is nearer to a universal demonstration in the strictest sense, will deliver knowledge of 2 R *through what 2 R is in itself* (“*kath’ hauto*”, 85a23–4). On the other side, Syllogism 1, as an example of particular demonstration, will deliver knowledge of 2 R in virtue of “something else” (“*kat’ allo*”, 85a24), namely, in virtue of two angles in a species of triangle being equal to each other, which is an irrelevant feature for 2 R to obtain and does not capture what 2 R is.

Now, what do these syllogisms have to do with the notion of incidental knowledge? The expression “*kata sumbebekos*” does not appear in these passages from *APo* I.24. So, one might object, what entitles me to claim that there is some connection between the comparison made at *T12* and incidental knowledge? The connection is given by Aristotle’s use of the opposition between “*kata sumbebekos*” and “*hei ekeino*” in *APo* I.9. My argument involves two reasonable assumptions. The first one is that, for Aristotle, this opposition is exhaustive in its proper context: if a middle term explains its explanandum *as such*, it thereby does not explain it *kata sumbebekos* (cf. 76a4–6), and if a middle term explains its explanandum *not as such*, it thereby explains it *kata sumbebekos* (cf. 76a1–2). The second assumption is that the expressions “*kath’ hauto*”, “*hei auto*” and “*hei ekeino*” work in the same way in contexts where explanatory appropriateness is the main concern. From this it follows that Syllogism 1 delivers *kata sumbebekos* knowledge inasmuch as it fails to explain its explanandum as such.
In APo I.9, attempted demonstrations such as Bryson’s squaring of the circle are described as cases that deliver kata sumbebekos knowledge instead of explaining the explanandum as such (hei ekeino). Now, the problem with such pseudo-demonstrations is not that they commit some logical fallacy, nor that they smuggle a false premise into the demonstration.\(^{43}\) Aristotle makes it clear that he is discussing cases in which the premises from which one attempts to demonstrate are true and even immediate and somehow primary (see 75b39–41 and 76a28–30). Why do they fail to deliver scientific knowledge of the explanandum as such? Why are they described as delivering incidental knowledge? I submit that they fail to deliver a scientific demonstration because the middle term they select as the explanatory factor (despite being true, or necessarily true, or essentially true, or primarily true, or definitionally true of its subject in the minor premise) does not capture the most appropriate feature that makes the explanandum what it is.

Now, kata sumbebekos knowledge is not mentioned in T12, but Syllogism 1 is clearly described as failing to explain its explanandum qua itself (“hei auto”). Since “qua itself” (“hei auto”) and “as such” (“hei ekeino”) can be taken as equivalent in these contexts, and since it is reasonable to infer from 76a1–7 that kata sumbebekos knowledge is described as a failure to know the explanandum as such (hei ekeino), I argue that Syllogism 1 from T12 is a good example of kata sumbebekos knowledge – a good example of the foil Aristotle is presupposing and alluding to when he defines scientific knowledge as the grasping of the most appropriate cause in TI.

4. “Kata sumbebekos knowledge” and necessary principles

Let me come back to the Necessity Requirement in the definition of scientific knowledge. My interpretation of step [i] of TI has clarified (in addition to other things) how step [iii] can be interpreted as demanding the most appropriate cause in scientific demonstrations. I have already argued that the most appropriate cause for a given explanandum can be called a “necessary cause” or a “necessary principle” in the sense that it is the required one to capture what the explanandum is as such. Now I will point out how my picture helps us to attain a consistent and interesting interpretation of Aristotle’s appeal to the opposition between “necessary” and “incidental” in APo I.6.

Aristotle begins APo I.6 as follows:

**TI3:** [i] Since demonstrative knowledge proceeds from necessary principles […], and [ii] necessary is what holds of an explanandum in itself […]\(^{44}\), then [iii] it is clear that a demonstrative

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\(^{43}\) I need not discuss the details of Bryson’s argument. The reconstruction found in Mueller 1982, 161–164, is followed by Fait 2007, 154. A more powerful reconstruction is found in Hasper 2013, 314–319. The trouble with Bryson’s squaring is that a sound deduction of the conclusion through common principles is not enough for scientific demonstration, for demonstration should involve the basic items that are the relevant principles.

\(^{44}\) There are many problems in step [ii], besides the meaning of the expression “kath’ hauta huparchonta” (which I will discuss in the text). First, I take “anankai” as the subject and “to kath’ hauta huparchonta” as the predicate. This is contrary to what is (taken to be) the most common syntax involving subject and predicate, but it is very common in statements which assert some sort of sameness (e.g., many definitional statements have the definiendum with no article and the definiens introduced by an article); besides, this construal, which is justified by the coextensiveness between subject and predicate, delivers a sound Barbara (instead of delivering an invalid argument as suggested by Barnes 1993, 126; see Mignucci 2007, 170, for the syntax I am preferring). Second,
syllogism will proceed from some items of this sort; for [iv] everything holds either in this way or incidentally, and [v] what is incidental is not necessary. (74b5–12, my translation)

I take “pragmata” at step [ii] in 74b7 as explananda qua explananda: a given predication to be explained in its most characteristic feature. Furthermore, I take the expression “huparchonta + dative” (which Barnes has felicitously translated as “hold of”, avoiding the “belongs to” terminology usually related to predications) in the special and stronger sense of “what is characteristically true of” or “what is an important characteristic of”. Since I take “kath’ hauto” with “pragmata”, the result is that the whole expression “kath’ hauto huparchonta tois pragmasin” picks out the characteristics that are the most important to the explananda as such.45 Thus, Aristotle is saying that the principles of demonstration (the ones called “necessary principles” at step [i] and referred to by the adjective “necessary” in step [iii]) are those terms that capture the most important characteristics of their explananda as such. Therefore, the adjective “anankaia” (“the necessary items”) at step [ii] and the expression “necessary principles” in step [i] pick out the principles that are “necessary” in the sense of being “the necessary ones” for the fully appropriate explanation, which must be in terms of what the explanandum is in itself or as such. These necessary principles are opposed to the sumbebekota in steps [iv]–[v]. If “sumbebekota” were taken in the sense of contingent predicates, Aristotle’s point would be a very weak and unmotivated one: you must not select contingent predicates as explanatory factors. True, but why would Aristotle have taken this option seriously as something deserving discussion and special attention? In my view, the term “sumbebekota” in steps [iv]–[v] introduces the notion of a middle term that (despite being necessarily true of or even essential to its subject) is not the appropriate explanatory factor that makes its explanandum what it is. The definiens account of isosceles triangle used as middle term in Syllogism 1 (from T11) counts as an example of what Aristotle has in mind when he uses “sumbebekos” in T13. Even being definiens account of its subject, it only “comes together with” 2R without making 2R what it is – without being even a sine qua non condition for 2R to obtain.

A good piece of evidence in favour of my proposal comes from a passage a few lines below, where Aristotle says that “necessary items” are enough for turning a sound syllogism into a demonstration. The text reads as follows:

T14: Therefore, the syllogism must proceed from necessary [items], for from true [items] you can deduce without demonstrating, but from necessary [items] you cannot deduce without demonstrating – this is precisely the mark of demonstration. (74b15–18, Barnes’s translation slightly modified)

If the “necessary items” are taken in the sense of necessarily true premises, Aristotle would be committing himself to a deplorable philosophical thesis, which, besides other things,

“tois pragmasin” can be taken as doing double duty: “necessary for explaining the explananda is what holds of the explananda as such”. Third, I have skipped the embarrassing parenthesis at 74b7–10, which can be taken as strong evidence against my view, since Aristotle seems to be talking about per se predicates, but not about my third use of “kath’ hauto”. My response is that I do not take the parenthetical remark to be giving a paraphrase or elucidation of “kath’ hauto huparchonta” in the previous sentence, but as an additional comment about the sorts of per se predicates that will be used to attain a necessary principle.

45 I suggest that “alethos huparchonta tois pragmasin” in Prior Analytics 46a25 can be taken in a similar way.
will be incompatible with several requirements for scientific demonstrations. As Barnes (1993, 126) has remarked, Aristotle seems to be committed to the false proposition that “if P is inferred from II, and II is necessary, then P is demonstrated” (see also Mignucci 2007, 171). Now, Barnes is absolutely right in calling this a false proposition. But my interpretation does not attribute this false proposition to Aristotle. In my view, taking “necessary items” in the sense of essence-based necessarily true premises – i. e., premises whose necessary truth is based on the essence of their subjects – does not help. For Syllogism I above is a counterexample. Its minor premise is a full definition: the middle term is the whole definiens of its subject. It might be argued that the major premise is also some sort of per se predication. Are those facts about each premise enough to turn Syllogism I into a demonstration? I have argued that they are not: what makes a syllogism a scientific demonstration is that it presents the most appropriate cause as middle term, and what makes a middle term the most appropriate cause is that it captures the exact feature that makes the explanandum what it is. Therefore, the “necessary items” should be taken in the sense of “necessary (or required) for the most appropriate explanation”, for, otherwise, they would not be enough for delivering a scientific demonstration.

5. Conclusion

There is another text from APo I.6 which makes much more sense on my interpretation. The passage reads as follows:

T15: From these considerations it is plain too that those people are naïve who think that they assume their principles correctly if the propositions are true and reputable (e. g., the sophists who assume that to understand is to possess understanding). For it is not what is reputable that we count as a principle, but rather what is primary for the thing with which the proof is concerned – and not every truth is appropriate. (74b21–27, Barnes’s translation modified)

This passage is not a “curious aside” with no clear connexion with its context (pace Barnes 1993, 126; Mignucci 2007, 171), but a remark that fits perfectly with what was previously said. Let me set aside the question of what “reputable” means in this context (since that would lead me too far afield) and concentrate on the opposition between truth (including necessary truth) and appropriateness. It is clear from the context that Aristotle is talking about explanatory appropriateness. Although he explicitly talks about true propositions and does not mention necessity any more, the sentence “to understand is to possess understanding” can fairly be taken not only as true but as necessarily true.

46 The deplorable thesis clashes with (at least) the following requirements: that premises must express per se predications (73b16–18, 75a29–31); that principles should be suggenes with their explananda (75b3–12, 76a8–9, 29–30), that scientific explanation should state the principles of the explanandum as such (75b36–40, 76a4–7), i. e., that scientific explanation requires appropriate principles (71b22–23, 72a5–6, 74b25–26) etc.

47 See my 2014 for close discussion of this issue.

48 Just one point: I do not see why “endoxon” should be related only to the truth-value of a predicative sentence. I suggest that “endoxon” can also be taken to be related to the appropriateness of an explanation. Thus, it might be said to be “reputable” or “fair” to explain a good harvest by the amount of rain in a season, and it might be said to be “adoxon” to explain your fatness by your moral character. Thus, it is true that there was a good harvest in this season, as well as it is true that there was a lot of rain in this season, but “endoxon” has its focus on the explanatory power of the latter sentence as an explanation of the former.
Aristotle has chosen it as an example because its trifling nature helps him to stress that the most important feature of the premises on which scientific demonstration depends is the notion of explanatory appropriateness, not the notion of necessary truth.

**T15** is also important because it brings the sophists back into the story again. This leads us back to step [i] of **T1**, in which incidental knowledge, as the foil of scientific knowledge, is associated with a sophistical mode of knowing. I will finish with a very brief characterisation of what the sophistical mode of knowing might be.⁴⁹

From Aristotle’s depiction of sophistical arguments in *Topics* and *Sophistical Refutations*, one might expect that the sophistical mode of knowing would have something to do with fallacious arguments or with valid arguments that smuggle into the inference some premise that only appears to be true without being so. I submit that Aristotle is not appealing to any of these features of sophistical arguments when he mentions the sophistical mode of knowing in step [i] of **T1**. What defines an argument as a sophism is after all its *purpose*: the sophist uses an argument with the purpose of producing a false semblance of wisdom.⁵⁰ Now, someone not trained in logic can produce a fallacy without being a sophist: many of Socrates’s logically naïve interlocutors in Plato’s dialogues do not turn into sophists just by advancing logical fallacies. Similarly, a scientist who has not investigated a subject very closely might accept as true a false proposition, but this does not turn him into a sophist. Similarly, in any given science, an attempted explanation might be like Syllogism 1 above without thereby being a sophism. It might be that, at a given stage in a careful investigation, the most appropriate explanatory factor has not yet been discovered and an attempted demonstration such as Syllogism 1 is the best that can be done on the available evidence. There is nothing wrong with attempted demonstrations such as Syllogism 1 *if they are taken as such*, namely, as the best that can be done on the available evidence. Syllogism 1 is not only valid, but also sound, and its propositions are necessarily (or even definitionally) true. It also gives some explanation for its explanandum, although not the most appropriate explanation that characterises scientific knowledge. However, Syllogism 1 might be used by someone in order to produce a semblance of scientific knowledge: in order to “(appear to) refute the scientists” (cf. 168b6–7; 169b28–29), in order to produce a false semblance of demonstration (171b29). This is what the sophist does. And precaution against such a procedure is what is involved in Aristotle’s remark at step [i] of **T1**. Thus, the sophistical way of knowing is called sophistical only because it purports to produce a false semblance of appropriate explanation, and it produces this false semblance because it takes a middle term that is just a *sumbebekos* (in the sense I have proposed) but not an appropriate cause.⁵¹

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⁴⁹ The sophistical mode of knowing is mentioned again in *APo* I.5, 74b28–9. I have avoided discussion of this passage because the chapter is very complex and would require a careful discussion of *per se* and *katholou* predications, for which there is no room in this paper. For a careful examination of *APo* I.5, see Hasper 2006, especially 270–273, on 74b28–9; McKirahan 1992, 171–176.


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