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What's Wrong With Logic?

Introduction

The truth functional account of conditional statements 'if A then B' is not only inadequate; it eliminates the very conditionality expressed by 'if'. Focusing only on the truth-values of the statements 'A' and 'B' and different combinations of these, one is bound to miss out on the conditional relation expressed between them. *All* approaches that treat conditionals as functions of their antecedents and consequents will end up in some sort of logical atomism where causal matters simply are reduced to the joint occurrence of A and B.

The material conditional

	A	**B**	**A ⊃ B**
(TTT)	T	T	T
(TFF)	T	F	F
(FTT)	F	T	T
(FFT)	F	F	T

Some similarities between '⊃' and 'if'

Contraposition	*Modus ponens*	*Modus tollens*
If A then B ⇔	If A then B	If A then B
If not-B then not-A	A	not-B
	∴ B	∴ not-A

Some dissimilarities between '⊃' and 'if'

A	¬ A	¬ A
B	¬ B	B
∴ A ⊃ B	∴ A ⊃ B	∴ A ⊃ B

The truth-value of 'if A then B' cannot possibly be fully determined by the truth-values of 'A' and 'B':

(TTT) If Socrates is a man, then he is mortal.
(TTF) If Socrates is mortal, then he is a man.
(FTT) If Socrates is a cat, then he is mortal.
(FTF) If Socrates is a stone, then he is mortal.
(FFT) If Socrates is a stone, then he is inanimate.
(FFF) If Socrates is inanimate, then he is a stone.

The inadequacy of '⊃'

'A ⊃ B' is true for *more* possible combinations of truth-values than 'if A then B'.

'¬(A ⊃ B)' is true of *fewer* possible combinations of truth-values than 'not (if A then B)'.

Standard reply: There might be a tiny divergence between '⊃' and 'if', but nothing that we can't handle. '⊃' is still useful for representing 'if' and the closest we get.

True of more possible circumstances	*True of fewer possible circumstances*
A ⊃ B	¬(A ⊃ B)
(A ⊃ B) ∨ C	¬[(A ⊃ B) ∨ C]
¬[(A ⊃ B) ⊃ C]	¬(A ⊃ B) ∨ C
¬(A ⊃ B) ⊃ C	(A ⊃ B) ⊃ C
C ⊃ (A ⊃ B)	C ⊃ ¬(A ⊃ B)
¬[C ⊃ ¬(A ⊃ B)]	¬(A ⊃ B) & ¬C

Consequences for validity

P1 If God doesn't exist, then it is not the case that if I pray, my prayers are heard
P2 I don't pray
C God exists

1.	¬A ⊃ ¬(B ⊃ C)	P1
2.	¬B	P2
3.	B ⊃ C	1, 2, T
4.	¬¬(B ⊃ C)	3, T
5.	¬¬A	1, 4, T
6.	A	5, T

Some sophisticated alternatives to '⊃'

<u>*Alternative 1: Modal strengthening - necessity*</u>

1. $\Box B \Rightarrow \Box(A \rightarrow B)$
2. $\neg\Diamond A \Rightarrow \Box(A \rightarrow B)$
3. $\Box(A\&B) \Rightarrow \Box(A \rightarrow B)$
4. $\Box(\neg A\&\neg B) \Rightarrow \Box(A \rightarrow B)$

<u>*Alternative 2: Modal weakening - probability*</u>

Adams' Thesis:
Pr (If A then B) = Pr(B/A) = Pr(A&B)/Pr(A)

1. Pr(B) = 1 ⇒ Pr(B/A) = 1
2. (P(A&B)=P(A)·P(B)) ⇒ Pr(B/A) = Pr(B)
3. (P(A&B) = ~1) ⇒ ((P(A|B) = ~1) & P(A|B)) = ~1)

<u>*Alternative 3: Modal realism – possible worlds*</u>

Lewis on counterfactuals:

1. 'A > B' is true (in the actual world) if in the closest possible world where 'A' is true, 'B' is true as well.
2. If A is true in no worlds, then 'A > B' is true.
3. If B is true in all worlds, then 'A > B' is true.
4. If both A and B are true in all worlds, then both 'A > B' and 'B > A' are true.

Within this system we have to buy into the possible worlds *in addition to* '⊃'. This is because the material conditional holds as a model for conditionals within all possible worlds. This means that in a possible or actual world, all true statements will form true conditionals.

Why a separate logic of counterfactuals?

1. (A & B) $\Rightarrow$ ((A $\supset$ B) & (B $\supset$ A))
2. ($\neg$A & $\neg$B) $\Rightarrow$ ((A $\supset$ B) & (B $\supset$ A))
3. B $\Rightarrow$ ((A $\supset$ B) & ($\neg$A $\supset$ B))
4. $\neg$A $\Rightarrow$ ((A $\supset$ B) & (A $\supset$ $\neg$B)

To propose a separate theory for case (4) is a solution to a problem that we should not accept unless we believed that the material conditional got everything else right with respect to the logic of conditionals.

Conditionals are not factual or counterfactual: they are *hypothetical*. The same conditional relation is expressed in all the following expressions:

a. If I drink a whole bottle of whisky, I'll get ill.
b. If I were to drink a whole bottle of whisky, I'd get ill.
c. If I had drunk a whole bottle of whisky, I would have gotten ill.

The truth conditions are the same in (a), (b) and (c), and won't change just because I decide not to drink any whisky.

To treat counterfactuals as special cases misses the point that the truth or falsity of 'A' or 'B' as such cannot determine whether or not there is a conditional relation between them.

Are we all Humeans and logical atomists?

If any of these logical systems were adequate representations of conditionals, then we should all just convert (if we haven't already) to a Humean metaphysics.

> The propositional calculus is well designed to serve as the logic of a language addressed to a world of which all one had to say was that certain events have or have not occurred or will or will not occur. And this arrangement would be acceptable if we really thought it the case that the world is a collection of discrete or unrelated events, the occurrence or nonoccurrence of which can be expressed by simple assertions and their denials. Because if reality consists wholly of elementary facts that can, in appropriate circumstances, be determined by observation to obtain or not, and if the sentence letters of the calculus are interpreted by statements or propositions that express these facts, then if the facts are known, all the truths about the world that are capable of rational representation can be embraced in a long conjunction of simple or atomic propositions and their negations… (Robert N. McLaughlin 1990: 2)

Assuming that our interest in the world is primarily an interest in particular occurrences or non-occurrences, so that our interest in relations between A and B is reduced to a question about whether or not we observe A and not-B, most of our conditionals would be useless and senseless.

1. If a body is not subject to any net external force, it will continue in a uniform movement or stay at rest.
2. If a body is not subject to any net external force, it will jump up and down until it turns into a green cat.

These conditionals are both true given that their antecedents are false. Of course, we could go to the closest possible world and just check if also the consequents are true (which is probably what Newton should have done).

Naïve verificationism

Any conditional claim that cannot be directly tested must be true, since it then cannot be proved false. All hypotheses are either verified (TT) or falsified (TF).

A consequence is that the truth of all causal conditionals of the form 'if C then E' comes down to the joint truth of 'C' and 'E'. (We could also infer from this that 'If E then C'.)

Not even the craziest philosopher of science would believe any of this, so how can all logicians?

In all the above mentioned logical systems 'If A then B' is taken as true, probable, necessary or whatever, whenever both the 'A' and 'B' are true, probable, necessary or whatever.

Principles to be rejected for all logical systems

The functionality principle: A conditional's truth, probability, assertability or modality is calculable from the truth, probability, assertability or modality of its antecedent and consequent.

The principle that anything follows from a contradiction: (A&$\neg$A) $\Rightarrow$ B is only valid if we accept the following inference:

$$(A \Rightarrow (A \vee B)) \Rightarrow (\neg A \supset B)$$

Why 'if'?

Why bother about the logic of 'if'? Well, first of all because so much is at stake if we *fail* to grasp the logic of conditionals. Without an adequate understanding of 'if', we cannot account for some of the most basic matters in life, such as causation, dispositions and laws.

Unless we know what it means that something *might* happen, or that something *would prevent* or *trigger* something else to happen, or that something happens *because* of something else; how would we even be able to hope, fear, expect or regret anything?

A language without conditionals cannot be the language of a world where we make predictions, choices, calculations or even explanations.

A language without conditionals would be the language of a world that is nothing but a collection of unrelated particulars (events, facts, properties, or whatever). But this is not the world as we know it. Our world is all about causal relations between such particulars, whether the particulars themselves are taken to be actual, potential or purely hypothetical.

In our world we need 'ifs', and we need them badly. If successful, a logic of conditionals can help us understand matters like causation, dispositions and laws. If failed, it can dissolve the very conception of conditionality.