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Bolzano's Theory of Science

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Note on Translation

There are two partial translations of Bolzano's *Wissenschaftslehre* (hereafter *WL*), the 1972 translation by Rolf George and the 1973 translation by Jan Berg. This thesis relies on the George translation, as being more readable, when the relevant section has been translated by George. When the relevant section has only been translated by Berg, the Berg translation is used. Each reference below identifies whether the George or Berg translation is being used. All italics in translations are those of Bolzano, who frequently used italics.

Introduction

Most histories of analytic philosophy begin with Bolzano. These histories share a common account of the reasons why analytic philosophy found its genesis in Austria, an account that can be summarized as the vacuum thesis. Specifically, according to Barry Smith, 19th century Austria lacked "any entrenched national philosophy of the Kantian or Hegelian sort".[1] Furthermore, according to Nyíri, the development of empirical science itself was stifled by the lack of funds to support serious scientific research, creating "a vacuum which the theory of a practice so attractively pursued elsewhere could then fill".[2] All such theories are ultimately derivative of the Neurath-Haller Thesis, that Austrian philosophy began a new tradition precisely because it "saved itself from Kantian interlude". In this picture of Bolzano's freedom from philosophical influences, "Bolzano stands as a lonely and atypical figure in his time".[3]

This thesis presents a Bolzano whose genius resulted not from his freedom from powerful philosophical influences, but precisely from his response to the powerful philosophical influences of his day. Chief amongst these were the voices engaged in debate about the nature of science. It was precisely Bolzano's commanding synthesis of the views of those advocating progress through mathematical physics with the views of those advocating a classical model of a science as the study of causes that set the stage for the development of Austrian and analytic philosophy in subsequent centuries. This thesis presents the synthesis achieved by Bolzano in five chapters.

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[1] Barry Smith, *Austrian Philosophy: The Legacy of Franz Brentano* (Chicago: Open Court Publishing, 1996), 12.
[2] J.C. Nyíri, "The Austrian Element in the Philosophy of Science", in *From Bolzano to Wittgenstein: The Tradition of Austrian Philosophy*, ed. J.C. Nyíri (Vienna: Hölder-Pichlet-Tempsky, 1986), 143.
[3] Jan Sebestik, "Bolzano, Exner and the Origins of Analytical Philosophy", in *Bolzano and Analytic Philosophy*, ed. Wolfgang Künne, Mark Siebel and Mark Textor (Amsterdam: Rodopi, 1997), 34.

First, Bolzano's concern to reintroduce a classical model of science is presented. Bolzano's purpose, it is clear from these early works as well as his sermons, was enlightenment of the people through scientific proofs that don't just prove with certainty but that *explain*, and thus enable people to identify what they need to learn in their respective roles in society and to deduce new truths on their own. As an advocate of the tremendous discoveries in all sciences being made in his time, Bolzano devoted his efforts not to directly furthering the work of science as a scientist himself, but to a philosophic reflection on scientific knowledge intended to organize its presentation for the good of the people at large. As will be seen, though, this was not a completely novel argument for Bolzano, who was engaging in a longstanding debate amongst Catholic academics on the status of mathematical science with respect to the classical model of science. This largely Catholic debate thus paved the way for the onset of analytic philosophy via the work of Bolzano.

The specific steps taken by Bolzano to present modern scientific achievements in classically organized treatises that explain these achievements for society at large is the subject of the second chapter of this thesis. This chapter presents Bolzano's model for the exposition of an *a priori* science in detail. Bolzano's model stresses objectivity in an axiomatic structure of axioms and ground-consequence relations, reflecting the first principles and syllogisms of a classically presented science. The objectivity of these ground-consequence relations required, as well, a reorganization of the contemporary division of the sciences along more classical lines, which Bolzano provided in 1810.

Bolzano contrasts his axiomatic, objective proofs with subjective proofs within a science that establish certainty but that do not explain. This contrast anticipated later critiques of psychologism by Husserl and Frege. Where do these subjective, psychologistic proofs come from? Bolzano's discussion of these subjective proofs, including their origins, is the subject of the third chapter. Subjective proofs establish their conclusions with certainty because they rely upon complex, intuitive propositions at the outset of their proofs, while proofs that explain must always begin with simple axioms. Bolzano's specific critiques of psychologistic proofs are aimed in two directions: proofs in mathematical sciences (e.g. geometry, analysis) that rely on concepts from more specific (thus more intuitive) mathematical sciences and proofs that analyze mental concepts based on the early modern model of *a priori* truths as analytic.

The fourth chapter presents Bolzano's method for constructing an axiomatic science from the subjective proofs given for that science. Bolzano presents a method for analyzing the subjective proofs which establish the obviousness of their conclusion in order to reorganize these proofs along the axiomatic model of an objective, *a priori* science. Having redefined, along classical lines, the division of the sciences, Bolzano intensely analyzed contemporary mathematical proofs to uncover any implicit reference within the proof to sciences foreign to that of the proof's conclusion. This intense analysis of scientific proofs led to the analytic step of uncovering the semantics of statements that would be a hallmark of analytic philosophy, but the purpose of this linguistic analysis was always, for Bolzano, the rephrasing of a proof along the classical model of objective causal steps within a single science.

Each step of Bolzano's project can, up to this point, be paired with a parallel step in Aristotelian logic and philosophy of science. (While it is clear that Bolzano was very aware of the many parallels with Aristotle's logic, it is less clear with which specific steps Bolzano felt an Aristotelian debt.) However, what distinguishes Bolzano's project from that of

Aristotle is his assumption that *a priori* science studies beings as possible and thus as thinkable, rather than as actual and existing. The influence of this possibilism, which seems to have been a given for all European philosophers after the 17th century, is the subject of the fifth and final chapter of this thesis. In his later work, primarily his *Theory of Science*, Bolzano introduced an entirely non-Aristotelian analytic step known as variation which was a "possible worlds" analysis of propositions. These deviations from classical analysis ultimately led to the formalism and analysis of language that came to characterize future philosophy in Austria and beyond, as is evident in the work of Carnap, Tarski and Wittgenstein, as examples. However, recent analytic philosophers such as Plantinga, Chisholm and Zimmerman have explicitly reacted against possibilism, anchoring their philosophy of science and metaphysics in doctrines known as actualism and presentism.

I. Bolzano's Revival of the Classical Model of Science

Clearly the most consistent theme throughout the works of Bolzano, from his first publication in 1804 to his magisterial *Theory of Science* (TS) in 1837, is the distinction between proofs that prove and proofs that explain.[4] While scientific and mathematical proofs in Bolzano's time proved scientific conclusions against standards of certainty and obviousness, Bolzano called for a new presentation of these conclusions that explained their grounds from a more objective point of view. Throughout, Bolzano mentions the classical model of science, with primary reference to Aristotle, as a key source for the types of scientific presentation needed in his day. (While Bolzano's subject matter is mathematics, he makes it clear that the scope of this subject extends to all of the natural sciences given their mathematical treatment since the 17th century, as is discussed in the second chapter below.)

The classical heritage of Bolzano's theory of science is denied by most commentators, despite the reception by Bolzano's contemporaries such as Menelaos who remarked of TS, "throughout, the author assumes the old, strictly objective or dogmatic, viewpoint, in contrast to the contemporary, which is based on the psychological self-consciousness of the thinking mind".[5] This chapter attempts to articulate this classical heritage based on Bolzano's own view of his work as well as the debate concerning the classical model of science in which he participated.

Bolzano and the Classical Model of Science

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[4] Paolo Mancosu's studies of mathematical explanation across the history of mathematics has done much to reveal the close relation between Aristotle and Bolzano in their discussions of explanatory, causal proofs. This is fundamentally because Mancosu "take[s] seriously Bolzano's assertion that the distinction between certain and grounding proofs is the same as that between *hoti* and *dioti* proofs in Aristotle." Paolo Mancosu, "Bolzano and Cournot on Mathematical Explanation," *Revue d'Histoire des Sciences* 52 (1999): 429-455. See Mancosu's *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* (New York, NY: Oxford University Press, 1996).

[5] Quoted in *WL*, p. xxix, George translation.

Bolzano opens the preface to his 1804 work, *Considerations on Some Objects of Elementary Geometry*, with a statement of the two rules that would govern his presentation. We find in these two rules the first entree into modern philosophy of the concern about psychologism as well as the response, an objective science of necessary connections grounded in axioms. "*Firstly*, I propose for myself the rule that the *obviousness of a proposition* does not free me from the obligation to continue searching for a proof of it…*Secondly*, I must point out that I believed I could never be satisfied with a completely strict proof *if it were not derived from the same concepts* which the thesis to be proved contained, but rather made use of some fortuitous, alien, *intermediate concept*, which is always an erroneous *metabasis eis allo genos* [crossing to another kind]".[6]

Bolzano's first rule is found in all of Bolzano's works including TS, which distinguishes "between proofs of the mere *hoti* (establishing certainty) and of the *dioti* (grounding)"[7]. This rule is clearly not intended to be a criticism of mathematical science which aims at certainty, but a concern that modern mathematical science is incomplete. Proofs that establish certainty call for a subsequent formulation that reveals their objective grounds. This becomes clear in Bolzano's development of his first rule, which contains a pregnant rebuke to Kant.

> At one time something might have seemed superfluous, as when Thales (or whoever discovered the first geometric proofs) took much trouble to prove that the angles at the base of an isosceles triangle are equal, for this is obvious to common sense. But Thales did not doubt that it was so, he only wanted to know why the mind makes this necessary judgment. And notice, by drawing out the elements of a hidden argument and making us clearly aware of them, he thereby obtained the key to new truths which

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[6] Bernard Bolzano, "Considerations on Some Objects of Elementary Geometry," in *The Mathematical Works of Bernard Bolzano*, ed. Steve Russ (Oxford: Oxford University Press, 2004), 31-32.
[7] Quoted in Paolo Mancosu, "On Mathematical Explanation," in *The Growth of Mathematical Knowledge*, ed. Emily Grosholz and Herbert Breger (Dordrecht, The Netherlands: Kluwer Academic Publishers, 2000), 115.

were not so clear to common sense.[8]

Kant argues in the preface to *The Critique of Pure Reason* that mathematics had been placed on the royal road of *a priori* science once "[a] new light flashed upon the mind of the first man (be he Thales or some other) who demonstrated the properties of the isosceles triangle", namely that "[i]f he is to know anything with *a priori* certainty he must not ascribe to the figure anything save what necessarily follows from what he has himself set into it in accordance with his concept".[9] According to Bolzano's first rule, however, the criterion of an *a priori* science is not certainty, but ideas that "are everywhere clear, correct and connected in the most perfect order".[10]

This objectivity criterion of any *a priori* science leads to Bolzano's second rule that proofs not cross to another kind. Bolzano's use of the Greek, in reference to Aristotle's requirement that "[o]ne cannot, therefore, prove by crossing from another kind – e.g. something geometrical by arithmetic"[11], is repeated in his subsequent works on mathematics in 1810 and 1817.[12] In other words, objective causal connections can only be made between axioms and propositions within the same science. Bolzano is not opposed to proofs that establish certainty by crossing from one science to another. As is seen in the next chapter, in fact,

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[8] Bolzano, "Considerations on Some Objects of Elementary Geometry," 31.
[9] Immanuel Kant, *Critique of Pure Reason*, trans. Norman Kemp Smith, (Boston/New York: MacMillan, 1965), B xii, 19.
[10] Bolzano, "Considerations on Some Objects of Elementary Geometry," 31.
[11] Aristotle, *Posterior Analytics*, 75a38. trans. Jonathan Barnes, in *The Complete Works of Aristotle Vol 1*, ed. Jonathan Barnes (Princeton: Princeton University Press, 1984).
[12] See Bernard Bolzano, "Contributions to a Better-Grounded Presentation of Mathematics," in *The Mathematical Works of Bernard Bolzano*, ed. Steve Russ (Oxford: Oxford University Press, 2004), 126 and Bernard Bolzano, "Purely Analytic Proof of the Theorem, that between any two Values, which give Results of Opposite Sign, there lies at least one real Root of the Equation," in *The Mathematical Works of Bernard Bolzano*, ed. Steve Russ (Oxford: Oxford University Press, 2004), 254. Russ points out that this is not an exact translation, as "Bolzano has έις (to), where the text in Barnes has εξ (from)" in Bolzano, "Considerations on Some Objects of Elementary Geometry," 32.

crossing from one science to another in a proof is characteristic of proofs that establish a fact with certainty.

The first complete statement of Bolzano's intention to reintroduce a classical model of science in order to complete the more psychologistic expositions in contemporary science is found in his 1810 *Contributions to a Better-Grounded Presentation of Mathematics.*[13] Bolzano would make the same type of statement in each of his subsequent works, including TS, in which he makes the further point that just as modern science often overlooks the importance of objective demonstrations, so classical science often overlooked the importance of preliminary proofs that establish certainty.

> Since one so far does not always distinguish clearly the objective ground of a truth from its subjective means of knowledge so it follows automatically that also the grounding proofs cannot always be distinguished exactly from the purely certain proofs. Indeed Aristotle (An. Post. I, 2 and I, 13) and the Scholastics very diligently advanced the division of proofs into those which only show *that* (hoti) something is, and the ones which also show *why* (dioti) something is. They also maintained, with some exaggeration, that only the latter produce a genuine science. However, the new logicians seem to observe this distinction very little.[14]

The claim that classical science exaggerates in asserting that only objectively grounded demonstrations constitute a science is presumably based on statements such as that by

[13] "But this much seems to me certain: in the realm of truth, i.e. in the collection of all true judgments, a certain *objective connection* prevails which is independent of our accidental and *subjective recognition* of it. As a consequence of this some of these judgments are the grounds of others and the latter are the consequences of the former. Presenting this objective connection of judgments, i.e. choosing a set of judgments and placing them one after another so that a consequence is represented as such and conversely, seems to me the be the real *purpose* to pursue in a scientific exposition. Instead of this, the purpose of a scientific exposition is *usually* imagined to be the greatest possible *certainty* and *strength of conviction.* It therefore happens that the obligation to prove propositions which, in themselves, are already completely certain, is discounted. This is a procedure which, where we are concerned with the practical purpose of certainty, is quite correct and praiseworthy; but it cannot possibly be tolerated in a scientific exposition because it contradicts its essential aim. However, I believe that *Euclid* and his predecessors were in agreement with me and they did not regard the mere *increase in certainty* as any part of the purpose of their method. This can be seen clearly enough from the trouble which these men took to provide many a proposition (which in itself had complete certainty) with a proper *proof,* although it did not thereby become any more certain." Bolzano, "Contributions to a Better-Grounded Presentation of Mathematics," 103.

[14] Quoted in Mancosu, "On Mathematical Explanation," 104.

Aristotle, who supposed that we "possess unqualified scientific knowledge of a thing, as opposed to knowing it in the accidental way in which the sophist knows, when we think that we know the cause on which the fact depends as the cause of the fact and of no other, and further, that the fact could not be other than it is."[15] Whether Bolzano's critique is balanced or not is not at issue in this thesis (that Bolzano's path to demonstrative science by analyzing subjective proofs parallels a very similar path in Aristotle's *Topics* speaks against Bolzano's critique). The points made here are that Bolzano sought to reintroduce the classical model of science by completing, but not replacing, contemporary scientific proofs with grounding proofs that demonstrate and explain modern scientific achievements.[16]

Bolzano's Classicism and the Catholic Debate on the Status of Mathematical Physics

The predominant histories of analytic philosophy do not locate its origin in an attempted revival of the classical model of science. While the texts of Bolzano clearly support this claim, it is buttressed once one considers Bolzano's motivations. What motivated an Austrian priest who occupied a chair in the Science of Religion in early 19th century Prague to look to classical philosophy in order to make such innovative and influential arguments in the philosophy of science and mathematics?[17] This question must be answered on two levels.

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[15] Aristotle, *Posterior Analytics*, 71b9-11.

[16] As Bolzano would say later in TS that "it is the case that intuitive truths (experiences) are often helpful in the discovery of a purely conceptual truth; but the objective ground of such a truth cannot lie in them; if they have any ground at all, it must lie in other conceptual truths." WL, Sec. 221, p. 284, George translation. .

[17] The importance of this question is stressed by Petr Dvořák and Jacob Schmutz in their article on the most prominent philosophical predecessor to Bolzano, Juan Caramuel Lobkowitz (1606-1682). "Recent historians of the philosophical tradition of *Mitteleuropa* have often considered the work of the Prague logician Bernard Bolzano (1781–1848) as its starting point. His work is at the origin of the various roads taken by authors such as Gottlob Frege, Franz Brentano, Edmund Husserl or Alexius Meinong up to the Vienna Circle, and it can be considered as the common root of the two major trends of contemporary philosophy, the so-called 'continental' tradition, mainly inspired by phenomenology, and the Anglo-American 'analytical' tradition. But this obliterates the fact that Bolzano himself, a Roman Catholic priest schooled in the very late scholastic tradition of the *Katholische Aufklärung*, was himself deeply indebted to numerous early-modern sources, and that most of the problems he is often acknowledged to have introduced into philosophy – negative state of affairs, the distinctions between the intensional and extensional analyses of concepts, the paradoxes of the infinite, and

First, Bolzano's ideas were not completely original in his era, for a debate concerning the status of mathematical science as a classical science had raged throughout Europe, particularly in Catholic areas such as Prague, for the past couple centuries. Second, Bolzano reveals in his books and sermons his reasons for choosing to participate in these debates.

As philosophers and scientists across Catholic Europe came to terms with the Scientific Revolution, a debate emerged that came to be known as the "Quaestio de Certitudine Mathematicarum".[18] The question debated by philosophers and scientists, primarily in Italy, Germany and Austria, was whether mathematics can supply explanatory causes, along the Aristotelian model, of material phenomena. Bolzano's position is that mathematics can supply, but in practice has not supplied, explanatory causes. While Bolzano only occasionally refers to these recent predecessors, preferring to refer directly to Aristotle and Euclid, their arguments are so similar that Bolzano's own support of the classical model actually appears conventional for his time. The conclusion drawn here is that it was precisely this debate on the status of mathematical science in Catholic Europe that paved the way for the work of Bolzano and subsequent analytic philosophers.[19]

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above all his attempt to vindicate realism against Kantian subjectivism – have all their sources in the tradition of Caramuel's Prague." Petr Dvořák and Jacob Schmutz, "Caramuel in Prague: The Intellectual Roots of *Mitteleuropa*," in *Juan Caramuel Lobkowitz: The Last Scholastic Polymath*, ed. Petr Dvořák and Jacob Schmutz (Prague: Filosofia, 2008), 26.

[18] Mancosu, "On Mathematical Explanation", 110.

[19] Neurath acknowledges that analytic thought took root in precisely Catholic parts of Europe and attributes this to the independence of logic from dogmatic metaphysics in Catholic philosophy, which enabled a transition to logical analysis of science. "Catholics accept a compact body of dogma and place it at the beginning of their reflections, [thus] they are sometimes able to devote themselves to systematic logical analysis, unburdened by any metaphysical details....Once someone in the Catholic camp begins to have doubts about a dogma, he can free himself with particular ease from the whole set of dogmas and is then left a very effective logical instrument in his possession. No so in the Lutheran camp, where...many philosophers and scholars from all disciplines, while avoiding a commitment to a clear body of dogma, have retained half-metaphysical or quarter-metaphysical turns of speech, the last remnants of a theology which has not yet been completely superseded....This may explain why the linguistic analysis of unified science prevailed least in countries where the Lutheran faith had dealt the hardest blows to the Catholic church, despite the fact that technology and the sciences that go along with it are highly developed in these countries." Quoted in Barry Smith, "Austria and the

Historically, the mathematical sciences central to the Scientific Revolution had been considered mathematical inasmuch as they explained immaterial, and thus immobile and quantitative, aspects of these domains. This approach to these mathematical sciences followed the three-fold division of the theoretical sciences presented by Aristotle, Boethius and Aquinas, according to which natural philosophy treats of what exists in matter, mathematics treats of what exists in matter but doesn't require matter to be understood and metaphysics treats of what exists without matter and motion.[20] Astronomy, optics and mechanics were considered middle sciences because they can be treated either at one level of abstraction in terms of principles of matter and motion (as natural philosophy), or at a further level of abstraction in terms of principles that don't include matter or motion (as mathematics).[21] While debates occurred as to whether these sciences were more properly considered natural philosophy or mathematics, the distinction between natural philosophy and mathematics was not in doubt (in fact, it was agreement concerning this distinction that sustained the debate over the proper placement of the middle sciences).

The momentous change that is primarily responsible for bringing about the Scientific Revolution, as historians of science such as Edward Grant have made clear, was the merging of the middle sciences into natural philosophy. This integration meant that mathematics was no longer restricted to seeking immaterial and immobile first principles, but was also applied

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Rise of Scientific Philosophy," in *Phenomenology and Analysis: Essays on Central European Philosophy*, ed. Arkadiusz Chrudzimski and Wolfgang Huemer (Frankfurt, Germany: Alexander von Humboldt Foundation, 2004), 47.

[20] This formulations of the threefold division of the theoretical sciences closely follows that described by Aquinas in his *The Division and Methods of the Sciences: Questions V and VII of his Commentary on the De Trinitate of Boetheus*, ed. Armand Maurer (Toronto: Pontifical Institute of Medieval Studies, 1986), 14-15.

[21] "For mathematics is about forms, for its objects are not said of any underlying subject – for even if geometrical objects are said of some underlying subject, still it is not *as* being said of an underlying subject that they are studied." Aristotle, *Posterior Analytics*, I 13, 79a8-10.

to what exists in virtue of material causes.[22] The result of this integration was innumerable scientific projects designed to reveal the causes of natural phenomena.

However, this integration also prompted Allessandro Piccolomini to deny in an influential 1547 book that mathematics can supply explanatory causes of material phenomena.[23] Jesuit scholars argued on both sides of the ensuing debate, a highpoint of which occurred at the Collegio Romano in the late 16th century between philosopher Benito Pereyra, who defended Piccololini, and mathematician Christoph Clavius.[24] Clavius' advocacy extended to the development of the *Ratio studiorum* that was developed in 1599, and that would govern Jesuit education in Austria and elsewhere, but which in the end represented a compromise between the philosophers and the mathematicians reflecting the unresolved nature of the debate.[25]

It stipulated, for example, that the *De caelo* was to be taught in mathematics courses, except for the sections dealing with the elements and the heavens, which were to be taught in physics courses.[26] These stipulations were not always adhered to, and the tensions between those Jesuits who viewed mathematical physics as a science of causes in the classical model (Clavius, Biancani, Barrow, Wallis and Arriaga) and those who denied this was possible (Piccolomini, Pereyra, the Coimbrian commentators and Gassendi)[27] ultimately erupted in

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[22] As Grant explains while describing what was revolutionary in Newton's *Mathematical Principles of Natural Philosophy*, "To devote a treatise to ascertaining the mathematical principles of natural philosophy qualified as a virtual contradiction in terms. Why? Because natural philosophy in the medieval Aristotelian tradition did not – and could not – have mathematical principles." Edward Grant, *A History of Natural Philosophy* (Cambridge: Cambridge University Press, 2007), 313.

[23] Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*, 12.

[24] While Pereyra argued that mathematics, while admired for its certainty, was limited to the accidental form of quantity and thus had no access to the explanatory causes of existing things, Clavius viewed mathematical physics as in complete harmony with classical science.

[25] Marcus Hellyer, *Catholic Physics* (Notre Dame, Indiana: University of Notre Dame Press, 1996), 120-121.

[26] Ibid., 125.

[27] Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*, 110

censurae of certain mathematicians from Rome.[28] Barrow, for example, argues in 1683 that Pereyra "attempt[s] to prove that Mathematical Ratiocinations are not *Scientific*, *Causal* and *Perfect*, because the Science of a Thing signifies to know it by its Cause" when in fact "Mathematical Demonstrations are eminently *Causal*, from whence, because they only fetch their Conclusions from Axioms which exhibit the principal and most universal Affections of all quantities, and from Definitions which declare the constitutive Generations and essential Passions of particular Magnitudes."[29]

These tensions were far less present in non-Catholic Europe, where it was more common to dismiss the classical model of natural philosophy as an impure form of science.[30] However, it is the presence of this debate in Catholic Europe that laid the groundwork for a future synthesis of the classical model of science and mathematical science in Bolzano's work and subsequent analytic philosophy.

These tensions continued into the generation of Bolzano's teachers and were reflected in the untenable positions maintained by the General Councils of the Jesuit Order. The Sixteenth General Congregation in 1731 decreed there to be no opposition between Aristotelian philosophy and "the more attractive style of learning in physics…with which the more notable natural phenomena are explained and illustrated by mathematical principles", a

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[28] "The Society of Jesus' teaching enterprise developed at a time when the status of mathematics and its relationship with physics were undergoing thorough reevaluation….[T]he integration of mathematics into the Society's pedagogical program did not by any means follow a smooth path. The tensions between Jesuit mathematicians and philosophers can be seen in numerous questions along the porous border between mathematics and natural philosophy. For example, when mathematicians treated natural philosophical questions, a task they increasingly took upon themselves in the seventeenth century, they often did not adhere to the strictures of peripatetic philosophy. Consequently, they encountered problems with the *censurae* of the Roman revisers." Hellyer, *Catholic Physics*, 121.

[29] Quoted in Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*, 20-21.

[30] "We have yet to find a pure natural philosophy; so far it has been infected and corrupted: in Aristotle's school by logic." *The Oxford Francis Bacon*, vol. xi (Oxford: Oxford University Press, 2004), 153-155.

decree which was repeated by the Seventeenth General Congregation in 1751. These tensions ultimately erupted in the Suppression of the Jesuit Order in 1773, which was permitted by a monarchy eager to invest in applied science over natural philosophy for the further legitimization of its rule.

In Prague itself, the pre-Suppression period witnessed this tension in large measure, beginning with a proliferation of work in what has come to be known as Jesuit mathematics. The groundwork for this was laid by the students of Suárez who were sent to Prague in the 17th century, particularly Arriaga, who viewed mathematical physics as an Aristotelian science of causes and wrote that such doctrines "are completely accepted here at the University of Prague".[31] In fact, Prague is considered by some to be unmatched in all of Europe in the advances made by Jesuit mathematicians before the Suppression.[32] Monarchical enthusiasm for the *Ratio studiorum* began to wane in the mid 18th century as the prospect of applied sciences to further legitimize the monarchy began to have a hold in Vienna[33]. Joseph Stepling, a philosophy instructor who began lecturing in analytical geometry and differential and integral calculus in 1754[34], was appointed "director of studies" in the philosophy faculty by the monarchy as an attempt to extend the influence of the court

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[31] Hellyer, *Catholic Physics*, 47.

[32] J. Kasparova & K. Macak, *Utilitas matheseos. Jesuit Mathematics in the Clementinum (1602-1773)* (Prague: National Library, 2002) 66.

[33] "When tracing the development of Czech technical and industrial education, the last third of the 18th century and the early 19th century stand out as the major hallmark era. Many legal and educational measures were then carried out to meet the needs of the manufacturing industries, trade and agriculture. During that period, at the beginning of the first industrial revolution, the Czech lands, recognized as the most economically advanced part of the Hapsburg monarchy, witnessed the formation of their iron-making, textile, glass-making, ceramic and porcelain manufactories, coupled with an upsurge in ore and coal mining and an emerging food processing industry." Marcela Efmertova , "Czech Technical Education: The Educational Reforms of Franz Joseph Von Gerstner and his Relationship with the Paris Ecole Polytechnique," *ICON* 3 (1998): 202-223.

[34] "Beginning in 1754, Stepling delivered lectures on analytical geometry and differential and integral calculus, subjects that had not traditionally found their way into the Jesuit university curriculum." Paul Shore, *The Eagle and the Cross: Jesuits in Late Baroque Prague*, (St Louis: Institute of Jesuit Sources, 2002) 188-189.

into philosophy education.[35] This influence was subsequently felt in the attempt to replace traditional philosophical curricula with the exact sciences.[36] While it is well documented that Prague Jesuit intellectuals attempted to support and assimilate the developments in mathematical physics[37], as is attested by the disputations Prague students conducted in 1766 on the works of Newton which had been translated into Czech[38], the ensuing faculty debates always concluded with a decision to retain the Aristotelian framework for study into the theoretical sciences.[39]

How does Bolzano's position relate to those of the interlocutors of this debate? His position is a unique synthesis of each side of the debate. Bolzano reveals himself to be a passionate advocate of the mathematical sciences in his affirmation of the discoveries of mathematicians and the type of proofs through which those discoveries are presented. His unique contribution to this debate, however, is to also affirm that these proofs are not explanatory of the causes of their objects. A subsequent presentation is required for this. His distinction between proofs that prove and proofs that explain is unique for its affirmation of both styles

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[35] Shore, *The Eagle and the Cross*, 170.

[36] "In the second half of the 18th century, the Czech lands, like other countries in Europe (other parts of the Hapsburg monarcy, German lands, Poland, etc), displayed efforts to integrate their university studies in natural and technical sciences with faculties at the existing universities. At Prague University's Faculty of Philosophy, lectures were given in practical applications, such as surveying and geodesy, mechanics, civil and military engineering, technology associated with natural sciences, mathematics and physics branches, as well as the fundamentals of agriculture, mining, cameralist theories, etc." (Efmertova, "Czech Technical Education", 207). In 1747 a professorship in experimental physics was established in Prague and in 1763 a chair in mining was created in the Philosophy faculty. The former position was that of professor *physicae experimentalis et metaphysicae*. Ibid. The latter is discussed in Shore, *The Eagle and the Cross,* 35.

[37] "As for the actual research activities of Jesuit natural scientists [in Bohemia], it is clear that many of them disregarded the Holy See's expectation that they confine themselves to Ptolemaic cosmology." Shore, 2001, p. 74.

[38] Shore 2002, p. 159 ff.

[39] "Nevertheless, the university was in a less than entirely secure position in the second half of the [eighteenth] century, in that its curriculum, particularly in the Faculty of Philosophy, was beginning to seem archaic and far removed from curricular developments embraced elsewhere. The decision of the Society, after a rancorous internal battle, to remain committed to Aristotelian philosophy and physics only added to the conviction that the Society's philosophical program was out of date and even hostile to the new developments." (Shore, "Universalism, Rationalism, and Nationalism", 172).

of proof in an overarching theory of science that remains consistent throughout his career. This distinction would eventually develop into the distinction in analytic philosophy between psychologistic explanations and axiomatic systems.

Bolzano's Motivations for Reviving a Classical Model of Science

While Bolzano is participating in a debate on the status of the mathematical sciences that had raged for centuries in Catholic Europe, his reasons for doing so are largely pastoral as he makes clear in his sermons and the introductions to his books. The presentation of mathematical science along the classical model is particularly desirable for the enlightenment of the people. For mathematical proofs that *explain* enable people to identify what they need to learn in their respective roles in society and to deduce new truths on their own.

In the Introduction to TS, Bolzano argues that "It is obviously not a matter of indifference how…these individual sciences are to be represented in particular treatises." This is obvious for the following four reasons.

> One need not overrate in the least the value of mere knowledge to see that mankind is beset with innumerable evils simply because of ignorance and error, and that we would be incomparably better off and happier on earth if only each of us could acquire exactly that information which would benefit us most under the circumstances….(a) Everyone who has the necessary preliminary knowledge could instruct himself in the most reliable and complete manner about any subject in which he needs information, and could learn everything that is known on that topic. (b) If everything contained in these treatises were represented as clearly and convincingly as possible, doubt and error could be expected to diminish even in disciplines where passion makes it difficult to accept the truth: in religion and morals, especially since (c) the more widespread study of certain sciences from well written treatises would engender greater skill in right reasoning. (d) The discoveries which have so far been made would certainly lead to many new discoveries, if they were only more widely known among us; hence it is obvious that the benefits from these measures will spread in the course of time rather than

diminish.[40]

This statement of purpose is an expression of what some have called the movement within the Catholic *Aufklarung* known as Bohemian Reform Catholicism, of which Bolzano is considered the paramount example.[41] The conflict between the state, for whom "The school is and remains a *Publicum*",[42] and the church which erupted in the 1773 Suppression gave way at the turn of the century to a progressive Catholic program in Bohemia that adopted the state's enlightenment ideal of universal education fused with a theological foundation that lack of enlightenment is the cause of all sin.[43] This is evident in several of Bolzano's sermons, such as an 1817 sermon entitled "Want of Enlightenment Must Be Seen as the True Cause of the Evils that Beset our Fatherland", in which he asserts "as a fundamental principle of Christianity" that "one can only be vicious through error; sin only arises from blindness".[44] This is precisely why "Jesus applied no other means, and commanded his disciples to employ no other means than *teaching, instruction, the spreading of better*

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[40] WL, Sec. 1, p. 2, George translation. A similar set of purposes for the classical presentation of science was presented by Bolzano in 1810: "the most immediate and direct purpose which all genuinely philosophical thinkers had in their scientific investigations was none other than the search for the ultimate grounds of their judgments. And this search then had the *further purpose*, *on the one hand*, of putting themselves in the position of deriving from these clearly recognized grounds *some* of our judgments, perhaps also some *new* judgments and truths;; and *on the other hand*, of providing an *exercise* in correct and orderly thinking which should then *indirectly* contribute to greater *certainty* and strength in *all* our convictions." Bolzano, "Contributions to a Better-Grounded Presentation of Mathematics," 104.

[41] "During the first half of the nineteenth century, Bolzano and his fellow Catholics sponsored a movement of philosophic Josephinism unique to Bohemia. Their efforts to found schools, to teach rationalistic theology, and to promote harmony between Czechs and Germans influenced thinkers like Adalbert Stifter and Anton Günther, as well as a host of Herbartian philosophers." William M. Johnston, *The Austrian Mind*, (Berkeley and Los Angeles: University of California Press, 2000), 278.

[42] Maria Theresia. Hugo Hantch, quoted in Shore, *The Eagle and the Cross*, 7.

[43] "In contrast to Austrian proper and to Hungary, under Metternich Catholicism played a progressive role in Bohemia. Because values of the Enlightenment flourished inside the church, anticlericalism remained at a minimum, with the result that religious romanticism could emerge as a fruit of the Enlightenment instead of as a protest against it." Johnston, *The Austrian Mind*, 278.

[44] Bernard Bolzano, "Want of Enlightenment (Ignorance and Evil) Must be Seen as the True Cause of the Evils that Beset our Fatherland," in *Selected Writing on Ethics and Politics*, ed. Paul Rusnock and Rolf George (Amsterdam: Rodopi Press, 2007), 49.

ideas."[45] For Bolzano, the axiomatic structure of a properly presented treatise, with objective grounds and consequences, is critical for the enlightenment of the people. His sermon continues in this vein: "And since all truths stand in a friendly union, and because once one of them is found, others usually follow, a single wise word from someone can often awake several in us, and lead us to proceed even further in the knowledge of beneficial truths than he."[46]

"At first glance", Bolzano tells his congregation, "many will perhaps feel inclined to claim the exact opposite, and to look for the causes of a significant portion of our present sufferings precisely in the enlightenment that has already been spread among us. For, they ask, were not our forefathers, although perhaps they knew less than we do, and were less enlightened, were they not better off all the same?" Bolzano's response is a homiletic presentation of his primary philosophical contribution that psychologistic proofs establishing certainty must subsequently be completed through a classically presented proof establishing objective grounds and consequences.

> Here we have a simple misunderstanding, my friends. Certainly our age knows many things of which previous ages were ignorant – but from this it follows by no means that we think more wisely and correctly than our forebears. Just as when a dark room is suddenly pierced by a bright ray of sunshine we often find ourselves blinded by the force of the light to which we are unaccustomed, and for a while cannot distinguish things well, in fact seeing less than we could before; so too is it with our judgment, when a new ray of truth illuminates the edifice of our previous knowledge. We indeed now see many new things – but many old things that we formerly knew, although only obscurely, become invisible, and it can often happen that the truths we reject in such moments are more important than the new one we have added to our store of knowledge. In that case, it would be wrong to say that we have become wise and more enlightened. Rather, we should say that we have become more foolish. And this is indeed the case with those of our enlightened

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[45] Ibid., 47.
[46] Ibid., 52.

> contemporaries that people complain about....Thus if we connect the proper concept with the word *enlightenment*, we can indeed reproach our age with a want of enlightenment, and say that it is precisely this lack that is responsible for a multitude of the most oppressive evils.[47]

Bolzano's pastoral purpose in engaging the largely Catholic debate on the status of modern science - the enlightenment of the people - is thus intimately connected with the particular position he advances in this debate. While the initial achievements of mathematical physicists, expressed through proofs that establish these achievements with certainty, are critical to bringing about enlightenment, true enlightenment of the people is only possible when these achievements are properly explained in objective terms according to the classical model of science.

[47] Ibid., 48-49.

II. Bolzano's Presentation of Science

The specific steps taken by Bolzano to present modern scientific achievements in classically organized treatises that explain these achievements for society at large is the subject of this chapter. First, Bolzano's rules for the exposition of an *a priori* science stress objectivity in an axiomatic structure of axioms and ground-consequence relations. Second, these objective ground-consequence relations required a reorganization of the contemporary division of the sciences along more classical lines, in which physical sciences study causes though they sometimes make use of general mathematical concepts. Both were presented in Bolzano's 1810 *Contributions to a Better-Grounded Presentation of Mathematics* (BD).

Rules for Presentation of an *A Priori* Science

BD has two parts – a reorganization of the division of the sciences that had evolved out of mathematical physics (On the Concept of Mathematics and its Classification) and a presentation of rules for exposition of an *a priori* science (On Mathematical Method). The latter details the elements of scientific exposition, the subject of this section, and begins with two revealing claims. Bolzano first "must admit at the outset that I am not completely clear myself on the true nature of scientific exposition". However, "this much seems to me certain: in the realm of truth, i.e. in the collection of all true judgments, a certain *objective connection* prevails which is independent of our accidental and *subjective recognition* of it." This objective connection between true judgments, referred to by Bolzano as the ground-consequence relation, is the constant standard of and most important element in scientific presentations throughout Bolzano's work.

Bolzano's discussion of the rules for scientific exposition makes it clear that proofs are rendered incapable of objective ground-consequence relations most often by having the wrong starting point. "It is usually said, that '*the mathematician must always begin with definitions*.'"[48] However, if a definition is "the *statement of the most immediate components* (two or more) *out of which a given concept is composed*",[49] then "[t]hey obviously *cannot* be *the first thing*, with which we begin".[50] Definitions are made of complex concepts, whose simple concepts, for example, are a genus and a specific difference. Thus, the simple concepts must be presented first, in order to ensure a reader has an understanding of the simple concepts relevant to a science. This is due to the more fundamental principle that, as Bolzano writes elsewhere, "*the simpler truth must be stated in advance of the more complex*"[51] in a ground-consequent relation.

How does one begin a scientific exposition with a statement of the simple concepts which, as simple, are indefinable? Bolzano solves this problem of primitive concepts with what some consider "his main innovation in axiomatics" in this work.[52] The first thing with which a scientific treatise should begin, "in so far as it has simple concepts",[53] is a list of propositions that use the simple concepts so that their unknown meaning can become known. "This is well known as the means by which we each came to know the first meanings of words in our mother tongue."[54] Bolzano refers to this starting point as descriptions (*Umschreibunden*, also

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[48] Bolzano, "Contributions to a Better-Grounded Presentation of Mathematics," 104.
[49] Ibid., 104.
[50] Ibid., 107.
[51] Bernard Bolzano, "On the Mathematical Method," in *On the Mathematical Method and Correspondence with Exner*, ed. Paul Rusnock and Rolf George (Amsterdam: Rodopi Press, 2004), 79.
[52] Jan Sebestik, "Bolzano, Exner and the Origins of Analytical Philosophy," in *Bolzano and Analytic Philosophy*, ed. Wolfgang Künne, Mark Siebel and Mark Textor (Amsterdam: Rodopi Press, 1997), 37.
[53] Bolzano, "Contributions to a Better-Grounded Presentation of Mathematics," 108.
[54] Ibid., 107.

translated as paraphrases),[55] and provides the following example in the case of the geometric simple concept of point.

> So, for example, from the propositions: the *point* is the *simple* [*object*] in space, it is the *boundary* of a line and itself no *part* of the line, it has neither extension in length, breadth, nor depth, etc., anyone can gather which concept is designated by the word 'point'.[56]

Once a scientific treatise has successfully clarified the simple concepts of a science, Bolzano's argument thus far would seem to lead to definitions as the next thing in a treatise. However, merely pointing out that a complex concept includes particular simple concepts is not essential to a scientific treatise. The requirement mentioned above, that in an objective scientific connection the ground is simpler than its more complex consequent, leads to Bolzano's agreement with Kant that scientific truths are actually synthetic and not analytic. Analytic judgments are judgments in which the predicate is contained in the subject, whereas all others are synthetic. Simply decomposing a complex subject to find predicates does not teach anything new about the subject, and thus analytic proofs cannot establish the objective ground for a judgment. Bolzano writes, "Indeed, in my opinion they do not even deserve the name of *judgments*, but only that of *propositions*, they teach us *something new* only as *propositions*, i.e. in so far as they are expressed in words, but not as *judgments*. In other words, the *new* [fact] which *one can learn from them* never *concerns concepts and things in themselves* but at most only their *designations*."[57] Thus, "they do not even deserve a place in a scientific system, and if they are used, it is only to recall the concept designated by a certain word".[58] As is shown in the next chapter, Bolzano would later identify in the modern

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[55] Ibid., 108.
[56] Ibid., 107.
[57] Ibid., 115.
[58] Ibid., 115.

doctrine of analytic truths the source of much of the psychologism in modern scientific presentations.

Thus, any exposition that includes a definition "should also enable us to realize the *purpose* for which this combination is made and considered". The purpose for including definitions in a scientific treatise is found in the objective connections between synthetic judgments that follow from the descriptions of simple concepts, through the following process. With only simple concepts available, the first of these judgments will combine simple subjects with simple predicates. These judgments, Bolzano makes clear, are axioms. When judgments with a complex subject or predicate (called theorems) are grounded in these axioms, then definitions may be useful to help clarify the complex concepts that are used in theorems. As summarized by Bolzano, "The domain of the axioms stretches as far as that of the purely simple concepts: where the latter end and the definitions begin, there also the axioms come to an end and the theorems begin."[59]

Bolzano's approach to axioms, the second thing in a scientific exposition, is, like his approach to definitions, animated by his opposition to any reliance on intuitiveness for the grounds of scientific proofs. Just as the first thing in subjective treatises are often definitions, rather than expositions of the less intuitive simple concepts which are their ground, so axioms are commonly presumed to be useful in scientific treatises due to their intuitiveness.

> In the usual mathematical textbooks and even in many logic books, it is said of axioms that 'they are propositions which on account of their intuitiveness (obviousness) require no proof, or whose truth is recognized as soon as their meaning is understood. According to this, therefore, the distinguishing

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[59] Ibid., 119.

> feature of an axiom would lie in its intuitiveness [*Anschaulichkeit*].[60]

However, intuitiveness is a highly problematic basis for distinguishing all propositions into axioms and non-axioms (theorems). First, intuitiveness is a matter of degree. Second, the intuitiveness of a proposition often depends on circumstances such as education or experience. Finally, "for this reason the degree of intuitiveness is also *very different* for different people".[61] Bolzano's most comprehensive attack on basing mathematical proofs on the intuitiveness of its concepts and propositions by relying on intuitiveness in definitions of complex concepts and axiomatic propositions is given here.

> If we consider that what we said about simple *concepts* in §8 [that they lack the intuitiveness of complex concepts and must be described] also holds for *axioms*, then it will not be expected that all axioms should appear in our minds with perfect vividness. On the contrary, our clearest and most vivid judgments are obviously *inferred.* The proposition that *a curved line between two points is longer than the straight line between the same points* is far clearer and more intuitive than some of those from which it must laboriously be derived. The proposition (to give, for once, an example from another science), '*You should not lie*', is far clearer and more obvious than that principle from which it follows, '*You should further the common good*'. Indeed, it could even be that an axiom may appear *suspicious* and *doubtful*, particularly from a misunderstanding of its words, or because we do not immediately see that the things we recognize at once as true can be derived from it.[62]

The suspicion and doubt surrounding axioms recalls the warning of Heraclitus, "Nature loves to hide", which was more formalized in Aristotle, for whom "those things which are furthest from sensation are the most universal".[63] Bolzano comments conclusively, "From this one

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[60] Ibid., 109.
[61] Ibid., 109.
[62] Ibid., 119.
[63] Aristotle, *Posterior Analytics*, I.2 72a2.

sees how wrong it is to say, as the usual textbooks of mathematics do, '*the axioms follow the definitions*'."[64]

Bolzano thus defines theorems as provable propositions that necessarily have a complex subject or predicate, and axioms as unprovable propositions that have a simple subject and predicate.[65] Not all propositions with a simple subject and predicate are axioms however. They must actually ground a theorem.[66] Bolzano employs the infinite regress argument first used by Aristotle to argue that there *are* axioms.[67]

The form of reasoning that begins with consequences that are obvious and ends with axioms that ultimately ground these consequences Bolzano refers to as derivation or deduction. "Axioms are therefore not *proved*, but they are *deduced*, and these *deductions are an essential part of a scientific exposition* because without them one could never be certain whether those propositions which are used as axioms actually are axioms....the deduction of the axiom must first instill in us a confidence in its truth and this will happen if it proceeds from some generally accepted and unmistakably clear propositions which are however basically nothing but *consequences*".[68]

Bolzano's Reclassification of the Sciences

If the objective relation of ground-consequence is the criterion of proofs that explain an *a priori* science, and crossing to another (usually more intuitive) science within a proof is a common source of proofs that only prove with intuitive certainty, then clarity on the division

[64] Bolzano, "Contributions to a Better-Grounded Presentation of Mathematics," 119.
[65] Ibid., 110, 117.
[66] Ibid., 119.
[67] Ibid., 112. Aristotle's infinite regress argument is in Aristotle, *Posterior Analytics*, I.3.
[68] Ibid., 119.

and proper subjects of the sciences is critical. Furthermore, the study and presentation of causes must not be marginal to the subjects of the sciences, but in fact must be central. Bolzano thus embarks on a reclassification of the sciences that attacks the modern subsumption of all the sciences into a poorly defined science of mathematics in mathematical physics, and reorganizes modern sciences along classical lines with a defining role given to the study of causes.

As discussed in Chapter One, the classical three-fold division of the theoretical sciences into metaphysics, mathematics and natural philosophy collapsed during the Scientific Revolution. Historically, the middle sciences of astronomy, mechanics, optics, etc, had been considered mathematical only inasmuch as they explained immaterial, and thus immobile and quantitative, aspects of these domains. These middle sciences treated their subjects either at one level of abstraction in terms of principles of matter and motion (as natural philosophy), or at a further level of abstraction in terms of principles that don't include matter or motion (as mathematics). The momentous change of the Scientific Revolution was the merging of the middle sciences into natural philosophy. This integration meant that mathematics was no longer restricted to seeking immaterial and immobile first principles, but was also applied to what exists in virtue of material causes.

As the middle sciences were merged into natural philosophy, there resulted two major shifts in the classification of the theoretical sciences. First, mathematics and natural philosophy were merged into one discipline that went by various names (mathematics, physics, mathematical physics). Second, the more general topics in natural philosophy (substance, accident, form, time, space, etc) were transferred to metaphysics, resulting in an even more mathematical approach to physics. The result was a two-fold division of theoretic sciences into mathematics, or, mathematical physics, and metaphysics which now included the general portions of physics.

The first change became evident in the numerous texts published in the 17th century under the title, *physico-mathematica*.[69] While this merging of mathematics and natural philosophy progressed with less debate outside of Catholic Europe, as is evident in the title of Newton's *Mathematical Principles of Natural Philosophy*, the advocates of this merger within Catholic academies were precisely those participants in the debate discussed in Chapter One who argued that mathematics can provide a classical science of causes with respect to the middle sciences. The Dillingen mathematician Christopher Haunold, a Jesuit, would write in 1645 that Plato restricted the Academy to those skilled in geometry

> since he understood that in many arguments of philosophical disputations excellent work could not be done without an understanding of the mathematical disciplines: for many things occur in physical speculations that one unskilled in mathematics cannot penetrate, nor can he explain or treat them according to their dignity. Because of this, we wish to add to these theses a treatment of these sciences so connected among themselves by nature, so that is may be clear how much philosophy is helped by mathematical knowledge.[70]

The second change is evident in textbooks such as Benedikt Stattler's 18th century *Philosophy Explained by the Method Appropriate to the Sciences*, whose three major parts are logic, metaphysics and physics. Stattler's division of metaphysics into ontology, cosmology, psychology and natural theology had become the predominant four-fold division of metaphysics in the 18th century. Ontology discussed quantity, extension, space, motion and time, while cosmology presented motion, heaviness, cohesion and impenetrability. In

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[69] An example is Grimaldi's *Physico-mathesis de lumine*. Dear discussed these texts in Peter Dear, *Discipline and Experience: The Mathematical Way in the Scientific Revolution* (Chicago: University of Chicago Press, 1995), 172-3.
[70] Quoted in Hellyer, *Catholic Physics*, 134.

other words, as one historian describes the change, "[m]uch of the first half of the traditional Physics curriculum, *physica generalis*, which had concerned itself with the topics covered by the first four books of Aristotle's *Physics*, such as matter, quantity and form, was removed from physics and into metaphysics."[71]

While Bolzano presumes, following the prevailing division of theoretic sciences, that mathematics and metaphysics are "the two main parts of our *a priori* knowledge",[72] the subjects of these two sciences were, in Bolzano's analysis, very different from those presented in the current textbooks. Bolzano begins by asking for a definition of mathematics, finding the definition found "in all *modern* textbooks of mathematics" – "*mathematics is the science of quantity*" – to be inadequate.[73] For any definition of quantity would radically alter the domain of mathematics beyond recognition. After surveying problems with some common definitions of quantity,[74] Bolzano then raises questions about the notion that mathematics is the "*science of those objects to which the concept of quantity is especially applicable*", the definition that is "basically intended" by those who define mathematics as the science of quantity.[75] This definition is "objectionable" because the "concept of quantity is *applicable* to *all* objects, even *objects of thought*". "Therefore if one wanted to consider the mere *applicability of the concept of quantity* to an object a sufficient reason for counting

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[71] Ibid., 219.
[72] Bolzano, "Contributions to a Better-Grounded Presentation of Mathematics," 94.
[73] Ibid., 91.
[74] "If a quantity is "something that exists and can be perceived by some sense", as one author claims, then all sciences become parts of mathematics, whereas if a quantity is a sensible object, then all immaterial things are excluded from mathematics, even if they are things that can be counted." Ibid., 92.
[75] Ibid., 92.

the theory of that object among the mathematical disciplines, all sciences would in fact have to count as mathematics".[76]

By extending the mathematization of all physical sciences to its logical conclusion Bolzano critiques the modern organization of the sciences which would simply replace every science, whether physical or not, with mathematics. Bolzano, noting that it "will not be as easy to put a better definition in the place of the usual one as it has been for us to criticize it and reject it", then embarks on a redefinition and corresponding reorganization of mathematics along much more classical lines.[77]

Bolzano begins by noting that those contemporary mathematicians who in their definitions of mathematics define quantity as "*something that exists and can be perceived by some sense*",[78] or simply "*that which is*", "seem to have felt that mathematics is concerned with *all* the forms of things, not merely with their *capacity to be compounded out of equal parts* (their *countability*)".[79] Bolzano agrees with this, and offers a new definition of mathematics as the "*science which deals with the general laws (forms) to which things must conform in their existence*".[80] This definition, which resembles much more the classical approach to science as a study of formal causes, then becomes the basis for Bolzano's project "to derive a *logical classification* of this science [mathematics] into several individual disciplines".[81] This classification is presented by Bolzano in a diagram, given below, that appears at the conclusion of his text.

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[76] Ibid., 92.
[77] Ibid., 92.
[78] Ibid., 91.
[79] Ibid., 95.
[80] Ibid., 94.
[81] Ibid., 95.

A. General mathesis (things in general)
B. Particular mathematical disciplines (particular things)
    I. *Aetiology* (things which are not free)
    II. (sensible things which are not free)
        a. (form of these things *in abstracto*)
            α. Theory of time (time)
            β. Theory of space (space)
        b. (sensible things *in concreto*)
            α. Temporal aetiology (sensible things in time)
            β. Pure natural science (sensible things in time and space)[82]

Bolzano's primary division is between general mathesis and particular mathematical disciplines. This distinction, as explicated by Bolzano, effectively reverses the merger of mathematics and physics in order to allow each to study the laws specific to its level of abstraction or universality. "Now these laws are either so general that they are applicable to *all things completely without exception*, or not. The former laws, put together and ordered scientifically, will accordingly constitute the *first* main part of mathematics. It can be called *general mathesis*; everything else is then *particular mathesis*."[83] The distinction between general mathesis and particular mathesis is thus a hierarchical one: particular mathematical sciences "are *subordinate* to the general mathesis as a whole, as species of the genus. And because the concept of *number* is one of those of the *general* mathesis it will also appear *frequently* in all these particular parts, but it will not exhaust their content."[84] In other words, one science "must precede" its subordinate science "because the latter appeal to certain theorems of the former".[85] Similarly, classical mathematics studies forms that, while found in physical reality, are so universal that they cannot be comprehended in terms of physical

[82] Ibid., 102.
[83] Ibid., 95.
[84] Ibid., 96.
[85] Ibid., 97.

reality. Classical physics, on the other hand, studies the same physical reality as mathematics, while restricting itself to forms that can be understood in physical terms.

The divisions within particular mathesis are also hierarchical for Bolzano. The most universal science within particular mathesis is "*the theory of grounds* [*Grundlehre*] or *aetiology*", which "contains the theorems of ground and consequence, some of which also used to be presented in *ontology*, e.g. that *similar grounds* have *similar consequences*."[86] Bolzano thus explicitly returns the study of causes from metaphysics (ontology) to the study of physics (which takes place under the name of particular mathesis).

That the most universal form to be studied in the particular mathematical sciences is that of grounds is derived by Bolzano's distinction of every existing thing as being "either *necessary or free* (i.e. not necessary)", with the latter being "subject to no conditions or laws in its *becoming*" and "therefore not an object of mathematics" (Bolzano has in mind the practical, as opposed to theoretic, sciences).[87] The former is either necessary in itself, which "is called *God* and is considered in *metaphysics*", or is "*hypothetically necessary* which we consider as *produced through some ground* [*Grund*]".[88]

The next two sciences, which are equally subordinate to the study of grounds, are "the *theory of time* (chronometry), and the *theory of space* (geometry)" as "*time* and *space* are the two *conditions* which must govern all things open to the *senses*".[89] Finally, if "time and space are not to be considered merely *in abstracto* but as occupied with *actual things* and indeed with such things as are not free in their existence but are subject to the laws of causality, then two

[86] Ibid., 97.
[87] Bolzano says in a footnote to this distinction: "But it is certainly the object of *morality* which investigates the question: *how that which happens* (or *is*) *freely, should happen* (or *be*)." Ibid., 97.
[88] Ibid., 97.
[89] Ibid., 97.

new sciences emerge", the theory of grounds which presume the condition of time ("*the theory of causes* [*Ursachenlehre*] or *temporal aetiology*") and the theory of grounds which presume space and time ("*pure natural science*, and otherwise *the theory of motion* or *mechanics*").[90] Bolzano provides as an example of a theorem from temporal aetiology, "every effect is simultaneous with its cause".[91]

Bolzano contrasts his organization of mathematics with competing classifications that would marginalize the concern for causes. First, he addresses those for whom mathematics studies "the general laws *of the possibility of motion without regard to a force producing the movement* – therefore the concepts of *time*, *space*, and *matter* without that of a *cause*". Bolzano replies that "such a science cannot even exist", "[f]or all propositions which have so far been put forward in it are in fact only provable with the aid of the concept of cause".[92] Second, the "*assumption*" which "many have adopted, even in the definitions of mathematics, is that it is the sole business of mathematics *to find quantities which are not given, from others which are given*. If this were correct, then all the propositions of mathematics should have the form of *problems*. *Axioms* and *theorems*, etc., could not actually appear in it at all."[93] It is important to notice here that Bolzano does not seek to replace the algebraic approach of mathematics as the solution of problems with an axiomatic approach to mathematics as the study of causes, but to complete the former with the latter. Bolzano's concern about this algebraic approach to proofs is presented in the discussion of the chemical analogy of concepts in the next chapter.

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[90] Ibid., 98.
[91] Ibid., 98.
[92] Ibid., 98.
[93] Ibid., 99.

Finally, Bolzano addresses the common understanding of "*applied* mathematics" as being "*essentially based on some propositions borrowed from experience*", further demonstrating the need for his reclassification of the sciences if objective ground-consequence relations are to be thematized. Bolzano says of such an applied mathematics that he "do[es] not believe its existence can be justified" because scientific propositions cannot be borrowed from experience. However, applied mathematics can and should be pursued as a practical mathematics, which "is *an exposition of the mathematical disciplines established particularly for useful application in everyday life*". The distinction, between a science whose purpose is "*perfection in scientific form*" and one whose purpose is "*direct usefulness for the needs of life*", is essentially the classical distinction between theoretic and practical science. While "most existing textbooks of mathematics are based on a certain *mixed* approach", Bolzano does not object as long as it is recognized that practical mathematics "cannot be achieved until the *purely scientific* system has been completed".[94]

[94] Ibid., 101.

III. Bolzano and Psychologism

In contrast to the axiomatic exposition of ground-consequence relations that is the hallmark of an *a priori* science, Bolzano viewed most contemporary expositions of *a priori* sciences as subjective proofs in which certainty, intuitiveness and obviousness are the hallmarks. "Proofs, on the contrary, which only produce conviction without giving objective grounds, one might call mere *certifications* or also *subjective proofs*."[95] Subjective proofs establish their conclusions with certainty because they rely upon complex, and thus more intuitive, propositions at the outset of their proofs. Proofs that explain, on the other hand, must always begin with simple, thus less intuitive, axioms as demonstrated in Bolzano's 1810 work presented in the previous chapter.

Bolzano's specific critiques of subjective proofs are aimed in two general directions: proofs in mathematical sciences (e.g. geometry, analysis) that rely on concepts from more specific (thus more intuitive) mathematical sciences and proofs that analyze mental concepts based on the early modern model of *a priori* truths as analytic. While there is no categorization of these two critiques by Bolzano, they are clearly distinct, with the former critique prominent in Bolzano's early work while the latter critique is prominent in his later work. This chapter addresses each critique in turn.

Bolzano's Early Critique: Crossing to Another Kind

Bolzano viewed many proofs in mathematics – particularly geometry and analysis – as relying on propositions from foreign sciences. When this "crossing to another kind" occurs, the foreign proposition usually imports concepts whose greater intuitiveness renders the proof more convincing, but in turn begs the question. This is because the more specific,

[95] Bolzano, "On the Mathematical Method," 71.

complex concept presumes the generic concept demonstrated by the proof. As mentioned in the first chapter, Bolzano refers to Aristotle in several of his works for this critique of crossing to another kind. In fact, Aristotle discusses mathematics in his primary injunction against such expositions.

> …he knows better whose knowledge is deduced from higher causes, for his knowledge is from prior premises when it derives from causes themselves uncaused: hence, if he knows better than others or best of all, his knowledge would be science in a higher or the highest degree. But, as things are, demonstration is not transferable to another genus, with such exceptions as we have mentioned of the application of geometrical demonstrations to theorems in mechanics or optics, or of arithmetical demonstrations to those of harmonics.[96]

Bolzano's first work, in 1804, addresses geometry and makes this critique in the Preface.

> In this respect I considered it an error in geometry that all propositions about angles and ratios of straight lines to one another (in triangles) are proved by means of *considerations of the plane* for which there is no cause in the *theses* to be proved. I also include here the concept of *motion* which some mathematicians have used to prove purely geometrical truths.[97]

Bolzano then attempts to objectively prove theorems about lines, angles and triangles with no recourse to the concepts of the plane and motion. This is because the plane, while it is a geometric concept, is more advanced and complex than that of a line. Motion is even more alien, as the idea of motion requires "the idea of a movable object in space", which is the subject of mechanics. "Now because the assumption of any motion presupposes for the proof of its possibility (which one has a duty to give), particular theorems of space, there

[96] Aristotle, *Posterior Analytics*, 76a19-25.
[97] Bolzano, "Considerations on Some Objects of Elementary Geometry," 32.

must be a science of the latter which precedes all concepts of the former." Bolzano calls this science pure geometry.[98]

Kästner, whose mathematics textbook was assigned to Bolzano by his professor Vydra, demonstrates that three points are required for the determination of an angle as follows.

> A straight line, of which two points are in a plane, lies completely within this plane. But since the plane, in which the straight line is, can rotate around it as an axis, three points determine the position of a plane, and therefore every plane angle, and every triangle is in a plane.[99]

Bolzano regards just such geometric proofs as crossing to another kind, particularly to a more intuitive kind to render the proof more certain. In contrast, Bolzano lays out the required structure of the geometric proofs that he would set out to present.

> It is obvious that for a proper theory of the straight line – I am thinking of the proofs of propositions such as: the possibility of a straight line, its determination by two points, the possibility of being infinitely extended, and some others – no considerations of *triangles or planes* can be used. On the contrary, the latter theory must only be based on the former. So I have set out in the *first part* an attempt to prove the *first propositions of the theory of triangles and parallels* only on the assumption of *the theory of the straight line*. As far as I am aware this has not been done before, because in all other places various *axioms of the plane* have been assumed, axioms which, if they had to be proved, would require precisely that theory of triangles. Therefore in my view the first theorems of geometry have been proved only *per petitionem principia* [begging the question]; and even if this were not so, a *probation per aliena et remota* [proof by alien and remote [ideas]] has still been given which (as already mentioned) is absolutely not permissible."[100]

Bolzano would, in his later work, refer to such begging of the question by appeal to intuitive propositions as the "fallacy of classification". The fallacy of classification occurs when a

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[98] Ibid., 33.
[99] Quoted in Ibid., 32.
[100] Ibid., 33.

concept is defined in terms of properties or specific differences of the concept which, as properties or specific differences, contain the concept to be defined unanalyzed.[101]

Bolzano makes the same critique of analysis. The binomial theorem, on which rests "almost the whole of the so-called differential and integral calculus", is generally proved using the concept of infinity to demonstrate that it is true for a values of the power to which (x+y) is raised unto infinity. However, Bolzano sees infinity as an alien concept that relies on intuitions of infinite time and space, and counsels against its use.[102]

Bolzano's critique extended as well to the Intermediate Value Theorem that was critical to contemporary analysis: the claim that a continuous function which yields values above and below zero must yield a value of zero as well. This claim had been proven through appeals to intuitions of space in geometry - "…that every continuous line of simple curvature of which the ordinates are first positive and then negative (or conversely) must necessarily intersect the x-axis somewhere at a point that lies in between those ordinates"[103] – and to intuitions of time.[104] While the application of mathematics to account for motion in mathematical physics led naturally to an appeal to spatial and temporal intuitions to

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[101] WL, Sec. 23, p. 27, George translation.

[102] "We can easily think what a moment is, or a duration, without thinking the idea of total infinite time. On the contrary, in order to think the latter, we must already have the idea of a moment, since infinity time is nothing but the class of all moments." WL, Sec. 79, p. 115, George translation.

[103] Bernard Bolzano, "Purely Analytic Proof of the Theorem that between any two Values, which give Results of Opposite Sign, there lies at least one real Root of the Equation", 254.

[104] "'If two functions,' they say, 'vary according to the law of continuity, and if for $x=\alpha$, $f\alpha<\varphi\alpha$, but for $x=\beta$, $f\beta>\varphi\beta$, then there must be some value μ lying between α and β for which $f\mu=\varphi\mu$. For if one imagines that the variable quantity x, in both these functions, gradually takes all values between α and β, and the same value is always taken by them both at the same moments, then at the *beginning* of this continuous change in x, $fx<\varphi x$, and at the end $fx>\varphi x$. But since both functions, by virtue of their continuity, must first go through all intermediate values before they can reach a higher value, there must be some *intermediate moment* at which they are both equal to one another.' This is further illustrated by the example of the motion of two bodies, one of which is initially *behind* the other and later *ahead* of the other. It necessarily follows that at one time it must have been going *beside* the other." Ibid., 255.

demonstrate the propositions required in calculus, such demonstrations could certainly not objectively ground these propositions for Bolzano.

> There is certainly nothing to be said against the *correctness*, nor against the *obviousness* of this geometrical proposition. But it is also equally clear that it is an unacceptable breach of *good method* to try to derive truths of *pure* (or general) mathematics (i.e. arithmetic, algebra, analysis) from considerations which belong to a merely *applied* (or special) part of it, name *geometry*. Indeed, have we not long felt, and acknowledged, the impropriety of such a *metabasis eis allo genos* [crossing to another kind]? Are there not a hundred other cases where a method of avoiding this [transition] has been discovered, and where the avoidance was considered a virtue? So if we wish to be consistent must we not strive to do the same here? In fact, anyone who considers that scientific proofs should not merely be *confirmations* [*Gewißmachungen*], but rather *groundings*, i.e. presentations of the objective reason for the truth to be proved, realizes at once that the strictly scientific proof, or the objective reason of a truth, which holds equally for *all* quantities, whether in space or not, cannot possibly lie in a truth which holds merely for quantities which are in *space*. If we adhere to this view we see instead that such a *geometrical* proof is, in this as in most cases, really circular.[105]

When one thus abstracts from the foreign intuitions and senses that adhere to what one actually intends in making this claim contained in the Intermediate Value Theorem, one is left with Bolzano's restatement of this theorem.

> [T]he expression that a function fx varies according to the law of continuity for all values of x inside or outside certain limits means only this: if x is some such value, the difference f(x+w)-fx can be made smaller than any given quantity provided that w can be taken as small as we please.[106]

Bolzano, it is generally recognized, thus succeeds in this instance in carrying out his larger mathematical project of ridding mathematical concepts of any appeal to spatiotemporal intuition.[107]

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[105] Ibid., 254.
[106] Ibid., 256.
[107] Bolzano makes clear in his Autobiography that this is in fact his larger mathematical project (speaking of himself in the third person): "From very early on he dared to contradict him [Kant] directly on the theory of

Bolzano's Later Critique: Chemical Theory of Concepts

An analogy often used in modern thought that had a profound influence on scientific proof is that of the decomposition of a chemical compound.[108] According to this analogy, we understand things algebraically by analyzing their concepts into constituent components. This is fundamentally an algebraic approach to proof, and was extremely influential at the level of working scientists and of philosophers in early modern thought, as is explained below. It was also unable, according to Bolzano in TS, to provide objective proofs that explain because its assumption that concepts somehow agree with their objects is an intrusion of psychologism into logic.

Discussion of Bolzano's critique of this decompositional mode of reasoning requires a thorough discussion of Bolzano's relationship to Kant, as Bolzano followed Kant in asserting that *a priori* science does not consist of analytic concepts to be decomposed. Thus, this section includes a brief history of this mode of scientific reasoning, its critique by Kant and the reaction of Bolzano.

A. Origin of Chemical Theory of Concepts

Key to this decompositional approach to scientific reasoning was the algebraic turn in analysis introduced by Viète in 1591, according to which known facts and unknown causes in traditional science could be assigned letter variables and then subjected to algebraic

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time and space, for he did not comprehend or grant that our synthetic a priori judgments must be mediated by intuition and, in particular, he did not believe that the intuition of time lies at the ground of the synthetic judgments of arithmetic, or that in the theorems of geometry it Is allowable to rest so much on the mere claim of the visual appearance, as in the Euclidean fashion. He was all the more reluctant to grant this, since very early on he found a way to derive from concepts many geometric truths that were known before only on the basis of mere visual appearance." Bernard Bolzano, "Zur Lebensbeschreibung," *Gesamtausgabe*, ser. 2A, vol. 12, pt 1, p. 68, quoted in Coffa, *The Semantic Tradition from Kant to Carnap to the Vienna Station*, 28.

[108] Ibid., 9.

decomposition until the unknown variables are defined, when possible. While Viète considered his "art of analysis" to be a restoration of ancient analysis using "a new vocabulary"[109], the historical connection was gradually lost to mathematicians and scientists enthralled by the potential to bring ancient mysteries of the physical world to the light of day though algebraic analysis.

Descartes' *Discourse on Method* was published in 1637 together with his *Geometry*, which explicated the method used in the *Discourse* and begins, "Any problem in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction." While the result of this approach to geometry was analytic geometry in which geometric problems are transformed into algebraic problems and thus more easily solved, the result of this approach to philosophy in general is a new form of philosophical analysis in which any problem is transformed into algebraic form and then subjected to algebraic decomposition.[110] Descartes applies this analytic method more specifically to our cognitions – our cognized propositions and ideas – until cognitive simples, "clear and distinct ideas", are uncovered which then serve as the first principles of philosophical synthesis.[111] This would become codified in the *Port-Royal Logic* textbook,

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[109] "Behold, the art which I present is new, but in truth so old, so spoiled and defiled by the barbarians, that I considered it necessary, in order to introduce an entirely new form into it, to think out and publish a new vocabulary, having gotten rid of all its pseudo-technical terms." Quoted in Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra* (New York: Dover Publications, 1992), 318.

[110] Descartes' Rule Thirteen describes this new form of analysis, "If we perfectly understand a problem we must abstract it from every superfluous conception, reduce it to its simplest terms and, by means of an enumeration, divide it up into the smallest possible parts". Rene Descartes, *The Philosophical Writings of Descartes*, I 51. trans. J. Cottingham *et al.* (Cambridge: Cambridge University Press, 1985).

[111] Descartes teaches his readers that analysis requires attention to the composition one's thoughts in Rule V: "The method consists entirely in an orderly arrangement of the objects upon which we must turn our mental vision in order to discover some truth. And we shall be observing this method exactly if we reduce complex and obscure propositions step by step to simpler ones, and then, by retracing our steps, try to rise from intuition of all of the simplest ones to knowledge of all the rest." Ibid.

which asserts that "we can have knowledge of what is outside us only through the mediation of ideas in us".[112]

Analysis for Leibniz *is* analysis of concepts into their parts until one arrives at simple concepts which have no parts, and which form the foundations of science.[113] Leibniz envisioned facilitating this analysis in the physical sciences with an even more broadly applicable symbolic logic that would transform the unknown and known concepts and propositions in any science into symbolic form.[114] Philosophers such as Locke who elevated this view to the status of logic used the analogy of the breakdown of chemical compounds to explain how reasoning proceeds. Decompositional analysis thus replaced classical logic as the primary logical foundation of a science intent on discovery rather than formal exposition.

B. Kant's Attack on Analytic *A Priori*

Immanuel Kant responded to this theory of science by establishing a framework for contemporary mathematics and natural philosophy that, in part, sought to protect the achievements of the Scientific Revolution by securing a foundation for mathematics and natural philosophy on the one hand and for metaphysics on the other.[115] For the application of early modern analysis to physical sciences, in Kant's estimation, is inconsistent with

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[112] A. Arnauld and P. Nicole, *La logique ou l'art de penser* (Paris: Flammarion, 1970), 63, quoted in J. Alberto Coffa, *The Semantic Tradition from Kant to Carnap* (Cambridge: Cambridge University Press, 1993), 9.
[113] "Whether men will ever be able to carry out a perfect analysis of concepts, that is, to reduce their thoughts to the first possible or to irreducible concepts , or (what is the same thing) to the absolute attributes of God themselves or the first causes and the final ends of things, I shall not now venture to decide." Gottfried Leibniz, "Meditations on Knowledge, Truth, and Ideas", in Gottfried Leibniz, *Philosophical Papers and Letters*, 2nd Ed, trans. L. Loemker (Dordrecht, Holland: Reidel Publishing Company, 1970), 293.
[114] His primary success in this regard was one shared with Newton, both of whom discovered a symbolic representation of motion as functions in what came to be called differential and integral calculus. With this discovery, the application of mathematical method to account for matter and motion was literally complete, as motion became the explicit subject of the new mathematical analysis.
[115] "It is, indeed, the common fate of human reason to complete its speculative structures as speedily as may be, and only afterwards to enquire whether the foundations are reliable." Immanuel Kant, *Critique of Pure Reason*, trans. Norman Kemp Smith, (Boston/New York: MacMillan, 1965), A5/B9, p. 47.

human freedom itself. That is, if *a priori* truth in natural philosophy is a result of the analytic discovery of a predicate contained as a part in its subject, through decomposition of the subject concept[116], then man would possess a faculty of intellectual intuition which is incompatible with freedom.[117]

Kant, nonetheless, affirms that the truths of mathematics and natural philosophy are necessary, *a priori* truths. This problem revealed, for Kant, the Copernican discovery that the truths of mathematics and natural philosophy are actually synthetic *a priori* truths, which are made possible by the pure intuitions through which reason constitutes our knowledge of objects. These pure intuitions of space, time, causation, substance, and so on reflect the central principles of classical natural philosophy which had provided unity to pre-modern science. Logic, as the set of rules that direct our deductive discovery of these synthetic *a priori* truths of mathematics and science, is no longer reflective of the subject matter of science (and thus should not be considered an organon), but is instead a canon for the correct use of reason.[118] Kant thus restores a unifying framework of science for mathematic and scientific research by placing this framework in the pure intuition of reason, thus reversing

[116] "in every affirmative true proposition, necessary or contingent, universal or singular, the notion of the predicate is contained in some way in that of the subject, *praedicatum inest subjecto*. Or else I do not know what truth is." Gottfried Leibniz, *Philosophical Writings*, trans. Mary Morris and G.H.R. Parkinson (London: J.M. Dent and Sons Ltd, 1973), 62.

[117] Kant describes "an ambiguity which may occasion serious misapprehension", namely that "[t]he understanding, when it entitles an object in a [certain] relation mere phenomenon, at the same time forms, apart from that relation, a representation of an *object in itself*, and so comes to represent itself as also being able to form *concepts* of such objects....But if we understand by [object in itself] an object of a *non-sensible intuition*, we thereby presuppose a special mode of intuition, namely, the intellectual, which is not that which we possess, and of which we cannot comprehend even the possibility." (Kant, *Critique of Pure Reason*, A250/B307, 268).

[118] "General logic analyzes the whole formal procedure of understanding and reason into its elements, and presents these as principles by which the logical validity of knowledge may be estimated. This part of logic, which is well called Analytic, supplies a negative touchstone of truth…but it does not enable us to determine positive anything in regard to objects. At the same time, there is something so seductive in an art that enables us to reduce all our knowledge to the form of understanding, however empty and poor in content it may be, that general logic, although it is merely a canon of judgment, is apt to be used as an organon by means of which new truth, or rather the specious appearance of new truth, may be obtained. When it is thus misused as a supposed organon, logic is called *Dialectic*." (Kant, *Critique of Pure Reason*, A60-61/B84-85, 98-99).

the traditional assumption “that all our knowledge must conform to objects” to the Copernican insight “that objects must conform to our knowledge”.[119]

Kant thus reclassifies the mathematical and physical discoveries of the Scientific Revolution as not analytical judgments but as synthetic *a priori* judgments, for judgments such as 1+3=4, or, all bodies are heavy, cannot be considered analytic as the predicate of these judgments simply is not thought when thinking the subject. Rather, “when I seek to go beyond the concept A, and to know that another concept B is connected with it”, this connection is *a priori* because it rests not on experience but on the principles of reason through which such connections are constituted.[120]

Kant’s Copernican revolution is a turning point in philosophy in one respect as it was the first significant attempt to ground the Scientific Revolution and thus secure its continued progress, and in another respect as it in turn raises the problem of reason’s ground. This latter problem would lead directly to Bolzano’s placement in the Chair of Religious Science that he held for 15 years.

That reason is self-ordering indicates Rousseau’s profound influence on Kant[121], and engendered the belief that freedom is something that must be realized by man through an act of will. Observing the French Revolution, many Kantians perceived this act of will to be genuinely possible, a perception central to Post-Kantian Idealism.[122] As a result, Kant would

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[119] Kant, *Critique of Pure Reason*, Bxvi, 22.
[120] Kant, *Critique of Pure Reason*, A9/B13, 50.
[121] “The most decisive moment in the prehistory of ‘criticism’ is Kant’s intense engagement with Rousseau’s writings in the early and middle 1760s…Rousseau brought Kant around to the true conception of the function of theoretical inquiry in human life: inquiry’s sole dignity lies in its contribution to the defense of the rights of humanity.” Richard Velkley, *Freedom and the End of Reason: On the Moral Foundation of Kant’s Critical Philosophy* (Chicago: University of Chicago Press, 1989), 6.
[122] *German Philosophy 1760-1860: The Legacy of Idealism*, ed. Terry P. Pinkard (Cambridge: University of Cambridge Press, 2002), 82.

become associated with the French Revolution by Hapsburg officials and his *Critique of Pure Reason* was placed on the Papal Index by the Church.[123] When Emperor Francis I created Chairs in Religious Doctrine at each Hapsburg university in 1805, his intent was to contain the influence of such revolutionary ideas.[124] Bernard Bolzano, upon taking this Chair at the University of Prague in 1805, had earlier "spent more than a year mastering Kant's *Critiques*", which he had studied under Professor Karl Heinrich Seibt.[125] The following section provides an account of Bolzano's historic response to Kant.

C. Bolzano's Reaction to Kant

What one finds in Bolzano, contrary to Neurath's declaration that Hapsburg philosophy skipped the "Kantian Interlude", is in fact agreement with Kant that the discoveries of the Scientific Revolution, and all true judgments in mathematics and science, are not logically analytic judgments arrived at through decomposition of concepts. Bolzano asserts his indebtedness to Kant repeatedly in his *Theory of Science*.

> Kant taught us that there are truths (analytic truths) in which a component of the subject-concept is predicated of that subject; and that there are other kinds of truth (synthetic truths) where something is predicated of a subject which is not present among the components of the subject idea. He goes as far as to claim that the major concern of every science is the recognition of such synthetic truths, and that all the theorems of mathematics, physics, etc. are such synthetic truths.[126]

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[123] "Kantianism, together with idealism, was always rejected by government and church circles who considered it an emanation of the French Revolution." Massimo Libardi, "Vienna 1870-1918", in *In Itinere: European Cities and the Birth of Modern Scientific Philosophy*, ed. Roberto Poli (Amsterdam: Rodopi Press, 1997), 66. Kant was "deemed suspicious by the Government as well as the Church, which considered his thought to be connected with the French Revolution". Ramon Cirera, *Vienna, The Circle and Otto Neurath*, (Amsterdam: Rodopi Press, 1994), 105.

[124] "Terrified by the French Revolution, harassed and humiliated by Napoleon, Francis wished above all to put an end to the reform movement, indeed to all *thought* of reform within his realm. The ideals of Enlightenment, now seen as fomenting revolution, were to be fought with all available means." Rolf George and Paul Rusnock, "Introduction", in *Selected Writings on Ethics and Politics by Bernard Bolzano* (Amsterdam: Rodopi Press, 2007), 4.

[125] William M. Johnston, *The Austrian Mind*, 275.

[126] WL, Sec. 65, p. 84, George translation.

Bolzano credits Kant's distinction between analytic and synthetic judgments for alerting him to a related distinction not found in Kant but which becomes central to Bolzano's logic. Specifically, "since the predicate of a synthetic truth can at best be an attribute of the subject but not a component of the subject idea", it must be the case that the parts of an object and the parts of a true idea of that object cannot be said to agree, or, correspond, with each other.[127] The assumption that such an agreement is possible reflects a psychologistic approach to logic. As an example, Bolzano criticizes Tetens, who maintained that truth is the correspondence of the relations between our ideas with the relations between the things they represent, as follows.

> Can one really say that the ideas of a house and of a garden stand in the same relation to one another as these objects, i.e. house and garden themselves? The ideas 'house' and 'garden' have always the same immutable relation to one another, while the objects, houses and gardens, can stand in a variety of relations, depending on their characteristics. The objects *God* and *world* are related to one another as cause to effect. God is the creator of the world. But who would want to maintain that the *concepts* of God and world stand in the same relation; that the concept 'God' is the creator of the concept 'world'?[128]

Searching for an account of objects in mathematics and the sciences by analyzing their concepts is thus as wrong-headed as Kant portrays it, according to Bolzano. Bolzano provides three arguments to support this claim that it is "erroneous" to say "that the constituents of an idea must be ideas of the parts of its object"[129]. First, "there are ideas that have no object at all", such as 'nothing' and 'round square'.[130] The doctrine of truth as agreement could thus only apply to existing objects. Second, Bolzano argues that complex ideas often include not just other ideas, but also include whole propositions. The idea of a

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[127] Ibid.
[128] *WL*, Sec 29, p. 36, George translation.
[129] *WL*, Sec 63, p. 78, George translation.
[130] Ibid.

right triangle, for instance, is the idea of a 'triangle which has a right angle' which contains the idea, triangle, and the proposition which includes, 'has a right angle'. Third, "there are objects that have no parts at all, while their idea is clearly complex", such as the object 'any mental being' which is simple but whose concept is complex.[131]

An equally psychologistic doctrine that follows from the erroneous doctrine of truth as agreement between idea and object is that of the inverse relation between the intension of an idea and its extension. Bolzano provides as a counterexample the idea 'of a color that can be prepared from blue plant juices' which has both more content and a broader extension (e.g. blue, green, red) than the idea 'blue'. This fallacious doctrine was found in most any logic textbook beginning with the *Port-Royal Logic*, according to Bolzano, because the author of the *Port-Royal Logic* "had already provided the occasion for the development of the basically erroneous view here by presenting the concept of content in such a way that he counted every property that necessarily belongs to the object of an idea (every *attributum*) within the idea's content."[132] Bolzano then concludes his refutation of this doctrine with further indebtedness to Kant, "If I am so fortunate as to have avoided a mistake here which remained unnoticed by others, I will openly acknowledge what I have to thank for it, namely it is only the distinction Kant made between analytic and synthetic judgments, which could not be if all of the properties of an object had to be components of its idea."[133]

However, Bolzano did not subscribe to Kant's notion of pure intuition, or the existence of synthetic *a priori* truths, for the problem created by Kant of reason's ground undermines any

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[131] *WL*, Sec 63, p. 79, George translation.
[132] *WL*, Sec 120, p. 162-163, Berg translation.
[133] *WL*, Sec 120, p. 163, Berg translation.

attempt to bring unity to science, according to Bolzano. Bolzano expresses three concerns over Kant's conception of logic, which Bolzano summarized by quoting from Kant's *Logik* (A4), "The science of the necessary laws of understanding and reason in general, or of the mere form of thinking, is logic."[134]. First, such a conception of logic includes any form of thinking whether in truth or in error and is thus "too wide". Second, this conception, Bolzano astutely observes of Kant's critical project, converts logic into a branch of ethics. If one attempts to save this conception of logic with the addendum that they should serve the purpose of thinking, "then we are concerned with moral laws whose development belongs to ethics. As a consequence of this definition, all of logic would be changed into a chapter of ethics concerning the dutiful use of the faculty of understanding. But how many rules of which so far in logic nobody has dreamed would he not have to admit into his exposition!"[135] Third, the real flaw of this conception, and other available conceptions of logic, is that it is afflicted with "great vagueness" and lack of "precision".[136] For logic as the form of thinking vaguely includes all thinking, and is thus no definition at all, unless it is defined as thinking for the purpose of knowing the truth, in which case any mental activity that improves our recognition of truth (study of grammar, becoming skilled at mathematics) is suddenly included in logic.

Vagueness is the charge leveled by Bolzano against another characterization of early modern logic which became central for Kant, namely that logic concerns the form, and not the content, of thinking. In this characterization the content of thoughts are received through

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[134] *WL*, Sec 7, p. 5, George translation.
[135] *WL*, Sec 7, p. 6, George translation.
[136] Ibid.

intuition, while the form of thoughts are imposed by reason through thinking. This characterization is “too loose” to settle the matter, for Bolzano.

> But in order to decide to what extent this boundary line is correct we must first know what precisely is meant by form and matter. But we do not get any satisfactory information. Kant, for example, merely says: ‘matter is the object, form the generality’. Thus he merely replaces one word with another, and for a vague word he gives one that is even more vague.”[137]

Bolzano is thus concerned about scientific reasoning that relies on analysis of one’s concepts of objects as much as reasoning that relies on intuitive but foreign concepts. Both forms of reasoning are subjective proofs, relying on subjective grounds, rather than objective proofs that explain via objective relations of ground and consequence.

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[137] WL, Sec 116, p. 164. George.

IV. From Subjective to Objective Proofs

As noted in chapter 2, Bolzano addresses the question of how one discovers which simple judgments are in fact the grounding axioms of a science through a "special consideration". Specifically, "the deduction of the axiom must first instill in us a confidence in its truth and this will happen if it proceeds from some generally accepted and unmistakably clear propositions which are however basically nothing but *consequences*".[138] This chapter articulates the path to an objective exposition of *a priori* science given by Bolzano, a path whose starting points are the merely subjective, psychologistic proofs that establish the certainty of their conclusions.

First, this chapter presents the evidence that incorporating subjective proofs into, and not expelling them from, logic was Bolzano's mission. This is contrary to most interpretations of Bolzano that see him attacking and expelling subjective proofs from logic, an interpretation that is more true of Frege than it is of Bolzano. Second, this chapter presents the semantic analysis through which Bolzano proceeds from subjective to objective proofs. Semantic analysis of subjective proofs resulted from Bolzano's intense analysis of mathematical proofs to uncover propositions with any implicit reference to a science foreign to the conclusion.

Bolzano's Incorporation of Subjective Proofs into *A Priori* Science

Bolzano writes of subjective proofs in 1810: "This is a procedure which, where we are concerned with the practical purpose of certainty, is quite correct and praiseworthy; but it cannot possibly be tolerated in a scientific exposition because it contradicts its essential aim."[139] Bolzano does not wish to expel subjective proofs, which are "quite correct and

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[138] Bolzano, "Contributions to a Better-Grounded Presentation of Mathematics," 119.
[139] Ibid., 103.

praiseworthy" when one's purpose is "certainty", from logic. One instead finds Bolzano making a distinction between the predominant mode of subjective proof and the "discounted" mode of objective proofs. This distinction, we have seen, is one Bolzano consciously parallels with the Aristotelian distinction between two modes of proof.

> I have been strengthened in the view that there is a special relation of ground and consequence between truths by noticing that some of the most penetrating thinkers have been of the same opinion. It is known that Aristotle already (*Analytica Posteriora*, 78a22) and for many centuries after him the scholastics have distinguished two kinds of proofs, namely those which show only the hoti, i.e. which show *that* something is the case, and those which show the dioti, i.e. *why* something is the case.[140]

Aristotle received the title 'Philosopher of Common Sense' precisely because he did *not* seek to expel proofs from science that fell short of the model of science as a demonstration of necessary causes. Rather, proofs which show *that* something is the case formed the starting point for his derivation of proofs which show *why* something is the case. Bolzano included this type of "derivation", or "special consideration", in his "Mathematical Method" in 1810 as discussed previously, and in his mathematical works. His objective proof of the Intermediate Value Theorem that was so critical to analysis, discussed above, is not intended to replace geometric demonstrations of the theorem, but to complete it.

> For while the geometrical truth to which we refer here is (as we have already admitted) extremely obvious and therefore needs no proof in the sense of confirmation, it none the less does need a grounding.[141]

Bolzano even favors the use of subjective proofs in objective expositions for less important propositions when such concessions would facilitate the explanation of the proof: "What I

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[140] WL, Sec 198, p. 272. George translation.

[141] Bolzano, "Purely Analytic Proof of the Theorem, that between any two Values, which give Results of Opposite Sign, there lies at least one real Root of the Equation," 254.

have set out to accomplish in this book is to present the most important theses in such a way that one will be able to recognize the objective connection between them; although in the case of many less important propositions I will content myself with mere certifications in order to avoid prolixity"[142]

Bolzano is often interpreted, in contrast to what has been said here, as attacking psychologistic proofs and arguing for their replacement with objective proofs. Rolf George begins one of his treatments of Bolzano and psychologism with the claim: "It must be granted that Bolzano did not think much of grounding logic in psychological self-consciousness".[143] While this would be consistent with what we know from Bolzano's writings, it is also true that, as George himself writes later in the article, "On the role of psychology in logic, he had remarkably little to say. In contrast to Frege and Husserl, there is no polemic to speak of against those who had thought to found logic upon psychology."[144]

George's contrast of Bolzano with Frege is revealing, for what one finds in secondary literature on Bolzano is often a Fregean Bolzano who allegedly attacked psychologism and in response postulated an independent realm of propositions-in-themselves (playing the same role as Frege's *Gedanken*) that exist whether they are thought or not. Drozdek, for example, argues that, "[i]n his battle against one-sidedness of subjectivism and psychologism, Bolzano fell into one-sidedness of objectivism by concentrating on the objectivity of truth, on its being truth in itself, and overlooking the fact that it should also be truth for us, known to us, and used by us." Nothing, it seems from Bolzano's texts, could be further from the truth. Such claims are difficult to square with remarks of Bolzano such as his 1810 claim, "The

[142] Bolzano, "On the Mathematical Method," 71.
[143] Rolf George, "Psychologism in Logic: Bacon to Bolzano," *Philosophy and Rhetoric* 30 (1997): 213.
[144] Ibid., 234.

history of mathematics shows increasingly that whatever has been accepted at first merely from experience is subsequently derived from concepts and therefore comes to be treated as a part of pure *a priori* mathesis".[145]

Bolzano's Semantic Analysis of Subjective Proofs

This section presents in greater detail the path that Bolzano advocates from subjective proofs to objective proofs in science. The hallmark of this transition from subjective to objective proofs is a semantic analysis of subjective proofs. This semantic analysis was not formalized until TS, but was carried out in all of Bolzano's works on mathematics.

The previous chapters have shown that the most important feature of an objective presentation of an *a priori* science is necessary connections of grounds and consequents, and that the most common sin against *a priori* science is proofs that cross to more intuitive, but alien, genera. Thus, as we have seen in Bolzano's mathematical works, the first step is to intensely analyze the prevailing proofs to uncover such foreign, intuitive elements. Bolzano describes this semantic analysis of subjective proofs in TS.

> We think a certain representation in itself, i.e. we have a corresponding mental representation, only if we think all the parts of which it consists, i.e. if we also have mental representations of these parts. But it is not necessarily the case that we are always clearly conscious of, and able to disclose, what we think. Thus it may occur that we think a complex representation in itself, and are conscious that we think it, without being conscious of the thinking of its individual parts or beings able to indicate them.[146]

Bolzano's description of this process of semantic analysis in TS, explicated in detail below, is presented as the transformation of propositions into the character of propositions-in-

[145] Bolzano, "Contributions to a Better-Grounded Presentation of Mathematics," 100.
[146] WL, Sec 56, p. 69. George translation.

themselves. Bolzano's semantic analysis would lay the broad outline for future axiomatic methods in analytic philosophy that begin with linguistic analysis of what is being said, and then proceed to situate a transformed, symbolic restatement of what is said into an axiomatic system of similarly transformed propositions.

Bolzano begins his exposition of this method with his famous distinction between thought propositions and propositions in themselves. By a propositions-in-itself, Bolzano means "any assertion that something is or is not the case, regardless whether or not somebody has put it into words, and regardless even whether or not it has been thought".[147]

> [T]he source of most errors in logic has been the lack of distinction between thought truths and truths in themselves, and between thought propositions and thought concepts on the one hand, and propositions and concepts in themselves on the other.[148]

When one speaks of spoken propositions, or of thought propositions, one tacitly recognizes propositions-in-themselves as differentiated from their articulation. This revelation is critical for Bolzano, for whom this distinction reveals that the intended meaning, and thus the basis in truth, of judgments is usually more than one is aware of.

Spoken propositions or thought propositions (judgments) are thus the subjective counterparts to propositions-in-themselves, which are objective propositions. Propositions-in-themselves consist of ideas-in-themselves but, indicating the primacy of propositions in Bolzano's logic, ideas are defined derivatively as anything that can be part of a proposition and is not itself a proposition.[149] Finally, for Bolzano, propositions and ideas-in-themselves constitute the matter [*stoff*] of subjective propositions and ideas, and are not multiplied when thought or

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[147] *WL*, Sec. 19, p. 20, George translation.
[148] *WL*, Sec. 12, p. 13, George translation.
[149] *WL*, Sec. 48, p. 61, George translation.

spoken by multiple people even as the subjective propositions and ideas are multiplied with each instance of being spoken or thought.[150]

The transformation of subjective propositions to propositions-in-themselves is a process of abstracting from figurative associations which adhere to a judgment in order to understand what is really being said, what is really intended. Essentially, according to Bolzano, all propositions can be rewritten in the objective form, 'Idea A has Idea B'. While this act of transformation is limited and, in fact, was also performed by Aristotle for whom the forms 'Idea A is Idea B' and 'Idea B is said of Idea A' can express all judgments, Bolzano also maintains that parts of propositions can themselves be propositions. The idea of a right triangle, for instance, is the idea of a 'triangle which has a right angle' which contains the idea, triangle, and the proposition, 'has a right angle'.[151] Bolzano demonstrates this act of transformation, and the fallacies of classification it exposes by revealing what one is really intending in a statement, in his examination of the definition of a proposition offered by Leibniz.

> The concept which is designated by Leibniz' expression *cogitation possibilis* (*loc. cit.*) is not composed of the two concepts *cogitatio* and *possibilis* in the same way in which many other concepts are generated by connecting two concepts, for example the concept of a golden candlestick. This is composed of two concepts designated, respectively, by a noun and an adjective, namely candlestick and something golden. The golden candlestick is a kind of candlestick, but the possible thought is not a kind of thought, but merely a kind of possibility. Therefore, if we wish to make the above expression more precise, we will have to say that, according to Leibniz, a proposition is "the possibility of a thought", or, even more clearly, "it is something that *can* be thought or *can* constitute the content of a thought." There is indeed no doubt that this thinkability is a property of any proposition, but it is also obvious that it does not form part of the concept of a proposition.[152]

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[150] *WL*, Sec. 48, p. 62, George translation.
[151] *WL*, Sec. 63, p. 79, George translation.
[152] *WL*, Sec. 23, p. 26, George translation.

Bolzano is often accused of Platonism with regard to his doctrine of propositions-in-themselves and ideas-in-themselves. This accusation misunderstands Bolzano's argument. First, the concept of an idea-in-itself is derivative of the concept of a proposition-in-itself for Bolzano, so any accusation of Platonism concerns only propositions-in-themselves. Second, the example just provided demonstrates that propositions-in-themselves are not different propositions from spoken propositions; rather, they are the intended meanings that constitute spoken propositions, whose full meanings are generally inaccessible to their speakers. In this regard, Bolzano answers this accusation as follows: "What is meant, then, when we assert that there are such truths? Nothing, I answer, but that certain propositions have the character of truths in themselves."[153] It is through the act of transformation that a spoken proposition takes on this character and discloses its full meaning. Third, while subjective propositions have existence, Bolzano goes out of his way to deny existence to propositions-in-themselves. They have neither *Sein*, nor *Dasein*, nor *Existenz*, nor *Wirklichkeit*.

This denial of the existence of propositions-in-themselves has perplexed interpreters of Bolzano who then ask what sense of 'are' is meant in the statement, 'Propositions-in-themselves are the matter of actual spoken propositions'. However, the Aristotelian and Thomistic tradition to which Bolzano often appeals walks a similar middle road between Platonism and nominalism, but with regard to the natures of things. Both substantial and accidental natures exist only in individuals for Aristotle and Thomas, yet the existence of such individuals is accidental to the natures as such.[154] While Bolzano does not appeal to

[153] *WL*, Sec. 30, p. 39, George translation.
[154] "That which is prior is always the reason for what is posterior, and when the posterior is removed, the prior remains but not conversely. Thence it is that what is attributed to a nature according to an absolute consideration is the reason for its being attributed to some nature according to the existence which it has in a singular, and not conversely. For Socrates is rational because man is rational, and not conversely. So if

Aristotle or Thomas in his denial that propositions-in-themselves exist, it seems consistent with Bolzano's Aristotelian approach to natures to suppose that Bolzano took the same approach to propositions-in-themselves.

Coffa, who viewed Bolzano as the first of a tradition of philosophers who employed semantic analysis, has presented the clearest articulation of Bolzano's purpose. Coffa answers the question of "the sense and purpose of foundationalist or reductionist projects such as the reduction of mathematics to arithmetic, or of arithmetic to logic".

> It is widely thought that the principle inspiring such reconstructive efforts was epistemological, that they were basically a search for certainty. This is a serious error. It is true, of course, that most of those engaging in these projects believed in the possibility of achieving something in the neighborhood of Cartesian certainty for those principles of logic or arithmetic on which a priori knowledge was to be based. But it would be a gross misunderstanding to see in this belief the basic aim of the enterprise. A no less important purpose was the clarification of what was being said.[155]

While it is very true that the essence of the grounding of a proposition is for Bolzano essentially a "clarification of what was being said", what is missing from this summary is the initial problem that led Bolzano to such semantic clarifications, avoidance of crossing to another kind within a proof. This Aristotelian impetus for semantic analysis was soon found missing in the tradition Bolzano founded, as it is from most interpretations of Bolzano himself. The reason for the non-classical character that Bolzano's revival of the classical model of science took in the form of analytic philosophy is the topic of the final chapter.

Socrates and Plato did not exist, rationality, would still be attributable to human nature." St. Thomas Aquinas, *Quodl.* VIII, I, I, ed. R Spiazzi (Marietti, 1956), 159.
[155] Coffa, "The Semantic Tradition from Kant to Carnap," 26.

V. Essentialism in Bolzano and in Analytic Philosophy

While Bolzano's methodology presented thus far appeals to classical logic at each stage, what distinguishes Bolzano's project from that of Aristotle and Aquinas is his assumption that *a priori* science studies beings as possible, rather than as actual and existing. Bolzano finds himself within an influential tradition beginning with and continuing through Wolff known as essentialism or possibilism, which views existence as univocal and philosophy as thus concerned with being as possible, shorn of considerations of existence.

In his later work, primarily his TS, Bolzano introduced an entirely non-Aristotelian step into his semantic analysis known as variation which was designed to generate variations of propositions across various possible worlds in order to uncover the conditions of possibility of different objects. Semantic analysis is primarily used by Bolzano to transform subjective into classically objective propositions. However, the influence of essentialism and possibilism on this otherwise traditional semantic analysis is the linchpin in the development of a revival of a classical model of science into analytic philosophy. This deviation from classical analysis ultimately led to the formalism and analysis of language that came to characterize future philosophy in Austria and beyond, though recent reactions to possibilism within analytic philosophy known as actualism and presentism are important to note.

Essentialism in Bolzano

Bolzano employed semantic analysis of subjective (thought or spoken) propositions to reveal the objective propositions that are intended in such thoughts or utterances. Bolzano's initial concern was to give subjective proofs the status of *a priori* science by eliminating appeals to intuitive, foreign concepts revealed through semantic analysis. Thus far in Bolzano's logic, parallel concerns and steps can be found in the classical logic of Aristotle and Aquinas.

Bolzano's departure from classical logic and theoretic science is found in his 1810 redefinition of the theoretic sciences discussed earlier in Chapter Two. Bolzano offers a new definition of mathematics as the "*science which deals with the general laws (forms) to which things must conform in their existence*",[156] which, we noted above, resembles the classical approach to science as a study of formal causes. However, Bolzano adds the following clarification: "Furthermore, if I say that mathematics deals with *the laws to which these things conform in their existence*, this indicates that our science is concerned not with the proof of the *existence* of these things but only with the *conditions of their possibility*."[157] That mathematics (which included all of natural philosophy for Bolzano) prescinds from any concern for "the existence of … things" appears to be an important point for Bolzano, as his subsequent section is devoted to developing it. After noting that his definition of mathematics is certainly not too narrow, he addresses whether this prescinding from existence raises the opposite concern.

> But I am all the more afraid that it might be found rather *too wide* and the objection might be made that it leaves too little for *philosophy* (metaphysics). The latter will be limited by my definition to the single concern of proving, from *a priori* concepts, the *actual existence* of certain objects. Mathematics and metaphysics, the two main parts of our *a priori* knowledge would, by this definition, be contrasted with each other so that the *former* would deal with the general conditions under which the existence of things is *possible*; the latter, on the other hand, would seek to prove *a priori* the *reality* of certain objects (such as the freedom of God and the immortality of the soul). Or, in other words, the *former* concerns itself with the question, *how must things be made in order that they should be possible*? The *latter* raises the question, *which things are real* – and indeed (because it is to be answered *a priori*) – *necessarily real*? Or still

[156] Bolzano, "Contributions to a Better-Grounded Presentation of Mathematics," 94.
[157] Ibid., 94.

> more briefly, *mathematics* would *deal* with *hypothetical necessity*, *metaphysics* with *absolute necessity*.[158]

Bolzano is here claiming for mathematics what 18th century philosophers such as Wolff and Stattler had moved into metaphysics under the name ontology (as discussed previously in Chapter Two, ontology addressed topics such as quantity, extension, space, motion and time). The metaphysical science of ontology, according to the first to use the term in 1647, Clauberg, is "that science which does not deal with this and that being, as distinct from the others owing to its special name or properties, but with being in general."[159] And being in ontology, according to Wolff, "is what can exist and, consequently, that with which existence is not incompatible.... What is possible is a being. [*Quod possible est, ens est.*]"[160] Thus, while Bolzano is on the one hand opposing Wolff's division of the sciences, he is more importantly continuing Wolff's study of being as possible being, free of existence, but under another name (mathematics instead of ontology). Why is this important? For an answer to that question, one must return to the time when this view of being as possible being was not taken for granted, as it had become in the 17th century.

Specifically, one must return to Suárez and, even further back, to Aquinas. For it is Suárez, whom Wolff deferentially refers to as "the Metaphysician", who closed the debate on the relationship between essence and existence in being for modern philosophy. This debate on the nature of being was a central dispute in medieval philosophy, and it is Aquinas who held the position opposite that of Suárez and Wolff.

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[158] Ibid., 94-95.
[159] Quoted in Etienne Gilson, *Being and Some Philosophers* (Toronto: Pontifical Institute of Mediaeval Studies, 1949), 112.
[160] Quoted in Gilson, *Being and Some Philosophers*, 114.

Aquinas argued that every finite being is constituted by two metaphysical co-principles, an act of existence and a limiting essence, which participates in and limits the act of existence, thus determining the mode of existence of a particular being. Existence is thus conceived by Aquinas in a maximal sense, not as a mere fact of existence tagged on as a property to essences. Thomas arrives at this position early in his career in his *De Ente*, where he begins by distinguishing between two meanings of *ens* (being) – *ens* as the truth of propositions and *ens* as that which is positive in reality. The primary sense of *ens*, according to Thomas, and thus the most fully real, is "that which is positive in reality".[161] This anticipates the discovery of the subject of metaphysics that Thomas describes soon after his completion of *De Ente* in his commentary on Boethius' *De Trinitate* which occurs after an initial discovery of concrete beings as real through a subsequent discovery of being as being (as existing, in the maximal sense of existence).[162]

Previous metaphysics in the Platonic tradition had identified form, or essence, as a primary sense of being, and were thus derivative of Plato's notion of forms as the primary sense of being.[163] These ultimately were various forms of essentialism. Thomas emphasized concrete *ens* (that which is "positive in reality") as the primary sense of being and thus overcame essentialism. In fact, immediately prior to his argument, the first argument of *De Ente*, that

[161] St. Thomas Aquinas, *On Being and Essence*, trans. Armand Maurer (Toronto: PIMS, 1968), 30. Thomas, with Aristotle, frequently identifies substance as the primary sense of being. But substance can itself be taken in the sense of substance as form or substance as concrete *ens*. In *De Ente*, Thomas makes clear that the primary sense of being is substance as concrete *ens*, for only it is "something positive in reality". St. Thomas Aquinas, *Commentary on Aristotle's* Metaphysics, trans. John P. Rowan (Notre Dame, Indiana: Dumb Ox Books, 1961), IV, c. 2.
[162] St. Thomas Aquinas, *The Division and Methods of the Sciences*, trans. Armand Maurer (Toronto: PIMS, 1986), Q. 5, a. 3.
[163] "I think that, if there is anything beautiful besides the Beautiful itself, it is beautiful for no other reason than that it shares in that Beautiful, and I say so with everything." Plato, *Phaedo*, 100b.

concrete *ens* is the primary meaning of *ens*, Thomas writes, "A slight initial error eventually grows to vast proportions".[164]

How is it that overlooking concrete *ens* as most fully real was for Thomas the "slight initial error" that had "grown to vast proportions"? For Thomas, if the primary sense of being is as concrete *ens* and not as form or essence, this reveals the necessity of *esse* (the 'to be') as the core of every real being such that the co-principles of *ens* could no longer be conceived as form and matter but instead were reconceived as *esse* and something else in virtue of which an *ens* has *esse*. This something-else-in-virtue-of-which-an-*ens*-has-*esse* is the primary meaning of essence for Aquinas.[165]

Suárez, in contrast, advocates a minimal notion of existence as signifying the bare fact of existence.

> [U]sed as a noun, *ens* signifies what has a real essence (*essential realis*), prescinding from actual existence, that is to say, neither excluding it nor denying it, but merely leaving it out of account by mode of abstraction; on the contrary, taken as a participle (namely, as a verb) *ens* signifies real being itself, that is, such a being as has both real essence and actual existence, and, in this sense, it signifies being as more contracted.[166]

This articulation of being, which Suárez inherited from Scotus and published in the 17th century, seems to have ended the debate on the constituent co-principles of being. As Gilson notes, "It seems then to be a fact that, in seventeenth-century classical metaphysics, essence reigns supreme."[167] This was particularly so in the German-speaking part of Europe where

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[164] Aquinas, *On Being and Essence*, 28.
[165] Thomas affirms this in the subsequent argument of *On Being and Essence*, Ch 1, which then turns from *ens* to essence to affirm that "essence is derived from *ens* in the first meaning of the term [as concrete *ens* positive in reality]" and thus the term "essence is used because through it, and in it, that which is has being". Aquinas, *On Being and Essence,* 30-32.
[166] Quoted in Gilson, *Being and Some Philosophers*, 98.
[167] Ibid., 111.

Spanish academics influenced by Suárez were sent by the Jesuits when they took over educational institutions following the Thirty Years War. The two most prominent 17th century philosophers in the University of Prague, Arriaga and Caramuel, were both Spanish disciples of Suárez.

It was thus an extension of the influence of these Spanish academics that Wolff came to be the preeminent philosophical authority in Austria into the eighteenth century. For Gilson, "Suárez begot Wolff" in the sense that after a century of Suárezian pronouncements of being as possible being, "the times [were] now ripe for some systematic science of 'being as being', as completely free from existence as being itself actually is".[168] Such was Wolff's metaphysical science of ontology. Gilson imagines that "[i]t is hardly possible to guess what would have happened to modern philosophy if, instead of teaching with Suárez that *operatio sequitur essentiam*, Wolff had taught with Thomas Aquinas that *operatio sequitur esse*."[169]

Bolzano, for whom mathematics addresses "the general conditions under which the existence of things is *possible*", is squarely within this tradition of essentialism. This is clearly in contrast to Aquinas and Aristotle, but how does such essentialism affect Bolzano's theory of science? Once one supposes that existence is merely a univocal fact, then diversity in reality, and the forms and laws governing diversity, can only be understood by reflection upon what is possible but may not be actual (not by reflection on the existing world). In other words, the discipline that now provides access to what is possible, not the narrower concern for what is actual, is not natural philosophy or empirical science but logic.

[168] Ibid., 112.
[169] Ibid., 118.

Bolzano anticipates this development in 1810, in which "a work on mathematical method would be basically nothing but logic".[170] Once having employed semantic analysis on subjective proofs within a science, the subsequent presentation of that science doesn't objectively demonstrate "what actually *takes place*" but instead "the *conditions* or *forms* which something must have *if it is to take place*".[171] Once one's concern is with the essence of an object, shorn of existence, then the axioms of an objective presentation are really hypothetical axioms of what an object could be were the property of existence added. These implications of essentialism are in tension with Bolzano's other statements asserting, in more classical terms, "that an axiom may appear *suspicious* and *doubtful*...because we do not immediately see that the things we recognize at once as true can be derived from it",[172] as discussed in Chapter Two. Nonetheless, Bolzano makes these essentialist implications explicit in the same 1810 work.

> For mathematics does not deal at all with what actually *takes place* but with the *conditions* or *forms* which something must have *if it is to take place*. Therefore it is only necessary to present those propositions offered by experience, purely hypothetically, and then to derive by *a priori* deductions, on the one hand, the *possibility* of these hypotheses, and on the other hand, the *consequences* which follow from them. Hence no empirical judgment appears in the whole exposition, and the science is therefore *a priori*. So, for example, in the exposition of optics there is no need for borrow from *experience* the law that light bends in going from air to glass in the ratio 3:2, and similar things; it is sufficient to make comprehensible *a priori* the mere *possibility* of a material like light and the fact that it bends in passing through different media. On this basis the statement is established in hypothetical form, '*if there is a material which etc....then this and that consequence must follow from it*'.[173]

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[170] Bolzano, "Contributions to a Better-Grounded Presentation of Mathematics," 103.
[171] Ibid., 100.
[172] Ibid., 119.
[173] Ibid., 100-101.

While this statement of methodology "present[s] those propositions offered by experience, purely hypothetically", by Bolzano's later work he had expanded logic in possibilist directions in order to uncover other possible forms of objects.

The evolution of logic in order to penetrate possible worlds began in Bolzano's later work, most primarily in TS. While semantic analysis was important to the early Bolzano in order to reveal spoken or thought propositions that cross to more intuitive, yet alien genera within a subjective proof, and thus ground the proof through more objective connections between propositions, it is to semantic analysis that Bolzano would add subsequent steps in TS to reveal the forms of things hiding in possible worlds. These subsequent steps begin to resemble analytic philosophy much more than do the methods of the early Bolzano. The reason for this is that these subsequent steps begin the process of untethering the axiomatic model of science from the existing world it was meant to isomorphically reflect in the classical model of science.

For Bolzano in TS, once one knows what a statement is really intending, then one is able to analyze the resultant semantically rich objective proposition, or, proposition-in-itself, in a manner previously impossible with only spoken propositions. Specifically, one is able to enquire into the satisfiability of a proposition-in-itself. Satisfiability is a property unique to propositions-in-themselves that is not possessed by subjective propositions. One discovers the satisfiability of a proposition-in-itself by varying one of its terms, as is explained below.

Bolzano argues that propositions-in-themselves are either true forever or false forever, unless we change some part of the proposition in which case we end up with a different proposition. "We do this frequently without being clearly aware of it; this is one of the reasons why it seems as if the same proposition could sometimes be true and sometimes false, depending on the different times, places or objects to which we relate it."[174] Thus, "the keg of wine costs 10 Thaler" is true in one place and time and false in another, as is "this flower has a pleasant fragrance". When considered as subjective propositions existing as thoughts or speech, such statements appear to be true in some cases and false in others. The critical role played by propositions-in-themselves which constitute spoken and thought propositions is the variation of parts of propositions-in-themselves that is possible since their full semantic content (including time, place, references for words such as 'this') is made explicit. Since we do implicitly vary the parts of our propositions very often, Bolzano asks what would happen if we brought this act of variation up to explicit awareness as well.

> These examples show that we often take certain ideas in a given proposition to be variable and, without being clearly aware of it, replace these variable parts by certain other ideas and observe the truth values which these propositions take on. Since we do this anyhow, it is worth the effort to undertake this procedure with full consciousness and with the intention of gaining more precise knowledge about the nature of such propositions by observing their behavior with respect to truth.[175]

For example, Bolzano considers the proposition 'The man Caius is mortal', and finds that by treating the idea 'Caius' as arbitrarily variable, "it becomes obvious that the new propositions which are thus generated are all true, without exception".[176] On the other hand, treating the idea 'Caius' as variable in the proposition 'The being Caius is mortal' generates propositions some of which are true and others false. "All this should make it clear that the truth-values of all the propositions which can be generated from a given proposition through the assumption of one of more arbitrarily variable parts may be envisaged as a quality which

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[174] WL, 147, p. 194, George translation.
[175] WL, Sec. 147, p. 194, George translation.
[176] WL, Sec. 147, p. 195, George translation.

makes better known to us the nature of the original proposition itself."[177] This quality, satisfiability [*Gültigkeit*], is an "inner attribute" of every proposition, and expresses "the relation of all true propositions to the total of all propositions which can be generated by treating certain ideas in a proposition as variables and replacing them with others according to a certain rule".[178] Satisfiability is clearly relative to the ideas in a proposition-in-itself which are taken to be variable. When all propositions generated by replacing an idea with other ideas with reference are true, as in the case of 'The man Caius is mortal' with reference to the idea 'Caius', then the proposition is *universally* or *fully* satisfiable with respect to that idea.[179]

Bolzano thus adds the step of variation to his semantic analysis. The result is a broadening of the scope of his objective propositions from the semantically-rich versions of what is fully intended in subjective propositions to also include the set of generated variations of such propositions that are analytically true in Bolzano's larger sense. For most propositions are such that the propositions generated by treating individual ideas as variable are neither all true nor all false.[180] The significance of propositions that are fully satisfiable with respect to at least one idea is unclear, though it constitutes for Bolzano an expansion of the domain of analytic truths.

> But it would be important enough to deserve notice if a proposition contained even a single idea which could be arbitrarily changed without altering the truth or falsity of the proposition; i.e., the propositions which could be obtained from it through the arbitrary alteration of this one idea would either all be true or all false, provided only they have reference. Borrowing this expression from Kant, I allow myself to call propositions of this kind *analytic*. All other propositions, i.e. all those which do not contain any ideas which can be changed without altering their truth or

[177] Ibid.
[178] *WL*, Sec. 147, p. 196, George translation.
[179] *WL*, Sec. 147, p. 197, George translation.
[180] *WL*, Sec. 147, p. 195, George translation.

falsity, I shall call *synthetic*.[181]

Bolzano thus broadens the scope of what Leibniz and Kant took to be analytic truths from those truths whose predicate is contained in its subject to now include those truths that are fully satisfiable with respect to at least one idea. The former type of analytic truths requires only logical knowledge to identify and is referred to as "logically analytic" truths. The latter type of analytic truths is referred to as "analytic in the broader sense".[182] Bolzano's logic thus seeks to undermine the form-content distinction that had animated "vague" conceptions of logic as a formal science by asserting that "[l]ogic must indeed *attend* to the differences between possible objects of thought in so far as they influence the rules for the investigation of these objects".[183] The *a priori* truths of mathematics and natural philosophy are "analytic in the broader sense", for Bolzano.

The form of a proposition, revealed through variation as discussed above, also helps us identify when relations of ground and consequence exist between propositions. This relation is present when (1) the variation of the same idea within two true propositions always generates two true propositions (such that they are deducible from each other), and (2) the truths generated by one of the propositions "are genuine consequences of the truths generated from" the other.[184] While Bolzano's second criteria for ground-consequent relations shows that he certainly does not treat ground-consequence as an external characteristic of propositions, Bolzano apparently does assume that "genuine" cases of ground-consequence

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[181] *WL*, Sec. 148, p. 198, George translation.
[182] *WL*, Sec. 148, p. 199, George translation.
[183] *WL*, Sec. 12, p. 14, George translation.
[184] *WL*, Sec. 162, p. 247, George translation.

are more easily identifiable once the full semantics of propositions, including their satisfiability, are made explicit in propositions-in-themselves.[185]

It is contended here that it is through expansions of logic to deal with possible things that subsequent analytic philosophy developed its distinctive character. The manner of this development is addressed below, but it suffices to mention at this point that Frege's truth, "The square root of Napoleon's mother is the false", is the type of truth, unhinged from positive reality, that is unique to analytic philosophy and is also analytically true in Bolzano's method of variation and thus part of *a priori* science for Bolzano. As one commentator notes, "The variation of ideas-as-such in propositions is an original and very fruitful conception. Bolzano exploits his innovation with great thoroughness and accuracy in a logical theory which, on decisive points, anticipates notions expounded in modern logical semantics and axiomatic."[186]

Essentialism in Analytic Philosophy

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[185] "It happens all too often that the words 'ground' and 'consequence' makes us think of merely subjective grounds and consequences in the order of knowing, i.e. of truths which, as premises, produce a cognition, or which follow from another cognition. By contrast, I will sometimes call the grounds and consequences in the present sense *objective* grounds and consequences, in order to indicate that this relation holds among truths *in themselves* independent of our recognition....It cannot be denied that this connection between truths in themselves has not often been discussed; but this is not surprising. First of all those who had framed the concept of a truth in itself at all did not spend a lot of time over it; secondly, the relation between ground and consequence has great similarity with a pair of other relations, namely the relation between truths that are deducible from each other, and the relation that holds between real things when one is the cause of the other. Hence it is not surprising that our relation was confused with one or the other of these and that this confusion did not become apparent since it did not lead to any obvious contradiction. The schoolmen already distorted the matter by using words ratio (*ground*), cause (*cause*) and principium (*origin*) as synonymous, and defining the concept of these words as that which *determines* another (*id, quod determinat*)." *WL*, Sec. 198, pp. 271-273, George translation.

[186] Jan Berg, "Bolzano, The Prescient Encyclopedist," in *Bolzano and Analytic Philosophy*, 15.

As noted in Chapter Four, Bolzano is generally assumed to ascribe some type of existence to propositions-in-themselves.[187] However, while this is more true of Frege than Bolzano and reveals another way in which a Fregean Bolzano infiltrates Bolzano studies, it also reveals the nature of Bolzano's legacy. For once our *a priori* propositions are untethered from what actually exists in favor of the broader ground of possible existence, then we are forced to assert the full-blown reality of either *a priori* propositions themselves or of possible worlds. Otherwise, we are using nothing to discuss nothing. The former leads to Frege, who very clearly asserted the existence of objective propositions, which he termed thoughts (*Gedanken*). Frege began a tradition of reducing ontology to logic that Smith has called Fantology, which includes Russell and the early Wittgenstein. The latter option – ascribing full-blown reality to possible worlds – leads to the possible worlds logic of Kripke, Carnap and Lewis, as well as the Bolzanian method of variation employed by Wittgenstein and Kripke. Both share the critical essentialist premise that we understand what is many and diverse in reality, and the necessary forms therein, by turning to what is possible and not to what is actually existing. These two options are discussed in turn.

Frege, like Bolzano, developed an axiomatic model of science based on a prior analysis and transformation of the propositions within that science, but Frege's transformation is more extensive than Bolzano's. Where Bolzano viewed predicate and subject as the basic form of a proposition-in-itself, Frege viewed function and object as this basic form. Frege's

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[187] "Bolzano's propositions in themselves are, of course, the equivalent of Frege's thoughts. He held, like Frege, that ideas and propositions in themselves are objective and do not depend upon our apprehension of them; but, precisely like Frege, he viewed them as not being 'actual', which he also expressed, as Frege did not, by denying 'existence' to them. By this he seems to have meant what Frege meant by speaking of thoughts and other objects as objective and not actual, namely that they do not enter into any causal transactions." Dummett, 23-24. Bolzano went well beyond this in denying existence to propositions in themselves. "What is meant, then, when we assert that there are such truths? Nothing, I answer, but that certain propositions have the character of truths in themselves." *WL*, Sec. 30, p. 39, George translation

functional approach to predication enabled him to transform statements into an abstract form that permitted a more general treatment of inferences than was possible in the logic of Aristotle or of Bolzano. The inferential systems enabled by the predicate calculus of Frege were complete axiomatic systems.

Interpreters of Bolzano thus frequently characterize Bolzano's analysis as lacking the more well-developed semantic transformation and axiomatic system of inferences found in Frege. According to Peter Simons, "[w]here he falls short of Frege is that he does not have the concept of a formal system, where axioms are laid down and theorems follow by precisely defined syntactic rules of inference. Bolzano on the other hand prefers to work throughout with semantic concepts....The basic form of proposition for Bolzano is *A has b*, where *A* is the subject-concept and *b* is an abstract name for a predicate-concept, e.g. instead of *This is red* he would say *This has redness*. He even thought every proposition could be tortured into this form."[188] J. Alberto Coffa is more direct: "The central ideal of logical analysis, the realization that language is an extraordinarily misleading guide to content, was still in the future. For Bolzano language was a rather reliable picture of the form of objective propositions."[189] Coffa quotes Frege in support this assessment, "The business of the logician is to conduct an ongoing struggle...in part against language and grammar insofar as they fail to give clear expression to the logical".[190] Rolf George summarizes this view, "Without doubt, Bolzano's insistence on a common form for all propositions was detrimental to the development of several aspects of the system".[191]

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188 Peter Simons, "Bolzano, Brentano and Meinong: Three Austrian Realists," in *German Philosophy since Kant*, ed. Anthony O'Hear (Cambridge: University of Cambridge Press, 1999) 113-14.
189 Coffa, *The Semantic Tradition from Kant to Carnap*, 39-40.
190 Frege, "Logik", quoted in Coffa, *The Semantic Tradition from Kant to Carnap*, 64.
191 Rolf George, "Editor's Introduction",in *WL*, p. xxxiii, George translation.

While Frege's logicist approach to ontology is a natural evolution of Bolzano's essentialism, as discussed below, it seems clear that Bolzano himself would not condone the non-linguistic, symbolic transformation of linguistic statements developed by later analytic philosophers beginning with Frege. For Bolzano's reason for beginning semantic analysis with the subjective proofs of others is rooted in his belief that these subjective proofs provide the essential context of given truths that necessarily precede analysis. In other words, subjective proofs provide an anchor in intuition, in what exists. While Bolzano agrees with Kant that all ideas are either intuitions or concepts, he vigorously disagrees with Kant's view that concepts are indirectly related to objects via mediate intuitions. "The concepts 'man', 'living being', and others represent something real just as immediately as any other idea, e.g. an intuition, such as 'Socrates'."[192] It is precisely because these judgments provide the givens of experience that analysis must attend to the formal structure of the relevant domain of experience rather than transform these judgments into a purely symbolic language. Symbolic language, for Bolzano, is unable to attend to specific domains of experience.

> Finally, I do not understand how it can be said that in logic we think the objects "altogether indeterminate as to their inner characteristics." For, if we think an object as altogether indeterminate, then we cannot assert anything of it. This whole definition probably stems only from the fact that in the examples which are used in logic, e.g. in the syllogism: all A or B, all B are C; therefore all A are C, the signs A, B and C may mean, as we say, "anything".[193]

Analysis after Bolzano is thus deprived of its essential context of a prior intuition of the whole that, while true, is pre-scientific and thus requiring of analysis to clarify its deductive structure. It is the loss of this context that marks the transition from Bolzano to the linguistic turn.

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[192] *WL*, Sec. 77, p. 105, George translation.
[193] *WL*, Sec. 7, p. 9, George translation.

Nonetheless, Frege's premise that the structure of a logical language is the key to the structure of reality is a natural development of Bolzano's possibilism, for while non-existing worlds cannot be analyzed, the logic to which they would need to conform can be analyzed. Smith thus refers to "the fantologists' assumption that 'existence' is univocal – it is in every case a property of what Frege called concepts or functions – and is captured in the E".[194] The result, for Smith, is that "Frege's object/function distinction rides roughshod over two traditional ontological distinctions, between *substance* and *property* and between *particular* and *universal*."[195]

Geach tries to save Frege on the issue of existence by asserting that "Frege was clear as the…difference between the two senses of 'is' as follows: An individual may be said to 'be', meaning that it is at present actually existing; on the other hand, when we say that 'there is' an X (where 'X' goes proxy for a general term), we are saying concerning a kind or description of things, Xs, that there is at least one thing of that kind or description." While Frege was clear on this distinction, according to Geach, "he rightly had no special interest, as a *mathematical* logician, in assertions of present actuality". However, the distinction that Geach asserts Frege to be clear on is a distinction between *sum*, on the one hand, and mere factual existence, on the other. No mention is made of existence in a maximal sense, *esse*, which is the constituting core of beings for Aquinas.

However, that even assertions of present actuality were too maximal for Russell is argued by Geach: "It is a great misfortune that Russell has dogmatically reiterated that the 'there is' sense of the substantive verb 'to be' is the only one that logic can recognize as legitimate; for

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[194] Barry Smith, "Against Fantology," in *Experience and Analysis*, ed. M.E. Reicher and J.C. Marek (Vienna: ÖBV & HPT, 2005), 157.
[195] Smith, "Against Fantology," 160.

the other meaning – present actuality – is of enormous importance in philosophy, and only harm can be done by a Procrustean treatment which either squeezes assertions of present actuality into the 'there is' form or lops them off as non-sensical".[196] We thus find Russell arguing that "logic is concerned with the real world just as truly as zoology, though with its more abstract and general features".[197] We see likewise in Wittgenstein ("Just as the only necessity that exists is logical necessity, so too the only impossibility that exists is logical impossibility")[198] the prescinding from existence to determine essential reality through logic.

While Frege, Russell and the early Wittgenstein ascribed some level of reality to objective propositions as displaying the logical form of potential reality, other analytic philosophers ascribed reality to possible worlds. Carnap followed Leibniz's conception of necessity in terms of possible worlds and Wittgenstein's characterisation of logical truth as tautological to advance the beginnings of a possible worlds analysis, by asserting that time consists of a series of static worlds. Tarski more explicitly asserted the full-blown reality of possible worlds, as did David Lewis and Jaakko Hintikka, introducing a neo-Leibnizianism into analytic philosophy that is the natural development of a possibilist approach to being and thus to *a priori* science.

The ascribing of full-blown reality to these domains (Frege's thoughts existed in a "third realm") is obviously unsatisfying to many.[199] Some reacted, as Dummett describes, by "look[ing] about…to find something non-mythological but objective and external to the

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[196] G.E.M. Anscombe and Peter Geach, *3 Philosophers: Aristotle, Aquinas, Frege* (Oxford: Basil, Blackwell & Mott, 1961), 90-91.
[197] Quoted in Smith, "Against Fantology," 155.
[198] Quoted in Smith, "Against Fantology," 154.
[199] "One may yet feel unhappy with the ontology mythology which, as we have seen, was already in some tension with Frege's more detailed accounts of particular senses". Michael Dummett, *Origins of Analytical Philosophy*, (Cambridge: Harvard University Press, 1993), 25.

individual mind to embody the thoughts which the individual subject grasps and may assent to or reject. Where better to find it than in the institution of a common language?" Dummett thus sees the linguistic turn a reaction to the "ontological mythology" that was erected by early analytic thinkers in order to extrude concepts and propositions from the mind. "Given the initial step taken by Bolzano, and followed by Frege, Meinong and Husserl, whereby thoughts were removed from the inner world of mental experience, the second step, of regarding them, not as merely transmitted, but as generated, by language, was virtually inevitable: it is puzzling only why it took so long".[200]

Others, however, reacted by making a distinction between the reality of possible worlds and the reality of actual worlds. Prior, Chisholm and Zimmerman, for example, expound a doctrine called presentism according to which reality is not temporally extended, and only what exists now truly exists. Plantinga is the most forceful voice for actualism, which rejects the reality given to possible worlds. Loux summarizes the significance of these positions within analytic philosophy.

> Whether the focus is the modal framework of possible worlds or the framework of times, if we accord full-blown reality to all the frames making up the framework, we seem committed to denying that an individual existing in one frame can be literally identical with items in any other frame, and we are forced to hold that the idea of an item that is stable across frames is the idea of something that is a kind of aggregate of numerically different items from different frames. But if we attribute special ontological status to just one frame in the framework and claim that its contents alone constitute what really exists, then we can accommodate the idea that an individual from that privileged frame can be literally identical with an individual from some other frame.[201]

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[200] Dummett, *Origins of Analytical Philosophy*, 25-26.
[201] Michael Loux, *Metaphysics: A Contemporary Introduction* (New York: Routledge, 2002), 237-238.

This attribution of "special ontological status to just one frame in the framework" of space and time is precisely what defenders of the Thomistic view of being held as the starting point of philosophy. For reflection on the "special ontological status" of what is "positive in reality", in Aquinas words, reveals existence in a maximal sense to be the constituting core of beings. It is the prevalence of essentialist approaches to being, prescinding from existence, which shaped the character of the revival of the classical model of science championed by Bolzano.

The two most frequently noted aims of analytic philosophy are an axiomatic system of knowledge and a philosophic account of language.[202] This thesis has argued that both of these aims are in fact derivative of Bolzano's 19th century reintroduction of the classical model of science as the study of causes within the context of modern mathematical science. The distinctive character, however, that the classical model of science took in the hands of Bolzano and subsequent analytic philosophers is due to the modern premise that any *a priori* science studies things as possible, and thus as thinkable (rather than studying things as existing). This premise detached the classical scientific formalisms of Aristotle, Euclid and Aquinas from the positive reality that they isomorphically reflected, leaving simply formalism and language. In the seminal works of Bolzano, we find both the explicit concern to reintroduce classical scientific formalism as well as an anachronistic possibilism inherited ultimately from Suárez.

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[202] "What distinguishes analytical philosophy, in its diverse manifestations, from other schools is the belief, first, that a philosophical account of thought can be attained through a philosophical account of language, and, secondly, that a comprehensive account can only be so attained." Michael Dummett, *Origins of Analytical Philosophy* (Cambridge, Massachusetts: Harvard University Press, 1993), 4.

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