Dynamic Semantics, Imperative Logic and Propositional Attitudes

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[1] **Appendix** by Berislav Žarnić and Damir Vukičević

1 'One move' eliminative semantics

Veltman's simple update system. The main features of the system are given in the following citation.

> Let W be the powerset of the set A of atomic sentences. σ is an information state iff $\sigma \subseteq W$; 0, the minimal state, is the information state given by W; 1, the absurd state is the information state given by the empty set;[...]
>
> $\sigma[p] = \sigma \cap \{w \in W \mid p \in w\}$
> $\sigma[\neg\varphi] = \sigma - \sigma[\varphi]$
> $\sigma[\varphi \wedge \psi] = \sigma[\varphi] \cap \sigma[\psi]$
> $\sigma[\varphi \vee \psi] = \sigma[\varphi] \cup \sigma[\psi]$
> $\sigma[might\ \varphi] = \sigma$if $\sigma[\varphi] \neq 1$
> $\sigma[might\ \varphi] = 1$ if $\sigma[\varphi] = 1$
>
> [...] If $\sigma[\varphi] \neq 1$, φ is acceptable in σ. If $\sigma[\varphi] \neq 1$, φ is not acceptable in σ and if $\sigma[\varphi] = \sigma$, φ is accepted in σ. [...] sequence of sentences $\varphi_1; ...; \varphi_n$ is consistent iff there is an information state σ such that $\sigma[\varphi_1] ... [\varphi_n] \neq 1$. [32] p.228.

1.1 Some remarks

In the dynamic semantical framework one is not supposed to think of meaning only as a relation between a sentence and the world (including possible ones). Rather meaning is conceived as result of an interpretation process. In this case, interpretation is understood as elimination process. In a metaphor, interpreter entertains different candidate pictures of yet unknown situation, sentences are picture fragments, and interpretation consists of comparing picture fragments with candidate pictures and removing the nonfitting ones. The number of remaining pictures measure the amount of information, the more pictures remain the less information a text conveys. The residual pictures create a context for the next step in interpretation process. The truth value of a sentence may be unsettled at a particular stage if the context contains both pictures having and pictures lacking the fragment. Sentence being true in a context means that the context is not empty and that the sentence does not have a power to eliminate any picture from it.

One-move system. The simple update system is basically a 'one move' system with basic instruction $\sigma[\varphi] = \sigma \cap \|\varphi\|^W$, where $\|\varphi\|^W$ is a set of valuations $v \in W$ verifying φ in the standard sense.

Context dependent intension. A sentence gets its meaning in the context created by other sentences. A context dependent intension $\sigma[\varphi]$ of sentence φ is the intersection of its context free intension $\|\varphi\|^W$ and a context σ.

Semantic value. Sentences are state or context transitions. Relative to a context σ a sentence φ may has one among four semantic values. A sentence is either (α) accepted and acceptable, or (β) not accepted and acceptable, or (γ) not accepted and not acceptable, or (δ) accepted and not acceptable in a given context.

$$V(\varphi, \sigma) = \begin{cases} \alpha \text{ if } \sigma[\varphi] = \sigma \wedge \sigma[\varphi] \neq 1 \\ \beta \text{ if } \sigma[\varphi] \neq \sigma \wedge \sigma[\varphi] \neq 1 \\ \gamma \text{ if } \sigma[\varphi] \neq \sigma \wedge \sigma[\varphi] = 1 \\ \delta \text{ if } \sigma[\varphi] = \sigma \wedge \sigma[\varphi] = 1 \end{cases}$$

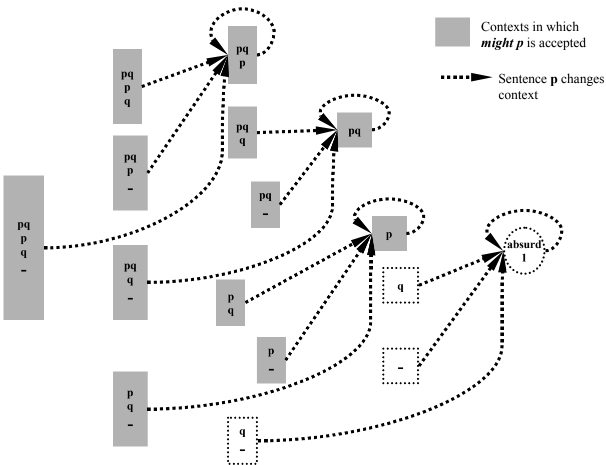
Update paths. We call a sentence φ basic if its meaning is defined only in terms of the basic intersective action (e.g. $p \vee q$ is basic since $\sigma[p \vee q] = \sigma \cap \|p \vee q\|^W$). The meaning of a complex sentence is defined in terms of executability of basic actions (e.g. *might* p is complex sentence).

Define meaning potential of a sentence φ in the simple update system *dyn* relative to the set of contexts Σ as a relation $\|\varphi\|^{\Sigma}_{dyn} = \{\langle\sigma, \sigma'\rangle \mid \sigma[\varphi] = \sigma'\}$, where $\Sigma = 2^W$.

transition type $\subseteq \|\varphi\|_{dyn}^{\Sigma}$		success	basic	test
$\alpha\,[\varphi]\,\alpha =$	$\left\{\langle\sigma,\sigma'\rangle \in \Sigma^2 \mid \begin{array}{c} V(\varphi,\sigma) = \alpha \wedge \\ V(\varphi,\sigma') = \alpha \end{array}\right\}$	+	+	+
$\beta\,[\varphi]\,\alpha =$	$\left\{\langle\sigma,\sigma'\rangle \in \Sigma^2 \mid \begin{array}{c} V(\varphi,\sigma) = \beta \wedge \\ V(\varphi,\sigma') = \alpha \end{array}\right\}$	+	+	-
$\gamma\,[\varphi]\,\delta =$	$\left\{\langle\sigma,\sigma'\rangle \in \Sigma^2 \mid \begin{array}{c} V(\varphi,\sigma) = \gamma \wedge \\ V(\varphi,\sigma') = \delta \end{array}\right\}$	-	+	-
$\delta\,[\varphi]\,\delta =$	$\left\{\langle\sigma,\sigma'\rangle \in \Sigma^2 \mid \begin{array}{c} V(\varphi,\sigma) = \delta \wedge \\ V(\varphi,\sigma') = \delta \end{array}\right\}$	-	+	+

Since the simple update system is eliminative, there are only four transition types. If one is interested in meaning relations rather then in modelling of cognitive dynamics, he may disregard 12 remaining transition types. E.g. revision transition type $\gamma\,[\varphi]\,\beta$ requires some worlds to be restored.

Distinction between basic and test sentences is visible in their transition types. Basic sentences allow of all four types, while tests allow only of two. Tautology is identity relation: $\|\top\|_{dyn}^{\Sigma} = \alpha\,[\top]\,\alpha \cup\, \delta\,[\top]\,\delta = \left\{\langle\sigma,\sigma'\rangle \in \Sigma^2 \mid \sigma = \sigma'\right\}$, while contradiction is all-to-one relation: $\|\bot\|_{dyn}^{\Sigma} = \gamma\,[\bot]\,\delta \cup\, \delta\,[\bot]\,\delta = \left\{\langle\sigma,\sigma'\rangle \in \Sigma^2 \mid \sigma' = \delta\right\}$.



Update paths for sentence p in the set of contexts $\Sigma = 2^W$.

A comparison with Łukasiewicz' three-valued system. The eliminative semantics provides a simple solution for the problem of unknown truth-value. In order to compare eliminative system with Łukasiewicz' three-valued system we can connect dynamic values with static ones: dynamic value α corresponds to 1, β to $\frac{1}{2}$, γ and δ (i.e. value in a nonabsurd state and value in the absurd state, respectively) correspond to 0. Counterintuitive principle $\Diamond\varphi \wedge \Diamond\psi \vDash \Diamond(\varphi \wedge \psi)$ is valid in Łukasiewicz' logic, but it is not valid in the simple update system. If both p and $\neg p$ are epistemically possible, then surely $p \wedge \neg p$ is not epistemically possible. In Łukasiewicz system, if $V(p) = \frac{1}{2}$ and $V(\neg p) = \frac{1}{2}$, then $V(p \wedge \neg p) = \frac{1}{2}$ and $V(\Diamond(p \wedge \neg p)) = 1$. In the simple update system, if $V(p, \sigma) = \beta$ and $V(\neg p, \sigma) = \beta$, then $V(p \wedge \neg p, \sigma) = \gamma$ and $V(\Diamond(p \wedge \neg p), \sigma) = \gamma$.

static

$\wedge$	1	$\frac{1}{2}$	0
1	1	$\frac{1}{2}$	0
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
0	0	0	0

$\neg$	
1	0
$\frac{1}{2}$	$\frac{1}{2}$
0	1

$\Diamond$	
1	1
$\frac{1}{2}$	1
0	0

dynamic[1]

$\wedge$	$\boldsymbol{\alpha}$	β	γ	δ
$\boldsymbol{\alpha}$	$\boldsymbol{\alpha}$	β	γ	■
β	β	?	γ	■
γ	γ	γ	γ	■
δ	■	■	■	δ

$\neg$	
α	γ
β	β
γ	α
δ	δ

might	
α	α
β	α
γ	γ
δ	δ

1.2 Semantic notions

Consequence relations. *No adding* metaphor of valid conclusion is explicated as *no removal* case. Within this framework one can define different variants of *no removal* consequence, three

[1] '■' indicates that a particular combination of semantic values is not possible. '?' indicates that value cannot be calculated for the general case; in some cases $\beta \wedge \beta = \beta$ and in others $\beta \wedge \beta = \gamma$.

variants are discussed in [32] (reproduced here in a slightly modified notation and denoted by $\vDash_{0-ut}$, $\vDash_{ut}$, $\vDash_{tt}$). It is important to note that using dynamic semantics one can define other meaning relations that resemble consequence relation, one of them is added here, namely $\vDash_{uu}$[5].

1. 0-update-to-test consequence: $\varphi_1; ...; \varphi_n \vDash_{0-ut} \psi$ iff

$$0\,[\varphi_1]\,...\,[\varphi_n] = 0\,[\varphi_1]\,...\,[\varphi_n]\,[\psi]$$

2. Update-to-test consequence: $\varphi_1, ..., \varphi_n \vDash_{ut} \psi$ iff

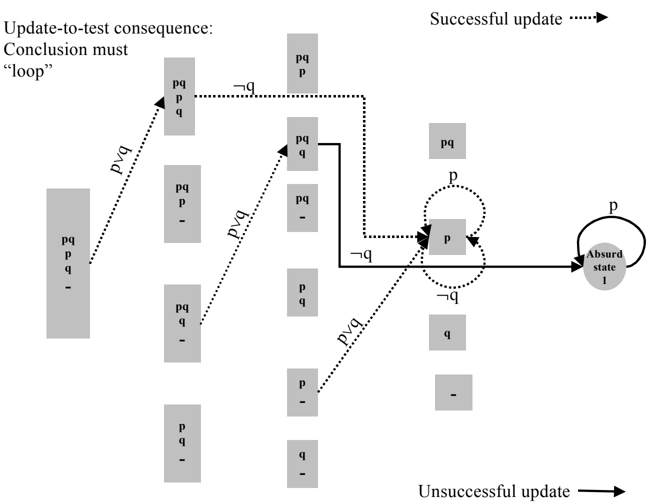
$$\forall \sigma : \sigma\,[\varphi_1]\,...\,[\varphi_n] = \sigma\,[\varphi_1]\,...\,[\varphi_n]\,[\psi]$$

3. Test-to-test consequence: $\varphi_1; ...; \varphi_n \vDash_{tt} \psi$ iff

$$\forall \sigma : \sigma\,[\varphi_1] = ... = \sigma\,[\varphi_n] \rightarrow \sigma\,[\varphi_1]\,[\psi] = ... = \sigma\,[\varphi_n]\,[\psi]$$

4. Update-to-update consequence: $\varphi_1; ...; \varphi_n \vDash_{uu} \psi$

$$\forall \sigma : \sigma\,[\varphi_1]\,...\,[\varphi_n] \neq 1 \rightarrow \sigma\,[\varphi_1]\,...\,[\varphi_n]\,[\psi] \neq 1$$



Some update paths for text $p \vee q; \neg q; p$. Sentence p ”adds nothing” in the contexts created by preceding two sentences. If the starting point is not 0, then some information has already been accepted.

First two notions of validity, $\vDash_{0-ut}$and $\vDash_{ut}$ may be useful in the imperative logic for semantic phenomena that exhibit order sensitivity and defeasibility. They do not have "structural properties" of classical consequence relation [5] [21](table below shows some properties).

	reflexivity	permutation	monotonicity
	$\dfrac{}{...,C,...\vDash C}$	$\dfrac{X,P_1,P_2,Y\vDash C}{X,P_2,P_1,Y\vDash C}$	$\dfrac{X,Y\models C}{X,P,Y\vDash C}$
0-update-to-test	-	-	-
Update-to-test	-	-	only left
Test-to-test	+	+	+

Given these weaker notions of validity, one may hope to discover cases of "weak inclusion" of meaning. Weak inclusion may be lost by changing the order of sentences or by adding new ones. Such a weaker notion of validity was implicitly present in number of works on practical logic (e.g. [14], [19], [24], [33]).

Another semantic notion that will be used here (see 5.1 below) is the notion of dynamic coherence of a text. It requires existence of a nonabsurd state in which every sentence from the text is accepted. Dynamic coherence corresponds to the static notion of satisfiability. Dynamic consistency, on the other hand, points to a transition path and it may be realized by an incoherent text.

Coherence. A sequence of sentences $\varphi_1;...;\varphi_n$ is coherent [20] iff $\exists\sigma : \sigma \neq 1 \wedge \sigma[\varphi_1] = ... = \sigma[\varphi_n]$.

Example 1 (Order sensitivity.) *"It might be raining...It is not raining" is a consistent and incoherent text, reversing the order gives an inconsistent text. "It is impossible to open this window...You should open this window" is an inconsistent text, reversing the order gives consistent, yet incoherent text.*

Example 2 (Defeasibility.) *"(i) Make people laugh! (ii) If you tell that joke, you will make them laugh. So, (iii)* ***maybe you should tell the joke.****" seems to be a valid argument. The*

conclusion *is defeated by additional premises: " (iv) If I tell that joke, I will ruin my wife's good mood. (v) Let it be the case that my wife's good mood is sustained!"*

The last example can be analyzed in the framework of Geach's characterization of defeasibility of practical inference..

> But even if a conclusion is validly drawn from the acceptable premises, we are not obliged to accept it if those premises are incomplete... (Geach [19] p. 77.)

If we follow his line of thought throughout the Example 2, then a person who accepts (i) and (ii) and who thinks that (iii) is their consequence nevertheless is "not obliged to accept" (iii) if he considers "the premises to be incomplete" since (iv) and (v) are missing.

Example 3 (Nonreflexivity) *"Make people laugh! If you tell that joke, you will make them laugh. Maybe you should tell the joke... If you tell that joke, you will ruin your wife's good mood. Sustain wife´s good mood! So,* ***maybe you should tell the joke****." does not seem to be a valid argument.*

2 Simple update system for imperatives

Moods and modal elements.

> Imagine a picture representing a boxer in a particular stance. Now, this picture can be used to tell someone how he should stand, should hold himself; or how a particular man did stand in such-and-such situation; and so on. One might (using the language of chemistry) call that picture a proposition-radical. [35] §23.

Speaking in terms of the picture-metaphor: the picture, or rather - picture fragment means something, but its meaning is *unsaturated* until it has been used in a certain way. The active part, *use* is modal element[1] of the sentence, the passive its sentence-radical. $M\varphi$ is a "saturated expression" having sentence-radical φ and modal element M. The sentence-radical may be used for different semantic actions, and the way of its use is determined by the modal element of sentence.

The dynamic semantics provides a natural way of thinking about meaning of natural language sentences: different types of semantic actions correspond to different moods. The number of conceivable types of semantic actions exceeds the number of natural language moods. Still the standard threefold division into indicatives, imperatives and interrogatives may (and perhaps should) be sustained. There are two obvious ways to do so. First, one may divide the class of generated states in such a

[1] This terminology is used in [30], but not in the identical sense. While Stenius takes modal element in the sense of mood indicator, here sentence moods are taken to be a category of modal elements.

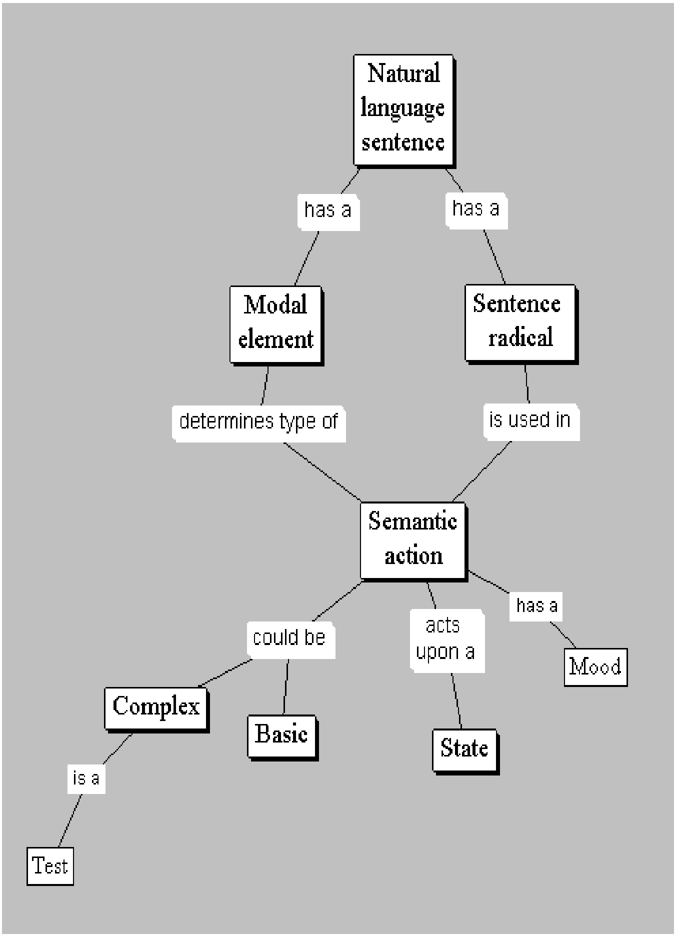


Figure 1 Modal element, sentence radical and semantic action.

way to define a characteristic property for each mood[2]. Second, one may define sentence mood in terms of inter-mood and intra-mood meaning relations between sentences having contradictory sentence radicals[3].

Two galleries metaphor. It seems that we can apply eliminative semantics approach in imperative logic, *mutatis mutandis.* Let us start with the metaphor of two galleries, one containing, as already stated, alternative pictures of yet unknown actual situation, the other gallery containing pictures of yet unknown ought-to-be situation. When both galleries contain all alternative pictures, then no information has flown in. If all the pictures are visible in upper, ought-to-be or goal gallery then there is nothing that should be done. If there is a strong discrepancy in the content of the galleries (*i.e.* when they have no common element), the system should change the actual situation.

Four moves system. While the simple elimination semantics is a one (re)move system, the imperative logic requires at least a four moves system. On the indicative side, besides sentences that give picture fragments for unraveling facts, one needs also sentences for expressing regularities, since they restrict the range of possible present and future situations. On the imperative side, the idea that imperatives solely act upon goal gallery seems to be straightforward. Still, if we take imperatives to be instructions for changing or sustaining a type of situation, then *change to* φ means *change actual* $\neg\varphi$ *situation into* φ *situation*, and *sustain* φ *situation* means *do not change actual* φ *situation into* $\neg\varphi$ *situation.* These double semantic moves, the asymmetric and symmetric one, seem to be more accurate account of natural language imperative semantics when compared to act-on-goals-only approach. There is another motivation for this kind of imperative semantics: in connecting

[2] In the system developed here the generated classes are not disjoint. Rather, the class of states $\mathcal{I}$ generated by indicative sentences is a superset of the class of states $\mathcal{M}$ generated by imperative sentences.

[3] For imperatives and indicatives it holds that if a text $M^a\varphi; M^b\neg\varphi$ is consistent, then modal elements M^a and M^b do not belong to the same mood category.

imperative logic with belief-desire logic one may characterize agent motivational state by the imperative the agent accepts (see Chapter 5). The asymmetric imperative can characterize the state of desire, since ”anyone...who desires, desires...what is not present” (Plato, Symposium, 200A-201A) .

Suggestions. If there is a degree of soundness in this imperative variant of eliminative semantics, then test sentences as phenomena visible through eliminative semantic lenses should become visible if present. It seems that the answer is positive: suggestion expressing sentences seem to be imperative sentences in a test-mode. The suggestions are complex test sentences. In the formal treatment, they test acceptability of certain imperatives and they also include a test whether a model belongs to the category of models generated by application of imperative sentences. The latter test reflects the fact that suggestions in the typical case propose a *relative goal* for acceptance or rejection.

2.1 ’Three moves’ language $\mathbf{L}_{!\bullet\boxdot}$ for imperative logic

Syntax. If φ, ψ are propositions in the standard language L_P of propositional logic, then $\bullet\varphi$, $\boxdot\varphi$, $!\varphi$ are sentences of the language $L_{!\bullet\boxdot}$ of practical update logic. Nothing else is in $L_{!\bullet\boxdot}$. A sequence of sentences $\boldsymbol{\varphi}_1, ..., \boldsymbol{\varphi}_n$ in $L_{!\bullet\boxdot}$ is called a text[4].

Semantics. Sentences are interpreted as functions from Σ to Σ. The set $\Sigma = \{\langle\gamma, \alpha\rangle \mid \gamma \subseteq W, \alpha \subseteq W\}$ is a family of states (*i.e.* model variations or contexts). The set $W = \mathcal{P}D$ is the set of situations *i.e.* valuations over the finite set D of all propositional letters in the part of language L_P under consideration. In the class Σ distinguished elements and subclasses are:

- $0 = \langle W, W\rangle$, initial or minimal state

[4] Bold Greek letters stand for sentences in $L_{!\bullet\boxdot}$.

- the class of final states $\mathcal{F} = \{\langle\gamma, \alpha\rangle \mid \gamma = \varnothing \vee \alpha = \varnothing\}$ including absurd state $1 \in \mathcal{F}$ defined as $1 = \langle\emptyset, \emptyset\rangle$.

Sentences.

Preliminary definitions. Truth set of sentence radical $\varphi \in L_P$ in W: $\|\varphi\|^W = \{w_{\in W} \mid w \vDash \varphi\}$. Truth set of sentence radical $\varphi \in L_P$ in $X \subseteq W$: $\|\varphi\|^X = \|\varphi\|^W \cap X$

- $\langle\gamma, \alpha\rangle [\boxdot\varphi] = \langle\|\varphi\|^\gamma, \|\varphi\|^\alpha\rangle$
- $\langle\gamma, \alpha\rangle [\bullet\varphi] = \langle\gamma, \|\varphi\|^\alpha\rangle$
- $\langle\gamma, \alpha\rangle [!\varphi] = \langle\|\varphi\|^\gamma, \|\neg\varphi\|^\alpha\rangle$

Text. $\sigma [\boldsymbol{\varphi}_1; ...; \boldsymbol{\varphi}_n] = (\sigma [\boldsymbol{\varphi}_1] ...) [\boldsymbol{\varphi}_n] = \sigma [\boldsymbol{\varphi}_1] ... [\boldsymbol{\varphi}_n]$, where $\boldsymbol{\varphi}_i \in L_{!\bullet\boxdot}$

The repertoire of semantic actions for imperative logic. The basic moves are: (1) information update about 'hard facts' which cannot be changed in any case (deterministic rule), (2) information about the current situation where one believes oneself to be (fact), (3) information about the desired non-actual state (goal and fact). The moves consist in removing points from the set γ of future and desired situations or removing points from the set α of situations which are indistinguishable with respect to their actuality.

The semantic impact of a deterministic rule cannot be identified with the impact of a factual sentence. Deterministic rule fixes that what cannot be the case neither now or in the future. Usually it is expressed by conditional *if φ is the case, then ψ* ***will be*** *the case.* Therefore, sentence-radical of a rule is projected on both sets of candidate situations. On the other hand, information on facts is projected on the set α leaving only those situations which are indistinguishable regarding their actuality.

Instructions to change situation in a certain way is projected both to the set γ of situations that are indistinguishable with respect to their desirability and to set α. Imperatives have double semantic impact. "Change to φ" thus consists of projection $\|\varphi\|^\gamma$ and $\|\neg\varphi\|^\alpha$: it sets a goal and reports on facts. This combined goal-fact approach seems to justified if one assumes a semantic basis for pragmatics. If a speaker suggests a goal she believes to be impossible or s/he commands that the actual state is to be brought about, then the logic of suggestions and commands is violated. When the Imperator is not sure wether φ is the case, s/he should say "Change to φ if φ is not the case." If one does not choose mixed, goal-and-fact semantics for imperatives, then s/he must endorse the idea that an imperative $!\varphi$ commands two actions: first, an epistemic one consisting in checking whether φ is the case, and the second, conditional one consisting in seeing to it that φ (if φ is not the case). On this approach, but not on ours, a conditional $\bullet\neg\varphi \rightarrow !\varphi$ is identical with $!\varphi$.

Semantic notions. Variants of ordinary semantic notions of eliminative semantics (see page 5).

Acceptance and acceptability A sentence $\boldsymbol{\varphi} \in L_{!\bullet\boxdot}$ is accepted in a state σ iff $\sigma[\boldsymbol{\varphi}] = \sigma$. A sentence $\boldsymbol{\varphi} \in L_{!\bullet\boxdot}$ is acceptable in a state σ iff $\sigma[\boldsymbol{\varphi}] \notin \mathcal{F}$.

Update-to-test consequence relation

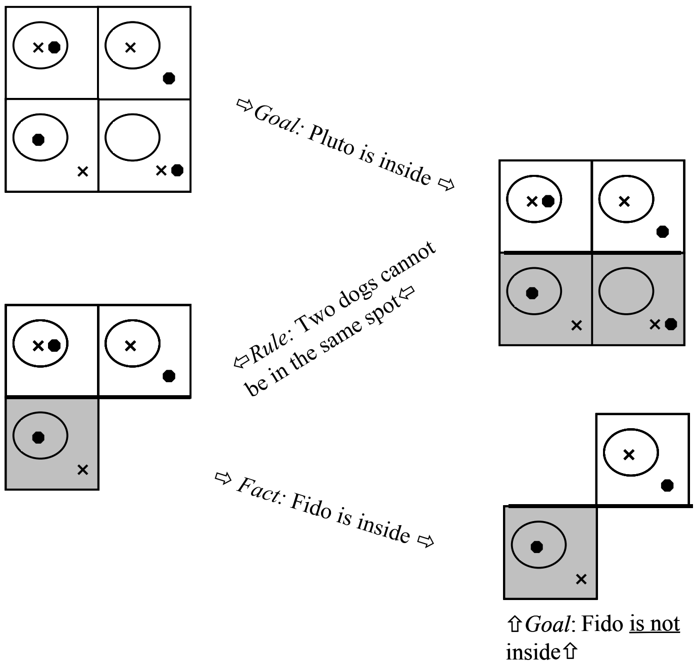
$$\boldsymbol{\varphi}_1; ...; \boldsymbol{\varphi}_n \vDash_{ut} \boldsymbol{\psi} \text{ iff } \forall\sigma : \sigma[\boldsymbol{\varphi}_1] ... [\boldsymbol{\varphi}_n] = \sigma[\boldsymbol{\varphi}_1] ... [\boldsymbol{\varphi}_n][\boldsymbol{\psi}]$$

Zero-update-to-test consequence relation

$$\boldsymbol{\varphi}_1; ...; \boldsymbol{\varphi}_n \vDash_{0-ut} \boldsymbol{\psi} \text{ iff } 0[\boldsymbol{\varphi}_1] ... [\boldsymbol{\varphi}_n] = 0[\boldsymbol{\varphi}_1] ... [\boldsymbol{\varphi}_n][\boldsymbol{\psi}]$$

Consistency and coherence. A sequence of sentences in $L_{!\bullet\boxdot}$ $\boldsymbol{\varphi}_1; ...; \boldsymbol{\varphi}_n$ is consistent iff $\exists\sigma : \sigma[\boldsymbol{\varphi}_1] ... [\boldsymbol{\varphi}_n] \notin \mathcal{F}$, and coherent iff $\exists\sigma_{\notin\mathcal{F}} : \sigma[\boldsymbol{\varphi}_1] = ... = \sigma[\boldsymbol{\varphi}_n]$

Example 4 ***Goal:*** *Pluto ($\times$) is inside;* ***Fact****: Fido ($\bullet$) is inside;* ***Deterministic rule****: Pluto will not be inside unless Fido is outside. What should be done?* ***Goal****: Pluto is inside and Fido is not inside.*



Reduction of uncertainty regarding goals (white area) and facts (grey area) may lead towards an emergence of a new relative goal.

2.2 Reduction of uncertainty in the practical setting

A way of explicating the notion of practical reasoning can be given in terms of 'reduction of uncertainty'. In theoretical reasoning an agent discovers the actual situation (i.e. the situation believed to be actual) by eliminating its possible descriptions. Parallel to this popular metaphor of information growth as uncertainity reduction, practical reasoning may be viewed as reduction of uncertainty with respect to facts and goals. In a successful case, 'practical uncertainty reduction' is an informational process in which determination of the actual situation and covering rules enables the determination of the minimal, feasible and permitted change needed for realization of an original goal. In a typical case reduction of motivational uncertainty evolves through the sequence: initial state 0 $\stackrel{imperative}{\Longrightarrow}$ extended motivational state $\stackrel{indicative}{\Longrightarrow}$ maximal motivational state. In an extended motivational state some goals are acceptable and not accepted i.e. they are rejectable (see Definition 8 and Proposition 10). On the other hand, maximal motivational states are complete in the sense that for any sentence in the part of language $L_{!\bullet\boxdot}$ under consideration it holds that the sentence is either accepted or not acceptable (see Definition 7 and Proposition 4). A maximal motivational state is a point of motivational certainty in so far it has been settled which situation is the actual one and which situation ought to be brought about. If there is more than one candidate for the goal situation, it is possible that one of them could turn out not to be a desired one. The information that the situation believed to be actual has changed leads to a final state. Within eliminative approach the notion of satisfaction should be modelled as a 'historical entity' having a genetic definition[5].

Some technical remarks Any maximal state can be reached in two steps *i.e.* by a two sentence text (Proposition 5). For any text there is a decision procedure which determines whether given amount of "information and duties" leads to maximal state (Proposition 6).

The 'interaction between theoretical and practical reasoning' is reflected in the possibility that a indicative-sentence update on a motivational state in which goal φ is accepted may lead towards a state that verifies goal ψ (where ψ is not equivalent to φ). In literature two types of relations between goals have drawn attention: the relation between a goal and its subgoals,

[5] The notion of satisfaction will not be discussed here since its requires additional structure.

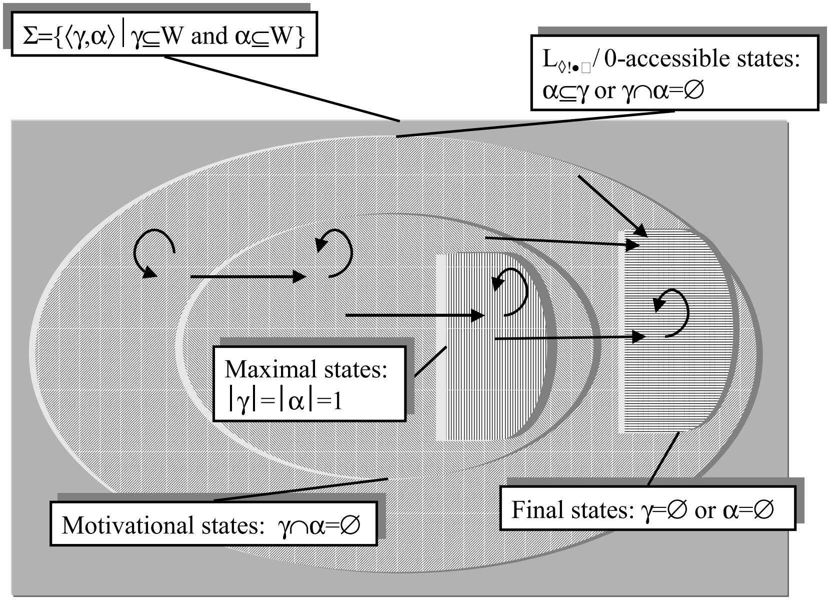


Figure 2 Update paths.

and the relation between an end and its means. Subgoal is usually understood as having the property of being entailed by goal: if φ is a goal and ψ is its subgoal, then $\varphi \models \psi$. On the other hand, ends and means are logically unrelated goals: if φ is an end and ψ is a means to it, then $\varphi \not\models \psi$ and $\psi \not\models \varphi$. It seems that a distinction between original and derived (relative) goal may be introduced with all reason: on that account subgoals and means are derived or relative goals.

3 Prima facie consequence

No adding metaphor. Suppose that a process of elimination by text T has created context C in which sentence φ makes no change. Shall we say that φ is a consequence of T since φ *adds (removes) nothing*? The answer is positive only provided that nothing else besides T has been accepted. How can we be sure that in this process of elimination only sentences from T have taken part? If all the models from the family are present at the initial point of elimination, the context created by T will be 'the largest'. One may call this context a minimal one: its informational content is the least in the class of contexts that verify T. Minimal context can be useful in formal semantics if there is a meaning relation that exists between a sentence and a text in the minimal context. A meaning relation of that kind seems to be a ground for the notion of *prima facie* consequence. The conclusion that cannot be detached from its grounds must be of a special kind. For if it is a model eliminating sentence, then it will hold in any context that verifies premises: if it can not remove models from the largest class, it surely will not be able to remove the models from any of its subclasses. So undetachable conclusion must be a noneliminative sentence. It seems that in natural language we encounter such sentences, which 'report on the *gallery* condition instead of removing pictures from it'. They are introspective sentences reporting on a context (state, model) status.

In next paragraphs we will discuss test-sentences in imperative mood[1]. They exhibit nonpersistence and in the framework of eliminative semantics they qualify as plausible candidates for the role of defeasible conclusion.

Extending language to $L_{\Diamond!\bullet\boxdot}$. For the purpose of accommodating test-sentences we will extend the language.

Syntax. A string $\boldsymbol{\varphi}$ of symbols is a sentence of the language of $L_{\Diamond!\bullet\boxdot}$ iff $\boldsymbol{\varphi} \in L_{!\bullet\boxdot}$, or $\boldsymbol{\varphi} = \boxed{\Diamond\bullet}\psi$ or $\boldsymbol{\varphi} = \boxed{\Diamond!}\psi$ or $\boldsymbol{\varphi} = \boxed{!\Diamond}\psi$,

[1] An idea similar to the here presented modalities within moods was hinted by Castañeda for deontic logic: "[...] statemental modal operators apply to deontic assertables" p.39.[8]

where $\psi \in L_P$.

Semantics for test sentences. Distinguished states and classes of states are: absurd state 1, class of nonfinal motivational states[2] $\mathcal{M}$ characterized by $\alpha \cap \gamma = \emptyset$, and class of final states $\mathcal{F}$.

- Indicative modalities
  - Epistemic possibility:

$$\langle \gamma, \alpha \rangle \left[\boxed{\Diamond \bullet} \varphi)\right] = \begin{cases} \langle \gamma, \alpha \rangle \text{ if } \langle \gamma, \alpha \rangle [\bullet \varphi] \notin \mathcal{F} \\ 1 \text{ otherwise} \end{cases}$$

- Imperative modalities
  - Buletic possibility

$$\langle \gamma, \alpha \rangle \left[\boxed{\Diamond !} \varphi)\right] = \begin{cases} \langle \gamma, \alpha \rangle \text{ if } \langle \gamma, \alpha \rangle [! \varphi)] \notin \mathcal{F} \\ 1 \text{ otherwise} \end{cases}$$

- – Pro(h)airetic possibility

$$\langle \gamma, \alpha \rangle \left[\boxed{! \Diamond} \varphi\right] = \begin{cases} \langle \gamma, \alpha \rangle \text{ if } \begin{cases} \langle \gamma, \alpha \rangle \in \mathcal{M} \text{ and} \\ \langle \gamma, \alpha \rangle [! \varphi] \notin \mathcal{F} \text{ and} \\ \langle \gamma, \alpha \rangle [! \neg \varphi)] \in \mathcal{F} \end{cases} \\ \qquad 1 \text{ otherwise} \end{cases}$$

Uncertainty provides a natural habitat for sentences such as "It might be the case that φ", "It might be good to bring it about that φ", "It seems that φ should be done". Sentences of such a kind are not intended to change the state of mind, rather they are put forward for consideration. In the typical case, they express some cognitive and volitive options which have not been eliminated yet, although they can be eliminated later.

In the formal sense the state/model σ corresponding to a state of mind not completely determined is not a maximal state

[2] Motivational states are generated if and only only an imperative sentence has been applied (see Proposition 16).

and therefore, it can be further refined by additional sentences[3]. That state of mind can be changed only with respect to relative goals and uncertain facts. In that sense some accepted sentence can be rejected later (see Propositions 8, 9 and Lemma 7).

3.1 Nonpersistent sentences and preferred model

It has been noted by several authors in philosophy of action and in philosophical logic that the notion of validity in practical logic is not classical one. Usually nonmonotonicity is recognized as a specific feature of consequence relation in practical logic. Expression 'prima facie' is used either to indicate that additional premises may defeat the conclusion (Wallace [33]) or that conclusion can not be detached from its premises (Davidson [14]).

The idea that there can be some kind of valid inference with defeasible conclusion can be modeled as a premises-conclusion meaning relation holding in the minimal model of the premises. In the set of states Σ^T for a sequence of sentences T there is a minimal state σ^{minT}such that for any state $\sigma' \in \Sigma^T, \sigma' \neq \sigma$ there is a sentence accepted in σ' and not accepted in σ^{minT}. Assume that "prima facie consequence" means "(relation of) conclusion being acceptable in the light of premises T but possibly rejectable by an extended text $a \oplus T$, where '$\oplus$' denotes operation of arbitrary positioning ". Since $\sigma^{minT} = 0\,[T]$, it follows that *prima facie* consequence relation can be explicated by notions of rejectable sentence (p. 19) and a consequence relation defined over the minimal state (p. 14).The informational content of the minimal state in the class of models for of the text T is the smallest content (see Propositions 8 and 9).

Reconciliation. The conflicting intuitions on the valid forms of imperative-indicative inferences seem to be reconcil-

[3] A state is accessible if there is an update path leading to it. Our attention is restricted here only to 0-accessible states; other states have a different genealogy pertaining to a richer language.

able to an degree. Typically, if a schema is taken to be invalid by an author, and valid by another, then the premises are incomplete (in the sense of Proposition 6) while the conclusion is a rejectable sentence accepted in the minimal state.

Example 5 (Practical syllogism) *Disagreement with respect to validity in imperative and belief-desire logic.*

Formulations of the allegedly valid principle:

> (1) S genuinely wants p to be the case for its own sake. (2) Only if S does X will p be the case. (1) and (2) constitute *prima facie* grounds for (C) S should do X. (Wallace [33])
>
> If X believes [knows] that (p implies q), then that X intends p implies that X intends q. (Castañeda [9])
>
> One wants to attain x. Unless y is done, x will not be attained. Therefore y must be done. (Von Wright [37])

Systems with explicit criteria[4]:

		Valid?
Segerberg [27]	$(!\delta p \wedge [\delta p]\, q) \rightarrow !\delta q$	no
Belnap, Perloff, Horty [4][23]	$([\alpha\ dstit : p] \wedge \Box(p \rightarrow q)) \rightarrow [\alpha\ stit : q]$	no
Cross [13]	$(\triangle(p) \wedge (p \rightarrow q)) \rightarrow \triangle q$	no
Chellas [11]	$(!p \wedge \boxdot\,(p \rightarrow q)) \rightarrow !q$	yes
Kenny [24]	$(Fiat(p) \wedge Est(p \rightarrow q) \rightarrow Fiat(q)$	no
Fulda [18]	$((p \rightarrow \mathfrak{s}) \wedge (p \rightarrow q)) \rightarrow (q \rightarrow \mathfrak{s})$	no
$L_{\Diamond ! \bullet \boxdot}$	$!p; \boxdot(p \rightarrow q) \vDash_{ut} !q$	no
$L_{\Diamond ! \bullet \boxdot}$	$!p; \boxdot(p \underset{\cdot}{\rightarrow} q) \vDash_{0-ut} \boxed{!\Diamond}\, q$	yes

Different results arise from different semantics. From our standpoint, the premises are incomplete in the sense that they

[4] Main features of formal semantic systems are given in the remark on the page**??**.

do not necessarily lead to the maximal state (in this case addition of $\bullet\neg q$ would suffice) thus giving grounds for acceptance of defeasible conclusion $\boxed{!\Diamond}q$ (which can be ruled out by $\bullet\neg q$).

Example 6

		Valid?
Segerberg [27]	$(!\delta\,(p \wedge q) \wedge p) \rightarrow !\delta q$	no
Belnap, Perloff, Horty [4][23]	$([\alpha\ dstit : p \wedge q] \wedge p)) \rightarrow [\alpha\ dstit : q]$	no
Cross [13]	$(\triangle(p \wedge q) \wedge p)) \rightarrow \triangle q$	no
Cross [13]	$(\triangle(p \wedge q) \wedge \oplus p)) \rightarrow \triangle q$	yes
Chellas [11]	$(!\,(p \wedge q) \wedge p) \rightarrow !q$	yes
Kenny [24]	$(Fiat(p \wedge q) \wedge Est(p) \rightarrow Fiat(q)$	yes
Fulda [18]	$(((p \wedge q) \rightarrow \mathfrak{s}) \wedge p) \rightarrow (q \rightarrow \mathfrak{s})$	yes
$L_{\Diamond!\bullet\boxdot}$	$!\,(p \wedge q)\,; \bullet p \vDash_{ut} !q$	yes
$L_{\Diamond!\bullet\boxdot}$	$!\,(p \wedge q)\,; \bullet p \vDash_{0-ut} \boxed{!\Diamond}q$	yes

In this example the premises determine the maximal state. Still the validity declarations do not converge. It could be useful to point out semantic peculiarities which give rise to negative results.

Context independency. The authority commands 'Close the door and open the window' in a situation which is believed both by the authority and the addressee as the situation where the door is closed (and each of them knows what the other believes). It is redundant then to issue the further command 'Open the window'. If it is nevertheless issued, it will not change or add anything to the initial command. In the sequence '(i) Close the door and open the window! (ii) The door is closed. Therefore, (iii) open the window!' the last sentence (iii) does not seem to be contextually independent. It seems natural to have alongside with the contextually independent reading[5], i.e. 'Do anything to bring it about that the window is open!' (as suggested in Segerberg [27]), a contextual dependent one 'Make the minimal

[5] Counterexample is an action $\langle w, v\rangle$ such that $w \in \|p\|$ and $v \in \|\neg p \wedge q\|$ which does not belong to a command set: $\|p \wedge q\| \in \Gamma_w$ but $\langle w, v\rangle \notin \{\langle w, y\rangle : y \in \|p \wedge q\|\}$, therefore, it is possible that $\|q\| \notin \Gamma_w$.

change of actual state that will bring about that the window is open'. In contextual dependent reading the imperative (iii) does not change the initial goal. Similar arguments apply to $STIT$ logic where the possibility that some agent does not have a choice to open the window makes this subgoal inference form invalid.

Relative notion of satisfaction. One translation for '$!(p \wedge q), \bullet p$ Therefore, $!q$' in MLD (Cross [13]) is a valid schema

$$(\triangle(p \wedge q) \wedge \oplus p) \rightarrow \triangle p$$

'$\triangle$' reads 'desire in the sense of incompatible goal-belief discrepancy' and '$\oplus p$' reads 'agent is satisfied that p'. The implicit idea that an agent is satisfied by realization of a subgoal seems counterintuitive. One would rather say that agent wants to preserve a subgoal if it happens that the actual situation belongs to the same situation type. The other reading

$$(\triangle(p \wedge q) \wedge p) \rightarrow \triangle p$$

gives an invalid inference. Motivational extension can be conceived of as a process in which the goal "remains fixed' while "subgoals change", but the assertion that every realization of a subgoal leads to the state of satisfaction (Cross, 1997) does not seem acceptable. The motivational force of a desire is exhausted by the realization of its content, and only that final point of motivation extension is a state of satisfaction, not an intermediate motivational state in which an agent desires to sustain something.

Remark 1 *Main semantic features of several systems.*

<table>
<tr><td>[13]</td><td>$\|!\varphi\|(w,t)=1$</td><td>$\|\varphi\|(w',t)=1$
for every w' such that $R_t(w,w')$</td></tr>
<tr><td>[23]</td><td>$M, m/h \vDash [\alpha\ dstit : \varphi]$</td><td>$\forall h' : h' \in Choice^m_\alpha(h) \rightarrow M, m/h' \vDash \varphi$
and
$\exists h'' : h'' \in H(m) \wedge M, m/h'' \nvDash \varphi$</td></tr>
<tr><td>[13]</td><td>$V_{M,w}(\triangle\varphi)=\top$</td><td>$\forall w' \left(wR_1w' \rightarrow \mathcal{V}_{M,w'}(\varphi)=\top\right)$ and
$\forall w'' \left(wR_2w'' \rightarrow \mathcal{V}_{M,w''}(\varphi)=\bot\right)$</td></tr>
<tr><td>[27]</td><td>$\Gamma \underset{requires}{\vDash_x} !\delta\varphi$</td><td>$\{\langle x,y\rangle : y \in \|\varphi\|\} \in \Gamma_x$</td></tr>
</table>

4 Negation of imperatives and notion of change

The task of dynamic semantics is to provide a connection between a structure that verifies a sentence and operations that produce verifying structural variation. In the simple update system an imperative is verified by a pair $\langle\gamma,\alpha\rangle$ of disjoint sets, $\gamma\cap\alpha=\emptyset$ and an asymmetric action is connected with imperatives in order to generate required structure. To update a structure with a sentence means to produce a minimal modification of the structure needed for the verification of the sentence. Following that line of thought, negation of a sentence can be understood as an operation that will prevent any successful[1] modification by that sentence. In that sense, negation changes a structure in a way that makes accepting of negated sentence impossible. There are different ways of making an imperative unacceptable. According to the semantics for imperatives as change directives the imperative $!\varphi$ is not acceptable in any state $\langle\gamma,\alpha\rangle$ such that $\gamma\cap\|\varphi\|=\emptyset$ or $\alpha-\|\varphi\|=\emptyset$. Since fulfillment of one condition suffices, indicative sentence $\bullet\varphi$ could be regarded as a variant of a dynamic negation (see $\overset{2}{\neg}!\varphi$ below). In general dynamic negation in a eliminative update system ('forward looking system') can be characterized by the following proposition .

Proposition 1 $\forall\sigma:\sigma\,[\neg\boldsymbol{\varphi}]\,[\boldsymbol{\varphi}]\in\mathcal{F}$.

It could be interesting to examine several sentential instructions that will satisfy the proposition.

- 1. $\langle\gamma,\alpha\rangle\left[\overset{1}{\neg}!\varphi\right]=\langle\|\neg\varphi\|^{\gamma},\alpha\rangle$ "Don't change to φ whatever the actual situation may be."
  2. $\langle\gamma,\alpha\rangle\left[\overset{2}{\neg}!\varphi\right]=\langle\gamma,\|\varphi\|^{\alpha}\rangle=\langle\gamma,\alpha\rangle\,[\bullet\varphi]$ "It is the case that φ."

[1] 'Successful' means 'not landing into a final state'.

3. $\langle \gamma, \alpha \rangle \left[\overset{3}{\neg} ! \varphi \right] = \langle \|\neg\varphi\|^{\gamma}, \|\varphi\|^{\alpha} \rangle = \langle \gamma, \alpha \rangle [!\neg\varphi]$ "Change to $\neg\varphi$."
4. $\langle \gamma, \alpha \rangle \left[\overset{4}{\neg} ! \varphi \right] = \langle \|\neg\varphi\|^{\gamma}, \|\neg\varphi\|^{\alpha} \rangle$ (i) "Preserve $\neg\varphi$", "Do not change $\neg\varphi$", "Do not change **to** φ", "Sustain $\neg\varphi$"
5. $\langle \gamma, \alpha \rangle \left[\overset{5}{\neg} ! \varphi \right] = \langle \|\varphi\|^{\gamma}, \|\varphi\|^{\alpha} \rangle$ (i) "Preserve φ", "Do not change φ", "Do not change **to** $\neg\varphi$"

We will assume that negation of an imperative is an imperative too, ruling out $\overset{2}{\neg}!\varphi$. The opposition $!\varphi$ *vs.* $\overset{3}{\neg}!\varphi$(or $!\neg\varphi$) is not a good candidate for the role of negated change instruction if it is intuitively acceptable that the 'negated change' amounts to 'no change'. The opposition $!\varphi$ *vs.* $\overset{5}{\neg}!\varphi$ is not a good candidate either in so far both commands share the same goal content. The remaining cases $\overset{1}{\neg}!\varphi$ and $\overset{4}{\neg}!\varphi$ give good candidates. The 'one-sided goal negation' has special attractiveness since, together with $\bullet$-type sentences, it can make all states $\sigma \in \Sigma$ reachable. On the other hand, within the proposed semantics for imperatives as change instructions negation $\overset{1}{\neg}$ turns out to be indeterminate between a variant of a conditional imperative

$$\langle \gamma, \alpha \rangle \left[\overset{1.1}{\neg} ! \varphi \right] = \begin{cases} \langle \|\neg\varphi\|^{\gamma}, \alpha \rangle \text{ if } \langle \gamma, \alpha \rangle [\bullet\varphi] = \langle \gamma, \alpha \rangle \\ \langle \gamma, \alpha \rangle \text{ otherwise} \end{cases}$$

and a command without informational content

$$\langle \gamma, \alpha \rangle \left[\overset{1.2}{\neg} ! \varphi \right] = \langle \|\neg\varphi\|^{\gamma}, \alpha \rangle$$

We opt for $\overset{4}{\neg}!\varphi$ as a plausible candidate for negated imperative since $\overset{4}{\neg}!\varphi$ and $!\varphi$ impose different imperative alternatives for the same actual situation. Still, the identical symmetric semantics given to the negations $\neg^4$ and $\neg^5$and regularity expressing indicatives ($\boxdot\neg\varphi$ and $\boxdot\varphi$, respectively) shows that the formal structure must be refined since it is obvious that e.g. $\boxdot\neg\varphi$ and $\overset{4}{\neg}!\varphi$ do not mean the same.

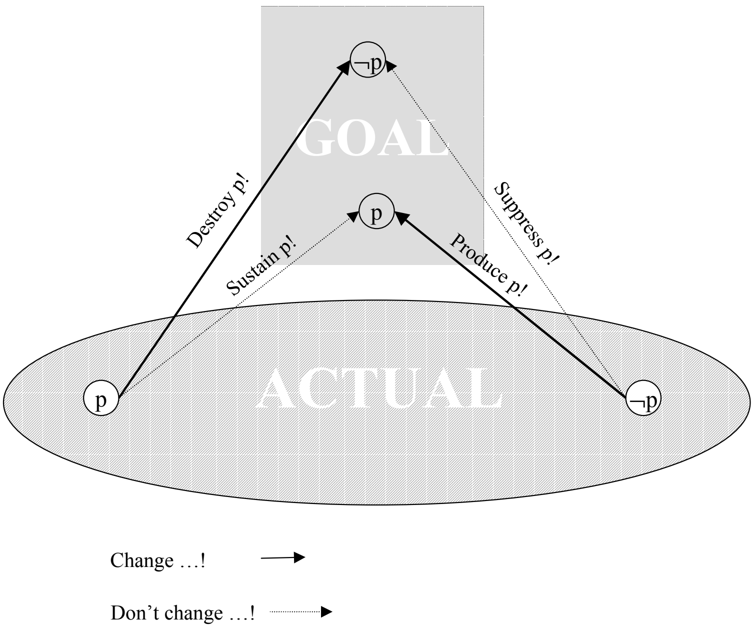


Figure 1 Negated and affirmed imperative: alternatives for the same situation.

4.1 Extended language $L_{\neg!\Diamond!\bullet\boxdot}$ and refined models

In order to accommodate negated imperatives, more refined models will be introduced (*i.e.* ordered triples of sets of valuations). The resulting system will be a 'four moves system'.

Semantics. Family of models

$$\Sigma^* = \{\langle\gamma, \alpha, \pi\rangle \mid \gamma \subseteq \pi, \alpha \subseteq \pi, \pi \subseteq W\}.$$

- $0 = \langle W, W, W\rangle$, initial or minimal model
- the class of final models $\mathcal{F} = \{\langle\gamma, \alpha, \pi\rangle \mid \gamma = \varnothing \vee \alpha = \varnothing\}$ including absurd model $1 \in \mathcal{F}$ defined as $1 = \langle\emptyset, \emptyset, \emptyset\rangle$
- the class of nonfinal models $\mathcal{S}$ characterized by $\gamma \neq \pi$
- the class of nonfinal models $\mathcal{M} \subset \mathcal{S}$ characterized by $\alpha \cap \gamma = \emptyset$

4.1.0.0.1 Sentences

Definition 1 (Truth set) $\|\varphi\|^X = X \cap \|\varphi\|^W$

Basic sentences.

- *necessary* φ
  - $\langle\gamma, \alpha, \pi\rangle [\boxdot\varphi] = \langle\|\varphi\|^\gamma, \|\varphi\|^\alpha, \|\varphi\|^\pi\rangle$
- *actually* φ
  - $\langle\gamma, \alpha, \pi\rangle [\bullet\varphi] = \langle\gamma, \|\varphi\|^\alpha, \pi\rangle$
- *change to* φ
  - $\langle\gamma, \alpha, \pi\rangle [!\varphi] = \langle\|\varphi\|^\gamma, \|\neg\varphi\|^\alpha, \pi\rangle$

- *sustain* $\neg\varphi$ (*do not change to* φ)

  – $$\langle \gamma, \alpha, \pi \rangle \left\llbracket \boxed{\neg !}\varphi \right\rrbracket = \begin{cases} \langle \|\neg\varphi\|^{\gamma}, \|\neg\varphi\|^{\alpha}, \pi \rangle \text{ if } \|\neg\varphi\|^{\gamma} \neq \pi \\ 1 \text{ otherwise} \end{cases}$$

Defined, complex sentences.

- *possibly* φ

  – $$\sigma \left\llbracket \boxed{\bullet\Diamond}\varphi \right\rrbracket = \begin{cases} \sigma \text{ if } \sigma\,[\boxdot\varphi] \neq 1 \\ 1 \text{ otherwise} \end{cases}$$

- *not necessary* φ (*possibly not* φ)

  – $$\sigma \left\llbracket \boxed{\neg\boxdot}\varphi \right\rrbracket = \sigma \left\llbracket \boxed{\bullet\Diamond}\neg\varphi \right\rrbracket$$

- *not actually* φ (*actually not* φ)

  – $$\sigma \left\llbracket \boxed{\neg\bullet}\varphi) \right\rrbracket = \sigma\,[\bullet\neg\varphi)]$$

- *might* φ

  – $$\sigma \left\llbracket \boxed{\Diamond\bullet}\varphi \right\rrbracket = \begin{cases} \sigma \text{ if } \sigma\,[\bullet\varphi] \notin \mathcal{F} \\ 1 \text{ otherwise} \end{cases}$$

- *might be good to produce* φ

  – $$\sigma \left\llbracket \boxed{\Diamond !}\varphi \right\rrbracket = \begin{cases} \sigma \text{ if } \sigma\,[!\varphi] \notin \mathcal{F} \\ 1 \text{ otherwise} \end{cases}$$

- *might be good to sustain* φ

  – $$\sigma \left\llbracket \boxed{\Diamond\neg !}\neg\varphi \right\rrbracket = \begin{cases} \sigma \text{ if } \boxed{\neg !}\neg\varphi \notin \mathcal{F} \\ 1 \text{ otherwise} \end{cases}$$

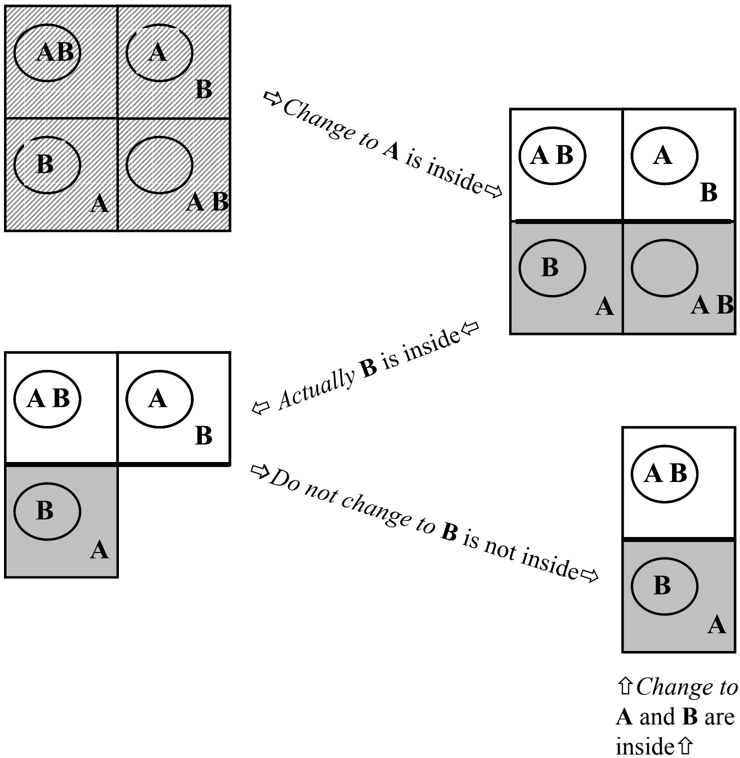
- *maybe* φ *should be brought about*

  – $$\sigma \left\llbracket \boxed{!\Diamond}\varphi) \right\rrbracket = \begin{cases} \sigma \text{ if } \begin{cases} \sigma \in \mathcal{M} \text{ and} \\ \sigma\,[!\varphi] \notin \mathcal{F} \text{ and} \\ \sigma\,[!\neg\varphi] \in \mathcal{F} \end{cases} \\ 1 \text{ otherwise} \end{cases}$$

- *maybe φ should be preserved*

$$- \ \sigma \left[\!\left[\boxed{\neg!\Diamond}\neg\varphi)\right]\!\right] = \begin{cases} \sigma \text{ if } \begin{cases} (\sigma \in \mathcal{M} \text{ or } \sigma \in \mathcal{S}) \text{ and} \\ \sigma \left[\!\left[\boxed{\neg!}\neg\varphi\right]\!\right] \notin \mathcal{F} \text{ and} \\ \sigma \left[\!\left[\boxed{\neg!}\varphi\right]\!\right] \in \mathcal{F} \end{cases} \\ 1 \text{ otherwise} \end{cases}$$

Remark 2 *We call '!', '$\boxed{\neg!}$', '$\boxed{\Diamond!}$', '$\boxed{\Diamond\neg!}$', '$\boxed{!\Diamond}$', '$\boxed{!\neg\Diamond}$' modal elements of imperative mood, and '$\bullet$', '$\boxed{\neg\bullet}$', '$\boxed{\Diamond\bullet}$', '$\boxdot$', '$\boxed{\bullet\Diamond}$' modal elements of indicative mood.*



4.1.1 Explanation of semantic moves: $\boxed{\neg!\Diamond}$ and $\boxed{!\Diamond}$

The sentences having form (i) 'Maybe φ should be brought about' and (ii) 'Maybe φ should be sustained' could be regarded as kind of imperatives in so far as their sentence radical denotes the type of an acceptable (yet defeasible) goal situation. Using sentences of the kind one usually express *practical* suggestions[2]. The sentence forms (i) and (ii) seem to be a complex test functions. The Speaker, having incomplete knowledge of Hearer's reasons and supposing an inclination for a suggested goal on the Hearer's side, proposes a relative goal for Hearer. A not rigorous extraction of semantic parts of suggestion sentences could give us the following decomposition. For (i): on Speaker's opinion *there is something x Hearer already wants + φ is related to that x so that Hearer does not want to have $\neg\varphi$ and Hearer may want to have φ*. For (ii): *there is something x Hearer already wants it+ φ is related to that x so that Hearer does not want to lose φ and Hearer may want to keep φ.*

Proposition 2 $\forall\sigma : \sigma\,[!\varphi] = \sigma \rightarrow \sigma\,[!\neg\varphi] \in \mathcal{F}$

Proposition 3 $\forall\sigma : \sigma \left[\boxed{\neg!}\varphi\right] = \sigma \rightarrow \sigma \left[\boxed{\neg!}\neg\varphi\right] \in \mathcal{F}$

Assume that there are only 2 permitted moves in giving the semantics for complex sentences: 1. use basic sentences, 2. use distinguished elements and subclasses of the family of models Σ^*.

We want to delineate two classes of states, SC for suggested changes and SS for suggested preservations.

$$\text{(i) } SC = \left\{\langle\gamma, \alpha, \pi\rangle \mid \gamma \subseteq \|\varphi\|^W \ \wedge \alpha - \|\varphi\|^W \neq \emptyset\right\}$$

and

$$\text{(ii) } SS = \left\{\langle\gamma, \alpha, \pi\rangle \mid \gamma \subseteq \|\varphi\|^W \ \wedge \alpha \cap \|\varphi\|^W \neq \emptyset\right\}$$

(i) 'Maybe φ should be brought about'
'+' stands for 'All worlds are...'
'+/-' stands for 'Some worlds are and some worlds are not...'
'-' stands for 'No world is...'
'@' marks states in which *produce* $\neg\varphi$ can not be accepted[3]

[2] **suggest** *v.* 2. to propose (*a plan* or theory) for acceptance or rejection (The Oxford Dictionary for Modern English)

[3] All goal situations are φ-situations or no actual situation is a φ-situation.

'*' marks states in which *produce* φ can be accepted

'M' marks states which are necessarily M-type states

double lines mark the classes of models that satisfy the conditions for $\boxed{!\Diamond}\varphi$ being accepted

Goal situation	Actual situation		$\sigma[!\neg\varphi] \in \mathcal{F}$	$\sigma[!\varphi] \notin \mathcal{F}$
$\varphi - worlds$	$\varphi - worlds$			
+	+		@	
+	+/-	M	@	*
+	-	M	@	*
+/-	-		@	*
-	-		@	

(ii) 'Maybe φ should be preserved'

'+' stands for 'All worlds are...'

'+/-' stands for 'Some worlds are and some worlds are not...'

'-' stands for 'No world is...'

'#' marks states in which *sustain* $\neg\varphi$ can not be accepted[4]

'$' marks states in which *sustain* φ can be accepted

'M' marks states which are necessarily $\mathcal{M}$-type states

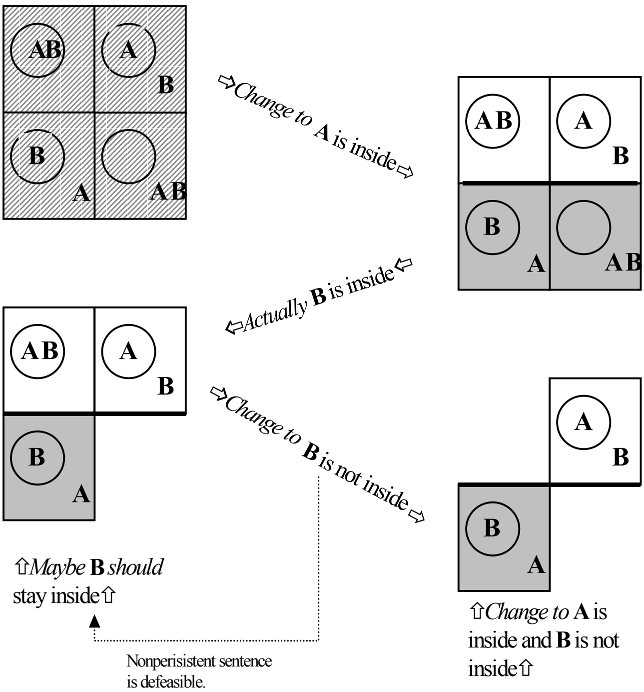
'S' marks states which are usually $\mathcal{S}$-type states

double lines mark the classes of models that usually satisfy the conditions for $\boxed{\neg!\Diamond}\neg\varphi$ being accepted

Goal situation $\varphi - worlds$	*Actual situation* $\varphi - worlds$		$\sigma\left[\boxed{\neg!}\varphi\right] \in \mathcal{F}$	$\sigma\left[\boxed{\neg!}\neg\varphi\right] \notin \mathcal{F}$
+	+	S*	#	$
+	+/-	M	#	$
+	-	M	#	
+/-	+		#	$
-	+		#	

Remark 3 *See appendix for a way to prove whether a class of states is a subset of $\mathcal{M}$-type states (an example covering second row in the table above is given in Proposition 17).*

[4] All goal situations are φ-situations or all actual situations are φ-situations.



Dotted arrow denotes the nullifying of nonpersistent sentence by additional sentence. Also note that a nonpersistent imperative sentence may be accepted even when suggested goal is not a relative one.

4.1.2 Conditional imperative

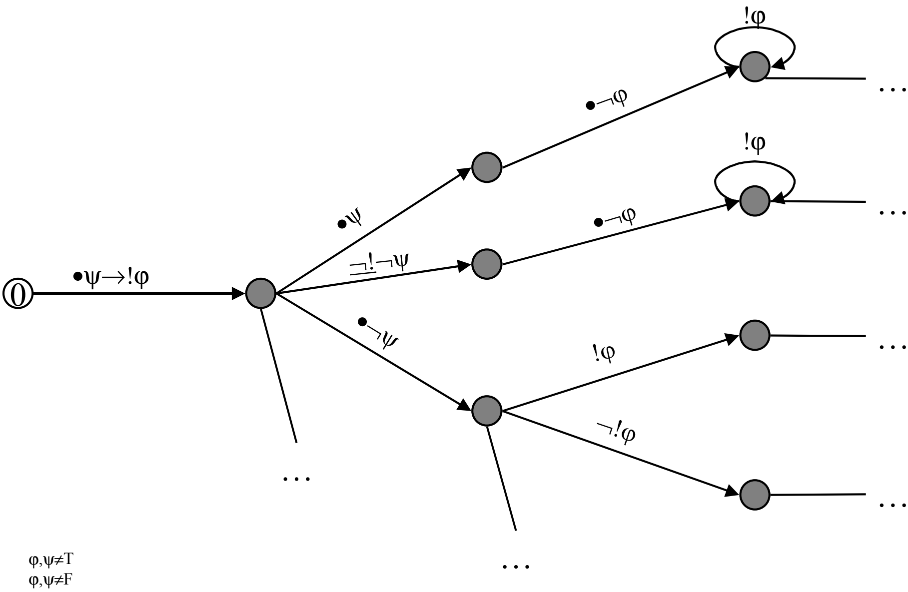
Conditional imperative anchors a type of a goal situation to a particular kind of actual situations. The simplest specimen is

$$\bullet\psi \rightarrow !\varphi$$

The simplest solution for the semantics of a conditional imperative within here proposed modelling would be to treat it as simple test function.

- $\sigma[\bullet\psi \rightarrow !\varphi] = \begin{cases} \sigma[!\varphi] \text{ if } \sigma[\bullet\psi] = \sigma \\ \sigma \text{ otherwise} \end{cases}$

We do not want semantics in which conditional imperative may be "forgotten" if the antecedent is not true (desired behavior is depicted in the picture below). On the contrary, the semantics of the conditional imperative should restrict the possible evolution of a model even when the indicative antecedent is not true. In order to meet that demand, the semantics of a conditional imperative must somehow encapsulate the semantic impact of different sequences of sentences in order to restrict the space of evolution paths.



Part of evolution tree for conditional imperatives.

Unfortunately the proposed semantics of ordered triples of sets of valuations can not fullfil that demand[5]. The solution requires different modelling like the one given in the example 7.

[5] If $0[\bullet\psi \rightarrow !\varphi; \bullet\psi; \bullet\neg\varphi] = \langle\gamma, \alpha, \pi\rangle$, then $\langle\gamma, \alpha, \pi\rangle[!\varphi] = \langle\gamma, \alpha, \pi\rangle$

Example 7

$$\Sigma = \left\{ \langle P, R \rangle \mid R \subseteq P \times P, P \subseteq W, W = 2^{\text{ set of propositional letters }} \right\}$$

$$\langle P, R \rangle [!\varphi] = \langle P, \{(w, v) \in R \mid w \vDash \varphi \wedge v \nvDash \varphi\} \rangle$$

$$\langle P, R \rangle [\bullet\varphi] = \langle P, \{(w, v) \in R \mid v \vDash \varphi\} \rangle$$

$$\langle P, R \rangle \left[\boxed{\neg !} \varphi \right] = \langle P, \{(w, v) \in R \mid w \nvDash \varphi \wedge v \nvDash \varphi\} \rangle$$

$$\textit{Operation } \Cup \; : \; \langle P_i, R_i \rangle \Cup \langle P_j, R_j \rangle = \langle P_i \cup P_j, R_i \cup R_j \rangle$$

The desired behavior is obtained by

$$\sigma [\bullet\psi \rightarrow !\varphi] = \begin{cases} \sigma [!\varphi] \; \textit{if} \; \sigma [\bullet\psi] = \sigma \\ \sigma [\bullet\psi] [!\varphi] \Cup \sigma [\bullet\neg\psi] \; \textit{otherwise} \end{cases}$$

---

should hold. The latter requires $\gamma \subseteq \|\varphi\|^W$. On the other hand, if $0 [\bullet\psi \rightarrow !\varphi; \bullet\neg\psi; !\neg\varphi] = \langle \gamma', \alpha', \pi \rangle$, then $\langle \gamma', \alpha', \pi \rangle \notin \mathcal{F}$ should hold. The latter requires that $\gamma' \cap \|\neg\varphi\|^W \neq \emptyset$. Since indicative texts $\bullet\psi; \bullet\neg\varphi$ and $\bullet\neg\psi$ cannot eliminate points from goal set, it follows that $0 [\bullet\varphi \rightarrow !\psi] = \langle \gamma'', \alpha'', \pi \rangle$ should satisfy contradictory conditions *i.e.* $\gamma'' \subseteq \|\varphi\|^W$ and $\gamma'' \cap \|\neg\varphi\|^W \neq \emptyset$.

5 Semantics of propositional attitudes reports

We use term 'propositional attitude report' to denote a class of sentences in which a psychological intentional verb is used. The term 'psychological -intentional verb' will be used in a informal sense and therefore we introduce it by way of examples: 'to desire', 'to think', 'to believe', 'to hope' are examples, and non-examples are: 'to touch', 'to taste', 'to suffer'. We adopt the tradition which treats propositional attitude reports as sentences (that can be rephrased as) having a **verb** of the kind followed by *an embedded sentence* describing a state of affairs that is the object of the attitude denoted by the verb , e.g. "She **desires** that *her art would have an emotional impact".*

Two-part anatomy and direction of fit. In analyzing intentional states, i.e. propositional attitudes, and sentence moods authors usually distinguish two components.

	`mental states`	
Husserl	noesis	noema
Russell	attitude	proposition
Anscombe	direction of fit	content
	`sentences`	
Hare	tropic	phrastic
Stenius	modal element	sentence radical
Kenny	mood indicator	descriptive content

This psychological - linguistic parallelism in anatomy is further emphasized by another similarity. Imperatives and desires are such that 'the world must fit with' them, while declaratives and beliefs are such that they 'must fit the world'. This twofold parallelism motivates an idea of establishing connection between mental states and sentences.

Dynamic semantics. It is not necessary to identify the changing models of dynamic semantics with the internal states of an computing system (be it a machine or a pre-linguistic child). We can treat dynamic semantics as just another kind of model theory which uses *variations* of Kripkean, Tarskian and Kripkean-Tarskian models to account for some semantic phenomena, like order sensitivity, non-classical consequence relation and nonpersistent sentences. Nevertheless, the idea of connecting models and intentional states seems to be natural in that framework: models are states of (an ideal) system and model evolution paths are histories of such states.

Representation free approach. By accepting the idea that the model changes are triggered by sentences we are not forced to adopt a variant of representationalistic metaphor according to which the sentences are components in internal functioning of a system. The metaphor was prominent in history of ideas (e.g. inner voice, cultural imperative, categorical imperative, language of thought, to name a few), but it is weak in number of points which have been discussed in the literature. Instead we adopt a non-realist position: by using propositional attitude predicates we adopt an interpretational stance (intentional stance, Dennett [16]) in which we exploit inferential relations between mood designated sentences (modified version of Davidson's measurement theoretic approach to the semantics of propositional attitude reports [15]). The position advocated here differs from the Davidson's position in so far as we take entities to be mood designated sentences and not mood free propositions.

> Just as in measuring weight we need a collection of entities which have structure in which we can reflect the relations between weighty objects, so in attributing states of belief (and other propositional attitudes) we need a collection of entities related in the ways that will allow us to keep track of the relevant properties and relations among the various psychological states. (Davidson, What is present to the mind in [15] p. 60.)

5.1 Characterization of intentional states

The approach to the propositional attitude report semantics, which we want to explore here, relies on two basic theoretical assumptions. Intentional states may be (i) theoretically identified with models of some sort and (ii) characterized by semantic relations between models and mood designated sentences. On this approach, the informational content of sentence type

$$[\textit{mental predicate}\ \mathrm{P}]\,([\textit{agent}\ \mathrm{a}]\,,[\textit{proposition}\ \varphi])$$

is identical with the informational content of the sentence 'there is a model σ such that agent's state is identical with it and σ belongs to the class of models satisfying *update condition* with respect to a mood M designated sentence(s) $M\varphi$'.

agent a is in a state σ s.t.	$\sigma \in \{\sigma \mid ...\textit{update condition}...\}$
(i) a wants to change to φ	$\sigma\,[!\varphi] = \sigma$
(ii) a wants to sustain φ	$\sigma\left[\boxed{\neg!}\neg\varphi\right] = \sigma$
(iii) a believes that φ is the case	$\sigma\,[\bullet\varphi] = \sigma$
(iv) a believes that φ might be the case	$\sigma\left[\boxed{\Diamond\bullet}\varphi\right] = \sigma$
(v) a believes that φ is necessary the case	$\sigma\,[\boxdot\varphi] = \sigma$
(vi) a opposes that $\neg\varphi$ will be produced	$\sigma\left[\boxed{!\Diamond}\varphi\right] = \sigma$
(vii) a opposes that $\neg\varphi$ will be sustained	$\sigma\left[\boxed{\neg!\Diamond}\neg\varphi\right] = \sigma$

The states verifying suggested changes or preservations of the situation type seem to be useful for modelling of a weaker sense of desire.

> [...] we may now distinguish what we might call various levels of want or desire, beginning at the lowest level: (i) He prefers A to not-A. (ii) He opposes not-A. (iii) He favors A. (iv) He favors A and opposes not-A. [...]. The terms 'want' and 'desire', in their ordinary use, may be used to refer to any of these various levels of want

> or desire. They are generally used in such a way that the objects of want and desire are restricted to objects that are future [...]and to objects such that the persons who wants or desires them does not believe them to be impossible. (Chisholm [12] 623)

Remark 4 *Cross conceptualizes state (i) as a state of 'desire in the sense of incompatible goal-belief discrepancy' and state (ii) as a state of satisfaction. Our position is quite different. See Section 3.1 for a discussion. Note that on Chisholmian reading 'opposing that $\neg\varphi$' means 'desiring that φ in a weaker sense'.*

Semantic holism and rationality assumption. The 'semantic holism' involved in use of mental vocabulary is explained by the assumption that propositional attitudes *characterize* mental states of an agent. Propositional attitude reports do not single out an intentional state, rather they characterize a class of states by a necessary condition. The use of mental vocabulary imposes 'rationality assumption' on *descriptum* [14][16].

> Any effort at increasing the accuracy and power of a theory of behavior forces us to bring more and more of the whole system of the agent's beliefs and motives directly into account. But in inferring this system from the evidence, we necessarily impose conditions of coherence, rationality, and consistency. (Davidson [14] p. 231.)

One of the ways the rationality assumption is introduced is reflected in coherence condition: only coherent text can characterize a non-final mental state.

Example 8 *The text "It might be raining...It is not raining" is dynamically or sequentially consistent. Still it cannot be used for characterization of an intentional state due to its incoherence.*

5.2 Validity of rationalizations

Founded and founding systems. The studies in the *logical form of intentional explanations* and in *meaning relations between sentences in different logical moods* may be given a unified treatment by distinguishing founded and founding system. The founding 'inferential relations' are meaning relations between sentences in imperative and declarative logical moods. The validity of rationalizations formulated within intentional language depends on the validity of corresponding inference formulated in the language using logical moods.

5.2.1 Two modes of validity for rationalizations

Rationalization and rationality. First we introduce a criterion of validity for intentional explanations (also called: rationalizations or reason explanations. *A rationalization is valid iff it exemplifies a norm of rationality.*

Two modes of rationality. We make a distinction between two modes of rationality. Horizontal rationality is internal relation between intentional states. Vertical rationality is a external relation between an intentional state and the real world (see [38]). Two modes of rationality correspond to the two Davidsonian principles of interpretation. Principle of Coherence corresponds to the horizontal mode, Principle of Correspondence to the vertical mode.

> The process of separating meaning and opinion invokes two key principles which must be applicable if a speaker is interpretable: the Principle of Coherence and the Principle of Correspondence. The Principle of Coherence prompts the interpreter to discover a degree of logical consistency in the thought of the speaker; the principle of Correspondence prompts the interpreter to take the speaker to be responding to the same features of the world that he (the interpreter) would be responding under similar circumstances. Both principles can be (and

have been) called the principles of charity: one principle endows the speaker with the modicum of logic, the other endows him with a degree of what interpreter takes to be true belief about the world. Successful interpretation necessarily invests the person interpreted with the basic rationality. It follows from the nature of correct interpretation that an interpersonal standard of consistency and correspondence to the facts applies to both the speaker and the speaker's interpreter, to their utterances and to their beliefs. (Davidson [15] p.211)

Horizontal mode.

Definition 2 *The sequence*

$$P^1(a,\varphi_1),...,P^n(a,\varphi_n),\ \textit{therefore, (should)}\ P^{n+1}(a,\varphi_{n+1})$$

exemplifies a prima facie norm of horizontal rationality iff

$$M^1(\varphi_1);...;M^n(\varphi_n)\models_{0-ut} M^{n+1}(\varphi_{n+1})$$

and

$$0\left[M^1(\varphi_1);...;M^n(\varphi_n)\right]\notin\mathcal{F}$$

where M^i is a modal element corresponding to psychological predicate P^i .

Definition 3 *The sequence*

$$P^1(a,\varphi_1),...,P^n(a,\varphi_n),\ \textit{therefore, (should)}\ P^{n+1}(a,\varphi_{n+1})$$

exemplifies a norm of horizontal rationality iff

$$M^1(\varphi_1);...;M^n(\varphi_n)\vDash_{ut} M^{n+1}(\varphi_{n+1})$$

and

$$M^1(\varphi_1);...;M^n(\varphi_n)\ \textit{is coherent}$$

where M^i is a modal element corresponding to psychological predicate P^i .

Example 9 *The statement 'The desire that p and the belief that p only if q can rationalize the desire that q, but cannot rationalize the belief that q' should be conceived as abbreviation for 'The desire that p and the belief that p only if q rationalize the desire that q only if* $!p; \boxdot (p \to q) \vDash !q$*, but cannot rationalize the belief that q since* $!p; \boxdot (p \to q) \nvDash \bullet q$*'.*

The definitions given above are simplified in order to point out the connection between belief-desire logic and imperative logic. In order to make the definitions applicable one should extract sentence radical and consequently treat intentional-psychological predicates generously. For example, the intentional predicate in sentence "a believes that it might be snowing' is 'believes that it might be the case'. The other option in which complete sentences rather then their radicals are taken to be the 'objects' of a propositional attitude seems more natural.

Example 10 *'a believes that it is necessary that it is cold if it snows. She believes that it might be snowing. Therefore, she (should believe) believes that it might be cold' is not formalized as* '$B(a, \boxdot(s \to c)), B(a, \boxed{\Diamond\bullet} s) \vDash B(a, \boxed{\Diamond\bullet} c)$'*, but rather as* '$B_{\boxdot}(a, s \to c), B_{\Diamond\bullet}(a, s) \vDash B_{\Diamond\bullet}(a, c)$'*.*

Vertical mode

The definition of validity in vertical mode requires additional notions. The intentional states are rational if connected to the real world in appropriate way. It is not enough for a belief to be justified by another beliefs, both its grounds and the belief itself should be true. The same goes for desires: if one desires the impossible, she should revise her beliefs. For example, while it may be horizontally rational to want to open the window believed to be closed, it is always vertically irrational to want to open it if it is already open.

Definition 4 $\underline{w}$ *is a model of the real situation .* $\underline{\mathsf{R}_{w}}$ *is the set of objectively (really) possible continuations of the real situation,*

$$\underline{\mathsf{R}_{w}} = \{v \mid v \text{ is really possible at } \underline{w}\} .$$

Definition 5 *Theoretical model* $\langle \gamma, \alpha, \pi \rangle$ *of an intentional state is (possibly* α*-incomplete) veridical state iff*

$$\forall v : v \in \gamma \rightarrow v \in \underline{R_w}$$

and

$$\underline{w} \in \alpha$$

Definition 6 *The sequence*

$$\text{'}P^1(a, \varphi_1), ..., P^n(a, \varphi_n), \text{ therefore, (should) } P^{n+1}(a, \varphi_{n+1})\text{'}$$

exemplifies a norm of vertical rationality iff (i) it exemplifies a norm of horizontal rationality and (ii) every $\sigma \notin \mathcal{F}$ *such that* $\sigma \left[M^1(\varphi_1); ...; M^n(\varphi_n) \right] = \sigma$ *is a veridical model (where* M^i *is a modal element corresponding to psychological predicate* P^i*).*

5.3 Application: anatomy of excuse

Making an excuse shows how two kinds of semantic relations are being used in a language game. In the typical case, Excuser shows that his intentional action would not have been preformed if his state of mind was a veridical one. In virtue of having a horizontal and vertical semantic dimensions, the language of intentionality makes it possible for a horizontally rational intentional state to be vertically irrational.

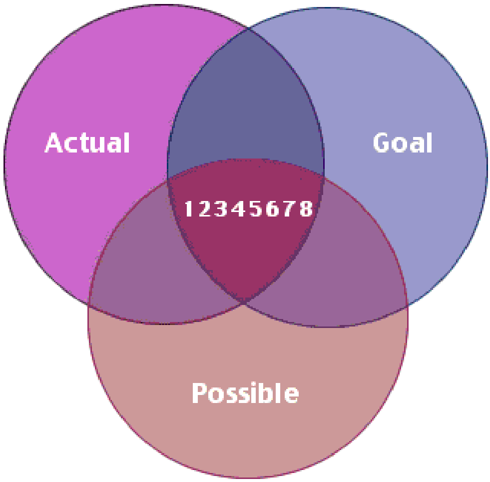
At a dull party, Husband wanted to make people laugh by telling a joke. Unfortunately, his telling a joke spoiled Wife's good mood. Husband apologizes: "I did not know that the joke would spoil your mood." In that way he shows that his desire to tell a joke was horizontally *prima facie* rational, but irrational in the vertical sense.

Full blown excuse:

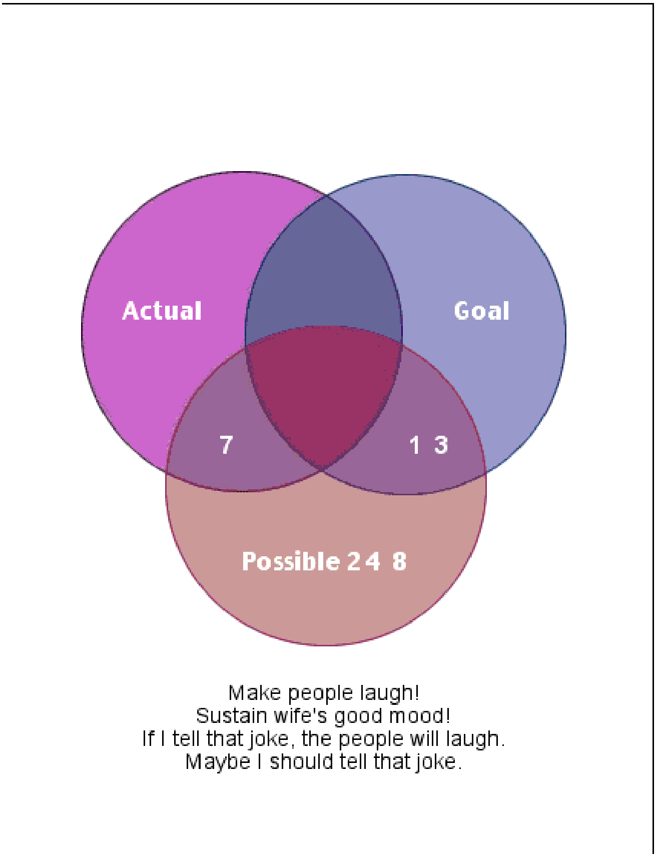
a) **realized horizontal part:**Wants$_!$(Husband, *The people are laughing*), Believes$_\Box$(Husband, *If Husband tells that joke,* *the people will laugh*), [1], therefore, *prima facie* $\text{Wants}_{!\Diamond}$(Husband, *Husband tells that joke*),

b) **failed vertical part**: $\text{Wants}_{!}$(Husband, *The people are laughing*), $\text{Believes}_{\boxdot}$(Husband, *If Husband tells that joke, the people will laugh*), $\text{Wants}_{\neg!\neg}$(Husband, *Wife is in a good mood*), $\text{Believes}_{\boxdot}$(Husband, *If Husband tells that joke, Wife will not be in a good mood*), therefore, $\text{Wants}_{!}$(Husband, *Husband does not tell the joke*).

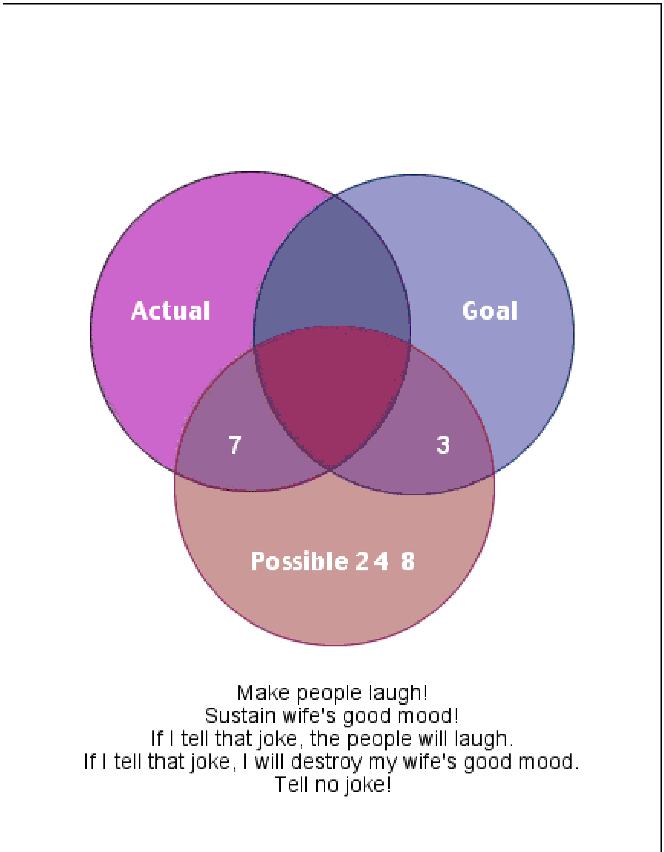
[1] 'Wants$_{\neg!\neg}$(Husband, *Wife is in a good mood*)'reads 'Husband wants to sustain Wife's good mood'.



Initial state, 0-state. L=People are laughing, J=Husband tells the joke, M=Wife is in a good mood. 1=$\{L, J, M\}$, 2=$\{L, J\}$, 3=$\{L, M\}$, 4=$\{L\}$, 5=$\{J, M\}$, 6=$\{J\}$, 7=$\{M\}$, 8=$\{\}$.



Defeasible conclusion holding in a non-veridical state (situation 1 is not really possible situation, $1 \notin \underline{R_w}$).



Vertically waranted conclusion: 'Don't tell that joke!'.

6 *Appendix*[1]

Remark 5 *Test sentences will be left out of consideration in the lemmata and propositions regarding properties of accessible states. Nevertheless, the results apply to test sentences in virtue of the fact that their application may generate only one change, namely change to the absurd state,* 1*. The results are given for* $L_{!\bullet\boxdot}$ *and* $L_{\Diamond!\bullet\boxdot}$*, still most of them apply to* $L_{\neg!\Diamond!\bullet\boxdot}$ *mutatis mutandis.*

Definition 7 $\langle\gamma,\alpha\rangle$ *is maximal state iff* $|\gamma| = |\alpha| = 1$ *and* $\gamma\cap\alpha = \emptyset$*.*

Proposition 4 σ *is maximal state iff* $\sigma\in\mathcal{M}$ *and* $\forall\varphi\in L_P : \sigma\left[\boxed{\Diamond!}\varphi\right] = \sigma \rightarrow \sigma\left[!\varphi\right] = \sigma$

Proof. Define function $nf : 2^W \rightarrow L_P$ which delivers a disjunctive normal form for a set of worlds X as $nf(X) = \bigvee_{w\in X}\left(\left(\bigwedge_{l\in w} l\right)\wedge\left(\bigwedge_{l\notin w}\neg l\right)\right)$. We have to prove that $|\gamma| = |\alpha| = 1$ and $\gamma\cap\alpha=\emptyset$, iff $\langle\gamma,\alpha\rangle\in\mathcal{M}$ and $\forall\varphi\in L_P : \langle\gamma,\alpha\rangle\left[\boxed{\Diamond!}\varphi\right] = \sigma \rightarrow \langle\gamma,\alpha\rangle\left[!\varphi\right] = \langle\gamma,\alpha\rangle$. Going from left to right, suppose for *reductio* that $\langle\gamma,\alpha\rangle\left[\boxed{\Diamond!}\varphi\right] = \langle\gamma,\alpha\rangle \wedge \langle\gamma,\alpha\rangle\left[!\varphi\right] \neq \langle\gamma,\alpha\rangle$. Then, either $\exists w\exists v : w\in\gamma\wedge v\in\gamma\wedge w\notin\|\varphi\|^W\wedge v\notin\|\varphi\|^W$ or $\exists w\exists v : w\in\alpha\wedge v\in\alpha\wedge w\notin\|\varphi\|^W\wedge v\in\|\varphi\|^W$. Therefore, either $|\gamma|\neq 1$ or $|\alpha|\neq 1$. Contradiction. Going from right to left, suppose for *reductio* that $|\gamma|\neq 1$ or $|\alpha|\neq 1$ or $\gamma\cap\alpha\neq\emptyset$. If $|\gamma| = 0$ or $|\alpha| = 0$, then $\langle\gamma,\alpha\rangle\in\mathcal{F}$. Contradiction. If $|\gamma| > 1$, then

$$\begin{aligned}\exists w &: w\in\gamma\wedge\gamma-\{w\}\neq\emptyset\wedge\\ \wedge\langle\gamma,\alpha\rangle\left[\boxed{\Diamond!}nf(\{w\})\right] &= \langle\gamma,\alpha\rangle\wedge\langle\gamma,\alpha\rangle\left[!nf(\{w\})\right] = \langle\{w\},\alpha\rangle.\end{aligned}$$

Therefore, some acceptable sentence is not accepted. Contradiction. If $|\alpha| > 1$ then

$$\exists w : w\in\alpha\wedge\alpha-\{w\}\neq\emptyset\wedge$$

[1] **Appendix** by Berislav Žarnić and Damir Vukičević

$$\wedge \langle\gamma,\alpha\rangle \left[\boxed{\Diamond!} nf(\{w\}\cup\gamma)\right] = \langle\gamma,\alpha\rangle \wedge$$
$$\wedge \langle\gamma,\alpha\rangle [!nf(\{w\}\cup\gamma)] = \langle\gamma,\alpha-\{w\}\rangle .$$

Contradiction. If $\gamma\cap\alpha\neq 0$, then $\langle\gamma,\alpha\rangle\notin\mathcal{M}$. Contradiction. ■

Definition 8 *σ is extended motivational state iff* $\exists\varphi\in L_P : \sigma\left[\boxed{\Diamond!}\varphi\right] = \sigma\wedge\sigma[!\varphi]\neq\sigma$

Proposition 5 *Any maximal state can be reached in two steps.*

Use function $nf : 2^W \to L_P$ which delivers a disjunctive normal form for a set of worlds X i.e. $nf(X) = \bigvee_{w\in X}\left(\left(\bigwedge_{l\in w} l\right)\wedge\left(\bigwedge_{l\notin w}\neg l\right)\right)$.

Let $\langle\gamma',\alpha'\rangle$ be a maximal state and $\gamma'\subseteq\gamma$, $\alpha'\subseteq\alpha$. Text $\bullet nf(\alpha');!nf(\gamma')$ is an instance of a complete two-sentence text since $\langle\gamma,\alpha\rangle[\bullet nf(\alpha')]\,[!nf(\gamma')] = \langle\gamma',\alpha'\rangle$

Proposition 6 *It is decidable whether a given text can reach a maximal state.*

For each sentence from a text $s_1;s_2;...;s_n$ extract sentence radical or its negation, and form a conjunction according to the following recipe:

$$\Phi_n = \bigwedge_{i\in\{1,...,n\}[(s_i=!\varphi)\vee(s_i=\boxdot\varphi)]} \varphi$$

and

$$\Psi_n = \bigwedge_{i\in\{1,...,n\}[(s_i=\bullet\psi)\vee(s_i=\boxdot\psi)\vee(s_i=!\neg\psi)]} \psi.$$

Text $s_1;s_2;...;s_n$ may reach a maximal state iff

$$\left|\|\Phi_n\|^W\right| = 1\wedge\left|\|\Psi_n\|^W\right| = 1\wedge\|\Phi_n\|^W\neq\|\Psi_n\|^W$$

Definition 9 $0/L_{\Diamond!\bullet\boxdot}$*-accessibility relation* $R_{0/L_{\Diamond!\bullet\boxdot}}\subseteq\Sigma^{L_{\Diamond!\bullet\boxdot}}\times\Sigma^{L_{\Diamond!\bullet\boxdot}}$ *:*

$$R_{0/L_{\Diamond!\bullet\boxdot}} =$$
$$R_{0/L_{\Diamond!\bullet\boxdot}} = = \left\{\langle\sigma,\sigma'\rangle \mid \begin{array}{c}\exists s_1...\exists s_n\exists s_{n+1}...\exists s_m : s_1,...,s_m\in L_{\Diamond!\bullet\boxdot}\wedge\\ \wedge 0[s_1;...;s_n]=\sigma\wedge\sigma[s_{n+1};...;s_m]=\sigma'\end{array}\right\}$$

Lemma 7 *Let $\langle \gamma, \alpha \rangle = 0\,[T]$. Then for any model $\langle \gamma', \alpha' \rangle$ such that $\langle \gamma', \alpha' \rangle\,[T] = \langle \gamma', \alpha' \rangle$ and $\langle \gamma', \alpha' \rangle \neq \langle \gamma, \alpha \rangle$ it holds that $\gamma' \subset \gamma$ or $\alpha' \subset \alpha$.*

Proof. Let $T = s_1; s_2; ...; s_n$. Use the recipe from Proposition 6 to obtain Φ_n and Ψ_n for text T. Given that $0\,[T] = \left\langle \|\Phi_n\|^W, \|\Psi_n\|^W \right\rangle$, $\gamma' \subseteq \|\Phi_n\|^W$, $\alpha' \subseteq \|\Psi_n\|^W$, lemma follows. ■

Proposition 8 *Each model for a text T is accessible from its minimal model.*

Proof. Let $\langle \gamma', \alpha' \rangle$ be a nonfinal, $0/L_{\Diamond!\bullet\boxdot}$-accessible model for a text T and $\sigma^{minT} = \langle \gamma, \alpha \rangle = 0\,[T]$, then

$$\langle \gamma, \alpha \rangle \left[\boxdot nf\left(\gamma' \cup \alpha'\right)\right] \left[\bullet nf\left(\alpha'\right)\right] = \left\langle \gamma', \alpha' \right\rangle.$$

Text $\boxdot nf\,(\gamma' \cup \alpha')\,; \bullet nf\,(\alpha')$ is an instance of a text that reaches $\langle \gamma', \alpha' \rangle$. Therefore, $\left\langle \sigma^{minT}, \langle \gamma', \alpha' \rangle \right\rangle \in R_{0/L_{\Diamond!\bullet\boxdot}}$. ■

Proposition 9 *For each nonminimal model $\langle \gamma', \alpha' \rangle$ for a text T there is a text T^* which is not accepted in the minimal model $\langle \gamma, \alpha \rangle = 0\,[T]$.*

Proof. Denote by $\|T\|^{\Sigma_{0/L_{\Diamond!\bullet\boxdot}}}$ a set of nonfinal states in which T is accepted. If $\gamma' \subset \gamma$ or $\alpha' \subset \alpha$, then $0\,[T]\,[\boxdot nf\,(\gamma' \cup \alpha')]\,[\bullet nf\,(\alpha')] \neq 0\,[T]$. Therefore, $T; \boxdot nf\,(\gamma' \cup \alpha')\,; \bullet nf\,(\alpha') = T^*$ ■

Proposition 10 *A sentence ψ, accepted in a model $\langle \gamma, \alpha \rangle$, is rejectable iff: (i) $\psi = \boxed{\Diamond\bullet}\varphi$ and $\|\varphi\|^\alpha \neq \emptyset \wedge \|\neg\varphi\|^\alpha \neq \emptyset$, or (ii) $\psi = \boxed{\Diamond!}\varphi$ and $\|\varphi\|^\gamma \neq \emptyset \wedge \|\neg\varphi\|^\alpha \neq \emptyset \wedge (\|\neg\varphi\|^\gamma \neq \emptyset \vee \|\varphi\|^\alpha \neq \emptyset)$, or (iii) $\psi = \boxed{!\Diamond}\varphi$ and $\gamma \cap \alpha = \emptyset \wedge \|\varphi\|^\gamma \neq \emptyset \wedge \|\neg\varphi\|^\alpha \neq \emptyset \wedge ((\|\neg\varphi\|^\gamma = \emptyset \wedge \|\varphi\|^\alpha \neq \emptyset) \vee (\|\neg\varphi\|^\gamma \neq \emptyset \wedge \|\varphi\|^\alpha = \emptyset))$.*

Proof. Direct calculation. ■

Lemma 11 (Elimination lemma) *For any γ, α, γ', α' such that $\langle \langle \gamma, \alpha \rangle, \langle \gamma', \alpha' \rangle \rangle \in R_{0/L_{\Diamond!\bullet\boxdot}}$: $\alpha' \subseteq \alpha$ and $\gamma' \subseteq \gamma$. If $\langle \gamma, \alpha \rangle \neq \langle \gamma', \alpha' \rangle$, then $\gamma' \subset \gamma$ or $\alpha' \subset \alpha$.*

Proof. Trivial. ■

Lemma 12 *Let* $T = s_1; ...; s_n$*, where* $s_i = \boxdot\varphi$ *or* $s_i = \bullet\varphi$ *,* $\langle\gamma,\alpha\rangle[T] = \langle\gamma',\alpha'\rangle$*,* $\alpha \subseteq \gamma$*. Then* $\alpha' \subseteq \gamma'$*.*

Proof. Induction. ■

Lemma 13 *Let* $\langle\gamma,\alpha\rangle[T] = \langle\gamma',\alpha'\rangle$*,*$\gamma\cap\alpha = \emptyset$*. Then* $\gamma'\cap\alpha' = \emptyset$*.*

Proof. Use Lemma 11 ■

Lemma 14 *Let* $\langle\gamma,\alpha\rangle[s_1; ...; s_n] = \langle\gamma',\alpha'\rangle$ *and at least one sentence* s_i *is* $!\varphi$ *for some* $\varphi \in L_P$*. Then* $\gamma' \cap \alpha' = \emptyset$*.*

Proof. It is obvious that for any γ and α that $\|\varphi\|^\gamma \cap \|\neg\varphi\|^\alpha = \emptyset$ and that property is preserved by Lemma 13. ■

Theorem 15 *The state* $\sigma = \langle\gamma,\alpha\rangle$ *is* $0/L_{\Diamond!\bullet\boxdot}$*-accessible if and only if* $\gamma \cap \alpha = \emptyset$ *or* $\alpha \subseteq \gamma$*.*

Proof. Going from right to left. First, assume $\gamma \cap \alpha = \emptyset$. Then

$$0\left[\boxdot\left(nf\left(\alpha\cup\gamma\right)\right)\right]\left[!\left(nf\left(\gamma\right)\right)\right] = \langle\gamma,\alpha\rangle.$$

Second, assume $\alpha \subseteq \gamma$. Then

$$0\left[\boxdot\left(nf\left(\gamma\right)\right)\right]\left[\bullet\left(nf\left(\alpha\right)\right)\right] = \langle\gamma,\alpha\rangle.$$

Going from left to right, we shall prove that a state is not $0/L_{\Diamond!\bullet\boxdot}$-accessible if $\gamma \cap \alpha \neq \emptyset$ and $\alpha \nsubseteq \gamma$. Let $\alpha \nsubseteq \gamma$ and $0[s_1; ...; s_n] = \langle\gamma,\alpha\rangle$. It implies that there is a sentence s_i such that $s_i \neq \boxdot\varphi$ and $s_i \neq \bullet\varphi$. Since there are only three types of basic sentences, then it must be the case that $s_i =!\varphi$. But Lemma 14 and the fact $\gamma \cap \alpha \neq \emptyset$ imply that there is no such sentence in $s_1; ...; s_n$. Contradiction. ■

Proposition 16 $\langle\gamma,\alpha\rangle \in \mathcal{M}$ *iff* $\exists\varphi : \varphi \in L_P \wedge \langle\gamma,\alpha\rangle[!\varphi] = \langle\gamma,\alpha\rangle$

Proof. Going from left to right. For reductio assume $\neg\exists\varphi : \varphi \in L_P \wedge \langle\gamma,\alpha\rangle [!\varphi] = \langle\gamma,\alpha\rangle$. But, $\langle\gamma,\alpha\rangle [!nf(\gamma)] = \langle\gamma,\alpha\rangle$. Contradiction. The other direction trivially follows from $\|\varphi\|^{\gamma} \cap \|\neg\varphi\|^{\alpha} = \emptyset$. ■

Proposition 17 *Let $\gamma \subseteq \|\varphi\|^{W}$ and $\alpha \cap \|\varphi\|^{W} \neq \emptyset$ and $\alpha \nsubseteq \|\varphi\|^{W}$. Then $\langle\gamma,\alpha\rangle \in \mathcal{M}$.*

Proof. For reductio suppose $\gamma \cap \alpha \neq \emptyset$. Use 15 to obtain a contradiction. ■

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