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Imperative change and obligation to do

In:
Krister Segerberg and Rysiek Sliwinski (eds.)

Logic, Law, Morality: Thirteen Essays in Practical Philosophy in Honour of Lennart Åqvist.

pp. 79-95.

Uppsala philosophical studies **51**
Uppsala: Department of Philosophy, Uppsala University
2003.

IMPERATIVE CHANGE AND OBLIGATION TO DO

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The aim of this paper is to provide a solution for the problem (quoted below) posed by von Wright. Let '(A/B)' stand for[1] 'a change from initial state A to resulting state B occurs (will occur)' How is it possible that it is obligatory that an agent performs an action that makes (¬A/A) come true while it is forbidden that s/he performs an action that makes (A/A) true?

> Let "p" stand for "the window is closed". This state can result from two different actions of a given agent. One is the act of closing the window if it is open. The other is the act of preventing the window from opening should that happen if the agent stays passive. The first action *produces* a state which is *not* there. The second *prevents* a state which *is* there from vanishing.
> The two actions may have different "deontic status". For example: closing the window, if open, may be obligatory, but preventing it from being open, should it be closed, may in fact be forbidden.
> (von Wright, 1999, p. 18)

We will try to approach the problem following the idea that imperative logic has a natural connection with deontic logic. Intuitively, by uttering an imperative the "imperator" makes an action obligatory or forbidden for the addressee.

The plan of the paper is as follows. First, we will try to find a solution using Lemmon's (1965) system of change logic connecting logic of orders being in force and deontic logic. Second, we will analyze Aqvist's (1965) approach to the problem of connecting imperative and deontic logic. Third, *commanded change/prescribed actions* approach to the semantics of imperatives (Lemmon, 1965, Segerberg, 1990, 1996) will be restricted to the natural language imperatives and Aqvist's way of connecting imperative and deontic logic will be modified accordingly. Fourth, a simple "global" semantics for deontic "agentives" will be developed using the notion of "opposite action". Finally, a solution for von Wright's problem will be given. In the closing sections some further topics for investigation will be touched on: one of them being the connection between Aqvist's *epistemic imperative* conception of interrogatives

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1 In the text '(A/B)' will always be used as the notation for change expression. Consequently, 'Ought(A/B)' will not stand for conditional obligation 'A is obligatory under condition B', instead, it will stand for 'it is obligatory that A changes to B'.

and "epistemic obligations", the other being formalization of the idea that imperatives create and re-create obligation patterns that can be described in deontic terms.

Connection: imperative and deontic logic

There is a widespread conjecture that imperative logic and deontic logic are somehow connected. According to Alf Ross (1941), “linguistically indicative sentences of «duty» and «rightness» with regard to action” should be counted as imperatives. Stig Kanger (1957/71) equated the valuation[2] for imperative sentence !*F* and the valuation for deontic statement Ought*F*. Opposing attitudes are less common. Peter Geach (1958) argued that “moral utterances” and imperatives “have essentially different logical features: in particular, as regards negation” (p. 49).

It seems that we can distinguish two matching imperative-deontic pairs. On one side, imperative as commanded action matches *ought to be done* deontic logic, while, on the other side, the notion of imperatives as a special kind of future tensed utterances complies with *ought to be* deontic logic. It may well be that “different logical features” of which Geach spoke pertain to the members of different pairs. For example, one should not expect to find a parallelism between *ought to be done* logic and imperative obligations and imperative permissions in the Chellas’ (1971) style[3]. As regards von Wright’s problem quoted above, the first imperative-deontic pair, or *it shall be the case*-*ought to be* pair is obviously not suitable for explaining the possibility of different deontic status of two actions resulting in mutually exclusive situations.

In order to examine the consistency of ‘Obligatory(¬A/A)∧Forbidden(A/A)’ following the proposed route, i.e. from imperative to deontic logic, one should take into consideration the second, *do!-ought to be done* pair. In the forthcoming sections, we shall examine Lemmon’s (1965) imperative logic and its deontic extension given by Åqvist (1966). It is our opinion that the examination will provide the reasons for a semantics that is not “in the world”, i.e. the reasons for the semantics that cannot be reduced to a valuation at a particular point.

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2 T(r,V,!F) = T(r,V,OughtF); T(r,V,OughtF) = 1 if and only if T(r′,V,OughtF) = 1 for each r′.such that ROught(r′,r), where r and r′ are “non-empty classes of individuals”, V is “primary valuation” which delivers an interpretation for non-logical symbols for each r, and T is “secondary valuation”

3 Chellas defines imperative obligation as follows: $||!\varphi||(w, t) = 1$ iff $||\varphi||(w', t) = 1$ for every w′ such that Rt(w, w′); imperative permission: $||¡\varphi||(w, t) = 1$ iff $||\varphi||(w', t) = 1$ for some w′ such that Rt(w, w′).

Commanded changes and orders being in force

In Lemmon's (1965) analysis imperatives are a kind of change expressions[4]. Change expression is an "expression of the form (A/B) where A and B are truth functional expressions" (p. 59). Semantics of imperatives may be given in terms of obedience and disobedience conditions: an imperative !(A/B) is obeyed if and only if the change from A to B takes place.

First	Next	Change expression	"Positive" imperative	Two negative imperatives ~!(A/B)	
				N1	N2
A	B	(A/B)	!(A/B)	!(A/~B)	!~(A/B)
$\top$	$\top$	$\top$	**O**	D	**D**
$\top$	$\bot$	$\bot$	D	**O**	**O**
$\bot$	$\top$	$\bot$	**D**	D	**O**
$\bot$	$\bot$	$\bot$	D	**D**	O

'O' stands for 'obeyed'
'D' stands for 'disobeyed'

Lemmon (1965) further developed the semantics of imperatives using the notion of an order being in force. In his opinion, the common ground between imperative and deontic logic is to be sought for within the logic of orders being in force rather than within the logic of imperatives.

Notation 'O!(A/B)' stands for 'the order to effect change (A/B) is in force'. Lemmon's exposition on the notion is very condensed and it is explicated only in syntactic terms. The meaning of the notion is given by the axiom schema (O4) and the rule of inference (RO2). Axiom schema (O4) $O(S \rightarrow C) \supset (O!S \supset OC)$ is to be read as "if one is ordered to do C in case of change S, then if one is ordered to effect S one is implicitly under orders to do C" (p.61). The rule of inference is "(RO2) from $S \supset T$ to derive $O!S \supset O!T$" (p. 62), where 'S' and 'T' stand for arbitrary change expressions.

Application of the *order being in force* logic on the von Wright's problem of different deontic status shows that it is possible for two orders !(~A/A) and !(A/~A) to be simultaneously in force. Alas, the price is too high. In that case every order is in force.

4 The notation that is used in this paper always follows the notation used in the papers that are being referred to.

(1) $O!(\sim A/A)$ premise

(2) $O!(A/\sim A)$ premise

(3) $(\sim A/A) \supset ((A/\sim A) \supset ((\sim A/A)\&(A/\sim A)))$ tautology

(4) $O!(\sim A/A) \supset O!((A/\sim A) \supset ((\sim A/A)\&(A/\sim A)))$ (3), rule RO2

(5) $O!((A/\sim A) \supset ((\sim A/A)\&(A/\sim A)))$ (1), (4), modus ponens

(6*) $O((A/\sim A) \rightarrow !((\sim A/A)\&(A/\sim A)))$ (5) definition for imperative conditional[5]

(7) $O((A/\sim A) \rightarrow !((\sim A/A)\&(A/\sim A))) \supset (O!(A/\sim A) \supset O!((\sim A/A)\&(A/\sim A)))$ axiom O4

(8) $O!(A/\sim A) \supset O!((\sim A/A)\&(A/\sim A))$ (6),(7), modus ponens

(9) $O!((\sim A/A)\&(A/\sim A))$ (2),(8), modus ponens

(10) $((\sim A/A)\&(A/\sim A)) \supset (\sim A/A\&\sim A)$ change logic

(11) $O!((\sim A/A)\&(A/\sim A)) \supset O!(\sim A/A\&\sim A)$ (10), rule RO2

(12) $O!(\sim A/A\&\sim A)$ (9), (11), modus ponens

(13) $(\sim A/A\&\sim A)\supset(C/D)$ change logic

(14) $O!(\sim A/A\&\sim A)\supset O!(C/D)$ (13), rule RO2

(15) $O!(C/D)$ (12), (14), modus ponens

Lemmon (1965) wanted to reconsider "the soundness of […] deontic principles when interpreted as concerning orders". If one follows the line of thought, s/he must conclude that the notion of order being in force does not correspond to the notion of deontic status of an action. One may easily agree with the theorem $(O!(\sim A/A)\&O!(A/\sim A)) \supset O!(C/D)$: if the order to effect change $(\sim A/A)$ is in force and the order to effect change $(A/\sim A)$ is in force, then every order is in force. On the other hand, it does not seem valid to infer that every action is obligatory given that it is forbidden to effect change (A/A) and that it is obligatory to effect change $(\sim A/A)$.

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5 It may be questioned whether step (6) is really licensed in Lemmon's system. In the system, the symbol '$\rightarrow$' is used for the special case of conditional imperative, having indicative antecedent and imperative consequent, e.g. $(A/B)\rightarrow!(C/D)$. In all other conditional compounds, the symbol '$\supset$' is used in the sense of material implication. One can doubt whether $(A/B)\rightarrow!(C/D)$ is equivalent to $!(A/B)\supset!(C/D)$. The example on p.63 confirms the equivalence: "…[order $O((p/p)\rightarrow!(q/\sim q))$] is equivalent to $!((p/\sim p)\vee(\sim p/p)\vee(\sim p/\sim p)\vee(q/\sim q))$…"

Tense and deontic operators

Åqvist (1966) investigated four[6] ways of rewriting imperative change expressions in deontic terms. For that purpose he introduced binary deontic operators $\mathbf{O}_x$- ($\mathbf{F}_x$-, $\mathbf{P}_x$-) for formalization of *It ought to be (it is forbidden, it is permitted) that A changes to B* and he used unary modal operator **O** that may be applied on a change expression. In other words, *It ought to be that A changes to B* is formalized as $\mathbf{O}_x(A,B)$ and analysed using unary "obligation-connective" **O.**

$$\mathbf{O_1}(A,B) =_{def} \mathbf{O}(A/B)$$
$$\mathbf{O_2}(A,B) =_{def} A\wedge\mathbf{O}(\top/B)$$

The difference between $\mathbf{O}_1$ and $\mathbf{O}_2$ pertains to the temporal location of A. In the first case A and subsequent B both lie within each ideal future. In the second case, A is true now and there is a future state in each ideal history followed by B.

$$\mathbf{F_1}(A,B) =_{def} \mathbf{O}(\sim A/\sim B)$$
$$\mathbf{F_2}(A,B) =_{def} A\wedge\mathbf{O}(\top/\sim B)$$

Deontic unary connectives are usually conceived as inter-defineable: $\mathbf{O}A \equiv \mathbf{F}\sim A \equiv \sim\mathbf{P}\sim A$. On the other hand, Åqvist's (1966) analysis clearly shows that binary deontic operators are not interdefineable in that way.

$$\mathbf{P_1}(A,B) =_{def} \sim\mathbf{O}\sim(A/B) \text{ where } \sim(A/B) \equiv ((A/\sim B)\vee(\sim A/B)\vee(\sim A/\sim B))$$
$$\mathbf{P_2}(A,B) =_{def} A\wedge\sim\mathbf{O}\sim(\top/B)\text{, i.e. } A\wedge\sim\mathbf{O}(\top/\sim B),$$
$$\mathbf{F_3}(A,B) =_{def} \mathbf{O}((A/\sim B)\vee(\sim A/B)\vee(\sim A/\sim B))$$
$$\mathbf{F_4}(A,B) =_{def} \sim A\vee\mathbf{O}\sim(\top/\sim B))\text{, i.e. } \sim A\vee\mathbf{O}(\top/B))$$

The duality of **O** and **F** is not disputable: both $\mathbf{O}_1(A,B) \equiv \mathbf{F}_1(\sim A,\sim B)$ and $\mathbf{O}_2(A,B) \equiv \mathbf{F}_2(\sim A,\sim B)$ are valid. On the other side, $\mathbf{P}_1$ cannot be straightforwardly defined in terms of $\mathbf{F}_1$ or $\mathbf{O}_1$. The definition of $\mathbf{P}_1(A,B)$ cannot be given as $\sim\mathbf{O}_1\sim(A,B)$ or $\sim\mathbf{F}_1\sim(\sim A,\sim B)$ since the latter two expressions are not well-formed. If one uses $\sim\mathbf{P}A$ for defining $\mathbf{F}A$ or $\mathbf{O}\sim A$, one is thrown out of any of the two triads, $\mathbf{O}_1$-$\mathbf{F}_1$-$\mathbf{P_1}$ and $\mathbf{O}_2$-$\mathbf{F}_2$-$\mathbf{P_2}$. So, $\sim\mathbf{P}_1(A/B)$ and $\sim\mathbf{P}_2(A,B)$ are not expressible in terms of $\mathbf{F}_1$ and $\mathbf{F}_2$, respectively.

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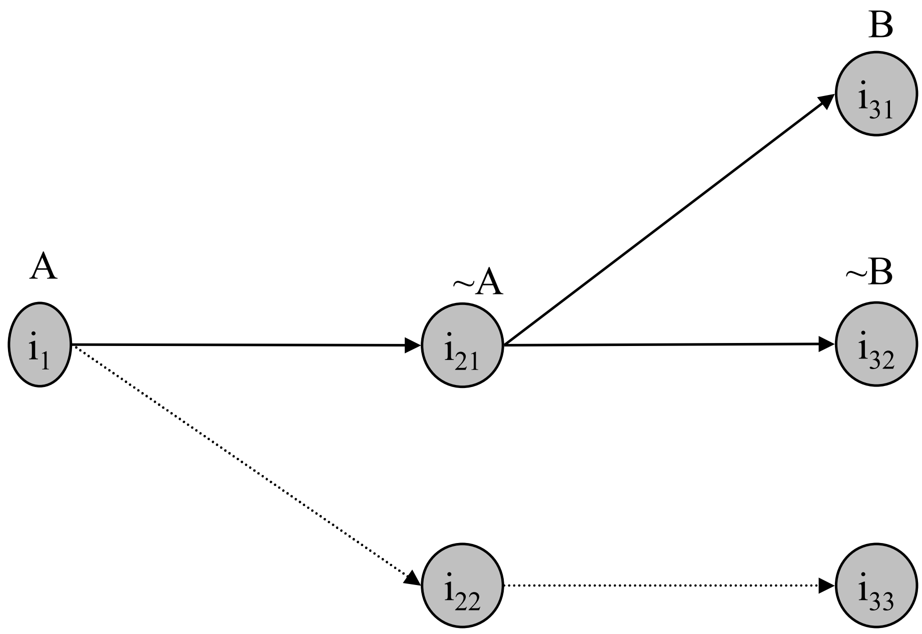
6 In deontic tense logic DDT (Åqvist, 1966) sentences are evaluated on a «DDT-system». Sentence $\top/A$ is true at the instant i and history h iff A is true at the successor i' lying on h. Sentence OA is evaluated at the instant i and the set histories, I that are ideal relative to i: OA is true at i iff A is true at each successor of i lying on any history that is ideal relative to i. The difference of $O\top/A$ from $\top/OA$ amounts to this: «$O\top/A$ speaks of what takes place at the next stage in a course of events supposed to be ideal relative to [the evaluation point]; whereas $\top/OA$ speaks of what takes place at the first stage of the course of events which is ideal relative to the next stage in the actual course of events», p.247.

$\sim\mathbf{P}_1(A,B) \equiv \mathbf{F}_3(A,B)$
$\sim\mathbf{P}_2(A,B) \equiv \mathbf{F}_4(A,B)$



Full lines mark the histories that are ideal relative to i_1. Dotted lines mark non-ideal ones. At the evaluation point i_1 only $\mathbf{F}_3(A,B)$ is true; $\mathbf{F}_1(A,B)$, $\mathbf{F}_2(A,B)$, and $\mathbf{F}_4(A,B)$ are all false.

Negative and positive imperatives

Let us pay attention to natural language imperatives. On the side of syntax, the general Lemmon change form, (A/B) may be restricted to the two forms. The change expressions that may occur in natural language imperative mood are limited to two elementary forms: symmetric, !(A/A) imperative form and complementary imperative form, !(¬A/A), both of which use the same base propositions. On the side of semantics, change expressions take two temporally located points of valuation: the initial and a resulting situation. In its informational part an imperative gives a description of the initial situation. The initial situation may be present or it may lie in the future. For example, in 'Go to a grocery store tomorrow!' we find a description of an initial future situation. The resulting situation always lies in the future, it is later than the initial situation, and it has a volitive or appetitive direction of fit (it is the resulting situation that should fit the imperative description and not *vice versa*).

In the next sections we shall try to apply the analysis of natural language imperatives partly following the style of Åqvist (1966) paper. A "one shot" semantics with two sets of valuation points will be chosen. The first evaluation point will check the left side of a change expression, the second evaluation point will check the right side, the *shall* part. In that way, the correspondent for $\mathbf{O}(A/\top)$ will be $\mathbf{O^*}(\top/A)$. For example, in evaluating '$(A/\top)\wedge\mathbf{O^*}(B/C)$' the valuation points for 'A' and 'B' will coincide and they will differ from the valuation point for 'C'.

The translation of natural language imperatives into a language of deontic terms should preserve the meaning relations between positive and negative imperatives. Positive and negative imperatives in natural language are equal in their binding force and informational layers. Negative imperative imposes obligation in the same way the positive imperative does. 'Do not open the door!' imposes obligation to keep the door closed in the way similar to the way in which 'Open the door!' imposes the obligation to open it. Negative and positive imperative take the same initial situation type but impose contradictory resulting states. The $\mathbf{O}_i$-F_i definitions will be modified in order to establish a correspondence with the concept of negative imperative.

The negation rule (Lemmon's N1 negation type):

$$\neg!(\neg A/A) \Leftrightarrow !(\neg A/\neg A)$$
$$\neg!(A/A) \Leftrightarrow !(A/\neg A)$$

Modified translations:

$$\mathbf{O_{1^*}}(A,B) =_{def} (\neg A/\top)\vee\mathbf{O^*}(\top/B)$$
$$\mathbf{F_{1^*}}(A,B) =_{def} (\neg A/\top)\vee\mathbf{O^*}(\top/\neg B)$$
$$\mathbf{O_{2^*}}(A,B) =_{def} \mathbf{O^*}(A/B)$$
$$\mathbf{F_{2^*}}(A,B) =_{def} \mathbf{O^*}(A/\neg B)$$
$$\mathbf{O^*}(A/B) =_{def} (A/\top)\wedge\mathbf{O^*}(\top/B)$$

Imperative conditional combines an imperative and an indicative sentence. The time of eventual occurrence of the situation described in the indicative part coincides with the time of the eventual occurrence of the initial situation described by the left side of the imperative change expression. For example: let 'P' stand for 'the window is closed' and 'Q' for 'it is raining'. 'Close the window if it is raining' is formalized as '$(Q/\top)\rightarrow!(\neg P/P)$'. 'Keep the window closed if it is raining' is in the same fashion formalized as '$(Q/\top)\rightarrow!(P/P)$'.

The law of contraposition:

$$!(\neg A/A)\rightarrow(B/\top) \quad \Leftrightarrow \quad (\neg B/\top)\rightarrow\neg!(\neg A/A)$$
$$\Leftrightarrow \quad (\neg B/\top)\rightarrow!(\neg A/\neg A)$$

For example, under the proposed interpretation the conditional imperative 'Unless it rains do not open the window' is equivalent to 'Open the window only if it rains'.

Deontic expressions		**Imperative expressions**	
$O_{1*}(A,A)$ It is obligatory that A does not change to $\neg$A.	$F_{1*}(A,\neg A)$ It is forbidden that A changes to $\neg$A	$(A/\top)\rightarrow!(A/A)$ Maintain A if A is the case.	$(A/\top)\rightarrow\neg!(A/\neg A)$ Do not produce $\neg$A if A is the case.
$O_{2*}(\neg A,A)$ It is the case that $\neg$A and it is obligatory that A (will be)..	$F_{2*}(\neg A,\neg A)$ It is the case that $\neg$A and it is forbidden that $\neg$A (will be)..	$!(\neg A/A)$ Produce A!	$\neg!(\neg A/\neg A)$ Do not preserve $\neg$A!
Two examples for translations (in the rows).			

Investigation by cases will not reveal the fact the two actions, producing P and preventing P, may "have different deontic status".

Case 1. $\mathbf{O}_{1*}$- ($\mathbf{F}_{1*}$-) translation.

(1) $\mathbf{O}_{1*}(\neg P,P)\wedge\mathbf{F}_{1*}(P,P)$	deontic premise
(2) $((\neg P/\top)\rightarrow\mathbf{O^*}(\top/P))\wedge((P/\top)\rightarrow\mathbf{O^*}(\top/\neg P))$	(1), $\mathbf{O}_{1*}/\mathbf{F}_{1*}$ definitions
(3) $\mathbf{O^*}(\top/P)\vee\mathbf{O^*}(\top/\neg P)$	(3), propositional logic

Case 2. $\mathbf{O}_{2*}$- ($\mathbf{F}_{2*}$-) translation..

(1) $\mathbf{O}_{2*}(\neg P,P)\wedge\mathbf{F}_{2*}(P,P)$	deontic premise
(2) $((\neg P/\top)\wedge\mathbf{O^*}(\top/P))\wedge((P/\top)\wedge\mathbf{O^*}(\top/\neg P))$	(1), $\mathbf{O}_{2*}/\mathbf{F}_{2*}$ translations
(3) $\perp$	(2), change logic

In local, or "truth at the point" semantics von Wright's problem cannot be solved. In the first case, $\mathbf{O}_{1*}(\neg P,P)\wedge\mathbf{F}_{1*}(P,P)$ is satisfiable. Nevertheless, what we need is $\mathbf{O^*}(\top/P)\wedge\mathbf{O^*}(\top/\neg P)$, and that is not satisfiable. In the second case, $\mathbf{O}_{2*}(\neg P,P)\wedge\mathbf{F}_{2*}(P,P)$ is not satisfiable. In order to explain von Wright's example (*i.e.* to show that 'Obligatory($\neg$P/P)$\wedge$Forbidden(P/P)' is consistent) one must turn to a different semantics.

Global semantics and opposite actions

Let us investigate an action deontic status within a simple global semantics that relies on some intuitive relations between the expressions: 'obligatory', 'forbidden' and 'permitted'. The formal model $<W,W^*,\Pi>$ is intended to model a cognitive-motivational state of an addressee with respect to the changes that s/he ought (ought not) to perform. Alternatively, we will use the term 'obligation pattern' for a cognitive motivational state.

> W is the set of valuations indexed by the time t
> W* is the set of valuations indexed by the time t* which is later than t
> $\Pi \subseteq W\times W^*$, the set of permitted performable changes

Intension of a sentence A of propositional logic given a set of valuations X is a subset of X:

$$|A|^{X} = \{w: w\in X \text{ and } w(A) = \mathrm{T}\}$$

Intension of a change expression (A/B) is an action:

$$\Vert A/B \Vert^{W\times W^*} = \{<w,v>: w\in W,\ v\in W^*,\ w\in|A|^{W} \text{ and } v\in|B|^{W^*}\}$$

An action $\Vert(A/B)\Vert^{W\times W^*}$ is performable from an agent's perspective only if s/he does not believe that $\neg$A is (will be) the case in the initial situation. If an agent knows that $\neg$A is the case in the initial situation, then s/he considers it to be impossible to perform an action that will make a change (A/B) come true. In the realm of uncertainty when an agent does not know whether A or $\neg$A is the case in the initial situation, both (A/B) and ($\neg$A/B) may be (doxastically) performable actions.

Opposite action:

$$\mathrm{OPPOSITE}(\Vert A/B \Vert^{W\times W^*}) = \Vert A/\neg B \Vert^{W\times W^*}$$

The notion of opposite action resembles the notion of negative imperative. For *positive imperative-negative imperative* pair and for *action-opposite action* pair the descriptions of initial situations coincide, while the descriptions of the resulting situations disagree.

Produce action $\Vert P/P\Vert^{W\times W^*}$	Complement action $\Vert\top/\top\Vert^{W\times W^*} - \Vert P/P\Vert^{W\times W^*}$	Opposite action $\Vert P/\neg P\Vert^{W\times W^*}$
Keeping the window closed		
	Opening the window	Opening the window
	Closing the window	
	Keeping the window open	
W and W* have all the valuations.		

In defining semantic conditions we will use the notion of ‘a formula being required by the model’. In that way we want to emphasize the fact that a valuation by the inclusion in the set of performable permitted actions will be used instead of valuation at a point

Obligatory action:

Model <W,W*,Π> requires **O***(X) if and only if $\|X\| \subseteq \Pi$ and $\text{OPPOSITE}(\|X\|) \cap \Pi = \varnothing$

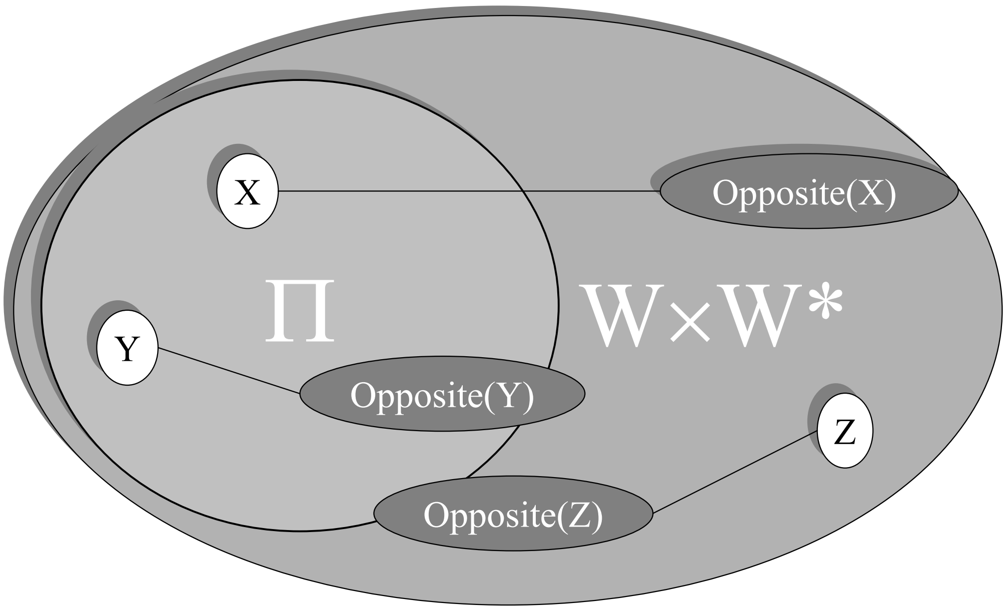
Permitted action:

Model <W,W*,Π> requires **P***(X) if and only if $\|X\| \cap \Pi \neq \varnothing$

Forbidden action:

F*(A/B) ⇔ **O***(A/¬B)

The main definitions rely on the meaning relations between ‘obligatory’, ‘forbidden’ and ‘permitted’. Interdefinability is preserved due to the fact that in a sense obligatory action is defined as the one which is permitted and whose opposite action is not permitted.



Deontic status of a generic action depends on the “location” of its opposite. Action X is obligatory and opposite of X is forbidden. Neither action Y nor opposite of Y are obligatory. Neither action Z nor opposite of Z are forbidden.

We shall formulate some principles that are sound with the respect to the given semantics.

Expansion principles:

(PE) If an action is obligatory, then any act that realizes it is permitted.

$$\mathbf{O}^*(A/B) \Rightarrow \mathbf{P}^*(A/B \wedge C)\text{, where } \neg C \text{ is not the consequence of B:}$$

(NE) If an action is obligatory, then no act failing to realize it is permitted.

$$\mathbf{O}^*(A/B) \Rightarrow \neg\mathbf{P}^*(A/\neg B \wedge C)$$

Using the two principles it can be proved that two actions equivalent with the respect to the initial situation and leading to logically independent states of affairs cannot be both obligatory.

$\neg(\mathbf{O}^*(A/B) \wedge \mathbf{O}^*(A/C))$, where $\neg C$ is not the consequence of B

Proof (*reductio ad absurdum*). Suppose that $\mathbf{O}^*(A/B) \wedge \mathbf{O}^*(A/C)$ holds. Then by propositional logic both $\mathbf{O}^*(A/B)$ and $\mathbf{O}^*(A/C)$ must hold. By (PE) expansion principle $\mathbf{O}^*(A/B)$ implies $\mathbf{P}^*(A/B \wedge \neg C)$. By (NE) expansion principle $\mathbf{O}(A/C)$ implies $\neg\mathbf{P}^*(A/B \wedge \neg C)$. Contradiction. Therefore, $\neg(\mathbf{O}^*(A/B) \wedge \mathbf{O}^*(A/C))$.

Using these definitions one can accommodate von Wright's example. It is consistent to assert that an agent may be obliged to open the closed window and s/he may be obliged to close the opened window. The two actions may have different deontic status. So, $\mathbf{O}^*(\neg P/P) \wedge \mathbf{O}^*(P/\neg P)$, or $\mathbf{O}^*(\neg P/P) \wedge \mathbf{F}^*(P/P)$ may define a consistent obligation pattern for an agent, but only under condition that s/he does not know which situation is actual. In order to prove that fact we shall introduce two types of sentences that can be used to describe the *belief side* of a cognitive motivational state. The initial situation sentence can be required by a model only if there is a description shared by all initial situations for any obligatory action. If there is no common feature in all initial situations, then the obligation pattern lacks information on the initial situation and the strongest initial situation sentence that can be used to characterize it is a "*might*-sentence".

Initial state sentence:

$$\text{Model } \langle W, W^*, \Pi \rangle \text{ requires } (A/\top) \text{ iff } \Pi \subseteq \|(A/\top)\|$$

Might sentence:

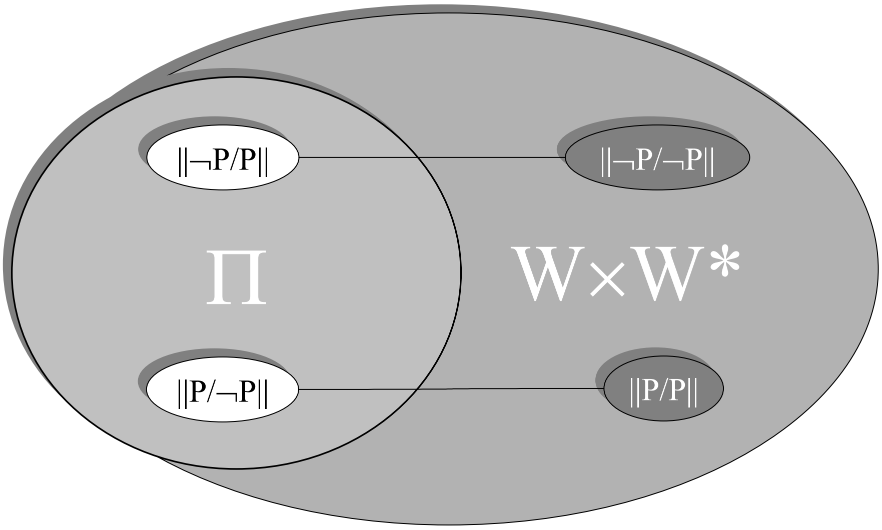
$$\text{Model } \langle W, W^*, \Pi \rangle \text{ requires } \mathit{Might}(A/\top) \text{ iff } \|(A/\top)\| \cap \Pi \neq \emptyset$$

Doxastic import of obligatory action:

$$\mathbf{O}^*(A/B) \Rightarrow \mathit{Might}(A/\top)$$

Doxastic implications of von Wright's example:

$$\mathbf{O^*}(\neg P/P)\wedge\mathbf{O^*}(P/\neg P) \Rightarrow \neg(P/\top)$$
$$\mathbf{O^*}(\neg P/P)\wedge\mathbf{O^*}(P/\neg P) \Rightarrow \mathit{Might}(P/\top)\wedge\mathit{Might}(\neg P/\top)$$



Von Wright's example:
$O(\neg P/P)\wedge F(P/P) \Leftrightarrow O(\neg P/P)\wedge O(P/\neg P)$
The simplest model for it:
$\langle W,W^*,\Pi\rangle \vDash_{requires} O(\neg P/P)\wedge F(P/P)$
$W=\{w_1,w_2\}$, $W^*=\{w^*_1,w^*_2\}$, where $w_1(P) = w^*_1(P) = \top$ and $w_2(P) = w^*_2(P) = \bot$;
$\Pi = \{\langle w_2,w^*_1\rangle,\langle w_1,w^*_2\rangle\}$;
$||\neg P/P||^{W\times W^*} = \{\langle w_2,w^*_1\rangle\}$; $\mathrm{OPPOSITE}(||\neg P/P||^{W\times W^*}) = ||\neg P/\neg P||^{W\times W^*} = \{\langle w_2,w^*_2\rangle\}$; $||P/\neg P||^{W\times W^*} = \{\langle w_1,w^*_2\rangle\}$; $\mathrm{OPPOSITE}(||P/\neg P||^{W\times W^*}) = ||P/P||^{W\times W^*} = \{\langle w_1,w^*_1\rangle\}$

Within an obligation pattern in which two generic actions, one resulting in P and the other resulting in ¬P are both obligatory it seems that another obligation arises; namely, the "epistemic obligation". In the absence of any information on the initial state the addressee should find out whether P is the case. The correspondence between imperative and deontic expressions that is argued for in this article can be briefly expressed as follows: imperatives create an obligation pattern that can be described in deontic terms.

It seems that Åqvist's (1975) analysis of interrogatives could be employed here to open further topics for investigation. He formalized questions in terms of two different modal operators: *imperative* operator and epistemic operator. For example, 'Is the window closed?' can be interpreted as '*Let it (turn out) to be* *the case that* either I know that the window is closed or I know that the window is not closed'. Following the idea that imperatives create an obligation pattern, one could say that (given a suitable social relation) a question obliges the Hearer to a specific action, namely the one resulting situation is defined in terms of Speaker's epistemic state.

On the other hand, in von Wright's example one should reverse the connection between epistemic imperative (interrogative) and deontic logic. By uttering the two conditional commands, 'Close the window if it is open, and if it is closed, open it' the Speaker creates an obligation pattern in which the action of closing the window is obligatory and the action of keeping the window closed is forbidden (i.e. the action of opening the window is obligatory). Such an obligation pattern may persist only in the absence of information on the initial state. There is an action that the Hearer ought to perform but it is impossible to perform both. The ignorant Hearer can satisfy the obligation pattern only by finding out which action is really (and not only doxastically) performable. So, an epistemic obligation arises for the Hearer.

One suggestion would be to may introduce a new composite, epistemic obligation operator $\mathbf{O}^{\mathbf{E}}$. The epistemic operator $\mathbf{O}^{\mathbf{E}}(A/\top)$ says that there is obligation to find out whether A is the case. The obligation $\mathbf{O}^{\mathbf{E}}(A/\top)$ is required by the given cognitive-motivational state (i) it is not settled whether A is (will be) the case in the initial situation, and (ii) there is an obligatory action triggered by an initial situation in which A holds or there is an obligatory action triggered by an initial situation in which $\neg A$ holds.

> Model $<W,W^*,\Pi>$ requires $\mathbf{O}^{\mathbf{E}}(A/\top)$ iff
> (i) $\|(A/\top)\| \cap \Pi \neq \varnothing$ and $\|(\neg A/\top)\| \cap \Pi \neq \varnothing$,
> and
> (ii) $\text{OPPOSITE}(\|(A/\top)\| \cap \Pi) \cap \Pi = \varnothing$ or $\text{OPPOSITE}(\|(\neg A/\top)\| \cap \Pi) \cap \Pi = \varnothing$

Some principles are obvious:

$$\mathbf{O}^{\mathbf{E}}(A/\top) \Leftrightarrow \mathbf{O}^{\mathbf{E}}(\neg A/\top)$$
$$\mathbf{O}^{\mathbf{E}}(A/\top) \Rightarrow \text{Might}(A/\top) \wedge \text{Might}(\neg A/\top)$$
$$\mathbf{O}^{\mathbf{E}}(A/\top) \Rightarrow \neg \mathbf{O}^*(\top/\top)$$

Epistemic obligation and Von Wright's example:

$$\mathbf{O}^*(\neg P/P) \wedge \mathbf{O}^*(P/\neg P) \Rightarrow \mathbf{O}^{\mathbf{E}}(P/\top)$$

From imperatives to obligations

> Here is how master shifts the boundary. From time to time he says to the slave that such-and-such courses of action are impermissible. Any such statement depends for its truth value on the boundary between what is permissible and what isn't. But if the master says that something is impermissible, and if that would be false if the boundary remained stationary, then straightway the boundary moves inward. The permissible range contracts so that what the master says is true after all. Thereby the master makes courses of action impermissible that used to be permissible. (Lewis, 1979, p. 340)

In this section we will try provide a sketch of a system of formal semantics that can explicate the way in which imperative sentences can shift the boundary between permissible and impermissible. The system will be restricted to "inward shift". Permissible range can contract but it cannot be expanded; no imperative can be derogated. Using the terminology of the preceding sections; we will try to describe a way which natural language sentences could create an obligation pattern. The main idea is that a sentence can bring about some change in the cognitive motivational state of a hearer. Going back to Lewis' quotation, the master utters sentences and changes cognitive motivational state of the slave and eventually controls his behavior in that way.

In giving (functional) dynamic semantics[7] one has to specify a language, a set of states, and an interpretation function, which takes a sentence and a state as input and delivers a state as output.

The language L_N that will be considered is intended to formalize natural language imperatives. For that reason, the syntax will be restricted. An atomic sentence of natural language imperative logic is composed out of one modal element and one sentence radical[8]. There are three modal elements in the language under consideration: one imperative modal element, '!' and two indicative modal elements,'$\bullet$' for 'it is the case in the initial situation that…' and '$\bullet^N$' for 'given the laws of the nature it is inevitable that …'. Sentence radicals in the language L_N are change expressions meeting the following restriction: change expression (A/B) is a sentence radical in L_N if and only if $A \Leftrightarrow B$ or $A \Leftrightarrow \neg B$ or $A \Leftrightarrow \top$ or $B \Leftrightarrow \top$. If A/B and C/D are sentence radicals in L_N, then $!(A/B)$, $\bullet(A/B)$, $\bullet^N(A/B)$, $\neg!(A/B)$, $\neg\bullet(A/B)$, $!(A/B)\rightarrow\bullet(C/D)$, $\bullet(A/B)\rightarrow!(C/D)$, are sentences in L_N.

7 Here developed formal semantics is a variant of "update semantics" of Veltman, 1996. The simple update system is basically a "one move" system with basic instruction $X[\varphi] = |\varphi|^X$, where $|\varphi|^X$ is a set of valuations verifying φ in the standard sense.

8 The terminology is derived from Stenius (1967).

Set of (slave's) cognitive-motivational states: $\Sigma=\{\langle W,W^*,\Pi\rangle: W\subseteq I, W^*\subseteq R, \Pi \subseteq W\times W^*\}$. W, W* and Π are defined as in the preceding section and I and R are sets containing all valuations for the propositional letters in the language under consideration indexed by the instants in which initial and resulting situation may occur. The subset $F=\{\langle W,W^*,\Pi\rangle: W = \varnothing \text{ or } W^* = \varnothing \text{ or } \Pi = \varnothing\}$ and in particular its element $1 = \langle\varnothing,\varnothing,\varnothing\rangle$ should be distinguished in order to define semantic notions. Since the notions of dynamic consequence will not be discussed here[9], we will use state 1 only as an indication that a contraction of permissible range has failed. Also we need the special state ("nothing believed, everything permitted") $0 = \langle I,R,I\times R\rangle$ to be the element in Σ.

In order to define the meaning of the sentences, conceived as their "potential" for changing states, we will use the notion of intension defined in the same way as in the preceding section. Intension of a sentence A of propositional logic given a set of valuations X is a subset of X: $|A|^X = \{w: w\in X \text{ and } w(A) = \top\}$. Intension of a sentence radical (A/B) is a subset of W×W*: $\|(A/B)\|^{W\times W^*}= \{\langle w,v\rangle: w\in W, v\in W, w\in|A|^W \text{ and } v\in|B|^{W^*}\}$.

Imperative:

$$\langle W,W^*,\Pi\rangle[!(A/B)] = \langle W,W^*,\|(A/B)\|^{\Pi}\rangle \text{ if } |B|^{W^*} \neq W^*$$
$$\langle W,W^*,\Pi\rangle[!(A/B)] = 1 \text{ if } |B|^{W^*} = W^*$$

Initial situation indicative:

$$\langle W,W^*,\Pi\rangle[\bullet(A/\top)] = \langle W,W^*,\|(A/\top)\|^{\Pi}\rangle$$

Inevitable situation indicative:

$$\langle W,W^*,\Pi\rangle[\bullet^N(A/A)] = \langle |A|^W,|A|^{W^*},\|(A/A)\|^{\Pi}\rangle$$

Conditional imperative:

(i) $\langle W,W^*,\Pi\rangle[\bullet(A/\top)\rightarrow!(B/C)] = \langle W,W^*,\Pi\rangle[!(B/C)]$ if $\langle W,W^*,\Pi\rangle[\bullet A] = \langle W,W^*,\Pi\rangle$

(ii) $\langle W,W^*,\Pi\rangle[\bullet(A/\top)\rightarrow!(B/C)] = \langle W,W^*,\|(A\wedge B/C)\|^{\Pi}\cup\|(\neg A\wedge B/\top)\|^{\Pi}\rangle$ if $\langle W,W^*,\Pi\rangle[\bullet(A/\top)] \neq \langle W,W^*,\Pi\rangle$

Negation of initial situation indicative:

$$\neg\bullet(B/\top) \Leftrightarrow \bullet(\neg B/\top)$$

Negation of imperative

$$\neg!(A/A) \Leftrightarrow !(A/\neg A)$$

9 An elaborate formal analysis of dynamic semantics is given in van Benthem, 1996.

Using the definitions one can easily prove some intuitive meaning relations. The equivalence between "result-oriented", *shall*-imperative and conditional imperatives is an interesting example of the natural language workings. 'Make sure that A' is equivalent to 'Produce A if $\neg$A is the case, and maintain A if A is the case'.

$$!(\top/A) \Leftrightarrow (\bullet(A/\top)\rightarrow !(A/A))\wedge(\bullet(\neg A/\top)\rightarrow !(\neg A/A))$$

It may be asked how it is possible to command a complex change using limited resources, i.e. two elementary imperative forms. For example, $!(\neg A\wedge B/A\wedge B)$ is neither a produce (complementary) nor a maintain (symmetric) imperative. Nevertheless, the sequence $!(\neg A/A);!(B/B)$ produces the same effect[10].

Let us turn our attention to von Wright's problem again. By uttering the two conditional commands, $((\neg P/\top)\rightarrow !(\neg P/P))\wedge((P/\top)\rightarrow !(P/\neg P))$ the master may make the two generic actions obligatory for the slave. Let the symbolic expression '$<S>\{O\}$' stand for 'there is a cognitive motivational state created by utterance *S* that requires obligation pattern *O*'.

$$<((\neg P/\top)\rightarrow !(\neg P/P))\wedge((P/\top)\rightarrow !(P/\neg P))>\ \{\mathbf{O}^*(\neg P/P)\wedge\mathbf{O}^*(P/\neg P)\}$$

Or, to put it another formalism in which $\sigma[\varphi]$ denotes the result of changing the model $\sigma\in\Sigma$ by the sentence $\varphi \in L_N$:

$$\exists\sigma:\quad \sigma[(\neg P/\top)\rightarrow !(\neg P/P);\quad ((P/\top)\rightarrow !(P/\neg P)] \models_{\text{requires}} \mathbf{O}^*(\neg P/P)\wedge\mathbf{O}^*(P/\neg P)$$

$$\neg\exists\sigma:\quad \sigma[(\neg P/\top)\rightarrow !(\neg P/P);\quad ((\neg P/\top)\rightarrow !(\neg P/\neg P)] \models_{\text{requires}} \mathbf{O}^*(\neg P/P)\wedge\mathbf{O}^*(\neg P/\neg P)$$

The second assertion shows that there is limit to the master's power. There is nothing that the master could say to make the opposite actions obligatory.

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[10] The translation of the full change expressions language into language L_N restricted to the two imperative forms can be constructed as follows. Let (X/Y) be any change expression: $!(X/Y)$ is equivalent to $\bullet(X/\top)\wedge !(\top/Y)$, and $!(\top/Y)$ is equivalent to $(\bullet(Y/\top)\rightarrow !(Y/Y))\wedge(\bullet(\neg Y/\top)\rightarrow !(\neg Y/Y))$

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