

Berislav Žarnić and Gabriela Bašić

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Berislav Žarnić*
Gabriela Bašić**

Metanormative Principles and Norm Governed Social Interaction

Critical examination of Alchourrón and Bulygin's set-theoretic definition of normative system shows that deductive closure is not an inevitable property. Following von Wright's conjecture that axioms of standard deontic logic describe perfection-properties of a norm-set, a translation algorithm from the modal to the set-theoretic language is introduced. The translations reveal that the plausibility of metanormative principles rests on different grounds. Using a methodological approach that distinguishes the actor roles in a norm governed interaction, it has been shown that metanormative principles are directed second-order obligations and, in particular, that the requirement related to deductive closure is directed to the norm-applier role rather than to the norm-giver role. The approach has been applied to the case of pure derogation yielding a new result, namely, that an independence property is a perfection-property of a norm-set in view of possible derogation. This paper in a polemical way touches upon several points raised by Kristan in his recent paper.

Keywords: normative system, standard deontic logic, metanormative principles, derogation, G. H. von Wright

1 THE NORMATIVE SYSTEM AS A SET OF NORMS

In his recent work on normative conflict resolution, Andrej Kristan (forthcoming) adopted the theoretical approach to normativity introduced by Alchourrón and Bulygin (1998). According to the set-theoretic approach presented in Alchourrón and Bulygin (1998), any sentence p describing “doable” states of affairs is a normative sentence: obligatory if p belongs to the set of logical consequences of “explicitly commanded propositions”, permitted if its negation $\neg p$ does not belong to the set, and prohibited if not permitted.¹ The metaphor for prescriptive use of language is that of putting something into a container (a proposition into the norm-set). The metaphor should not be stretched too far since sets unlike containers have no identity other than what is

* bzarnic@ffst.hr | Professor in Philosophy at the University of Split, Faculty of Humanities and Social Sciences.

** gbasic@ffst.hr | Assistant in Philosophy at the University of Split, Faculty of Humanities and Social Sciences.

1 The term ‘doable states of affairs’ is taken from von Wright (1999) and denotes ‘states of affairs which can come to obtain as the result of human action’.

given to them by their membership and, consequently, adding sentences to an existing norm-set creates a new set.

The range of properties that a set of propositions can have is vast and there are two ways to define them: descriptively and normatively. For example, on the descriptive side, a norm-set need not exemplify consistency as a matter of fact but, on the normative side, it may be subordinated to the consistency requirement as a matter of value. Kristan, following Alchourrón and Bulygin, defines the normative system \mathcal{N} in a descriptive way as a set of logical consequences of explicitly commanded propositions A : $\mathcal{N} = Cn(A)$. There are, however, some problems with this definition which will need to be resolved.

1.1 Consistency and deductive closure

In classical logic, a set \mathcal{T} of propositions is deductively closed just in case the negation of any non-member of the original set can be consistently added to it.² This fact is symbolically represented by the formula (1.1).³

$$\mathcal{T} = Cn(\mathcal{T}) \text{ iff } \perp \notin Cn(\mathcal{T} \cup \{\neg p\}) \text{ for all } p \notin \mathcal{T} \quad (1.1)$$

The notions of consequence and consistency are interdefinable and are both about desirable properties. Is there a reason to regard deductive closure as a property more fundamental than consistency? In this paper we will try to show that there is no order of precedence between these properties. Let the set of contents of explicit commands be the starting point of our analysis. This set is devoid of any inherent logical properties and its creation is an *empirical fact* brought about by the use of language.⁴

Example 1. Goble (2009: 484–5) and Broome (2013: 121–2) disagree on the question whether a normative system containing the explicitly commanded proposition (i) ‘There shall be no camping at any time on public streets’ must also include the proposition (ii) ‘There shall be no camping on public streets on Thursday night’. Only if the normative system is defined as a set of all logical consequences of explicit commands, must the answer be in the affirmative, but

- 2 For the left-to-right direction, suppose, for the purpose of a *reductio ad absurdum*, that the negation of a non-member cannot be consistently added to the deductively closed set. If so, then the non-member of the set is a consequence of it, which is impossible since it is deductively closed. For the right-to-left direction, suppose, for the purpose of a *reductio ad absurdum*, that an arbitrary sentence is a consequence of the set but not its member. If so, the negation of the sentence can be consistently added to the set, which is impossible if the sentence is a consequence of the set.
- 3 The formula $\perp \in \mathcal{X}$ says that falsum \perp is an element in \mathcal{X} or, in other words, that \mathcal{X} is inconsistent. The negation of the former formula is $\perp \notin \mathcal{X}$ and it says that \mathcal{X} is consistent.
- 4 In Broome’s (2013) theory of requirements, a code delivers a set of propositions closed under congruence, i.e., if a proposition belongs to the set, then so does any proposition equivalent to it. In our approach, all properties, including congruence, are abstracted away.

there are compelling reasons against it, as Broome shows. On the other hand, as Goble notes, the relation between (i) and (ii) concerns “one’s reasoning with ought-statements”. We will try to show here that both positions are correct.

1.2 Perfection properties and norm-sets

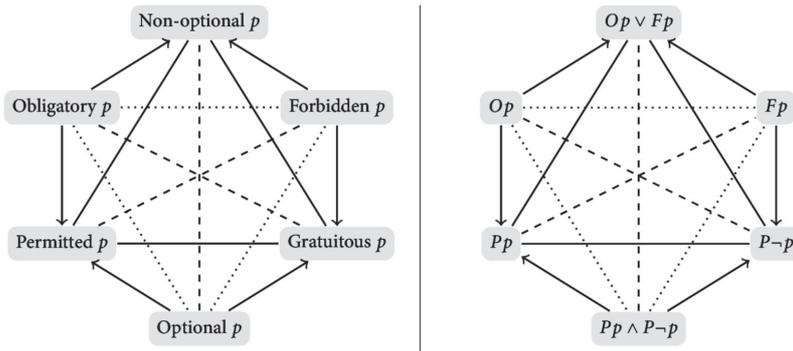
Extending von Wright’s line of thought, we will show that deductive closure can be understood as one among other perfection-properties.

[C]lassic deontic logic, on the descriptive interpretation of its formulas, pictures a gapless and contradiction-free system of norms. A factual normative order *may* have these properties, and it may be thought desirable that it *should* have them. But can it be a *truth of logic* that a normative order has (“must have”) these “perfection”-properties? Von Wright (1999: 32)

Standard or classical KD deontic logic accepts the interdefinability of the modal operators of obligation **O**, prohibition **F** and permission **P** as stated in (Def.) and graphically represented in Figure 1.⁵ KD deontic logic extends propositional logic with the necessitation rule (RN) and axiom schemata (K) and (D).

- $Pp \text{ iff } \neg O\neg p \text{ iff } \neg Fp.$ (Def.)
- If $\vdash p$, then $\vdash Op.$ (RN)
- $O(p \rightarrow q) \rightarrow (Op \rightarrow Oq)$ (K)
- $Op \rightarrow Pp$ (D)

Figure 1: The hexagon of logical relations holding in standard KD deontic logic. The dotted line represents the contrariety relation, the dashed line represents contradiction, the full line represents subcontrariety, and the arrows represent subalternation (implication). Deontic concepts are expressed in natural language in the left hexagon while the right hexagon presents the corresponding formulas.



5 Woleński (2008) has given a generalized account of squares, hexagons and octagons of logical relations, along with their applications to different domains of philosophical analysis.

1.2.1 Translating modal language to set-theoretic language

In Žarnić (2010), translation from the language of standard deontic logic without iterated operators is defined and it is proved that translated conditions of standard deontic logic describe the gapless, deductively closed and consistent type of norm-set \mathcal{N} . Our basic translations are similar to Alchourrón and Bulygin (1998) but with a slight difference in the definitia since ‘membership in $Cn(\mathcal{N})$ ’ is now replaced by ‘membership in the (possibly deductively unclosed) norm-set \mathcal{N} ’: ‘ $p \in \mathcal{N}$ ’ for ‘ \mathbf{Op} ’, ‘ $\neg p \notin \mathcal{N}$ ’ for ‘ \mathbf{Pp} ’, ‘ $\neg p \in \mathcal{N}$ ’ for ‘ \mathbf{Fp} ’. In short, the connection between the set-theoretic notion of norm-set and the modal notion of obligation is given by the simple equation: $\mathcal{N} = \{p \mid \mathbf{Op}\}$. The following correspondences hold:

1. It follows from the translation of principle (Def.) on the interdefinability of deontic notions that any norm-set is gapless (complete), $\mathbf{Pp} \vee \mathbf{Op}$, making each doable state of affairs either permitted or forbidden. When translated to set-theoretic language the definition (Def.) expresses a logical truth: $\neg p \notin \mathcal{N}$ or $\neg p \in \mathcal{N}$.⁶

2. The translation of the necessitation rule (RN) gives the claim that logical truths are included in a norm-set, $Cn(\emptyset) \subseteq \mathcal{N}$, while the translation of (K) axiom schema requires closure under modus ponens: *If $p \rightarrow q \in \mathcal{N}$ and $p \in \mathcal{N}$, then $q \in \mathcal{N}$* . Taken together, these two conditions are fulfilled iff a norm-set is deductively closed: $\mathcal{N} = Cn(\mathcal{N})$.

3. The translation of the (D) axiom schema gives: *If $p \in \mathcal{N}$, then $\neg p \notin \mathcal{N}$* . This condition is fulfilled if a norm-set is free of contradiction, i.e. if it is consistent: $\perp \notin Cn(\mathcal{N})$.

On the descriptive side, it is empirically evident that the exemplification of any of these properties is a contingent matter. On the normative side, we must employ meta-normative principles or intuitions in order to evaluate whether some property ought to be encoded by a norm-set.

1.2.2 On the possibility of creating a norm-system by norm-promulgation

By its own definition, theory \mathcal{T} is a deductively closed set, or symbolically: $\mathcal{T} = Cn(\mathcal{T})$. Any deductively closed set \mathcal{T} is infinite thanks to the inclusion of logical truths, whose number is not finite, or symbolically: $Cn(\emptyset) \subseteq \mathcal{T}$ and $|\mathbb{N}| \leq |Cn(\emptyset)|$. Consequently, one who agrees to define a normative system as a set of command contents has an ontological obligation to concede that infinite objects exist, since normative systems are infinite sets, and also an epistemolo-

⁶ This condition can be rewritten in its general form as $p \notin \mathcal{N}$ or $p \in \mathcal{N}$.

gical obligation to investigate their knowability. If logical truths are subtracted from the set of consequences $Cn^*(\mathcal{T}) = Cn(\mathcal{T}) - Cn(\emptyset)$, the resulting set $Cn^*(\mathcal{T})$ in addition to \mathcal{T} will contain relevant consequences, i.e., those elements whose deduction relies on a content from \mathcal{T} .

Example 2. Imagine that the normative system \mathcal{M} is created by the single command ‘It is forbidden to see to it that something is the case if one desires it not to be the case’ directed to a single actor i .⁷ Can actor i become aware of all and each norm from the set $Cn^*(\mathcal{M})$? Modal logic translation yields a conditional prohibition ‘An actor is forbidden to see to it that something is the case (F_i :stit p) if she desires it not to be the case ($D_i\neg p$)’, or symbolically: (1.2). In the set-theoretic approach, the deontic operator is replaced by the membership relation relation between a command content and its norm-set. The content of the command is: ‘If actor i desires that $\neg p$ then i does not see to it that p is the case’, or symbolically: (1.3). The content describes what conformation with the norm looks like and so it belongs to the single command norm-set \mathcal{M} , or symbolically: (1.4).⁸

$$D_i\neg p \rightarrow F_i\text{:stit } p \quad (1.2)$$

$$D_i\neg p \rightarrow \neg i\text{:stit } p \quad (1.3)$$

$$\ulcorner D_i\neg p \rightarrow \neg i\text{:stit } p \urcorner \in \mathcal{M} \quad (1.4)$$

If the variable p ranges over sentences of an infinite language \mathcal{L} , then it provides infinitely many sentences that can replace p in (1.4). So, the number of sentences in the set $Cn^*(\mathcal{M})$ will be infinite.

It is obvious that no normative source can complete the syntactic creation of an infinite set of command contents. Is it necessary to assume the existence of logical objects as infinite, deductively closed sets? The additional problem of deductive closure arises on the side of logic: is the consequence relation which defines a theory identical to the relation that defines the deductive closure of a norm-set? The thesis on the existence of a sui generis consequence relation in imperative language use (Žarnić 2011: 95) supports the rejection of the reduction of the consequence relation to the ‘logic of observance’ in the language of indicatives, just as Hans Kelsen claimed (Kelsen 1973: 254).

One can easily avoid ontological commitment to the existence of infinite norm-sets or logical commitment to the reduction of imperative-logic to indicative-logic by adopting the definition that a norm-set is merely a set of con-

7 This normative system can be interpreted as founded on Nietzsche’s maxim “Be thyself!”.

8 ‘Quine quotes’ are used for forming the name of an expression. Their use can be omitted if there is no possibility of confusion, but in cases where the same formula is both used and mentioned, Quine quotes will be used.

tents of explicit commands, and the thesis that the deductive closure of a norm-set under some logic is a contingent property.

1.3 Metanormative principles

Von Wright (1999: 33) introduced the notion of “normative demands on normative systems” or the notion of ‘the metanormative principle’, as it will be called hereafter

[another way] /.../ is to view the ideas of completeness and freedom of contradiction as themselves *normative* ideas, as normative demands on normative systems. They could be called *meta-normative* principles. They are norms of *higher order*.

At first sight, it seems possible to understand meta-normative principles as claims that a norm-set ought to have a certain property and to express these claims in formal language by allowing embedded KD modalities. For the purpose of analysis of metanormative principles, the expressive power of formal language will be enriched by introducing S5 alethic modalities of necessity \Box and possibility \Diamond .⁹ Alethic modalities can be interpreted in different ways: as logical, as nomological, and as historical possibilities.¹⁰ The concepts are ordered by inclusion: historical possibility is a nomological possibility and nomological possibility is a logical possibility.

The following list contains some prima facie plausible metanormative principles, denoted by tags of their formal translations given below:

- A norm-set ought to be gapless: $(\mathbb{O}.def.)$.
- A norm-set ought to be deductively closed: $(\mathbb{O}.RN)$ with $(\mathbb{O}.K)$.¹¹
- A norm-set ought to be consistent: $(\mathbb{O}.D)$.
- A norm-set ought to be realizable: $(\mathbb{O}.O\Diamond)$.¹²
- A norm-set ought to be realized: $(\mathbb{O}.T)$.

The following formal modal expressions for the listed metanormative principles are obtained using new symbol \mathbb{O} for the second-order obligation:

$$\begin{aligned} \mathbb{O}(Pp \vee O\neg p) & \qquad \qquad \qquad (\mathbb{O}.def.) \\ \mathbb{O}(\Box p \rightarrow Op) & \qquad \qquad \qquad (\mathbb{O}.RN) \\ \mathbb{O}(O(p \rightarrow q) \rightarrow (Op \rightarrow Oq)) & \qquad \qquad \qquad (\mathbb{O}.K) \end{aligned}$$

9 S5 logic can be axiomatized by rule of necessity: If $\vdash p$, then $\vdash \Box p$, axiom schemata: (K) $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$, (T) $\Box p \rightarrow p$, (4) $\Box p \rightarrow \Box \Box p$, (5) $\Diamond p \rightarrow \Box \Diamond p$, and the definition: $\Diamond p \leftrightarrow \neg \Box \neg p$.

10 Logical possibility is a world where laws of logic hold, nomological possibility is a world where logical and natural laws hold, and historical possibility is a nomological possibility that lies in the future of another nomological possibility.

11 If modality \Box is interpreted as logical necessity, then the meta-principle says that a given norm-set ought to include all logical truths.

12 Modality \Diamond can be interpreted as historical possibility in the remainder of the text.

$\mathbb{O}(\mathbf{O}p \rightarrow \mathbf{P}p)$	$(\mathbb{O}.D)$
$\mathbb{O}(\mathbf{O}p \rightarrow \diamond p)$	$(\mathbb{O}.O\diamond)$
$\mathbb{O}(\mathbf{O}p \rightarrow p)$	$(\mathbb{O}.T)$

Since in the set-theoretic approach only first-order translations can be given, the approach will have to be extended in order to accommodate metanormative expressions. One suggestive solution is to treat metanormative expressions as claims that a given norm-set type belongs to a certain class of norm-set types. The translation for the first-order obligation $\mathbf{O}p$ is ‘ p is a member of the norm-set \mathcal{N} ’. Almost analogously, the statement ‘property p is a perfection property’ and its extensional reformulation ‘the set of norm-sets satisfying condition p is a member of the perfection-set’ seem to provide a viable translation for the second-order obligation claim $\mathbb{O}p$. Let’s call Perfect the set of sets of norm-sets sharing certain perfection properties. To say that a property p of norm-sets is a perfection property means to say that ‘the set of norm-sets that satisfy condition p is an element in Perfect’, or symbolically ‘ $\{\mathcal{N} \mid \mathcal{N} \text{ satisfies condition } p\} \in \text{Perfect}$ ’.

Remark 3. If one accepts Gödel’s assumption that the second order property of being a positive property creates an ultrafilter, then a set of norm-sets having all perfection properties must be non-empty. Let’s call it Ideal. Let a be the set of norm-sets having a certain perfection property. Then the expression ‘ $a \in \text{Perfect}$ ’ means the same as ‘ $\text{Ideal} \subseteq a$ ’. An ultrafilter of a given set is a set of subsets that is closed under intersection and superset relation, the empty set is not its element and for any set either the set or its complement is a member of the ultrafilter.¹³

Remark 4. Can a norm-set have the property of making each doable state of affairs either obligatory or forbidden? Let’s call this property *the property of non-optionality* since it leaves no place for optional acts and forbearances. If a normative system is conceived as generated by deduction from a norm-set, then it should be noted that Gödel’s incompleteness theorem implies the unsatisfiability of the condition $p \in \text{Cn}(\mathcal{N}) \vee \neg p \in \text{Cn}(\mathcal{N})$ for a norm-set formulated in a language that is rich enough to express its own syntax (e.g., natural language). Since no normative system can satisfy this requirement, the property of non-optionality cannot be a perfection property under the Gödelian ontology of positive properties.

Next we proceed to the translation algorithm for the formulas where iterated deontic modalities of the same type are not allowed while first-order deontic modalities are allowed to occur within the scope of second-order ones.

¹³ For an investigation into Gödel’s ontology of properties, see Kovač (2003).

Definition 5. Let $\mathcal{L}_{\square PL}$ be the language of alethic modal logic. Function τ^1 translates formulas with first-order deontic modalities:

$$\begin{aligned}\tau^1(p) &= p \text{ if } p \in \mathcal{L}_{\square PL} \\ \tau^1(\mathbf{O}p) &= \ulcorner \tau^1(p) \urcorner \in \mathcal{N} \\ \tau^1(\mathbf{P}p) &= \ulcorner \tau^1(\neg p) \urcorner \notin \mathcal{N} \\ \tau^1(\neg p) &= \neg \tau^1(p) \\ \tau^1((p \rightarrow q)) &= (\tau^1(p) \rightarrow \tau^1(q))\end{aligned}$$

Definition 6. Function τ^2 translates those formulas whose main operator is a second-order deontic modality:

$$\begin{aligned}\tau^2(\mathbf{O}p) &= \{\mathcal{N} \mid \tau^1(p)\} \in \text{Perfect} \\ \tau^2(\mathbf{P}p) &= \{\mathcal{N} \mid \neg \tau^1(p)\} \notin \text{Perfect}\end{aligned}$$

Example 7. Let p be a sentence with no occurrence of first-order or second-order modalities.

$$\begin{aligned}\tau^2(\mathbf{O}(\mathbf{O}p \rightarrow \diamond p)) &= \{\mathcal{N} \mid \tau^1(\mathbf{O}p \rightarrow \diamond p)\} \in \text{Perfect} \\ &= \{\mathcal{N} \mid \tau^1(\mathbf{O}p) \rightarrow \tau^1(\diamond p)\} \in \text{Perfect} \\ &= \{\mathcal{N} \mid \tau^1(p) \in \mathcal{N} \rightarrow \diamond p\} \in \text{Perfect} \\ &= \{\mathcal{N} \mid \ulcorner p \urcorner \in \mathcal{N} \rightarrow \diamond p\} \in \text{Perfect}\end{aligned}$$

The translation for the condition $(\mathbf{O}, \mathbf{O}\diamond)$ says that requiring only that which is possible is a perfection property of norm-sets.

Example 8. The translation for the condition (\mathbf{O}, \mathbf{T}) is much less plausible.

$$\tau^2(\mathbf{O}(\mathbf{O}p \rightarrow p)) = \{\mathcal{N} \mid \ulcorner p \urcorner \in \mathcal{N} \rightarrow p\} \in \text{Perfect}$$

This says that requiring only that which is the case is a perfection property of norm-sets and that is obviously not the intended translation for the principle *a norm-set ought to be realized*.

The unequal plausibility of the translations in examples (7) and (8) shows that second order obligations designated by the homonymous expression — ‘ought to be’ in ‘a norm-set ought to be realizable’ and in ‘a norm-set ought to be realized’ — do not belong to the same category.

1.3.1 Roman Law principle as a norm for the norm-giver

We aim to draw a conceptual distinction between types of second order obligations with respect to the roles of actors involved in norm promulgation, norm realization and norm application. First, let our attention be drawn to the first

type, namely to the normative context of norm promulgation, the obligations for the norm-giver. The so-called ‘Roman Law principle’ forbids the norm-giver to require non-doable acts since no-one can be obliged to do the impossible. It will be shown that from the standpoint of standard deontic logic the use of the term ‘principle’ is unjustified because the Roman Law principle will be satisfied by a normative system whose norms consistently select only that which is possible.

The content $Op \rightarrow \Diamond p$ of the metanormative principle ($\textcircled{O.O}\Diamond$) has played an important role in normativity theory. Aristotle’s claim that in deliberation “If people meet with an impossibility, they give up” (Aristotle, *Nicomachean Ethics*, 1112b) can be understood as a contrapositive formulation of a related principle. In metanormative interpretation, the Aristotelian deliberation principle states that the impossible ought not to be the content of an intention. Closer to the ($\textcircled{O.O}\Diamond$) principle comes the Roman Law principle *ultra posse nemo obligatur* (*ad impossibilia nemo tenetur, impossibilium nulla obligatio*), itself a predecessor of the ‘ought’ implies ‘can’ principle that Kant formulated and for which $Op \rightarrow \Diamond p$ seems to be the direct translation.¹⁴ Nevertheless, the logic of the actor’s ability differs from the logic of alethic possibility. Some theorems of alethic logic fail in the logic of ability. For example, the thesis *If something is the case, then it is possible*, $p \rightarrow \Diamond p$, is valid in alethic logic but its ability counterpart is not: the thesis *If something is done, then it can be done* fails in the logic of ability.¹⁵ The metanormative principle with alethic modality is an over-generalization of these principles: whatever is forbidden because of alethic impossibility is also forbidden by the ‘ought’ implies ‘can’ principle, but the converse does not hold.

Terminologically speaking, the use of the term ‘principle’ is not correct in the context of principles ($\textcircled{O.D}$) and ($\textcircled{O.RN}$) since $Op \rightarrow \Diamond p$ is a theorem that follows from $\Box p \rightarrow Op$ in conjunction with $Op \rightarrow Pp$, i.e. from the contents of ($\textcircled{O.D}$) and ($\textcircled{O.RN}$). Two proofs, different in style, will be given for the fact.

Theorem 9. $Op \rightarrow \Diamond p \in Cn(\{\Box p \rightarrow Op, Op \rightarrow Pp\})$

Proof. First, let us give a deduction proof relying on the syntax of the language. From $\Box p \rightarrow Op$ and the definitions of deontic and alethic modalities we obtain the corollary: *If a state of affairs is permitted to be the case, then it is possible for it to be the case*, $Pp \rightarrow \Diamond p$. Assume that p is obligatory, Op . Then p is permitted, Pp , according to axiom D. From the corollary it follows that p is possible, $\Diamond p$. Therefore, if a state of affairs is obligatory, then it is possible, $Op \rightarrow \Diamond p$. Q.E.D.

14 There are numerous passages in Kant’s works dealing with the principle. For example, in *Religion Within the Boundaries of Mere Reason* (1793), a succinct formulation is given as “duty commands nothing but what we can do” Kant (1998: 68).

15 Picking the queen of hearts out of a card deck does not imply the ability to do so; see Brown (1992).

Proof. Second, let us give a proof in semantic terms! The basic semantic idea of modal logic is that the truth value of a formula at a point of valuation depends on the formula's truth values at other valuation points accessible via an appropriate relation. The deontic accessibility relation, D_{wv} , connects the world w to its normative alternatives v in which the norm-set is realized, $\mathcal{N} \subseteq v$ for all $v \in \{v \mid D_{wx}\}$. Similarly, the alethic accessibility relation interpreted, say, as nomological possibility, N_{wv} , connects the world w to any of its alternatives v in which all logical and natural laws hold. For some modal formulas (Sahlqvist formulas), the corresponding first-order property of the accessibility relation can be computed using the Sahlqvist-van Benthem algorithm.¹⁶ It is known that $\mathbf{Op} \rightarrow \mathbf{Pp}$ determines the seriality property of the deontic relation, $\forall x \exists y Dxy$. As stated above, this means that the given norm-set is consistent. Using the algorithm the following interrelation properties can be computed:

- $\mathbf{Op} \rightarrow \diamond p$ determines $\forall x \exists y (Dxy \wedge Nxy)$ interrelation property. It could be termed as the 'convergent seriality property of a relation pair' and it says that there is always a deontically accessible situation which is also nomologically possible. Or to use Professor's Segerberg's metaphor, *there are no tragic dilemmas* (Segerberg, 2003). A set of norms which exemplifies this property provides a possible and legal way out of any situation.

- $\Box p \rightarrow \mathbf{Op}$ determines the subordination of the deontic relation under nomological $\forall x \forall y (Dxy \rightarrow Nxy)$. A set of norms can be realized only in nomologically possible situations: if there is a legal way out of a situation, then this is also a possible way out.¹⁷

It is easy to see that if the deontic relation is serial and subordinated to the nomological, it must always have a point of convergence with it, a point where norms are realized in a nomologically possible world.¹⁸ Therefore, 'ought' implies 'can' is not a self-justifying principle but a consequence of other principles. Q.E.D.

16 Van Benthem defines the set of formulas algorithmically translatable to their first order equivalents in the following theorem: "Theorem 19. There exists an effective algorithm which translates all modal axioms of the form $A \rightarrow B$ into corresponding first-order properties, where A is constructed from basic formulas $\Box \dots \Box p$ using only \wedge, \vee, \diamond , B is 'positive': constructed from proposition letters with only $\wedge, \vee, \diamond, \Box$," (van Benthem 2010: 106).

17 For the purpose of illustration let us use the Sahlqvist-van Benthem algorithm to determine correspondences. We start with (i) $\Box p \rightarrow \mathbf{Op}$ and apply standard translation in two steps: (ii) $\forall P ST_x(\Box p \rightarrow \mathbf{Op})$; (iii) $\forall P(\forall y(R_Nxy \rightarrow Py) \rightarrow \forall y(R_Oxy \rightarrow Py))$. Then we determine the minimal valuation (iv) $Pu := R_Nxu$ and perform substitution: (v) $\forall y(R_Nxy \rightarrow R_Nxy) \rightarrow \forall y(R_Oxy \rightarrow R_Nxy)$. By simplification we get (vi) $\top \rightarrow \forall y(R_Oxy \rightarrow R_Nxy)$ and, finally, (vii) $\forall x \forall y(R_Oxy \rightarrow R_Nxy)$.

18 The formula $(\forall x \exists y Dxy \wedge \forall x \forall y (Dxy \rightarrow Nxy)) \rightarrow \forall x \exists y (Dxy \wedge Nxy)$ is a first-order logical truth.

2 NORMS AND SOCIAL INTERACTION

Two actor roles in communication are commonly recognized: the role of sender and the role of receiver, but, in a norm governed social interaction, besides the roles of norm-giver and norm-subject there is an additional role, the role of norm-applier. Communication is a kind of action, and that, according to Parsons' (1937) definition, means that the sender has an aim in a situation whose conditions and means are subordinated to normative requirements.¹⁹ The last condition in Parsons' definition of action points to its normative dimension. Similarly, Habermas equates the *social world* with the *normative context*.²⁰ The acts related to norms (promulgation, observance, application), as acts and social facts, must have their own normative contexts which, according to our hypothesis, are made explicit in their metanormative principles.

2.1 Normative contexts for norm related acts

As noted above, in a norm governed interaction there are three actor roles: the norm-giver, the norm-subject and the norm-applier role; and there are three types of norm related actions: norm-promulgation, norm-regulated action, norm-based judgement. In this kind of interaction, a norm-giver by norm-promulgation regulates the actions of a norm-subject whose observance of the norms is judged by a norm-applier.

First, we turn to the normative context of the norm-promulgation act. According to our interpretation, the language of KD logic is a description language and its axioms describe properties of norm-sets: the axiom *K* defines consequentality and the axiom *D* defines consistency. If the promulgation of a norm-set is an act (in Parsons' sense) or a social fact (in Habermas' sense), then at least one of its properties is either permitted or forbidden. For example, if it is not considered desirable that a promulgated norm-set is inconsistent, then desirability of the consistency property constitutes the normative context for norm promulgation. This desirable property can be interpreted as a second-

19 Parsons' definition of action: "...an 'act' involves logically the following: (1) It implies an agent, an 'actor'. (2) For purposes of definition the act must have an 'end,' a future state of affairs toward which the process of action is oriented. (3) It must be initiated in a 'situation' of which the trends of development differ in one or more important respects from the state of affairs to which the action is oriented, the end. This situation is in turn analyzable into two elements: those over which the actor has no control, that is which he cannot alter, or prevent from being altered, in conformity with his end, and those over which he has such control. The former may be termed the 'conditions' of action, the latter the 'means.' Finally (4) there is inherent in the conception of this unit, in its analytical uses, a certain mode of relationship between these elements. That is, in the choice of alternative means to the end, in so far as the situation allows alternatives, there is a 'normative orientation' of action" Parsons (1937: 44).

20 Habermas writes: "A social world consists of a normative context that lays down which interactions belong to the totality of legitimate interpersonal relations" Habermas (1984: 88).

order obligation and can be expressed by the claim that a norm-set ought to be consistent, as stated in (O.D) above. As regards the question whether it is desirable that a norm-set has all of its deductive consequences, a negative answer seems inevitable since the production of an infinite text is not a doable act. Therefore, the desirability of consistency belongs to a category different from the desirability of consequentiality or deductive closure.

Second, let us investigate the normative context of norm observance. A specific type of desirability appears in the metanormative thesis (O.T), a thesis which can be plausibly interpreted as *Conformation to norms is desirable, Duty must be done, Norms ought to be realized*, and so on.²¹ As noted above, it is not plausible, however, to interpret the thesis as a claim about a desirable property of a norm-set since the claim *It is desirable that norms require only what is the case* results in a kind of normative collapse. Rather, the thesis can be understood as an observance principle since it shows that a norm is that which ought to be observed. From this perspective, there is an important difference between the two meta-norms: unlike the norm-giver, the norm-subject has no obligations with respect to the properties of norm-sets, and unlike the norm-subject, the norm-giver has no obligations with respect to the observance of norms.

Third, let us discuss the normative context of norm application. The norm-applier or judge decides on the deontic status of a state of affairs brought about by the norm-subject's act. Suppose that a norm-subject has brought about that p . The norm-applier has to determine the deontic status of p with respect to some norm-set \mathcal{N} and can do so by two logically equivalent methods: either by adding p to \mathcal{N} and testing the consistency of the extended set $\mathcal{N} \cup \{p\}$ or by examining whether $\neg p$ is a consequence of \mathcal{N} . According to the first method, if $\mathcal{N} \cup \{p\}$ is not consistent, then p is forbidden, and if it is consistent, then p is permitted, as shown in (2.5) and (2.6). A similar case holds for the second method, as shown in (2.7) and (2.8).

$$\text{If } \perp \in (\mathcal{N} \cup \{p\}), \text{ then } \mathbf{F}p. \quad (2.5)$$

$$\text{If } \perp \notin (\mathcal{N} \cup \{p\}), \text{ then } \mathbf{P}p. \quad (2.6)$$

$$\text{If } \neg p \in \text{Cn}(\mathcal{N}), \text{ then } \mathbf{F}p. \quad (2.7)$$

$$\text{If } \neg p \notin \text{Cn}(\mathcal{N}), \text{ then } \mathbf{P}p. \quad (2.8)$$

The norm-applier performs deduction but there is no “normative system”, no deductively closed set $\text{Cn}(\mathcal{N})$ that needs to precede or can result from the thus

21 B. Chellas approves the use of the second order obligation within the thesis OU. $\mathbf{O}(\mathbf{O}A \rightarrow A)$ or O.T in our notation: “Note that OU is a theorem of deontic S5 /.../ The schema expresses the thesis that it ought to be the case that whatever ought to be the case be the case. It is a much discussed principle in deontic logic, because it is one of the few plausible cases of a theorem of the form $\mathbf{O}A$ in which A is non-trivial ...” (Chellas 1980: 193).

obtained determination of the deontic status of the state of affairs brought about by a norm-subject act or by forbearance. Although the second-order requirement of deductive closure or the consequentiality principle does not define the perfection-property of an empirical norm-set, it does define the metanormative context for the norm-applier. The consequentiality principle shows that normative judgements ought to obey the laws of logic.

Directed second-order obligations. Different metanormative principles are attached to different roles in norm-governed interaction. While norms are always directed to norm-subjects, second-order obligations can be differentiated by their addressees as shown in Table 1. This fact indicates the need to reformulate the metanormative principles discussed in Section 1.3: second-order obligations \mathbb{O} must be indexed by their holders' names.²² If the norm-giver role is denoted by the index g , the norm-subject role by s and the norm-applier (judge) role by j , the selection of metanormative principles obtains the following reformulation:

$$\begin{array}{ll} \mathbb{O}_g (\mathbf{O}_s p \rightarrow \mathbf{P}_s p) & (\mathbb{O}_g.D) \\ \mathbb{O}_s (\mathbf{O}_s p \rightarrow p) & (\mathbb{O}_s.T) \\ \mathbb{O}_j (\mathbf{O}_s(p \rightarrow q) \rightarrow (\mathbf{O}_s p \rightarrow \mathbf{O}_s q)) & (\mathbb{O}_j.K) \end{array}$$

The reading of reformulated metanormative principles can be given in terms of modal semantics. For example, $(\mathbb{O}_j.K)$ reads 'The logical consequences of the norm-subject's obligations are norm-subject obligations in all the worlds where the norm-applier's obligations are satisfied'.

Table 1: The different roles in norm governed interaction and their second-order obligations.

ROLES in norm governed interaction	Their second-order obligations:
NORM-GIVER g	ought to create norm-sets with perfection properties
NORM-SUBJECT s	ought to observe norms
JUDGE j	ought to apply norms

²² The same point has been made by Yamada (2011: 63): "The formula of the form $O_i\varphi$ means that it is obligatory upon agent i to see to it that φ . Although indexing of deontic operators with a set of agents is not standard in deontic logic, we need to be able to distinguish agents to whom commands are given from other agents if we are to use deontic logic to reason about how acts of commanding change situations".

2.2 A perfection-property related to derogation

The dynamic phenomenon of theory revision has been first and foremost recognized within the legal tradition. Several principles for the resolution of normative inconsistency have been established thanks to the determination of hierarchical relations between norm-sets on the grounds of their generality level (*lex specialis derogat legi generali*), temporal precedence (*lex posterior derogat legi priori*) and legal subordination (*lex superior derogat legi inferiori*).²³ According to Kristan (forthcoming), the principles of normative conflict are “rules about rules” which are generated by promulgation and thus belong to a norm-set. Viewed from the perspective of norm-governed interaction, these rules address the role of the norm-applier, giving a method for consistency restoration. Normative conflict shows that the norm-giver has failed to satisfy the higher-order principle of consistency or, more precisely, external consistency between norm-sets, but the requirement of consistency still holds for the norm-applier.

In the simplest case of pure derogation “the validity of a legal norm is repealed and no new one takes its place,” to use Kelsen’s (1973: 269) description. Kristan claims that in this case where a single norm x of a normative system is derogated “the new sets A and $Cn(A)$ are composed of all the elements of the previous ones, except x ” (and the consequences depending on x). This claim is not generally valid.

The simplest derogation corresponds to the contraction operation in AGM theory (Alchourrón, Gärdenfors, and Makison, 1985). Applying the notion of AGM contraction to the normative context, the following definition for the operation of pure derogation is obtained: a norm-content p of a norm-set \mathcal{N} is derogated iff the operation results in a new set $\mathcal{N} \div p$ which is a maximal subset of \mathcal{N} that does not entail p . The operation of pure derogation is sub-determined since, typically, there will be more than one maximal subset of \mathcal{N} not entailing p . The set of such sets can be called the remainder set of \mathcal{N} by p , $\mathcal{N} \perp p$. It contains all and only those sets \mathcal{M} that satisfy the following conditions:

1. The preservation condition: a new norm-set resulting from derogation is a subset of the original set, $\mathcal{M} \subseteq \mathcal{N}$.
2. The non-entailment condition: a new set does not entail the derogated norm, $p \notin Cn(\mathcal{M})$.
3. The maximality condition: a new set retains the maximal number of norms from the original set, there is no \mathcal{M}' such that $\mathcal{M} \subset \mathcal{M}' \subseteq \mathcal{N}$ and $p \notin Cn(\mathcal{M}')$.

²³ The axioms for complex hierarchical relations resulting from combinations of grounds are given in Malec (2001).

Analogously to the contraction operation, the operation $\mathcal{N} \div p$ of pure derogation needs an additional choice operation γ to pick a member of the remainder set: $\mathcal{N} \div p = \gamma(\mathcal{N} \perp p)$. The special and neat case of pure derogation arises when the norms of the initial norm-set are independent, i. e. when no norm from the set is entailed by the rest, i. e. $p \notin Cn(\mathcal{N} - \{p\})$ for all $p \in \mathcal{N}$. Only in this special case does it hold that pure derogation imposes no need to choose since there is exactly one member in the remainder set, namely $\mathcal{N} - \{p\}$, (2.9).

If $p \notin Cn(\mathcal{N} - \{p\})$ for all $p \in \mathcal{N}$, then $\mathcal{N} \div p = \mathcal{N} - \{p\}$ (2.9)

In view of possible derogation, independence turns out to be another perfection property of a norm-set, one that by relieving the burden of choice from the norm-applier enables “uniformity of judicial practice”. If a norm-set does not have the independence property, then pure derogation could lead to the switching of roles: by being forced to choose between the elements of the remainder set, the norm-applier actually becomes the norm-giver.

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