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Ontology of Sentential Moods

In this paper ontological implications of the Barcan formula and its converse will be discussed at the conceptual and technical level. The thesis that will be defended is that sentential moods are not ontologically neutral since the rejection of ontological implications of Barcan formula and its converse is a condition of a possibility of the imperative mood.

The paper is divided into four sections. In the first section a systematization of semantical systems of quantified modal logic is introduced for the purpose of making explicit their ontological presuppositions. In this context Jadacki's ontological difference between being and existence is discussed and analyzed within the framework of hereby proposed system of quantified modal logic. The second section discusses ontological implications of the Barcan formula and its converse within the system accommodating the difference between being and existence. The third section presents a proof of incompatibility of the Barcan formula and its converse with the use of imperatives. In the concluding section, a thesis on logical pragmatics foreclosing the dilemma between necessitism and contingentism is put forward and defended against some objections.

1 Ontologies of quantified modal logic

Quantified modal logic opens up a series of questions regarding the interpretation of singular terms and predicates, as well as the range of quantification. There is a significant number of semantical systems proposed by the most influential philosophical logicians in the second half of the 20th century, such as Rudolf Carnap (1891–1970), Jaakko Hintikka (1929–2015), Stig Kanger (1924–1988), Richard Montague (1930–1971), Saul Kripke (b. 1940). A comparative overview of the semantical systems of quantified modal and related ontological problems has been given in (Lindström and Segerberg 2007). It seems to be unquestionable that a predicate extension can change across worlds. So, the interpretation of an n-place predicate is a two-place function that delivers the predicate's extension at a world, $I(w, P)$. The controversial decision regards the range of predicate extensions. Should it be restricted to objects existing in a world, i.e., to the domain $D(w)$, so that $I(w, P^n) \subseteq D(w)^n$? Or should it also include the objects existing in some other world, i.e., to the domain $\mathrm{U} = \bigcup_{w \in W} D(w)$, so that $I(w, P^n) \subseteq \mathrm{U}$? Or might it also include objects that do not exist in any world, i.e., those belonging the domain $\mathfrak{U}$ such ast $\mathfrak{U} - \mathrm{U} \neq \emptyset$ so that $I(w, P^n) \subseteq \mathfrak{U}^n$? For example, suppose that the object denoted by an individual constant t does not exist in a possible world w, and that P is a monadic predicate. Can sentence Pt be true in w? can it be false? can it have the value of being neither true nor false? The answer depends upon the chosen semantical system. A systematization of semantical options is given in Table 1, while Table 2 presents the positioning of chosen exemplars within the systematization. The scope of quantifying can vary. Global

Table 1: Modelling options under presupposition that each object exists in some world. It is redundant to treat the assignement as a two-place function in rigid options.

option	**assignment of objects to individual variables**	**interpretation of predicates**	**range of quantification**
universal	$g(w,c) \in \mathfrak{U}$	$I(w,P^n) \subseteq \mathfrak{U}^n$	$\mathfrak{U}$
global	$g(w,c) \in \mathrm{U}$	$I(w,P^n) \subseteq \mathrm{U}^n$	$\mathrm{U} = \bigcup_{w\in W} D(w)$
local	$g(w,c) \in D(w)$	$I(w,P^n) \subseteq D(w)^n$	$D(w)$
rigid	$g(w,c) = g(v,c)$ for all w and v	$I(w,P^n) = I(v,P^n)$ for all w and v	
flexible	$g(w,c) \neq g(v,c)$ for some w and v	$I(w,P^n) \neq I(v,P^n)$ for some w and v	

Table 2: Semantical systems: [K63] denotes the system of (Kripke 1963), [BE] denotes the system introduced in this section, [L-Z] denotes the system of (Linsky and Zalta 1994), [JvB] denotes the system of (van Benthem 2010). The range of quantification is universal if $M(w,\forall) = \mathfrak{U}$, global if $M(w,\forall) = \bigcup_{w\in W} D(w)$, and local if $M(w,\forall) = D(w)$. Rigidity means that values are the same in all worlds: forall v such that Rwv, $g(w,x) = g(v,x)$, $V(w,x) = V(v,x)$, $M(w,\forall) = M(v,\forall)$. Flexibility is non-rigidity.

option	**assignment of objects to individual variables** $g(w,x)$	**interpretation of predicates** $V(w,P)$	**range of quantification** $M(w,\forall)$
universal	[BE]	[BE] except for E	[BE]
global	[K63], [L-Z]	[K63], [L-Z]	[L-Z]
local	[JvB]	[BE] only for E, [JvB]?	[K63], [JvB]
rigid	[K63], [BE], [L-Z],[JvB]		[K63], [BE], [L-Z],[JvB]
flexible		[K63], [BE], [L-Z],[JvB]	

interpretation of predicates goes together with the quantification over all possible objects. So, the "possibilist" gives the following truth-definition of quantified formula $\exists x\varphi$ in a model M at a world w under an assignment g: '$M,w \models \exists x\varphi \quad [g]$ iff for some $d \in \mathrm{D}$, $M,w \models \exists x\varphi \quad [g(d/x)]$'. On the other hand, the "actualist" restricts the quantification range to the objects actually existing in the domain of

w: '$M, w \models \exists x\varphi\ [g]$ iff for some $d \in D(w)$, $M, w \models \exists x\varphi\ [g(d/x)]$'.[1] The most permissive position in semantics does not require of objects to exist in any possible world and takes $\mathfrak{U}$ as the quantification range: '$M, w \models \exists x\varphi\ [g]$ iff for some $d \in \mathfrak{U}$, $M, w \models \exists x\varphi\ [g(d/x)]$'. For example, the semantical system of (Kripke 1963) has global and rigid assignment, global and flexible interpretation of predicates and the local range of quantification.

Example 1.1. Let $\mathrm{U} = \{a, b\}$, $D(w) = \{b\}$, $g = \{\langle x, a\rangle, \langle y, b\rangle\}$, Rww, in a Kripke's system (1963). Then $M, w \models x = x\ [g]$, but $M, w \not\models \exists y\, x = y\ [g]$ since for no $d \in D(w)$, $M, w \models \exists y\, x = y\ [g(d/y)]$. Therefore, $M, w \not\models \Box\exists y\, x = y\ [g]$. Nevertheless, $M, w \models \forall x \exists y\, x = y\ [g]$ since for all $d_1 \in D(w)$ there is some $d_2 \in D(w)$ such that $M, w \models x = y\ [g(d_1/x; d_2/y)]$, namely, $a = d_1 = d_2$.

1.1 Jadacki on being and existence

Summing up his ontological views Professor Jadacki states as follows:

2. The correct formulation of metaphysical problems requires introducing the notion of various modes of being.
   2.1. There is a difference between "being" and "existence".
      2.11. "Existence" is a predicate.
      2.22. "To exist" means the same as "to be real".
      (...)
3. Every object is (i.e. has being) but not every object exists.

(Jadacki 2003, 26-27)

Professor Jadacki's views can be observed within the theoretical framework of quantified modal logic with a slight modification of the system (Kripke 1963) with respect to the range of quantification.

1.1.1 Semantical and deductive system for the difference between being and existence

The system for modelling the difference between **b**eing and **e**xistence will be denoted by [BE]. The language $\mathcal{L}_{BE}$ has the following syntax:

$$\varphi ::= t_1 = t_2 \mid E(t) \mid P^n(t_1, \ldots, t_n) \mid \neg\varphi \mid (\varphi_1 \vee \varphi_2) \mid \forall x\ \varphi \mid \Box\varphi, \qquad (\mathcal{L}_{BE})$$

where t_i is an individual constant or individual variable, P^n is an n-place predicate, and E is the existence predicate.

A **frame** for the language $\mathcal{L}_{BE}$ is a quintuple $F = \langle W, \mathfrak{U}, R, D, w_0\rangle$ such that $W \neq \emptyset$, $\mathfrak{U} \neq \emptyset$, $R \subseteq W \times W$, $D : W \mapsto \mathfrak{U}$, and $w_0 \in W$ plays the role of the real world. A [BE] **model** is an ordered pair $M = \langle F, I\rangle$ where I is an **interpretation** function such that $I(w, P^n) \subseteq \mathfrak{U}^n$ for each n-place predicate P, and $I(w, c) \in \mathfrak{U}$

[1]The terms 'possibilist' and 'actualist' are taken from Linsky and Zalta (1994).

for each individual constant c. In particular, $I(w, =) = \{\langle d, d\rangle \mid d \in \mathfrak{U}\}$, and $I(w, E) = \{d \mid d \in D(w)\}$. An **assignment** g is a possibly partial function g : Variables $\mapsto \mathfrak{U}$; $g_\emptyset$ is the empty function that does not assign objects to any variables; $g(d_1/x_1; \ldots ; d_n/x_n)$ is the assignment that assigns the same values as g, with possible exception of assigning d_1 to $x_1, \ldots,$ and d_n to x_n; g is appropriate for φ iff all free variables of φ are in the domain of g. The denotation of a singular term $[\![t]\!]_g^M$ in the model M under the assignment g is $[\![t]\!]_g^M = I(t)$ if t is an individual constant, and $[\![t]\!]_g^M = g(t)$ if t is an individual variable. The formula $M, w \models \varphi\ \ [g]$ stands for 'assignment g satisfies formula φ in model M'.

Definition 1.1 (Satisfaction)**.** Let g be an assignment in M which is appropriate for each formula being evaluated.

- $M, w \models t_1 = t_2\ \ [g]$ iff $[\![t_1]\!]_g^M = [\![t_2]\!]_g^M$,
- $M, w \models E(t)\ \ [g]$ iff $[\![t]\!]_g^M \in D(w)$,
- $M, w \models P^n(t_1, \ldots, t_n)\ \ [g]$ iff $\langle [\![t_1]\!]_g^M, \ldots . [\![t_n]\!]_g^M\rangle \in I(w, P^n)$,
- $M, w \models \neg\varphi\ \ [g]$ iff $M, w \not\models \varphi\ \ [g]$,
- $M, w \models \varphi_1 \vee \varphi_2\ \ [g]$ iff $M, w \models \varphi_1\ \ [g]$ or $M, w \models \varphi_2\ \ [g]$,
- $M, w \models \forall x\varphi\ \ [g]$ iff for all $d \in \mathfrak{U}$, $M, w \models \varphi\ \ [g(d/x)]$.

Definition 1.2 (Truth in a world)**.** $M, w \models \varphi$ iff $M, w \models \varphi\ \ [g_\emptyset]$

Definition 1.3 (Truth in the real world)**.** $M, w_0 \models \varphi$ iff $M, w_0 \models \varphi\ \ [g_\emptyset]$

Definition 1.4 (Truth in a model)**.** A formula φ is true in the model M iff for all w, $M, w \models \varphi$.

Definition 1.5 (Truth in a frame)**.** A formula φ is true in the frame F iff for all I, φ is true in the model $\langle F, I\rangle$.

Definition 1.6 (General validity)**.** A sentence φ is generally valid in class K of frames iff for all $F \in K$, φ is true in the frame F.

1.1.2 Deductive system [BE]

Basin, Matthews and Vigano (1998) have developed a system of deduction for unimodal normal logics lying within "Geach's hierarchy" (i.e., those whose relational theory is representable in first order language).[2] The rules are the same for any type of universal and existential modality. The differences between logics are introduced using relational theory which describes frame properties. At each w we use classical rules except for negation, whose rules are "global". Table 3 presents the labelled deduction rules for the system [BE].

[2] A labeled deduction for bimodal, deontic-praxeological logic has been given (Žarnić 2006).

Table 3: Deduction system for [BE].

Logical sign	Introduction rule	Elimination rule
$\neg$	$\Gamma, w : \varphi \vdash v : \bot \Rightarrow \Gamma \vdash w : \neg\varphi$	$\Gamma, w : \neg\varphi \vdash v : \bot \Rightarrow \Gamma \vdash w : \varphi$
$\wedge$	$\Gamma \vdash w : \varphi, w : \psi \Rightarrow \Gamma \vdash w : \varphi \wedge \psi$	$\Gamma \vdash \varphi \wedge \psi \Rightarrow \Gamma \vdash w : \varphi, w : \psi$
$\vee$	$\Gamma \vdash w : \varphi \Rightarrow \Gamma \vdash w : \varphi \vee \psi, w : \psi \vee \varphi$	$\Gamma, w : \varphi \vdash w : \theta$ and $\Gamma, w : \psi \vdash w : \theta \Rightarrow \Gamma, w : \varphi \vee \psi \vdash w : \theta$
$\rightarrow$	$\Gamma, w : \varphi \vdash w : \psi \Rightarrow \Gamma \vdash w : \varphi \rightarrow \psi$	$\Gamma \vdash w : \varphi \rightarrow \psi, w : \varphi \Rightarrow \Gamma \vdash w : \psi$
$\forall$	$\Gamma \vdash w : \varphi(c) \Rightarrow \Gamma \vdash w : \forall x \varphi(x)$ provided that individual constant c does not occur in Γ and φ	$\Gamma \vdash w : \forall x \varphi(x) \Rightarrow \Gamma \vdash w : \varphi(c)$
$\exists$	$\Gamma \vdash w : \varphi(c) \Rightarrow \Gamma \vdash w : \exists x \varphi(x)$	$\Gamma \vdash w : \exists x \phi(x)$ and $\Gamma, w : \phi(c) \vdash \psi \Rightarrow \Gamma \vdash \psi$ provided that individual constant c does not occur in Γ , ϕ, and ψ
$=$	$\vdash w : c_1 = c_2$	$\Gamma \vdash w : \varphi(c_1), c_1 = c_2 \Rightarrow \Gamma \vdash w : \varphi(c_2)$
$\Box$	$\Gamma, Rwv \vdash v : \varphi \Rightarrow \Gamma \vdash w : \Box\varphi$ provided that v does not occur in Γ	$\Gamma \vdash Rwv,\ w : \Box\varphi \Rightarrow \Gamma \vdash v : \varphi$
$\Diamond$	$\Gamma \vdash Rwv,\ v : \varphi \Rightarrow \Gamma \vdash w : \Diamond\varphi$	$\Gamma, Rwv,\ v : \varphi \vdash \psi \Rightarrow \Gamma, w : \Diamond\varphi \vdash \psi$ provided that v does not occur in $\Gamma \cup \{\psi\}$

1.1.3 Application

In this subsection the possibility of formalization of thesis (BE.3) (Jadacki 2003, 27) will be examined.

(BE.3) Every object is (i.e. has being) but not every object exists.

The translation for (BE.3) is a conjunction whose candidate conjuncts for the left-hand side are: (1), (2), (3), and for the right-hand side: (4), (5), (6), (7).

$$\forall x\ x = x \tag{1}$$
$$\Box\forall x\ x = x \tag{2}$$
$$\forall x\Box\ x = x \tag{3}$$
$$\exists x\neg \mathrm{E}x \tag{4}$$
$$\exists x\Diamond\neg \mathrm{E}x \tag{5}$$
$$\exists x\Box\neg \mathrm{E}x \tag{6}$$

$$\Box\exists x\neg \mathrm{E}x \tag{7}$$

Nevertheless, not all translations are equally suitable. Propositions (1) and (2) are valid in any semantical system. Proposition (3) is more restrictive since it might fail if the range of quantification and interpretation of the identity predicate are local. In system [BE] proposition (3) is valid, cf. Proposition 1.1 below. So, (3) comes closest to the requirement of a universal domain, which is wider than the union of local domains. In order to capture the intended meaning of the second conjunct, the suitable translations would be those that make sure that the range of quantification includes objects outside the "worldly existence"; outside the spatiotemporal realm, $M(\forall) = \mathfrak{U}$ and $\mathfrak{U} - \mathrm{U} \neq \emptyset$, and that the interpretation of the identity predicate is defined for all objects, both spatiotemporal and non-spatiotemporal ones, $I(w, =) = \{\langle x, x\rangle \mid x \in \mathfrak{U}\}$. The translations (4) and (5) cannot be considered adequate since their truth does not depend on the fact that $\mathfrak{U} - \mathrm{U} \neq \emptyset$. If the accessibility relation is reflexive, then the translation (6) guarantees that there will be some non-worldly object d in worlds accessible from w ($d \notin \bigcup_{v\in\{v|Rwv\}} D(v)$) but cannot exclude the possibility of the same object occurring in worlds other than those accessible from w.[3] If, in addition to being reflexive, the accessibility relation is also transitive, then $\mathfrak{U} - \bigcup_{v\in\{v|R^*wv\}} \neq \emptyset$ where R^* is the reflexive transitive closure of R. Proposition (7) is implied by (6) (see Proposition 1.2) but not vice versa. So, the closest translation for (BE) within [BE] would be the conjunction of (3) and (6).

Proposition 1.1. $\vdash_{BE} \forall x\Box x = x$

Proof.

1	a	v	Rwv		
2				$v : a = a$	Intro=
3			$w : \Box a = a$		1–2/ Intro$\Box$
4		$w : \forall x\Box x = x$			1–3/ Intro$\forall$

$\Box$

Proposition 1.2. $\vdash_{BE} \exists x\Box\neg \mathrm{E}x \to \Box\exists x\neg \mathrm{E}x$

Proof.

1	$w :\ \exists x\Box\neg \mathrm{E}x$	
2	a | $w : \Box\neg \mathrm{E}a$	
3	v | Rwv	
4	$v : \neg \mathrm{E}a$	2, 3/ Elim$\Box$
5	$v : \exists x\neg \mathrm{E}x$	4/ Intro$\exists$
6	$w : \Box\exists x\neg \mathrm{E}x$	3–5/ Intro$\Box$
7	$w : \Box\exists x\neg \mathrm{E}x$	1, 2–6/ Elim$\exists$
8	$w : \exists x\Box\neg \mathrm{E}x \to \Box\exists x\neg \mathrm{E}x$	1–7/ Intro$\to$

$\Box$

[3]For example, $d \notin \bigcup_{v\in\{v|Rwv\}} D(v)$ but for some $v \in \{v \mid Rwv\}$ and some v' such that Rvv' it might be the case that $d \in D(v')$.

2 The Barcan formula and the converse Barcan formula

The first axiomatization of quantified modal logic has been introduced by Barcan (1946). The characteristic axiom schema 11 (Barcan 1946, 2) is the formula '$\Diamond\exists x\varphi \rightarrow \exists x\Diamond\varphi$', and it covers the relation between the two categories of logical operators; a modality operator and a quantifier. Later, the axiom schema is known in its contrapositive form under the name 'Barcan formula' (BF) and is usually discussed together with its converse (CBF):

$$\forall x\Box\varphi \rightarrow \Box\forall x\varphi \tag{BF}$$

$$\Box\forall x\varphi \rightarrow \forall x\Box\varphi \tag{CBF}$$

Although there is no semantical system in the original work (Barcan 1946), semantic intuitions are implicit. If the universal range of quantification had been presupposed, there would have been no need to introduce axiom schema 11 (BF) since it can be obtained in the deduction system [BE] embedding universal quantification range, as shown by Proposition 2.1.

Proposition 2.1. $\Diamond\exists xPx \rightarrow \exists x\Diamond Px$

Proof.

1	$\Diamond\exists xPx$	
2	$v \mid Rwv$	
3	$\exists xPx$	
4	$a \mid v : Pa$	
5	$w : \Diamond Pa$	2, 4/ Intro$\Diamond$
6	$w : \exists x\Diamond Px$	5/ Intro$\exists$
7	$w : \exists x\Diamond Px$	3, 4–6/ Elim$\exists$
8	$w : \exists x\Diamond Px$	1, 2–7/ Elim$\Diamond$
9	$\Diamond\exists xPx \rightarrow \exists x\Diamond Px$	1–8/ Intro$\rightarrow$

$\Box$

The Barcan formula and its converse do convey important metaphysical information. Namely, the information as to whether an object, not existing at w, might exist in an accessible v, and whether the object, existing at w, must also exist in an accessible world v. This information becomes directly visible in semantical systems where the range of quantification is local, i.e., where the quantifier $\exists$ is "existentially loaded" (Linsky and Zalta 1994). The systems of (Kripke 1963) and (van Benthem 2010) are of this kind.

Example 2.1. In the system [JvB] the assignment and quantification range are local. The interpretation for (BF) and (CBF) can be obtained using "standard translation" ST from modal to first-order language and additionally introducing Exw for 'x is an object in the domain of w'.

$$\begin{aligned}&ST(\forall x\Box Px \to \Box\forall xPx)\\&\qquad = \forall x(Exw \to \forall v(Rwv \to Pxv)) \to \forall v(Rwv \to \forall x(Exv \to Pxv))\\&\qquad \Leftrightarrow \forall v\forall x(Rwv \to (Exw \to Pxv)) \to \forall v(Rwv \to \forall x(Exv \to Pxv))\end{aligned}$$

The "minimal way of making antecedent true" is to let P be the property satisfied at v just by elements of $D(w)$: $Pxv := Exw$. The substitution of the minimal property gives the following formulas:

$$\begin{aligned}&\forall v\forall x(Rwv \to (Exw \to Exw)) \to \forall v(Rwv \to \forall x(Exv \to Exw)) \Leftrightarrow\\&\qquad \Leftrightarrow \top \to \forall v(Rwv \to \forall x(Exv \to Exw))\\&\qquad \Leftrightarrow \forall v(Rwv \to \forall x(Exv \to Exw))\end{aligned}$$

Using the set-theoretic language we finally obtain (DLD) the "decreasing local domain"- or "no object growth"-property characterized by (BF).

$$Rwv \to D(v) \subseteq D(w) \tag{DLD}$$

The converse Barcan formula characterizes cumulative or increasing local domains ("no object loss") where objects existing in a world persist in an accessible world.

$$\begin{aligned}&ST(\Box\forall xPx \to \forall x\Box Px) =\\&\qquad = \forall v(Rwv \to \forall x(Exv \to Pxv)) \to \forall x(Exw \to \forall v(Rwv \to Pxv))\\&\qquad \Leftrightarrow \forall x\forall v(Rwv \to (Exv \to Pxv)) \to \forall v\forall x(Exw \to (Rwv \to Pxv))\end{aligned}$$

Let P be the property satisfied at v just by elements of $D(v)$: $Pxv := Exv$.

$$\begin{aligned}&\forall x\forall v(Rwv \to (Exv \to Exv)) \to \forall v\forall x(Exw \to (Rwv \to Exv))\\&\qquad \Leftrightarrow \top \to \forall v\forall x(Exw \to (Rwv \to Exv))\\&\qquad \Leftrightarrow \forall v(Rwv \to \forall x(Exw \to Exv))\end{aligned}$$

Using the set-theoretic language finally we obtain (ILD) the "increasing local domain" property characterized by (CBF).

$$Rwv \to D(w) \subseteq D(v) \tag{ILD}$$

2.1 The Barcan formula and its converse within the universal quantification range system

Thanks to having the "existence predicate", the language of [BE] can express the properties of local domains defined by (BF) and (CFB).

$$\forall x(\mathrm{E}x \to \Box\varphi) \to \Box\forall x(\mathrm{E}x \to \varphi) \tag{BF*}$$

$$\Box\forall x(\mathrm{E}x \to \varphi) \to \forall x(\mathrm{E}x \to \Box\varphi) \tag{CBF*}$$

The counterexamples for (BF*) and (CBF*) are the same as those that have been described in the literature dealing with the local quantification range systems, and they will be omitted here. On the confirmation side, it must be proved that (BF*) and (CBF*) hold on decreasing and increasing local domains, respectively. For this purpose, the labeled deduction system [BE] must be enlarged with the "domain theory" as described in Table 4.

Table 4: Relational theory and domain theory. In this paper only the domain theory rules will be used. The relational theory rules adequate for S5 modal logic are given hereby only for the purpose of an illustration.

relational theory	
axiom T	$\vdash Rww$
axiom 4	$Rwv, Rvz \vdash Rwz$
axiom 5	$Rwv \vdash Rvw$
domain theory	
decreasing local domain [DLD]	$Rwv, v : \mathrm{E}c \vdash w : \mathrm{E}c$
increasing local domain [ILD]	$Rwv, w : \mathrm{E}c \vdash v : \mathrm{E}c$

Proposition 2.2. $\vdash_{BE} \forall x(\mathrm{E}x \to \Box Px) \to \Box\forall x(\mathrm{E}x \to Px)$

Proof.

1	$w : \forall x(\mathrm{E}x \to \Box Px)$	
2	v Rwv	
3	a $v : \mathrm{E}a$	
4	$w : \mathrm{E}a \to \Box Pa$	1/ Elim$\forall$
5	$w : \mathrm{E}a$	2, 3/ DLD
6	$w : \Box Pa$	4, 5/ Elim$\to$
7	$v : Pa$	2, 6/ Elim$\Box$
8	$v : \mathrm{E}a \to Pa$	3–7/ Intro$\to$
9	$v : \forall x(\mathrm{E}x \to Px)$	3–8/ Intro$\forall$
10	$w : \Box\forall x(\mathrm{E}x \to Px)$	2–9/ Intro$\Box$
11	$w : \forall x(\mathrm{E}x \to \Box Px) \to \Box\forall x(\mathrm{E}x \to Px)$	1–10/ Intro$\to$

$\Box$

Proposition 2.3. $\vdash_{BE} \Box\forall x(\mathrm{E}x \to Px) \to \forall x(\mathrm{E}x \to \Box Px)$

Proof.

1		$w : \Box\forall x(\mathrm{E}x \to Px)$	
2	a	$w : \mathrm{E}a$	
3	v	Rwv	
4		$v : \forall x(\mathrm{E}x \to Px)$	1, 3/ Elim$\Box$
5		$v : \mathrm{E}a$	2, 3/ ILD
6		$v : \mathrm{E}a \to Pa$	4/ Elim$\forall$
7		$v : Pa$	5, 6/ Elim$\to$
8		$w : \Box Pa$	3–7/ Intro$\Box$
9		$w : \mathrm{E}a \to \Box Pa$	2–8/ Intro$\to$
10		$w : \forall x(\mathrm{E}x \to \Box Px)$	2–9/ Intro$\forall$
11		$w : \Box\forall x(\mathrm{E}x \to Px) \to \forall x(\mathrm{E}x \to \Box Px)$	1–10/ Intro$\to$

$\Box$

2.2 Is quantified modal logic a metaphysical theory?

Among many semantical systems of quantified modal logic, the preference should be given to the one that has the greatest expressive power. For example, consider the sentence (SA) "Necessarily, if c is P, then c exists", which according to Linsky and Zalta (1994) expresses the thesis of "serious actualism". If a semantical system has the local quantification range and local valuation of predicates, then (8) captures the meaning of (SA) since in such system $\mathrm{E}c$ means the same as $\exists x\ c = x$, while $V(w, P^n) \subseteq D(w)^n$. In systems with global or universal quantification (8) and (9) differ in meaning. In particular, (8) is valid in [BE], while (9) is not. Nevertheless, the postulate of "serious actualism" can be added thus introducing a restriction on admissible models. For example, the addition of the rule $Pc \vdash \mathrm{E}c$ to the deduction system [BE] turns it into an actualist's system.

$$\Box(Pc \to \exists x\ c = x) \tag{8}$$

$$\Box(Pc \to \mathrm{E}c) \tag{9}$$

If the problem of quantification range is understood as the question of proper reading of quantifiers, then within the systems with universal or global quantification range only "existentially unloaded" readings are appropriate for $\exists$: 'for some objects it is true that ...' is acceptable reading, but not 'there exists an object such that ...'. For example, Professor Jadacki's thesis 1.3., quoted below, forecloses the possibility of existentially loaded reading of the existential quantifier.

> 1.3. The object domain, which any language refers to, is not identical with the real world.
>
> (Jadacki 2003, 26)

The implicit definition of metaphysical modalities is not completed by making explicit a logical system for $\Box$ and $\Diamond$ that is being employed in some theory. It is their relations to other modalities that constitute their meaning. For example, how do they relate to nomological, historical and deontic possibilities? The real challenge does not lie in combining metaphysical modalities with other modalities, but in their interaction with quantifiers. The problem of interaction of metaphysical modalities with quantifiers does not appear if quantifiers occur within the scope of modalities (in *de dicto* mode).[4] The *de dicto* mode of combination $\Box\ \forall$ is unproblematic since all that it requires is to move along the paths of accessibility relation and to apply first-order logic at evaluation points. The real problem appears in *de re* mode where quantifiers range over modalities: $\forall\ \Box$. A theoretical option must be chosen in order to solve the problem of the interaction between metaphysical modalities and quantifiers. The choice is not theoretically inert but implies a commitment to metaphysical theses.[5]

If the question of "worldly" existence of objects across metaphysical alternatives is regarded as a logical question, then the choice of a logical axiom implies the metaphysical choice.

> For a mixture of technical and philosophical reasons, any such separation of logic and metaphysics became increasingly hard to maintain, especially for principles like the Barcan formula and its converse. (Williamson 2013, 30-31)

The fact of inseparability of logical and metaphysical theoretical choices does not oppose Thesis 1 as put forward by Professor Jadacki.

> 1. No logical formula forecloses metaphysical problems.
>
> (Jadacki 2003, 26)

Within the framework of quantified modal logic, postulates of logic are conceived in view of the background of metaphysical choices.

The axioms or deductive rules of quantified modal logic have strong metaphysical consequences.

> Call the proposition that it is necessary what there is necessitism, and its negation contingentism. In slightly less compressed form, necessitism

[4](Hughes and Cresswell 1996, 250) write: "Whether or not the Latin descriptions are accurate the fact remains that wff of modal predicate logic divide into those called *de dicto*, in which no variable occurs free within the scope of a modal operator, and those called *de re*, in which some do".

[5]An example with the Barcan formula in an epistemic context shows that our intuitions are much clearer in this case. The rejection of the epistemic Barcan formula is straightforward since the possible truth of *Romeo knows that someone likes him, but he doesn't know who* := $K_r\exists xLxr \wedge \neg\exists xK_rLxr$ shows that $K_r\exists xLxr \rightarrow \exists xK_rLxr$ is not a truth of epistemic logic.

> says that necessarily everything is necessarily something; still more longwindedly: it is necessary that everything is such that it is necessary that something is identical with it. In a slogan: ontology is necessary. Contingentism denies that necessarily everything is necessarily something. In a slogan: ontology is contingent. (Williamson 2013, 2)

In local quantification systems, the necessitism thesis can be expressed by (N). On the other hand, if the quantification is either global or universal, the necessitism thesis can be expressed by (N*).

$$\forall x \Box \exists y x = y \tag{N}$$

$$\forall x(\mathrm{E}x \to \Box \exists y(\mathrm{E}y \land x = y)) \tag{N*}$$

The necessitism thesis holds on increasing local domains. The contingentism thesis is the negation of the necessitism thesis, and it can be formulated either as (C) or (C*) depending on the semantical system.

$$\exists x \Diamond \forall y\ x \neq y \tag{C}$$

$$\exists x(\mathrm{E}x \land \Diamond \forall y(\mathrm{E}y \to x \neq y)) \tag{C*}$$

The contingentist believes that at least one object that is contingent exists in the real world; the object whose existence is not necessary. Metaphysical necessitism, i.e., the denial of the contingentism, is a consequence of the converse Barcan formula, as shown by Proposition 2.5. For many philosophers, the necessitism is an unacceptable consequence.

> This conclusion [$\forall x \Box \exists y\ x = y$], however, is extremely counterintuitive (provided we read quantifiers in the normal way as ranging over ordinary objects). Intuitively, it is simple false that everything there is exists necessarily. (Lindström and Segerberg 2007, 1167)

Lemma 2.4. $\vdash_{BE} \Box \forall x(\mathrm{E}x \to \exists y(\mathrm{E}y \land x = y))$

Proof.

$$
\begin{array}{l|l|l|l|ll}
1 & v & Rwv & & & \\
2 & & a & v : \mathrm{E}a & & \\
3 & & & v : a = a & & \text{Intro}= \\
4 & & & v : \mathrm{E}a \land a = a & & 2, 3/\ \text{Intro}\land \\
5 & & & v : \exists y(\mathrm{E}y \land a = y) & & 4/\ \text{Intro}\exists \\
6 & & \multicolumn{2}{l}{v : \forall x(\mathrm{E}x \to \exists y(\mathrm{E}y \land x = y))} & & 2\text{–}5/\ \text{Intro}\forall \\
7 & \multicolumn{3}{l}{w : \Box \forall x(\mathrm{E}x \to \exists y(\mathrm{E}y \land x = y))} & & 1\text{–}6/\ \text{Intro}\Box
\end{array}
$$

□

Proposition 2.5. $\Box\forall x(\mathrm{E}x \rightarrow \exists y(\mathrm{E}y \wedge x = y)) \rightarrow \forall x(\mathrm{E}x \rightarrow \Box\exists y(\mathrm{E}y \wedge x = y)) \vdash_{BE} \forall x(\mathrm{E}x \rightarrow \Box\exists y(\mathrm{E}y \wedge x = y))$

Proof. Use the instance of (CBF*) with $Px := \exists y(\mathrm{E}y \wedge x = y)$. The right-hand side of this instance is exactly the necessitism thesis (N*). Using Lema 2.4, (N*) follows by modus ponens. □

3 Contingentism as the ontology of imperatives

There is a long and noteworthy tradition in logic of imperatives where imperatives are treated as requested acts, e.g., Lemmon (1965), Segerberg (1990), Belnap et al. (2001). In short, as is put forward (Belnap et al. 2001, 10), "the content of every imperative is agentive". This idea can be easily explained using von Wright's (1966) typology of acts and forbearances and his simple semantics of acts.

> Generally speaking, for a description of action in terms of states and transformations (changes), three items are required:
>
> (a) First, we must be told the state in which the world is at the moment, when action is initiated. I shall call this the *initial* state.
>
> (b) Secondly, we must be told the state, in which the world is, when action has been completed. I shall call it the *end*-state.
>
> (c) Thirdly, we must be told the state, in which the world would be, had it not been for the presence of agency in it, or, as I shall also say, "independently of the agent".
>
> (von Wright 1966, 123)

The original Von Wright's notation for acts and forbearances will be modified and extended to represent the eight elementary types of imperatives. Also, following Segerberg (1992), the term 'counter-state' will be used for "the state, in which the world would be, had it not been for the presence of agency in it". In Table 5, the three-part formula (TPF) is utilized for the purpose.

$$! \left(w_i : \text{initial state} \;\middle|\; \frac{w_e : \text{end state}}{w_c : \text{counter state}} \right) \tag{TPF}$$

Let us analyze a token of produce-imperative type expressed by formula (10).

$$! \left(\forall x \neg Px \;\middle|\; \frac{\exists x Px}{\forall x \neg Px} \right) \tag{10}$$

There are two ways for this imperative to become satisfied by means of the two sub-types of productive act. Firstly, it can be satisfied by the world rearrangement. Speaking in terms of semantics, the sufficient condition for the satisfaction is that of the change of the valuation of P, from $V(P, w_i) = \emptyset$ at w_i to $V(P, w_e) \neq \emptyset$ at w_e. Secondly, imperative (10) can be satisfied by an object creation. A new object d, which is not in the domain of world w_i, appears in the domain of world w_e but remains absent in the domain of world w_c: $d \notin D(w_i), d \in D(w_e), d \notin D(w_c)$.

Table 5: Eight elementary types of imperatives corresponding to Von Wright's eight elementary types of acts and forbearances. In the orginal Von Wright's notation the formula is written down as '$\varphi_1 T(\varphi_2 I \varphi_3)$'. In this table, indexed worlds — initial world w_i, end world w_e, counter world w_c — are added for the purpose of easy reading, but are redundant otherwise.

act imperatives		forbearance imperatives	
Produce φ!	$!\left(w_i : \neg\varphi \,\middle\vert\, \frac{w_e : \varphi}{w_c : \neg\varphi}\right)$	Let φ remain absent!	$!\left(w_i : \neg\varphi \,\middle\vert\, \frac{w_e : \neg\varphi}{w_c : \neg\varphi}\right)$
Maintain φ!	$!\left(w_i : \varphi \,\middle\vert\, \frac{w_e : \varphi}{w_c : \neg\varphi}\right)$	Let φ vanish!	$!\left(w_i : \varphi \,\middle\vert\, \frac{w_e : \neg\varphi}{w_c : \neg\varphi}\right)$
Suppress φ!	$!\left(w_i : \neg\varphi \,\middle\vert\, \frac{w_e : \neg\varphi}{w_c : \varphi}\right)$	Let φ appear!	$!\left(w_i : \neg\varphi \,\middle\vert\, \frac{w_e : \varphi}{w_c : \varphi}\right)$
Destroy φ!	$!\left(w_i : \varphi \,\middle\vert\, \frac{w_e : \neg\varphi}{w_c : \varphi}\right)$	Let φ continue!	$!\left(w_i : \varphi \,\middle\vert\, \frac{w_e : \varphi}{w_c : \varphi}\right)$

Example 3.1. The imperative 'Write the term paper' directed to the student r is a produce-imperative. Its form is depicted below in (11).

$$!\left(w_i : \text{There is no term paper of } r. \,\middle\vert\, \frac{w_e : \text{There is a term paper of } r.}{w_c : \text{There is no term paper of } r.}\right) \tag{11}$$

The sole permitted way of satisfying imperative (11) is by object creation: an object not existing in $D(w_i)$ ought to come into existence in w_e. On the other hand, the satisfaction of imperative (11) by means of the rearrangement of the world is forbidden, e.g., by r submission of an "object already existing at w_i" but not written by r.

Local range of quantification type of semantics does not make it possible to distinguish world rearrangement- from object creation-imperatives. On the other hand, within the semantics of global or universal quantification range, this distinction becomes expressible. In this kind of systems, besides imperative (10,) which is ambiguous with respect to modes of satisfaction, there is also formula type (12) which can only be satisfied by domain variation, since, as sentence (13) claims, no object from the domain of w_i can satisfy the condition P.

$$!\left(\forall x(\mathrm{E}x \to \Box\neg Px) \,\middle\vert\, \frac{\exists x(\mathrm{E}x \wedge \Diamond Px)}{\forall x(\mathrm{E}x \to \Box\neg Px)}\right) \tag{12}$$

$$\forall x(\mathrm{E}x \to \Box\neg Px) \tag{13}$$

The conjunction of (10) and (13) gives the intended reading for a produce-imperative that can only be satisfied by an object creation, not by world rearrangement.

3.1 The Barcan Formula and the Converse Barcan Formula in the context of imperatives

Suppose: (i) that the instance of the Barcan formula (BF*.1) $\forall x(\mathrm{E}x \rightarrow \Box\neg Px) \rightarrow \Box\forall x(\mathrm{E}x \rightarrow \neg Px)$ is valid; (ii) that historical alternatives are metaphysically possible, i.e., that Rw_iw_e and Rw_iw_c (where R is the relation of metaphysical accessibility). Further assume that an object-creation imperative, i.e., the conjunction of (10) and (13), has been successfully issued.

Proposition 3.1. *The success of object-creation imperative excludes the validity of the Barcan formula.*

Proof. In (1)-(3) it is assumed that indicative (13) is true and imperative (10) is successful.

(1) $M, w_i \models \forall x(\mathrm{E}x \rightarrow \Box\neg Px)$, assumption;

(2) $M, w_e \models \exists x(\mathrm{E}x \wedge Px)$, assumption;

(3) Rw_iw_e, assumption;

(4) $M, w_i \models \forall x(\mathrm{E}x \rightarrow \Box\neg Px) \rightarrow \Box\forall x(\mathrm{E}x \rightarrow \neg Px)$, the instance of Barcan formula;

(5) $M, w_i \models \Box\forall x(\mathrm{E}x \rightarrow \neg Px)$, by modus ponens, form (1) and (4);

(6) $M, w_e \models \forall x(\mathrm{E}x \rightarrow \neg Px)$, by the semantic definition of $\Box$, form (3) and (5);

(7) $M, w_e \models \bot$, form (2) and (6).

□

Mutatis mutandis the same result holds for the relation between the object-destruction imperative and the converse Barcan formula. The conjunction of (14) and (15) excludes the possibility of satisfaction by "shrinking" extension $V(w_e, P)$ of P at w_e (which is accessible from w_i), since by (15) no object existing in w_e satisfies condition P.

$$! \left(\exists x Px \,\middle|\, \frac{\neg\exists x Px)}{\exists x Px} \right) \tag{14}$$

$$\forall x(\mathrm{E}x \rightarrow \Box\neg Px) \tag{15}$$

Proposition 3.2. *The success of object-destruction imperative excludes the validity of the converse Barcan formula.*

Proof. Similar to proof of Proposition 3.1. □

4 Conclusion and a reply to objections

The use of imperatives presupposes the metaphysics of varying domains, which may be rightfully called the ontology of imperative mood. The ontology of imperative mood rejects both the Barcan formula and the converse Barcan formula. The implicit definition of metaphysical modality cannot be completed within a unimodal propositional logical system. In order to see what 'necessity' means, one must investigate in which way metaphysical modality interacts with all logical terms and elements, such as other modalities (e.g., deontic modality), quantifiers or sentential moods (e.g., imperative mood). In the context opened by the use of indicatives, it is meaningful to dispute whether the Barcan formula or the converse Barcan formula hold, but not so in the context of imperatives. One should either reject both (BF) and (CBF) or refrain from using imperatives. In short, *the ontology of imperatives is contingent*. Thus, logical pragmatics settles some questions of metaphysics.

There are several objections that can be raised against these theses on the primacy of logical pragmatics. According to the first objection, the semantics of imperatives involves time as well as metaphysical possibility. New objects come into existence in expanding domains or old objects cease to exist in shrinking domains on temporal rather than on modal grounds. According to the model of quantified modal logic, used in this paper, and imperative semantics based on Von Wright's approach (Žarnić 2011, 109-111), any historical possibility is a metaphysical possibility but not vice versa. In model [BE], worlds are conceived as atemporal points the sequence of which represents time. The objection presupposes a historical concept of the world as stretched through time. It is only under this presupposition that contingentism, implied by the use of imperatives, can be replaced with temporaryism according to which not everything exists eternally.[6] The fact that the historical concept of the world is not compatible with the branching time structure — which is indispensable for understanding of the human world — is a sufficient reason to refute the fusion of necessitism and temporaryism. According to the second objection, both necessitism and contingentism are compatible with the use of imperatives, and consequently, logical pragmatics does not decide on this theoretical question of metaphysics. For instance, the necessitist may claim that the appearance of new and the disappearance of old objects is just an issue of temporality or that any imperative can be satisfied by a rearrangement of the world. The first claim has been already refuted on the grounds of presupposing the historical concept of the world. The second claim depends on an untenable notion of object according to which the identity of a composed object is entirely determined by its minimal parts and not by their structural arrangement.

Finally, it may well be the case that no formula of logical syntax and logical semantics forecloses metaphysical problems, but this need not be the case with logical pragmatics.

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[6] Williamson (2013) gives an extensive exposition on the relation between necessitism and contigentism, on one side, and permanentism and temporaryism, on the other.

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