

Luciano Boi
Carlos Lobo
Editors

When Form Becomes Substance

Power of Gestures, Diagrammatical
Intuition and Phenomenology
of Space

 Birkhäuser

Contents

Part I Logic, Forms and Diagrams

The Semiotics of Laws of Form	3
Louis H. Kauffman	
Can We “Show” the Correctness of Reasoning? On the Role of Diagrammatic Spatialization in Logical Justification	27
Julien Bernard	
Catégorification et méthode	65
Franck Jedrzejewski	

Part II Geometrical Spaces and Topological Knots, Old and New

Which Came First, the Circle or the Wheel? From Idea ($\iota\delta\varepsilon\alpha$) to Concrete Construction	83
Marco Andreatta	
The Classical Style in Contemporary Geometry: Views from a Person Working in the Field	107
Alessandro Verra	
Knots, Diagrams and Kids’ Shoelaces. On Space and their Forms	125
Luciano Boi	

Part III Diagrams, Graphs and Representation

Diagrammes planaires qui représentent des objets de dimension 1, 2, 3 et 4	215
Carlo Petronio	
From Singularities to Graphs	231
Patrick Popescu-Pampu	

Part IV Diagrams, Physical Forces and Path Integrals

Mathematical Aspects of Feynman Path Integrals, Divergences, Quantum Fields and Diagrams, and Some More General Reflections	267
Sergio Albeverio	
Some Remarks on Penrose Diagrams	283
Jean-Jacques Szczeciniarz	

Part V Phenomenology in and of Mathematical Diagrams

Phénoménologie, représentations, combinatoire	315
Frédéric Patras	
Husserl, Intentionality and Mathematics: Geometry and Category Theory	327
Arturo Romero Contreras	
Diagrams of Time and Syntaxes of Consciousness: A Contribution to the Phenomenology of Visualization	359
Carlos Lobo	

Part VI Diagrams, Gestures and Subjectivity

A Topological Analysis of Space-Time-Consciousness: Self, Self-Self, Self-Other	427
Hye Young Kim	
Gestes, diagrammes et subjectivité	451
Philippe Roy	
Some Prolegomena for a Contemporary “Critique of Imagination”	473
Filipe Varela	
Le langage diagrammatique au-delà de la différence phénoménologique	485
Fabien Ferri	

Part VII Diagrams: from Mathematics to Aesthetics

Ars diagrammaticae	499
Charles Alunni	
Grid Diagram: Deleuze’s Aesthetics Applied to Maggs’s Photographs	525
Jakub Zdebik	
Les jeux de l’unilatère	547
Amélie de Beaufort	
Le diagramme à l’œuvre	565
Farah Khelil	

Part VIII Poetics and Politics of Diagrams

Diagrammes du possible : de l'espace des phases au sujet politique 579
Tatiana Roque

**La dimension diagrammatique de l'écriture littéraire : un
formalisme dynamique inscrit dans la sensorialité du langage 593**
Noëlle Batt

Preprint

Husserl, intentionality and mathematics:

Geometry and category theory

Arturo Romero Contreras, University of Puebla (BUAP)
rcarturo@gmail.com

Foucault cite un texte de Borges [...] où il est écrit que « les animaux se divisent en a) appartenant à l'Empereur, b) embaumés, c) apprivoisés, d) cochons de lait, e) sirènes, f) fabuleux, g) chiens en liberté, h) inclus dans la présente classification, i) qui s'agitent comme des fous, j) innombrables, k) dessinés avec un pinceau très fin en poils de chameau, l) etcetera, m) qui viennent de casser la cruche, n) qui de loin semblent des mouches ». [...] La monstruosité [...] consiste [...] en ceci que l'espace commun des rencontres s'y trouve lui-même ruiné. Ce qui est impossible, ce n'est pas le voisinage des choses, c'est **le site lui-même où elles pourraient voisiner**

Foucault. *Les paroles et les choses*.

What pattern connects the crab to the lobster and the orchid to the primrose and all the four of them to me? And me to you? And all the six of us to the amoeba **in one direction and the backward** schizophrenic in another?

Bateson. *Mind and Nature*.

Un ensemble variant est globalement $f: X_0 \rightarrow X_1$. Et sous-jacente à cela, comme un réel dont on ne parle pas, il y a la flèche $0 \rightarrow 1$, la flèche du temps, ou de mouvement, ou de changement pur. **Au commencement était la flèche.**

René Lavendhomme. *Lieux du sujet*.

Summary

The following text is divided in four parts. The first presents the inner relation between the phenomenological concept of intentionality and space in a general mathematical sense. Following this train of thought the second part briefly characterizes the use of the geometrical concept of manifold (*Mannigfaltigkeit*) in Husserl's work. In the third part we present some examples of the use of the concept in Husserl's analyses of space, time and intersubjectivity, pointing out some difficulties in his endeavor. In the fourth and final part we offer some points of coincidence between phenomenology and category theory suggesting that the latter

can work as a formal frame for ontology in the former. Our thesis is that intentionality operates in different levels as a morphism, functor and natural transformation.

I. Intentionality and space

Intentionality is the core concept of phenomenology, it describes the essential mode of being of consciousness, its mode of experiencing: “*Die Grundeigenschaft der Bewußtseinsweisen, in denen ich als Ich lebe, ist die sogenannte Intentionalität, ist jeweiliges Bewußthaben von etwas. Zu diesem Was des Bewußtseins gehören auch die Seinsmodi wie daseiend, vermutlich seiend, nichtig seiend, aber auch die Modi des Schein-seiend, gut-, wert-seiend usw.*“ (Husserl 1950, p.13). This is an early definition of intentionality but has already the ingredients for all further phenomenological investigations. First, one identifies two poles: the subjective (consciousness) and the objective (something). Second, it is clear that these poles are impossible to dissociate, they constitute the fundamental being of consciousness. The I *lives as* being conscious of something. Third, being conscious of something is a condition for all modes of being (*Seinsmodi*), but also other modalities of existence, like presumption, appearance and even not-being (given that every not-being is stated *through* some positive “something”).

Intentionality operates as the “absolute stage” in which experience takes place. It is a “space” in a wide sense. Phenomenological investigation can be effectuated only in a particular region called by Husserl *my own sphere* or originary region (*Urregion*, Husserl 1977, p. 158). We gain *access* (*Zugang*) to this absolute region of phenomenological investigation through a suspension of judgement on the existence of the world (i.e., its naïve transcendence) through the so-called epoché (Husserl 1977, p. 68). The relevant issue at stake here is the *nature* of this “sphere”, of this “region”, where the I lives, and where the theater of cogitations takes place and being presents itself as sense (*Sinn*). Concepts like world (*Welt*), world of life (*Lebenswelt*) or horizon (*Horizont*), which Husserl developed later in his oeuvre, will hint also at an ever-growing sphere of *being*, structured throughout levels or strata of foundation and concurrence of different ontological regions.

But intentionality is not only a space or region, but also the very *mode* how things *appear* or *show themselves*. Being modalizes itself not *in* but also *as* the correlation noesis-noema (corresponding to the subjective and the objective poles). It would not be correct of speaking of a neutral space “where” cogitations would occur, and different objects present a parade. On the contrary, it is impossible to separate the formal space (or formal ontology, which for the Husserl of the *Logical Investigations* is a *mereology*) the matter (*hyle*) of the experience, and the mode of appearance (the how).

We claim in this paper that intentionality *can* be read in mathematical, specifically, geometrical terms. Husserl will constantly refer to the mathematical concept of manifold (*Mannigfaltigkeit*). But even the very concept of intentionality already includes a spatial dimension. Brentano rescues the concept from its use in the Middle Ages and writes:

Jedes psychische Phänomen ist durch das charakterisiert, was die Scholastiker des Mittelalters die intentionale (auch wohl mentale) Inexistenz eines Gegenstandes genannt haben [...] die **Beziehung** auf einen Inhalt, die **Richtung auf ein Objekt** (worum hier nicht eine Realität zu verstehen ist) oder die immanente Gegenständlichkeit [...] Jedes enthält etwas als Objekt in sich, obwohl nicht jedes in gleicher Weise. In der Vorstellung ist etwas vorgestellt, in dem Urteile ist etwas anerkannt oder verworfen, in der Liebe geliebt, in dem Hasse gehaßt, in dem Begehren begehrt usw. (Brentano 1874, 124-125).

Brentano claims that every mental phenomenon includes its object *immanently*. However, immanence does not mean indifference or indistinguishability, since here is a *tension*, a pointing *towards*, a sort of *arrow* between two “subregions”. In fig 1 we see the unity of a mental phenomenon and its corresponding poles, a mental act and its content, as well as the arrow corresponding to the particular mode of their *relationship*:

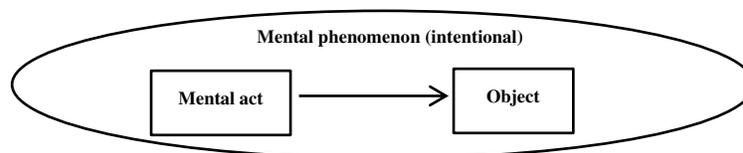


Fig 1.

Important here is not that there *is* a correlation between acts and their objects, but the *nature* of such correlation. To show the spatial character of intentionality and of consciousness, we

will allow ourselves a historical reference to Augustine of Hippo insofar as he anticipates the problematic relationship between mind, space and time in phenomenology. It suffices to remember that Husserl's lessons on the *Phenomenology of the Consciousness of Internal Time (PCIT)* begin with a quote of the Book XI the *Confessions*, acknowledging how he discovered fundamental paradoxes on the nature of time. Augustin exposes in this book the perplexity of time: the past is not anymore, the future is not yet, and present seems to be an ever-evanescent point. How is internal time possible? Augustin will argue that we could not have the slightest experience if our soul would not last *in itself*. He thus speaks of a *distention of soul (distentio animi)*, which inevitably leads us to spatialization of the soul. I do not know myself because I am an extended place which has to be traversed in order to find God in me.

Magnavacca highlights in her dictionary of Medieval philosophy the etymological and conceptual proximity of space and soul in the concept of *distentio*:

This notion, especially important in Augustine stems from the verb *distendo* meaning to extend or spread [*estirar*], [...] both to put in tension and to distract [...] Speaking of temporality [...] Augustine characterized -not defined- *distentio* as *distentio animi* in Conf. XI, 26, 33. In this context [...] we may translate the expression, for example, as 'extension' or 'distension'. In this regard it is interesting to appreciate the crossings between distention of soul [*alma, anima*], *extensio* as the realm of spatiality and *intentio*, (that trait of consciousness which will be interpreted in Medieval philosophy as consciousness-of-something). (Magnavacca 2005, p.230).

As we will see, there is also in Husserl's concept of intentionality, a need of "spatial" or "geometrical" approach to explain the "distention" (and differentiated *structure*) of time. The three Latin terms contain the root *tendere*: i.e. to stretch, with three different prefixes: *dis*, *ex*, and *in*. Such prefixes, as most prepositions, add a "spatial" dimension to words. The term *distentio* implies thus a sort of stretching *away* or *apart*, but without tearing. The term *intentio* shows a tension and a *direction* of soul as it points *towards* an object (we recognize a *Meinung*, in phenomenological terms) and, eventually, a presence (*Erfüllung*, also in the language of phenomenology). In the case of Augustine, this tension points towards a transcendence *of myself* (God) but *in myself* (interiority).

For Augustine the soul has no form, it does not belong to the world of objects and yet, because it is extended, one must ascribe to it some spatial traits, like continuity, indivisibility,

wholeness, etc. Also, soul is not *in* time, it is the very condition of lived time or temporality. In this sense, the soul is *stricto sensu* neither spatial nor timely, but the *origin* of *lived* time and space (temporality and spatiality). Now, if there is some self-constitution, a temporal soul must *return* to itself through memory to assure some *simultaneity*, only possible in *space*.

II. The idea of a *Mannigfaltigkeitslehre*

Let us now return to Husserl and evaluate to which extent consciousness but, above all, intentionality, has a spatial or a geometric dimension (intertwined with the temporal) in phenomenology. Concerned with ideal objects, like those of mathematics and with logical necessity (entailment in judgements) Husserl considers, unlike Brentano, impossible to ground them adopting a psychological stance. Logic and mathematics correspond to sciences of *principles*, while psychology (and history) corresponds to *fact* sciences. The empirical is contingent and, as Hume taught, it delivers no necessity, leading to skepticism. Kant's solution grounds causality also in human mind, but it ascribes to it a transcendental value, capable of assuring its inner necessity.

Husserl will demand an *access* to a pure ego capable of granting necessity and universality, but at the same time with no other ground than experience itself. The intentional ego must become the *absolute space of presentation of being* which will take the form of objective apprehensions. To gain access to such a pure *region*, Husserl will call for the necessity of a fundamental "bracketing" (*Einklammerung*), called the *epoché* (suspension of judgment on the transcendence of the world), to move from that world considered as transcendent and empirically existent, to a position in which it appears merely in its essential traits. The world of pure ego-immanence, governed by intentionality, presents essences through reflection on the structure of concrete experiences (*Erlebnisse*).

The concept of intentionality will change along Husserl's work. We will remain for the moment in the early formulations. In *Ideas I*, he describes the whole "spatio-temporal" world of realities as merely intentional being [*bloßes intentionales Sein*] as the absolute region,

where all possible *manifolds of appearance* [*Erscheinungsmannigfaltigkeiten*] can show themselves and be determined (Husserl 1976, p. 106).

What calls the attention in the given quote is the concept of manifold as *what is given* in general, and the concept of intentionality as the *common element or ground* for the appearance of the former. Husserl introduces the concept of manifold in phenomenological sense as early as his *Logical Investigations* (Husserl, 1901). With the focus in the *grounding* of logic, the first meaning ascribed to the concept is oriented towards the construction of *theories*. Different theories correspond to different *regions of experience*, but the emphasis is given to explicit knowledge. He defines thus a *Mannigfaltigkeitslehre* (science of manifolds) as the science of theories or a theory of theories, capable of providing *a priori* the conditions of possibility of *theories* (I stress the *plural* form) in general. This enterprise can be read as Husserl's project of a *mathesis universalis* (Husserl 1979, p.26). The aim here it to depart from purely categorial concepts to obtain multiple concepts of possible theories, to construct pure forms of theories and their reciprocal connections (*Beziehungen*). More precisely Husserl seeks the:

[...] möglichen Formen zu construieren, ihre gesetzlichen Zusammenhänge zu überschauen, also auch die Einen durch Variation bestimmender Grundfaktoren in die Anderen überzuführen vermögen [...] Es wird, wenn auch nicht überhaupt, so doch für Theorienformen bestimmt definierter Gattungen, allgemeine Sätze geben, welche in dem abgesteckten Umfange die gesetzmäßige Auseinanderentwicklung, Verknüpfung und Umwandlung der Formen beherrschen. (Husserl 1901, p. 247).

As we can see, the *Mannigfaltigkeitslehre* pursues a science of science (i.e. a *Wissenschaftslehre*) capable of showing the form of different theories and their interconnections, both dynamical (reciprocal development) and static (connectivity and transfers, or *connections* and *translations* among forms, including ideal variation or *deformation*). But why equate science in general with the study of manifolds? Husserl writes:

Das gegenständliche Correlat des Begriffes der möglichen, nur der Form nach bestimmten, Theorie ist der Begriff eines möglichen, durch eine Theorie solcher Form zu beherrschenden Erkenntnisgebietes überhaupt. Ein solches Gebiet nennt aber der Mathematiker (in seinem Kreise) eine Mannigfaltigkeit.

Es ist also ein Gebiet, welches einzig und allein dadurch bestimmt ist, daß es einer Theorie solcher Form untersteht, d. h. daß für seine Objecte gewisse Verknüpfungen möglich sind, die unter gewissen Grundgesetzen der und der bestimmten Form (hier das einzig Bestimmende) stehen. (Husserl 1901, p. 249).

As we can appreciate, a general theory of manifolds should deliver, first, the objects and connections of different areas of knowledge (which are always referred to areas of experience) and, later, in a more general theory, the connections between different manifolds, taken now as objects to find connections of a higher order. In his own words: “*Die allgemeinste Idee einer Mannigfaltigkeitslehre ist es, eine Wissenschaft zu sein, welche die wesentlichen Typen möglicher Theorien bestimmt ausgestaltet und ihre gesetzmäßigen Beziehungen zu einander erforscht*” (Husserl 1901, p. 249). Husserl takes the concept of *Mannigfaltigkeit* to be the highest achievement of modern mathematics. But his definition surprises if one considers his own words about the sources¹ of the concept:

Wenn ich oben von Mannigfaltigkeitslehren spreche, die aus Verallgemeinerungen der geometrischen Theorie erwachsen sind, so meine ich natürlich die Lehre von den n -dimensionalen, sei es Euklidischen, sei es nicht - Euklidischen Mannigfaltigkeiten, ferner Grassmann's Ausdehnungslehre und die verwandten von allem Geometrischen leicht abzulösenden Theorien eines W. R. Hamilton u. A. Auch Lies Lehre von den Transformationsgruppen, G. Cantor's Forschungen über Zahlen und Mannigfaltigkeiten gehören, neben vielen Anderen, hierher. (Husserl 1901, p. 250).

It is important to note that Husserl was influenced in his early conceptions by Cantor's set theory. However, Riemann was a constant figure along Husserl's oeuvre and provided phenomenology with a more “geometrical” approach, where relationships among elements are not “imposed” from the outside on a more primitive notion of set, but they belong to the very mathematical object at stake.² There isn't either any need to accept *a priori* the implicit

¹ Ortiz Hill (2002) writes about Husserl's sources on the concept: “Cantor used the terms ‘Menge’, ‘Mannigfaltigkeit’ and ‘Inbegriff’ interchangeably [...] Although Husserl did use the various terms for set interchangeably in the late 1880s, [...] he only began to use more frequently in posthumous writings of the 1890s, when he particularly studied geometrical manifolds [...] (p. 80)”, “by Mannigfaltigkeit Cantor merely meant an aggregate of any elements combined into a whole [by a law] [...] [but for Husserl] a Mannigfaltigkeit is an aggregate of elements that are not just combined into a whole, but are ordered and continuously interdependent” (p. 86).

² Some contest the importance of the usual references of Husserl's early work, like Frege or Cantor, arguing how Riemann played a key role in all his works. See, for example: (Rosado 2017).

ontology of set-theory, like the existence of points as the ultimate constituents of varieties. Another important element to note is the wide mathematical spectrum included by Husserl. Next to set-theory we find n-dimensional manifolds (Euclidian and non-Euclidian), but also Lie groups, approaching thus to abstract algebra.³

In the second volume of the *Logical Investigations*, we find another approach to the pure form of objects, i.e., a general theory of something in general (*etwas überhaupt*). This objectivity concerns the most elementary forms of objects in general and how they are grounded (i.e. if they are concrete, abstract, grounded or grounding). In the *third logical investigation* Husserl lays the ideas for a general theory of objects (*Gegenstände*) considering different classes of relationships like those of parts and wholes, (*a mereology*), subject and constitution (*Beschaffenheit*), individuum and species (*Spezies*), genus and species (*Art*), relation and correlation, unity (*Einheit*), number (*Anzahl*), series (*Reihe*), ordinal number, size (*Größe*), etc. (Husserl 1913, p. 225). Husserl will privilege, however, the relationship part-whole *because it allows to define relations of foundation*, replacing Stump's use of dependent and independent contents.⁴ Husserl's theory should then be considered a *formal ontology* or the presentation of the fundamental *formal ontological categories*. But how should we understand Husserl's mereology, i.e., his formal ontology?

In the §10 and §15 of the third investigation he uses again the concept of manifold. Objects in general *are also manifolds* of some kind, included the relationships of wholes and parts. So, one may ask, what is the difference between object-manifolds and theory-manifolds? To answer these questions, we need to introduce a fundamental distinction in Husserl between ground (*Begründung*) and foundation (*Fundierung*). *Begründung* can be translated as the

³ It is indeed possible to think phenomenological ideal *variation* of abstract objects through continuous maps (in topological sense), through functions among sets or even as group homomorphisms. A concrete determination of correspondences between Husserl's mathematical and phenomenological concepts is a hard task and would depend to great extent on speculation. One can try, however, to follow the spirit of his own words extracting the proper consequences. See, for example: (Tieszen 2005), who claims that essences can be thought of as invariants through a set of transformations.

⁴ This approach could have also been expressed in terms of set-theory by the relationship of membership and the usual operators. Yet, it may be *speculated* that Husserl preferred mereology, since it requires some structure from the outset. Mere aggregates do not constitute meaning or phenomenological experience. A set-theoretical approach would face the problem of introducing form (or structure) from outside and its ultimate elements, ideal points, would be indifferent to their organization in further structures.

operation of offering a principle from which concrete sentences can be derived or proved (like a theorem). The inverse of this type of foundation or grounding is instantiation. To ground means to offer a general concept or principle capable of *subsuming* particular cases. *Fundierung* means the ontological or logical relationship of *dependence* of some element on another. Husserl writes: “Kann wesensgesetzlich ein α als solches nur existieren in einer umfassenden Einheit, die es mit einem μ verknüpft, so sagen wir, *es bedürfe ein α als solches der Fundierung durch ein μ* ” (Husserl 1913, p. 261). Husserl’s formal ontology aims at defining every possible form of objectivity whatsoever. Such objects are of very general nature and are given by mathematics, considering that he uses the term of manifold. There are relationships of order or, more generally, of structure in such objects. But then, what is the link between objects and theories? *The essential question here is, clearly, what does it mean to ground (fundieren) and what is the Grund: meaning in German both fundament and reason, i.e. if there is something like a last instance a bottom of being.* It wouldn’t be precipitate to affirm that the whole understanding and development of phenomenology revolves around this issue. We should ask to which extent the concept implies a dual (clear-cut opposition ground vs grounded and without one term ever one passing onto the other), vertical (the ground is absolutely first, the grounded is derived) and unidirectional (the arrow flows from the ground to the grounded, but never in the opposite direction). But, as we will see, this concept of grounding, close to how set-theory intended to ground the whole of mathematics, will show insufficient in Husserl, offering in its place a structure with horizontal relationships (among regions) and irreducible layers (i.e. it is not the same to be a part of some wider instance, than belonging to a new level of organization, like when going from perception of objects, to categorical intuition), being the concept of *totality* displaced by those of *connectivity* and *translation* and making “unity” non-simple and distributed in different regions from the outset. But we will come back to this later.

In the first volume of the *Logical Investigations*, it becomes clear that a theory in eminent sense must constitute a *systematic unity of truths* (considered as contents of thought: *Denkinhalte*). Now, such unity is based on laws and principles, which assure a basic relationship of *Fundierung*. Husserl offers an image and characterizes theories as a

“systematic tissue of foundations” (*systematisches Gewebe von Begründungen*⁵, Husserl 1913, p. 25). *Objects* already show relationships of foundation, but a *theory* offers a *universe of objects together with their reciprocal relationships*. Considering the *wide* range of mathematical *relationships* offered by Husserl (parts and wholes, subject and constitution, individuum and species, genus and species, relation and correlation, unity, number, series, ordinal number, size) the operation of *Fundierung* is hard to define in unitary fashion. A field of mathematics is rather founded in a structure given by elements and relationships, which constitute “concrete universes”, even if they are infinite. We recognize a myriad of relationships among objects, *not necessarily* (or not only) of foundation, which we could equate with the mathematical concept of *morphism*.⁶ We could say that intentionality at this level is always a morphism. All this objects and relationships would then “live” or “sit” in a particular space of phenomenological region. Finally, the task of a *Wissenschaftlehre* would consist in providing the *highest (or lowest) level of foundation*, taking theories as its objects and their connections as their fundamental relations. As we see, there are *progressive levels of foundation* which should end in the highest unity. And this unity, who would grant it but the ego itself? If the ego operates always in an intentional structure, this structure should give us the *ultimate space* in which all theories *literally take place* (though not in a psychological sense) relating to each other. This would be the intentionality of higher-order and could be compared in category theory with functors or natural transformations.⁷ Jocelyn Benoist (2007) has called the attention to the fact that Husserl did not separate intuition and *categorical* form, as it is stated in the *Logical Investigations* in the concept of categorial intuition. Benoist tries to ground Husserl’s views on mathematics in an anticipation of category theory. However, if we are to take his claim seriously, then category theory should

⁵ Husserl is not always consistent in the use of the words *Begründung* and *Fundierung*. Important is the meaning implied. In this section both meanings could be correct.

⁶ The concept of morphism is very abstract and general. In category theory it is defined as a “structure preserving mapping”. It is a very general name for “relationship” though *specified* in some context of objects, a category. Morphisms are presented by arrows. Arrows may indicate “static” relationships among collections of objects or dynamic relationships, like transformations or sequences of states.

⁷ Functors are morphisms of second order, when categories operate as their objects. Natural transformations repeat the same operation but at an even higher level. MacLane claims to have taken the term “functor” from Carnap (Krömmmer 2007, p. 69), where it means a *grammatical function*, a mapping between categorematic expressions Belnap (2005). But it might be interesting to consider Hjemslev’s, a knower of Husserl, concept of *functor* as an organizer of relations of dependency and *form* as the composition of such dependencies.

structure all categorial apprehension and in a pre-linguistic way. Or rather, language would be legible only as a structure in the categorial sense.

We took here the *Logical Investigations* as the main framework to understand Husserl's use of the concept of *Mannigfaltigkeit*. A *Mannigfaltigkeitslehre* should provide, as we have seen, the general forms of theories and their connections. But as Husserl advanced in his phenomenological investigations, we appreciate a move from the general aim of founding sciences through an analysis of logic to a general theory of *experience* in general. In this sense, manifolds will not involve theories, but their *ideal*-lived correlates. In the late text of the *Krisis* (§9), Husserl resorts to the concept of manifold but in reference to the world:

„Mannigfaltigkeiten“ sind also in sich *kompossibile Allheiten von Gegenständen überhaupt*, die nur in leerformaler Allgemeinheit als ‚gewisse‘, und zwar als durch bestimmte Modalitäten des Etwas-überhaupt definierte gedacht sind. [...] mit der, wie man sagen kann, die *formal-logische Idee einer ‚Welt überhaupt‘ konstruiert wird*. [...] (Husserl 1976, p. 45).

Here Husserl is not concerned anymore with theories, but with the form of experienced world. The latter appears to us through totalities of objects, i.e., or general forms of something. It has been alleged that Husserl privileged the objective and thematic side of experience, as if science would grant the most originary access to the world. But as he moved backwards (through *reduction* and the insistent *Rückfrage*) from general scientific theories to fundamental experience, he placed his focus of analysis to a more and more indeterminate “background” out of which objective experience would emerge (in the sense of a relationship of foundation). But this dark, passive, non-thematic ground, never fell into absolute philosophical silence. There is, indeed, a tendency in phenomenology to pursue reduction of every positive instance, in order to conduce it to its genesis. Genesis of form itself cannot have any form, and we move towards an indeterminate ground, such as the concept of horizon (a central concept of transcendental phenomenology), over which concrete objects stand out. The very idea of founding requires, to avoid an infinite regress, a last instance, a bottom line, from which everything else is in relation of *some* last dependency.

But Husserl never gave up on the possibility of penetrating phenomenologically this dark ground, resorting to mathematics rather than poetry. The first examples of mathematical areas given by Husserl certainly point towards very determinate forms of objectivity, namely, fields of objects with several axioms to be fulfilled. But mathematics moved from the 19th century onwards towards generalization and brought with it the discovery of very abstract and conceptual fields. From the phenomenological point of view a transcendental grounding of mathematics requires a theory of subjectivity involving a locus (the ego) and a set of operations (intentional acts). But it is also true that to describe the structure of the ego requires to speak *of* and *through* some structure or form, mathematics being the most general and powerful language in this respect. Its abstraction becomes its weakness and strength at the same time. There is a certain circularity that did not escape Husserl. At stake is a subject (a space) grounding itself non-thematically but already sitting in some structured “space” (or structure). In this way, the most fundamental space of subjectivity is not a night for thought, but a *minimally* structured space. With this space we mean *almost* a void (some presuppositionless point of departure), *but* with some *restrictions*, which assure a *minimal structure*. But at the same time, such a minimum would not suffice to apprehend the world, but to move across different spaces or *modes* of being. The world would be such a space of spaces, a multiplicity of variously interlaced multiplicities. This is precisely what mathematical generalization achieves, to think the most general *forms* of being. But we may now ask, aren’t these forms or collections of forms closed domains, already constituted universes, derived forms to be brought back to a more originary base? Is there one last instance, the bottom line of all possible constitution? And if there is, does it have the form of a simple unity or does it have “parts”? Or, are there many different “spaces” whose interconnection constitute the common world?

III. Space and time in phenomenology

We devoted the pages above to show how Husserl understood objectivity in general. Now, Husserl gave preeminence to perception over other forms of objective apprehension. Space and time become thus fundamental to grasp objectivity in general. We will not dwell in the details of his analyses of time and space. Regarding space we want to show that a)

constitution of spatiality introduces the mathematical notion of transformation (and thus variation); b) objects are not “lose” things, but they appear in a “region”, which provides a priori not only its possible objects, but also the operations (and more generally: morphisms) on and among them (i.e. operations on objects and operations to combine objects). Regarding time, we want to point out that: a) time consciousness has a “form” or an inner relationship to space; b) time “appears” both as a continuous stream and as a discrete structure. We will thus content with providing the examples of the key issues we want to highlight.

In *Ideen I* we find a first approximation to the region of spatial things (*Raumdinge*):

[...] zu ihrem Wesen gehört die ideale Möglichkeit, in bestimmt geordnete **kontinuierliche Wahrnehmungsmannigfaltigkeiten** überzugehen, die immer wieder fortsetzbar, also nie abgeschlossen sind. Im Wesensbau dieser **Mannigfaltigkeiten** liegt es dann, daß sie Einheit eines einstimmig gebenden Bewußtseins herstellen, und zwar von dem einen, immer vollkommener, von immer neuen Seiten, nach immer reicheren Bestimmungen erscheinenden Wahrnehmungsdinge. Andererseits ist Raumding nichts anderes als eine intentionale Einheit [...] (Husserl 1977, p. 89).

Spatial things appear in a continuum. Farther, each one appears only in perspectives (*Abschattungen*, or adumbrations), but they “add up” to constitute a unity. It is clear that this quality belongs not to this or that object, but to all three-dimensional objects. This creates a *class* of possible objects. As I vary the position of my body respect to the object, there is covariation of its appearing perspectives, but I can also rotate it ideally in my mind (or see an object from one perspective and complete it though ideal continuation-deformation in my mind). Mathematically said (see Boi 2004), I can *continuously deform* an object without altering its structure (conserving its unity). This rather simple example points already at the general notion of a category: a collection of abstract objects and morphisms (relationships or transformations), respecting some additional axioms (like associativity, identity and composition).⁸

⁸ We won't offer definitions here. For excellent introductory presentations of the subject we remit the reader to Marquis (2009), Goldblatt (1984), Awodey (2010) and, with a psychoanalytic emphasis (Lavendhomme (2001).

Spatiality does not only deliver objects and their relative visible perspectives. It is rather a *system of relationships* that allows to identify an object through its variations, to create classes or subclasses of objects (according to dimension, for example), but also to translate one system of relations to another.⁹ For Husserl *there is only one world*, not despite, but *thanks* to its different perspectives. It belongs to the experience of a spatial world (but not only) to be given in adumbrations¹⁰, i.e., partially, incomplete, in the midst of an indeterminate yet infinitely determinable horizon. In this sense, there are not only different possible perspectives of an object for me, but many *actual and different but coexisting* perspectives of the same world, exhibiting various *degrees* of matching. Without *individual subjects* perceiving it, the world would be homogenous. There is an experience of multiplicity and experience as multiplicity. Now, *intersubjective experience* is possible because I can move (i.e. transform) different positions in a system of reference of space thanks to idealization:

Jedes Subjekt hat seinen ‚Orientierungsraum‘, sein ‚Hier‘ und sein mögliches ‚Dort‘, dieses Dort sich bestimmend nach dem Richtungssystem des Rechts-Links, Oben-Unten, Vorn-Hinten. Aber so ist die Grundform aller Identifizierung von intersubjektiven Gegebenheiten sinnlichen Gehalts, daß sie notwendig einem und demselben *Ortssystem* angehören [...], das sich nicht sinnlich sehen läßt, aber verstehbar, in einer höheren Anschauungsart, gegründet auf Ortswechsel und Einföhlung, ‚erschauubar‘ ist. (Husserl 1952, p.83)

⁹ For example, I have to translate into and “glue” together my visual to my bodily experience of an object to create a unitary experience. When I *see* the lines of a railroad intersecting at infinity, I operate under a projective geometry framework (non-Euclidian). But when I *walk* along it, I move in the familiar three-dimensional Euclidian space. It is not only possible, but a constant activity, to translate and merge information from two different types of spaces simultaneously lived. We glue information from different sources or “spaces”, like sound, kinesthesias, flavors, odors of the same object or complex situation (*komplexe Sachverhalte*). A further point in this train of thought can be illustrated by the famous art piece “One and three chairs” from Joseph Kosuth. It presents a chair of wood next to a real-sized photograph of it next to a cardboard with the word “chair” and its dictionary definition written on it. A chair is not any of the three objects isolated, but the “knot” tying the three different registers (real, image and language in this case) and the bundle of intentional rays involved in each register. This multiplicity of spaces does not lead to a more fundamental and unitary “ground”, on the contrary, *it constitutes the ground itself*.

¹⁰ As early as in the *Logical Investigations*, non-plenitude was part of experience. Significant intentions are *in certain way prior* to intuitions, for the later fulfil the former. On another sense, intuitions are first, because every possible act is related to some intuition. We can say that experience is always a mixture of empty mentions intentions and actual intuitions, but also that *firstness is not univocally given*.

I can transform my “here” in a “there”, which in turn is your “here”, exactly as I do with the pronouns “me” and “you”, or the temporal indexes “now” and “then”. All relative spatial and temporal (deictic terms) but also grammatical indexes can be reversed, interchanged, or translated into different positions.¹¹ This already complex conception of space and spatial things is expanded by Husserl to the most general notion of a (single) *world*:

Ich kann meine Aufmerksamkeit wandern lassen [...] zu all den [abwesenden] Objekten, von denen ich gerade ‚weiß‘, [...] ein Wissen, das nichts vom begrifflichen Denken hat und sich erst mit der Zuwendung der Aufmerksamkeit [...] nur partiell und meist sehr unvollkommen in ein klares Anschauen verwandelt [...] Aber auch nicht mit dem Bereiche dieses anschaulich klar oder dunkel [...] erschöpft sich die Welt [...] Sie reicht vielmehr in einer festen Seinsordnung ins Unbegrenzte [...] umgeben von einem dunkel bewußten Horizont unbestimmter Wirklichkeit. Ich kann **Strahlen des aufhellenden Blickes der Aufmerksamkeit in ihn hineinsenden** [...] der Kreis der Bestimmtheit erweitert sich immer mehr und ev. so weit, daß der **Zusammenhang** mit dem aktuellen Wahrnehmungsfelde, als der zentralen Umgebung, hergestellt ist. Im allgemeinen ist der Erfolg aber ein anderer: ein leerer Nebel der dunkeln Unbestimmtheit bevölkert sich mit anschaulichen Möglichkeiten oder Vermutlichkeiten, und nur die ‚Form‘ der Welt, eben als ‚Welt‘, ist vorgezeichnet. Die unbestimmte Umgebung ist im übrigen unendlich. Der nebelhafte und nie voll zu bestimmende Horizont ist notwendig da. (Husserl 1977, p. 57-58).

As we stated above, Husserl is ambivalent regarding the world as a totality. On the one hand, it seems to be already contained in the a priori forms of objectivity. Its infinity is really a “bad infinity”, just a multiplication of already available (*vorhanden*) objectivities. We should ask: what does it mean to be something? As we saw, it is not the objects that matter, but the “space of possibilities” in which they sit. Space and time can be thought of as infinite manifolds, but it is something different to take infinity *as a form of objectivity or a quality of*

¹¹ Husserl is however ambiguous here. He considers often that objectivity, subjective and intersubjective constitution actually render the *same* objectivity. The question is if “same” means “identical”, which would render intersubjectivity superfluous. This issue can be expanded to animals. Husserl ascribed mental activity to “superior” animals, i.e. capacity to constitute a world. We confront here a problem raised by Konraz Lorenz (1941) between the empirical and the transcendental. In phenomenology, my *experience* of things is linked to my bodily constitution, but phenomenological *essences* cannot be determined by my biological constrains. Even though different animals have different apprehensions of space, we could not speak of that diversity if we could not count with a more abstract concept of space in which we can *translate* such different space-constitutions. At the same time, I do not only constitute the world, but I am *part*, a *member (Mitglied)* of it (Husserl 1977, p. 58) and thus a part of nature and its history. When considering animals, we must accept that the *transcendental possibility of constitution*, even if it does not depend *directly* from bodily constitutions, it does evolve from nature itself. This would enlarge the very idea of intersubjectivity.

some region. There are limit-concepts (totality, nothing, unity, multiplicity, infinity, differential, incompleteness, incommensurability, etc.) that are neither empirical concepts, nor categories, but that present the very *mode* of appearing of somethingness. Take for instance the concept of cardinality coined by Cantor. After him infinity cannot be taken as a form of indeterminacy, an “excess” (regarding natural numbers, for the latter is countable and the former is not), but as a positive property of some sets. Even if we cannot experience cardinalities higher than \mathbb{R}^3 , it belongs to our ideal world the possibility achieving it. In this sense, we can think different “types” of infinity, or “indetermination” of “openness”. The idea of an obscure horizon could for example admit different interpretations: as a finite space with fuzzy boundaries, as a non-compact or as a non-simply connected space.

The “world” actually is not directly defined by Husserl. It is more a limit-idea. From the outset Husserl insisted in thinking manifolds as something more than mere aggregates of things. The world should integrate not only time-space manifolds, but also complex states of affairs, aesthetic objects, values, etc. different perspectives, both from different persons (belonging or not to the same tradition) and from animals, in both historical and natural¹² historical perspective. It is obvious that a concept of the world as a single, simply and univocally connected space is impossible. But here we should ask ourselves if there is still place for *intentionality* within the idea of a horizon. “Horizon” is a term than points towards potentiality, openness and indeterminacy, something very different to constituted, present objects. However, as Husserl states, a horizon operates like intentional empty intentions, characterized by limit-concepts like the stated above. A horizon is like a bundle of arrows pointing to potential areas of objectivity. The idea of intentionality as pointing-towards becomes thus more flexible and richer. Even if we can determine objectivities *locally* or by region, the form in which they are *combined* (i.e., the global form of the world) and *arranged* remains open.¹³ In the quote provided above about the world there is an interesting concept that plays a key role in his lessons on time (*PCIT*) (Husserl 1969): the “intentional rays”.

¹² For a study of topological forms in nature, both biology and physics, see: (Boi 2005).

¹³ Here we can remember Jakobson’s claim that language as a structure has *different levels of order* in which the degree of freedom for the speaker grows together with complexity. We are determined at the level of phonemes, but we are freer at the level of sentences. It is not potentiality and indetermination, but differentiation and accumulated orders of relative rigidity what grants more degrees of freedom. This is also true for higher orders of experience: for example, from perception of objects to categorial intuition. Phenomenological

In *PCIT* we come across the concept of intentional rays as *constituting* the fundamental stream of time-consciousness. We find again a structure of empty and fulfilled, which together constitute a “space” in its own right. Husserl presents the idea of an absolute stream of consciousness, self-constituting and non-thematic (i.e. *passive*). But here Husserl will rely again on mathematical ideas. First, Husserl claims that time-consciousness, not *time-objects*, is a manifold. As he writes, time has the “Charakter einer einseitig begrenzten orthoiden Mannigfaltigkeit” (Husserl 1969, p.99). This was, of course, not Husserl’s last stance on the form of time-consciousness, but it reveals the extensive use of the concept of *Mannigfaltigkeit* when grasping abstract and ideal forms, even the alleged *basis* of all experience. Husserl’s analyses of inner time-consciousness begin with *PCIT* but were continued in other important unpublished writings, like the *C-Manuscripts* and the *Bernau Manuscripts*. His aim, as with the phenomenology of space, was to characterize the *form of appearance of time-objects*. But it became clear that what is really lasting along objects is *consciousness itself*. Once we have reduced every content of time-consciousness we end up with its pure flow apprehending itself non-thematically. As we know, Husserl will identify a basic structure of *retention, present and protention*, which constitute a series of moments or phases (*Zeitphasen*). We cannot *separate* this flow in parts, but we can, however, *distinguish* different moments or phases (past, present and future). Time is a continuous manifold but with some additional rules (sequence and irreversibility, while space is simultaneous and path-capable). Husserl says that, if every moment were to definitively pass, without retention, we would forget it, and it would be nothing for us; we would have no continuity of experience. On the other hand, if things would absolutely remain in us, we would listen a “cluster chord”: all notes at the same time. Time must pass by and be retained at the same time. Time must include the emptiness of the “already gone” and its virtual presence in memory. Husserl will distinguish between primary and secondary memory. Primary memory is not a voluntary act of bringing into consciousness past events. On the contrary it belongs

reduction does not lead to a last instance, but to the original modes of givenness with their corresponding objectivities and degrees of subjective freedom. But, precisely, because there is no last instance, we should consider a *back and forth* movement between ground and determination. The individual, monadic ego is not a beginning but a *result*, embedded in the intersubjective tissue. But at the same time, it becomes a *new beginning*, *subtracting* or *separating* himself from the common, so that intersubjectivity takes distance from itself.

to presence (it is the relative absence in presence) like a “comet tail”. Present could be thought of as a point and memory as a line. But we can’t build the continuum by adding points. There is a constant and continuous transit (*Übergang*) and penetration (*Ineinandergehen*) from one moment into the other. In mathematical terms we could say that lived present is *like an open set* around an ideal point called pointwise-present, including the neighborhoods of past and present. Now, to distinguish the three main times within a fundamental flow, Husserl *maps* the continuous structure of time with a discrete structure of phases¹⁴. It is true that Husserl shifts his analysis of time constantly between a discrete and a continuous framework, but this should not be read as an inconsistency, but as a complementary mode of viewing. Time is both continuous and discrete, depending on the perspective assumed. There is no need of trying to ground one mode in the other.

These diagrams drawn by Husserl show that time flows at least in *two directions*: forwards, leaving events behind in a straight line, and downwards (or upwards, depending on perspective), sinking into memory.

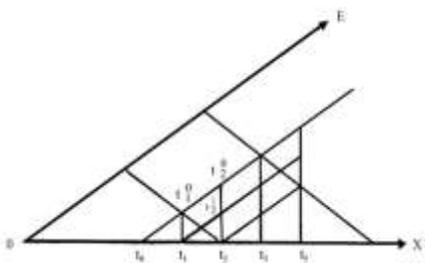


Fig. 2¹⁵

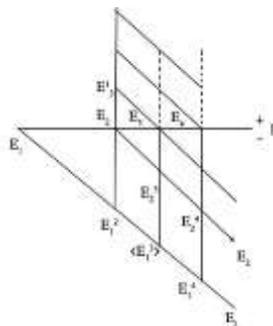


Fig 3¹⁶

In figure 2 we appreciate the time series (the horizontal) flowing from t_0 to $t_5 \dots t_n$. But we have also the retention of those past moments in memory: in t_1 we remember (the vertical) t_0 ; in t_2 we remember both t_0 and t_1 , etc. But we do not only remember time, we also have *expectations* of it. The future or anticipation flows first into present and then to past, as we

¹⁴ It is true that Husserl conceived of time as essentially continuous. However, he also wanted to stress its inner separation needed so that time actually is “cut” into past, present and future. Time must be continuous and introduce qualitative cuts. We claim he proposes a non-trivial continuity of time. Derrida (2010) also showed in his early engagement with Husserl’s theory of time, some important homologies between it and Saussure’s ordering of signs in a syntagmatic chain.

¹⁵ Nach: (Husserl 1966, p. 330).

¹⁶ Nach (Husserl 2001, p. 22).

see in figure 3. Husserl should have added a third axis to draw future. But he didn't. One can speculate that this decision would have complicated the diagram. But the *product* of the two lines of past and present can deliver a *surface*, an idea that Husserl incorporated in a late diagram called “edge-consciousness” (*Kantenbewusstsein*).

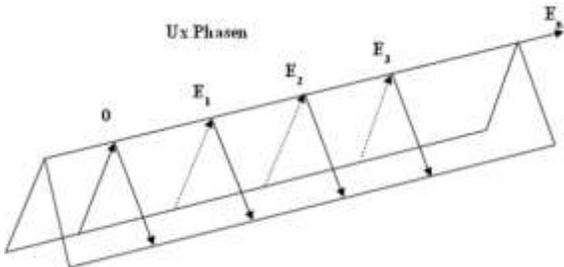
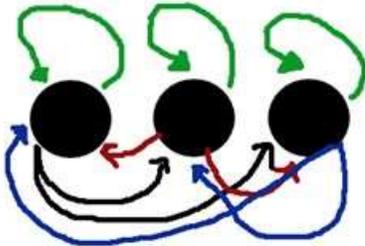
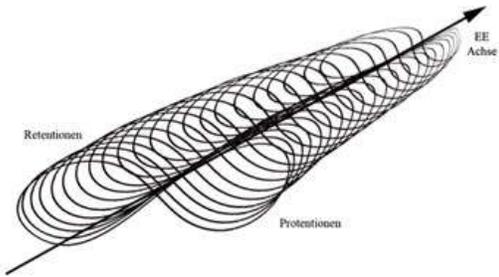


Fig. 4¹⁷

This is the surface-model of time-consciousness, but as we already said, Husserl described time also in terms of intentional rays (Husserl 1969, p. 29) in interlacement or entanglement (*Verflechtung* Husserl 1969, p. 83) forming a sort of braid or even a knot¹⁸. But how is this possible? Husserl explains that future is an empty mention that is fulfilled in the present, just to be emptied again as it sinks into the past (Husserl 1969, p. 83). But it is also true that the past conditions expectations just as expectations condition the past, and that all three phases obey a structure or reciprocal remissions: there are intentional rays going from every time to every other time. Following the image of time as a ray, we could give both a continuous (fig. 5) and a discrete (fig. 6) interpretation. In figure 5 we see a single curved line (a *lemniscate*) integrating past, present and future in a *single* flow. In figure 6 we appreciate the three time-phases and arrows representing intentional rays going from every time, to every other, included self-reference arrows.



¹⁷ Nach (Husserl 2001, p. 34-35). The diagram is only described in Husserl's text.

¹⁸ See the paper of Hye Young Kim in this volume: *A topological analysis of space-time consciousness: self, self-self, self-other.*

Fig. 5

Fig. 6

The diagram provided by Husserl where time constitutes a surface has the problem of not being able to show the strict *continuity* of the arrows. We have an infinite number of arrows “climbing” from future, crossing the axis of present (E_x) and sinking into past. But this diagram cannot show how the end of an arrow connects with the beginning of another. In figure 5 lines are connected in a lemniscate moving along the arrow of flowing present. A diagram is, of course, only a *projection* of a more complex structure. But we can analyze such a structure *choosing* which trait to show. We can also consider either a topological or an algebraic structure, a continuous or a discrete one to describe time, and *map one with the other* instead of choosing one over the other. There is some *hinted* but inconsistent use of an “algebraic geometry” where some types of structures are mapped by others, like when assigning discrete patterns to varieties. But in a second step it becomes possible to establish transits between very different universes of interpretation, (topology, set-theory, logic, groups, etc.). This looks not only close but also very promising regarding what Husserl understood as a *Mannigfaltigkeitslehre*, which now could be interpreted in a phenomenological-categorical approach (see: Peruzzi 1989 and 2006).

IV. Issues of foundation in phenomenology through category theory

The central concept of all phenomenology is that of *intentionality*, later conceived of as the *a priori of correlation*. We have seen that the side of objectivity always points at *mathematical* concepts.¹⁹ But the ego exhibits also a mathematical as we saw in the constitution of inner time-consciousness. Generally speaking, a subject is only the “who” of experience, the one *living* objectivities (and through its own relative objectivity) in the originary stream of consciousness, whose main *function* is to *ground* the series of objects and

¹⁹ But such concepts do not transform phenomenology into mathematics. It provides only the most abstract and general consideration for objects and relations. Mathematics as a discipline requires phenomenological foundation, but the structure of foundation is philosophico-mathematical. At this point phenomenology seems very abstract, far from every-day life, our body experience, or our emotions. It is not grounded in sensibility or language and does not choose art to express itself. Yet, this abstraction is also a *liberation from* particularity. But we do not impose instead an anonymous generality. There is no such encompassing point of view, but a combination of abstract regions and *modes to transform and connect them*.

relationships in an absolute time-flow. But this isolated ego shares the world with other egos in an intersubjective tissue (there are several perspectives, as we recognized in space constitution and a shared time), and this intersubjective humanity is distributed in different traditions, which are subject to a transcendental history of humanity, which also shares (co-constitutes) the earth with other animals, both belonging to a natural history. At this point, we obtain a complicated intentional structure, where it is not easy to set a last *instance* or a simple structure of layers of experience, one on top of the other, not even a clear directed hierarchy of foundation, not to speak about the absolute relationship of parthood presented in the third logical investigation. We should now ask how to think this *common space of things* and the *imbrication of several spaces* of experience.

Husserl identified in this *Logical Investigations* historicism and *psychologism* with *empiricism* (as sciences of *facts*, not of *principles*) and both with skepticism. He also understood the universality of laws of logic, which in turn could gain nothing with temporality. But the “natural” progress of phenomenology demanded to conduce the static to the dynamic (self)constituting ego, this to intersubjectivity and to historical intersubjectivity, and human intersubjectivity came to be embedded in the wide concept of earth. Husserl’s late concept of earth (*Erde*, Husserl 1940) should be read as the last expansion of the concept of “world” (with its *horizon*), but also as a general space grounded in nature. Earth does not move, says Husserl. It is, instead, an absolute *point of reference*. In this sense, the earth founds every possible material body relationship that can appear to us. Earth is both an instance that founds natural bodies (place, time and movement) and a ground for our own living body (*Leib*). But it is not clear anymore what does it mean to ground, because this earth only has *sense* as it *appears to us* in its *form*, and yet, we, as humans with our bodies and cogitations, stem from this very earth. This does not mean that we should naturalize phenomenology, but to trace the transcendental at play and its genesis in nature. Grounding becomes a “multidirected” arrow.

We see ourselves naturally confronted with different meanings of grounding (always in the sense of *Fundierung*) and thus with different types of arrows. Heidegger’s relationship between being and humans was defined in his late work as “belonging to-” (*Zugehörigkeit*),

but there is, despite everything, a main arrow starting in being and flowing in the direction of beings: $B \rightarrow b$. Coming into being, “desocultation” (*Entbergen, Offenbaren*), his translation of the Greek’s *aletheia*, all these terms imply this hierarchic and one-directional arrow. This is beginning itself, being *speaking* to humans and only to them. Despite everything, Heidegger seems to be caught in a *simple* relationship of ground-grounded, even if the first means an abyss (*Abgrund*). Heidegger speaks in *Being and Time* about the shared experience of the world coining the term being-with (*Mitdasein*), but this mention is not enough to display the whole problematic of intersubjectivity, where continuities and discontinuities (obstructions) give rise to a complex intentional structure in Husserl. This is why Heidegger insisted in the return to the source (*Quelle*) of phenomena. Like him, many phenomenologists devoted their efforts to bring the constituted to the constituting, to go from beings to being, from consciousness to being or to nature or to some primordial abyssal origin. However, as Fink²⁰ insisted, this is only the “half” of phenomenology, because the true enigma is not that things stem from an obscure ground of possibilities, but first, *how*, and second, how do they *last*, not alone, but *in a complex structure of remissions*. How do forms emerge (*morphogenesis*), how they last (*structural stability*)²¹, how they transform (*metamorphosis*), how they merge with each other (gluing or patching different spaces), how being distributes in different spaces (types, modes, layers, centers of experience), how these spaces evolve into, connect with and translate into each other. In this sense there is no being in general (B), but several spaces of being ($B_1, B_2, B_3 \dots$) or universes²², which link to each other through

²⁰ This is precisely what concerned Eugen Fink in his early writings. In his 6th *Cartesian Meditation* (Fink 1988) he addresses what he considers to be the last non-questioned supposition of phenomenology: the ego. But to reduce the ego introduces a shortcut, since it is the ego the only figure entitled to bracket the existence of the world. If this happens, argues Fink, we are not led to being (as Heidegger thinks), nor to a more fundamental space, like the world, but to nothingness. Phenomenology leads to a *meontology* (from the Greek *mé on*: non-being). This is the real transcendental subject. But, to avoid radical silence, the *task* of phenomenology consists in inventing concepts capable of bringing nothingness back to being. These are the *Entnichtungsbegriffe* (concepts that revert nothingness). Now, such concepts necessarily *ontify* being, offer *forms*. Now, these new concepts are not regular categories. They belong to a *logos hamártikos* or logos of “failure”. But failure means here creating limit-concepts, capable of unveiling unthought complexities but always from some perspective. *Entnichtungsbegriffe* are thus paradoxical, self-referenced, ironic, complex or multi-layered (See Fink: 1988 and 2006). This is precisely what contemporary mathematics can offer to phenomenology.

²¹ We borrow these concepts from René Thom (1975).

²² There is of course, the problem that we could simply be dealing with different senses of being. But if being is distributed in spaces, then, a *conditio sine qua non* of a world would be that they “touch” each other, i.e. if there is no totality, no ultimate ground but several interlaced spaces, we need a principle of non-trivial connectivity. Now, the point of thinking in terms of spaces in the mathematical sense of the term is that being does not dissolve in its meaning and in Dasein’s apprehension. Being always implies some sort of structure. Givenness is always formed. There is no duration outside form in general. Now the structure of remissions in

different types of arrows (some indicate transformations in time, other functions, other deformations, i.e. morphisms).

We ask now, how should we understand then the main concept of phenomenology, namely intentionality in the light of the former discussion? Intentionality is only the name for an arrow called *correlation*. It is essential to understand that the link between noesis and noema is not that of a fixed presence. There is *variation from both sides*, which assures the richness of the object and a richness of modes of access to it. The shift in Husserl from a static to a dynamic phenomenology revealed that objects are constituted in their becoming, but also that consciousness is constantly shifting or moving through different acts (but also through different regions, modes of being and strata of constitution). There is some type of *abstract* “function” leading from noesis to noema, or at least some *covariation*, in which some invariance is stated. Intentionality is constantly actualized in the correlation noesis-noema and recognized in the §48 of the *Krisis* as the fundamental *a priori of correlation*:

sobald wir nur anfangen, das Wie des Aussehens eines Dinges in seinem wirklichen und möglichen Wandel genauer zu verfolgen und konsequent auf die in ihm selbst liegende **Korrelation von Aussehen und Ausgehendem** als solchen zu achten, sowie wir dabei den Wandel auch als **Geltungswandel der in den Ichsubjekten und in ihrer Vergemeinschaftung verlaufenden Intentionalität** betrachten, drängt sich uns eine feste, sich immer mehr **verzweigende** Typik auf [...] [durch] **Weisen der Selbstgegebenheit** [...] [und] **Weisen der Intention in Modis der Geltung**, [...] in ihren **Synthesen der Einstimmigkeit und Unstimmigkeit, einzelsubjektiver und intersubjektiver** [...] [man erkennt] [ein] gewaltiges System neuartiger und höchst erstaunlicher apriorischer Wahrheiten [...] [Das Subjekt gilt als] Index seiner **systematischen Mannigfaltigkeiten**. (pp-168-169) [Aber] Impliziert ist [...] ein ganzer ‚Horizont‘ **nichtaktueller** und doch **mitfungierender** Erscheinungsweisen und Geltungssynthesen [...] bald stehen wir auch vor den **Schwierigkeiten einer konkreten Entfaltung dieses Korrelationsapriori**. Es kann nur in einer **Relativität** aufgewiesen werden, [...] daß unbeachtete Beschränkungen, **manche nicht fühlbar gewordene Horizonte zur Befragung neuer Korrelationen hindrängen**, die mit den schon aufgewiesenen **untrennbar zusammenhängen**. (p. 162) (Husserl 1976).

the world of the Dasein could couple (or be inserted) with (or in) other structures. Husserl recognized that animals are capable of intentional relationships and, as Uexküll and some Gestalt psychologists like Köhler showed it, they may also have an *Umwelt*, which touches the human world in several points and not only through human meaning.

This quotation confirms the *correlative* character of intentionality, i.e., the fact that all being must be *given* to me (as sense) in some way. Things, even if they cannot be exhausted, must be given in some sense to consciousness, and the subject, even if he is opaque (unconscious) to himself, must also be relative accessible to itself to allow self-constitution (the most basic mode of self-reference or self-relationship). Intentionality does not exhaust any of its poles, but recognises that there are no things at all if they are not for someone, in some sense and manner. There is no what (*Was*) without a who (*Wer*), and both must be brought together *in a context* to which it corresponds a mode of givennes (*Wie*). *What, who* and *how* constitute the most elementary set of elements related to each other making sense possible, but not taking it for being. Next to change or transformation (*Wandel*) of things and experiences, Husserl acknowledges the necessary relationship between presence (or actuality) and non-presence (or non-actuality), but always claiming that there *must be some* presence, givennes.²³

The personal ego does not exhaust the vast intersubjective experience. Simple presence does not exhaust the complex constitution of time consciousness, which includes the non-actuality of past and future (retentional and protentional *original* consciousness). Language includes empty intentions and present signs, such that sense is not exhausted by perception. But signs work in abstract systems and are not exhausted (fulfilled) in intuitions. Since animals also constitute their world, humanity does not exhaust being. The seen face of a three-dimensional object does not exhaust the object. There is always some non-presence, some surplus in presence and presentation, but this means, that there is *also* always a relationship to presence in otherness. The task of phenomenology is not to describe pure presence, but *the general space of being through different types of presence* together with the *implied forms of non-presence or non-actuality*. This is the enigma of the link operating in intentionality. As Barbaras points out, the apriori of correlation should not be interpreted in

²³ The same is valid for intersubjective experience. If there is some common, it also must appear in some way (direct or indirect, even as sign or symptom) in my *personal* experience. I am not *everything*, but a point of contact, a site for appearance or hearing (in the legal sense), a point of intersection of rays stemming from relationships that start before/outside the ego and continue after/outside it. But once something appears, it gains a life for itself in the “mental” space, where it intertwines with objects in bundles of relationships.

an idealist wake²⁴. It is also clear that realism is not refuted, but *mediated* and *complicated* by subjective givenness.²⁵ The relationship subject-object is not the absolute space of being, its main locus. It is, rather, *a* place, where intentional rays, surfaces, groups of abstract objects *touch* the subject (that means givenness).²⁶ Husserl himself spoke of time as a *braid* of intentional rays (*Verflechtung intentionaler Strahlen*). Objects are structures, like rays or surfaces or groups, and the subject is the place in which they “knot” in “bundles” and “braids” or other structures of relationships, but not their absolute origin. Barbaras writes:

En termes husserliens, la corrélation entre l'étant transcendant et ses modes subjectifs de donnée est un a priori universel [...] on ne préjuge en rien du statut exact du sujet de l'apparaître ou, plutôt, on souligne que **la référence de l'apparaître à un sujet ne compromet pas l'autonomie de cet apparaître**. Que l'apparaître soit destiné à un sujet ne signifie pas encore que **ce qui apparaît soit constitué au sein de ce sujet** et que sa teneur d'être propre soit finalement celle de la conscience et de ses vécus. (Barbaras 2010, p. 49).

This means that some *degrees of freedom* exist *both* in the subjective *and* in the objective side of the correlation, i.e. the subject is “more” than the object because it can apprehend it in different forms, or ideally vary in several ways, allowing constructions of higher order (and also of creative re-ordering); but the object is also “more” than the subject because it never gives itself in totality (it belongs to my experience the inexhaustibility of my horizon of meaning). Intentionality is thus a correlation (some sort of partial or local interaction), or more precisely, an abstract function relating subjective acts (including its “spaces” or regions, its temporal character and the variation and association of different acts) to appearances of actualities (modes of objectivity) in particular spaces (regions). There is nothing “ontological” said about the nature of the subject or of the object, for being as sense is always

²⁴ This is clearly a contentious issue in phenomenology. Speculative realism has objected phenomenology in this point. But we should here remain attentive to Husserl's own path to avoid both naïve idealism and realism. Transcendental idealism constitutes Husserl's position. The transcendental approach is needed to avoid a fall in empiricism and to confuse a *quid juri* with a *quid facti*. What remains open if there is something like a transcendental realism, as Schelling characterized his later philosophy, supplementing transcendental philosophy with *Naturphilosophy*.

²⁵ *Speculative realism* has accused that the *a priori of correlation* constitutes a circle that prohibits an access to the real. However, as Graham Harman (2002) has pointed out, a realist should not *eliminate* the correlation, but *extend it to all* beings to include object-object relationships but affirming, as Barbaras, that no correlation (no encounter or relationship) can exhaust things. And we would add ... nor the subject.

²⁶ In this sense we could we establish a parallel between Heidegger's and Husserl's notion of givenness as inseparable form otherness. The “es gibt” could be precisely represented by an arrow of unilateral giving.

only a *how* of one relating to the other. We do not affirm that there is nothing but representations, but that every being “testifies” through some (re)presentation, and this means an *encounter*. Subjective *constitution* means a *place* in which different intentional relationships *gather* in a particular context or space. But again, intentionality does not exhaust either what a subject (or an object) “is” or *can* be. It is not that subject and object are something in themselves, absolutely separate from correlation, but rather that there is a *multiplicity of objects (modes of “somethingness”)* and *types of subjective acts, a multiplicity of correlations* (between objects, between subjects, between objects, within subjects and within objects) and a *multiplicity of spaces, which in term may relate to each other through a multiplicity of relations*.

Following some key reflections on Husserl’s phenomenology, we have intuitively arrived at core ideas of category theory. We have made scattered references to mathematical concepts throughout the text. We made special emphasis in the notion of manifold in Husserl, involving not only Riemann’s geometrical interpretation, but also set and group theory, as he himself acknowledged. It was clear from the very beginning that Husserl tried to bring together different branches of mathematics to offer a general concept of objectivity in general. But now, *the aimed unity of objectivity cannot be granted without the unity of mathematics in which it rests*. This leads us to the historical context in which Husserl begun his phenomenological project, namely the crisis on foundations of mathematics, which *mutatis mutandis* entailed a crisis in the foundations of science in general. A ground should grant firstness (difference ground-grounded), unity (difference unity-multiplicity) and a structure of foundation of multiplicities (an order of being). Husserl advanced the ego as a transcendental solution to the problem of foundation but, as we saw, the very idea of grounding (*Fundierung*) depends of ideal forms which should assure objectivity. In every effort to surmount constituted (i.e. scientific) objectivity, as it is the case of the constitution of inner time-consciousness, Husserl resorts to the concept of manifold. The reason is not that science should grant the main access to being, but that being in general is always given in some *form*, without which there would be no phenomenology, but only its shadow. The central misunderstanding here lies in the concept of objectivity. It is normally understood as a figure, capable of being manipulated and positively presented. Against this,

phenomenology directed its efforts to unravel the realm of the non- or pre-objective. But in the end objectivity is nothing but the form of presentation, even if this form is vague, without clear borders and indeterminate, it is the structure without which matter (*hyle*) would sink in darkness and silence. Giving up on objectivity means to give up on presentation, intersubjectivity and above all, *form*. Category theory is the most ambitious enterprise to expand the concept of structure in mathematics. It does not deliver an ontology, i.e. a set of objects and relationships, but a multiplicity of them and modes to relate each other.

Husserl distanced from Kant in a very precise point: he would not accept a set of constituted ideal forms pertaining to mind or understanding that would apply to sensible matter (*hyle*) from the *outside*. However, Husserl seems to claim that material ontologies only deliver the places of instantiation of pure forms, pertaining to formal ontology. This means that although experience is always material, because it takes place in a particular region, with its own rules and modes of givenness, it can always eventually be *subsumed* in ideal forms of objectivity regardless of the region involved. This would render materiality phenomenologically irrelevant and it might even reintroduce the classical dualism form-matter. In the same line of thought, intersubjectivity seems to pose an important problem for objective constitution, since it involves different perspectives, mediation of signs and transmission along history. Husserl acknowledges the problem of otherness and the impossibility of constituting the alter ego originally, such that intersubjectivity has the need of *mediation*. But he claims also that every ego constitutes the world in *identical* ideal manner and that changes of position and perspective of observers all belong to a single system, so that objectivity and mutual agreement suffer no risk. This is already contentious, but a fundamental issue considered in the *Logical Investigations*, signs, would hardly be explained by this reasoning, and, as we move from temporal and spatial beings to other layers of constitution involving values and opinions, arriving to what Husserl considers more important: ethics, it seems impossible to hold the same objectivity claims, making multiplicity superfluous.

In mathematics, it was Hilbert's axiomatic view that imposed as an answer to the crisis of foundations. Later, it would be set-theory the mathematical language in charge of grounding the whole field of mathematics. Here, founding meant choosing a universe of objects (sets),

relationships (functions) and a small number of axioms, with which all mathematics could be *derived*. We don't find here any strict ontological definitions (although there are ontological *implications*) of the primitives, like "set"; signs are devoid of all meaning outside a formal system; and there is no claim that axioms are "evident" or "universal" but just a convention. Husserl reacted against this reasoning and reintroduced a transcendental subject to grounding mathematics in experience but retaining from the logical school, its critique of psychologism and historicism. We showed at the beginning of this article how set-theory could not serve the purpose of phenomenology for it is devoid of all meaningful content for a *subject*. But the idea of grounding was put in peril by the discoveries of logic itself. Already Cantor had discovered the paradoxes of infinity, which Russell only found to be operating in (naïve) set-theory appearing in the case of self-reference (i.e. sets counting as members of themselves). The last chapter in this history is to be found in Gödel's theorem of incompleteness of arithmetic, where he proved axiomatic mathematical systems to be either inconsistent or incomplete.²⁷ Next to the inherent problems of set-theory inherited by its axiomatic constitution, it also proved to be insufficient to *encompass* all the mathematical universe. Category theory appeared in the mid-twentieth century to contest the centrality of set-theory in the foundation of mathematics. But this is possible because *the meaning of foundation* changes when moving from one theory to the other. Set-theory allows to define notions precisely once the axioms are *decided*. There is no ground for deciding axioms, only their *usefulness* in a mathematical field. We then seek to express different regions of mathematics in the common language of sets and functions. But category theory operates on another level of abstraction and constructions may sometimes be arduous and complicated. We already said that we won't explain the main concepts of category theory, we will rather

²⁷ We may remember here Alain Badiou, who also draws on mathematical concepts to ground his ontology. However, he seems to be fixed to a *particular region* of mathematics, precisely set-theory *and* to classical logic. He claims that ontology must assert multiplicity as its most fundamental concept (a formless void). But if we start with a formless being, a "pure multiple" as he states in *Being and Event* (Badiou 2005), not being grasped by any unity whatsoever ("the one"), we have to explain how concrete beings emerge, how determination takes place from the outside. Remaining faithful to Heidegger's ontological difference, being must be pre-objective and pre-subjective, indeterminate but determinable in different manners (in concrete beings). But in this view concrete beings have no subsistence at all outside the contingent field of interpretation. Moreover, Badiou's ontology (re)introduces "the one" at many levels. It ties ontology to *one* single logic and to a *single* field of objects (those from set theory). It *unifies* ontology in *one* formal system. It establishes the ideal points of set theory as the *ultimate single* constituents, etc. Husserl's ontology, on the contrary, retains multiplicity on another level, namely as a *plurality of regions and levels of constitution*, a plurality of spaces, instead of a plurality of points in a set. See Plotnisky (2012) for a related discussion.

characterize it in a philosophical fashion to show how phenomenology sometimes approached to it and how sometimes it *could* and *should*.

Category theory does not rely on an (quasi)ontology as set-theory does. The simplest category counts only with a collection of objects and a collection of morphisms between them (respecting the very general axioms of associativity, composition and identity), without having to state *what* that objects and morphisms *are*. Objects are defined by what and only insofar as they are *for another* (or many other) object(s). This being-for-another is specified by a particular *morphism* or a collection of *several* morphisms in a single category. In this manner, “what there is in the world” remains open, but always related to some encounter or correlation. Categories are like universes, in which certain objects populate a space, obeying certain rules.²⁸ This variety of categories or spaces and not the sheer and unstructured “multiple” of set-theory seems suitable to do justice to the multiplicity of being. Category theory states that at first glance very different domains actually obey the same categorical structure. This makes the afore mentioned domains to be *relatively commensurable* applying the same categorical structure, but at another level, called functors. The development of category theory owes much to algebraic geometry, while trying to associate algebraic structures to topological spaces in order to extract information. Phenomenology must fulfill the *double* task of finding *common essences* of objectivity across different regions and *grounding* essences in particular regions. Category theory would provide phenomenology with a general frame to think objectivity along very different concrete domains.

We stated above that in category theory objects are not defined from the inside, it is not important how they are “internally constituted”. An object is only “revealed” through its relationships (morphisms) with other objects. This seems a rather precise definition of manifestation or appearing. However, objects retain possibilities from them, not always expressed in a category, but to appear in correlation with other objects. This possibility expresses in other terms Barbara’s concern of avoiding a reification of consciousness. An

²⁸ To explain category theory, one usually gives some example: in the category of sets objects are sets and morphisms are functions; in the category of topological spaces objects are topological spaces and morphisms continuous functions, in the category of groups objects are groups and morphisms are group homomorphisms, etc.

object may appear in several ways and establish several relationships with other objects. By the same token, a “subject” is always revealed not by its abstract “possibilities” but by the objects and subjects he relates to and by the concrete relationships established in the concrete fields that serve as “stages” of experience. Phenomenology may thus be compatible both with realism (an object is always more than the concrete correlation in which it appears), with a procedure (it investigates the varied modes of givenness and their correlations, without further assumptions) or with ontological pluralism (modes of appearance are modes of reality and a multiplicity of the former implies a multiplicity of the latter).

We can now advance the thesis that a category *expresses* the most general form of intentionality (and it can be complicated or scaled in complexity *ad libitum*). To achieve this, one must step out of the exact mathematical definition and extend the idea to a conceptual correlation, i.e. Categories show universes of objectivity (regions) and seem to obscure the underlying subject. But a category, with its objects and relationships is more the *result* of the interplay of subjective acts and modes of being, which is continued in higher levels (functors and natural transformations). Objectivity does not mean simple objects, but a set of possible transformations of the object and between objects. Since categories imply relationships between objects (which may be spaces, sets, groups, etc.) we always count with *structured universes of objects*. This means also that not every universe of objects is *reduced* to its categorical structure. Every region remains singular. This is precisely what phenomenology looked after: a general theory of objectivity capable of respecting the peculiar modes of appearance within the limits of a certain region or “universe”. The idea of a structure-preserving map played already a role in Husserl’s analysis of space and intersubjectivity: objects vary according to perspective, but this change preserves the structure of the object, both in me, when I change position respect to the object, and among persons, who despite never being in exactly the same place, ideally variate positions in a system to make common experience possible.

As we find in the first volume of Husserl’s *Logical Investigations*, a *Mannigfaltigkeitslehre* should deal with theories, not with direct objects. And since theories are linked to (directly or indirectly, but always grounded in) regions of experience, a phenomenological theory of

objectivity should analyze the region of regions, the space of spaces. Now, this raises the question about the *possibility* of such a *unity*. If we said that the unity of mathematics cannot be granted by set-theory, what does category theory has to offer in this respect? Category theory does not offer a super-theory of mathematical objects, capable of encompassing every thinkable domain, nor does it seek a theory of ultimate elements. It is an abstract theory that offers a *common language* for the most distant domains of mathematics to establish equivalences that not only allow to identify common features, but also to solve problems of one domain by recourse of another. The unity of mathematics is not achieved neither top-down (a theory capable of subsuming all others as mere cases) nor bottom-up (defining absolute elementary constituents), but “horizontally”. Making abstraction of some features, we may render two mathematical (or objectivity) universes comparable to perform different operations that would be impossible from the outset. In the first formulations of phenomenology the ego, or better, the relationship noesis-noema in its constituted ideality was meant to be the absolute space of manifestation. Later intersubjectivity pointed at a *distributed* (and not reiterated or repeated) subjectivity. Also, genetic phenomenology had to accept not only constituted forms, but also their *process* of constitution (emergence, genesis) (and not only of objects, but of forms of objectivity) and, eventually, a form of time and a form of genesis or change. Correlation, translation, gluing, transformation, etc., may be conceived of as morphisms. Locality, perspective, incompleteness, and multiplicity are part of the world, but they are not unsurmountable differences. Phenomenology traces the “graph” of a back and forth movement from equivalence and obstruction, identity and difference.

Of course, one thing is to be locally bounded and another to have the *concept* of boundedness, to be incapable of uniting a multiplicity and to have the concepts of unity, multiplicity and impossibility. Now, phenomenology must do *both*: it must explain what multiplicity is in terms of objectivity and show how this is *experienced* by synthetic acts. In this sense, phenomenology not only needs ways of transforming objects both in space and time preserving their structure, but also connect different regions of experience to effectuate translations and equivalences. Husserl insisted on identity and on how it assures objects to be identical for everyone. But mathematical objects as well as values depend heavily on the category or *context* they operate. If we move from one domain or region to another, or from

one mode of givenness to another, equivalences are not always possible. This is another important feature of category theory derived to great extent from topology and function-theory: there are degrees of likeness. Just as functions may be bijective, injective or surjective, we may find in categories different types of relationships (morphisms) between classes or objects, like monomorphisms, epimorphisms, bimorphisms, retractions, sections, isomorphisms, endomorphisms, automorphisms. The idea is to formalize the *degree* to which two objects or categories are “alike” or how “similar” is their structure and under which criteria. Instead of the metaphysical idea of unity (of the cosmos, God or the subject), phenomenology could investigate the local sites of manifestation and then show how they relate to each other through laws of *equivalence* and not through a simple all-encompassing unity. Identity and difference appear as the two poles of isomorphism and radical obstruction. This mode of approaching equivalence allow to address two phenomenological subjects: analogy (for example between bodies in intersubjectivity) and translation (from one domain to another, but also from one language to another). Let’s not forget that objectivities include not only isolated objects, but their combinations in complex situations, i.e. a “grammar”. We may very well share the “objects” we are speaking about (a river, for example), but all the connections of that river with my beliefs, my expectations, my social class, etc., are part of complex objectivity, which can never coincide exactly with that of others. We understand partially each other not because we share some objects underlying our discourse, but because we can achieve or produce partial overlappings of our universes of discourse and experience. The so-called “fusion of horizons” from which Gadamer speaks can be better understood in category theory, especially through the concept of sheaves.

Sheaves are an abstract concept in mathematics, developed (and expanded into topos theory) in the language of category theory by Serres and Grothendieck. Sheaves are method to obtain global from local information associating some rich objects to the open sets of a topological space. We may, for example, associate rings to the open sets of a topological space to apply the properties of the former, to better know the latter. We have a sheaf when we apply an arrow (an inverse) going from, in this case, the open sets of an algebraic structure like rings to points of the topological space. In this way we can introduce, for example, order relationships in the latter. A point ceases to be a dimensionless object without structure.

Being associated to an open set of another structure, our space becomes richer. We combine geometry (relations of proximity and distribution of points in space) and algebraic structures (richer structure, operations). We can thus construct a space with arrows.

One of the important uses of sheaves in philosophical sense relies on the possibility of obtaining global from local information. After we associate an algebraic structure to a topological space, we can glue isolated parts to produce a global, smoother section. We are not *subsuming* a complicated space into a simpler one, nor we reduce information to force global coherence. The procedure, intuitively speaking relies on associating richer structures to local parts of a space and the gluing that parts in a complex global and richer space. Fernando Zalamea defines sheaves in a philosophical wake as follows:

The ancient philosophical question “how to move from the multiple to the one” [...] (phenomenological transit) becomes the mathematical question “how to move from the local to the global?” (technical transit), which subdivides in turn into the questions: a) “how to register differentially the global?” and b) “how to integrate globally these registers”. When addressing question *a)* **analytically** we obtain the natural mathematical concepts of neighborhood, covering, coherence and gluing, while, when addressing **synthetically** question *b)* we obtain the natural mathematical concepts of restriction, projections, presheaves and sections. Presheaves (term coined by Grothendieck) cover the combinatory of **discrete** links neighborhood/restriction and covering/projection, while sheaves cover the **continuous** combinatory linked to the pairs coherence-preservation and gluing-section. (Zalamea 2009, p. 161).

It seems possible, for example, to glue together different first-person perspectives though association with other structures like language, thanks to local overlappings, possible by an ideal system of space. We do not exchange science directly through evidences but by linguistically structured reports. But at the same time, we cannot aim at global concepts, without being faithful to things themselves. Zalamea proposes a back and forth movement between difference and synthesis, which could be complemented by Fink's back and forth movement between reduction (following *ontological difference*) and expression (following a movement of *ontification*).

We arrive finally to the difficult concept of *topos of Grothendieck*. As noted above, this is not the place to mathematically consider the subject. We want, however, to highlight a big possibility reserved for phenomenology in this regard. We referred to the notion of sheaf. A topos generalizes the ideas behind sheaves and pre-sheaves, making possible to associate not only algebraic structures to topological spaces, but to relate via functors virtually any object of mathematics to any other. One of the most surprising results of Grothendieck was the possibility to associate mathematical structures to different logics. A topos is equivalent, for example, to intuitionistic logic. If we further generalize the concept of topos, we can also generate structures with different sub-object classifiers allowing thus other types of logics. While Husserl aimed at grounding “logic” in general, as it follows from his attempts in the *Logical Investigations*, the question of genesis also affected the *contents* of both logic and mathematics. This means that even if “the formal” operates in intuition from the outset, concrete mathematical and logical structures are historically grounded and may also experiment further modifications. However, it is not clear whether “genesis” meant also a multiplicity of valid coexistent frameworks. His reflections were constantly led by *one* single logic, as geometry had been led by one single geometry. Just like geometry experimented a revolution in the hands of Gauss and Riemann or Lobachevsky by suspending (not adding or inventing) an axiom (the one referring to the parallels) it was considered the possibility of expanding the domain of the reasonable by suspending in a regulated manner the principles of non-contradiction and excluded middle. And just as non-Euclidian geometry supposed any space to be *locally* Euclidian, emerging from “patches” of it, it is natural to defend the hypothesis that classical logic may be *locally valid* (i.e., bounded to some abstract space or domain), while some phenomena may exhibit more complicated forms. Phenomenology was constantly caught between two poles: on the one hand, the need of find universal forms capable of assuring the univocity of the world at least in its formal aspect; and, in the other, the need of grounding even the most general laws in concrete experience and this in concrete spaces and moments of history. Topos theory allows to do justice to this double exigency (see: Caramello, 2016). Topos theory bounds logic to a “context” and the material constrains and possibilities of a particular region, but at the same time, thanks to its degree of abstraction, it allows to effectuate *transits* between regions and logics. But Husserl’s ideas of some “protogeometry”

operating in perception may well demand a “protologic” to explain categorial intuition, both operating like a “formal landscape” of possible determinations.

When one takes a closer look to the history of phenomenology, it is easy to find a clear tendency against all forms of objectivity and formal thought. It was tacitly accepted that the task of phenomenology consisted in moving “back” towards an open origin which could deliver us from the closed and exhausted forms of scientific and objectifying thought in general. It became common sense to claim that the “constituted” forms of thought constrain a more fluid and indeterminate state, which could be seen as “constituting” and of more originary nature. For this reason, time became the model for openness, indeterminacy, and possibility, but also of subjectivity, while space was interpreted as the realm of exteriority, quantification and rigid structures. But this tendency only shows how philosophy distanced from the most powerful insights of contemporary mathematics. Already topology and its associated flexibility to conceive space made it a suitable tool to study dynamical systems and change. If we think of a meagre definition of space, like that of topology, fluidity, indeterminacy and openness can be formally addressed and articulated. But this is possible thanks to *formal and exact approaches*. The fluid becomes thinkable through formal inventions of thought. It is rather the constant movement from fluidity to structure and back what provides thought its power. For the same token we should not give more privilege to the possible and amorphous (or poorly structured) than to the actual and formed, but rather to the *transits* between those poles. Rigidity makes more complex systems possible, but less structure also allows freer transformations, deformations and types of motion. The actual carries its own possibilities not despite, but *thanks to its constraints*. To change form, a constraint must be left behind to find another. In phenomenology, mathematical tools opened deep possibilities of analysis. This choice turned later into an obstacle for its development. Now category theory and topology offer new means to reconsider Husserl’s ambitious task.

Bibliography

Awodey, S. (2010). *Category Theory*. 2nd Ed. Oxford University Press. Oxford.

- Badiou, A. (2005). *Being and Event*, transl. by Oliver Feltham. Continuum. New York.
- Barbaras, R. (2012). Sauver d'une réification de la conscience la tâche de la phénoménologie. In: *Les Études philosophiques* 2012/1 (nr. 100), pp. 49-63. Presses Universitaires de France. Paris.
- Benoist, J. (2007). Mettre les structures en mouvement : la phénoménologie et la dynamique de l'intuition conceptuelle. Sur la pertinence phénoménologique de la théorie des catégories. In : L. Boi et al. (eds.), *Rediscovering Phenomenology*, Springer, 339–355.
- Boi, L (2004). Questions Regarding Husserlian Geometry and Phenomenology. A Study of the Concept of Manifold and Spatial Perception. *Husserl Studies*. December 2004, Vol. 20, nr. 3, pp. 207–267.
- Boi, L (2005). Topological knots models in physics and biology: Mathematical ideas for explaining inanimate and living matter. In: Boi, L. *Geometries of Nature, Living Systems and Human Cognition New Interactions of Mathematics with Natural Sciences and Humanities*. World Scientific.
- Brentano, F. (1874). *Psychologie vom empirischen Standpunkte*. I. Vol. Duncker und Humblot. Leipzig.
- Derida, J. (2010). *L'origine de la géométrie de Husserl*. PUF. Paris.
- Fink, E. (1988). VI. *Cartesiansche Meditation: Teil 1. Die Idee einer transzendentalen Methodenlehre*. (Ebeling, H., Hrsg.). Kluwer. Dordrecht, Boston, London.
- Fink, E. (2006). *Phänomenologische Werkstatt* 1, 2: Eugen Fink Gesamtausgabe. Band 1. Karl Alber. Freiburg.
- Caramello, O. (2016). The Theory of Topos-Theoretic 'Bridges'—A Conceptual Introduction. *Glass Bead*. Online: <https://www.glass-bead.org/article/the-theory-of-topos-theoretic-bridges-a-conceptual-introduction/?lang=enview> Consulted on 7 January 2021.
- Goldblatt, R. (1984). *The Categorical Analysis of Logic*. Elsevier. North Holland, Amsterdam, New York, Oxford.
- Harman, G. (2002). *Tool-Being: Heidegger and the Metaphysics of Objects*. Open Court. Chicago and La Salle, Illinois.
- Husserl, E. (1901). *Logische Untersuchungen. Erster Theil. Prolegomena zur Reinen Logik*. Verlag von Veit & Comp. Leipzig.
- Husserl, E. (1913). *Logische Untersuchungen. Zweiter Theil. Untersuchung zur Phänomenologie und Theorie der Erkenntnis*. Max Niemeyer. Halle.

- Husserl, E. (1940). Grundlegende Untersuchungen zum phänomenologischen Ursprung der Räumlichkeit der Natur. In: Farber, M. ed. (1940). *Philosophical essays in memory of Edmund Husserl*. Harvard University Press. Cambridge.
- Husserl, E. (1950). *Husserliana I. Cartesianische Meditationen; Pariser Vorträge*. Martinus Nijhoff. The Hague.
- Husserl, E. (1952). *Husserliana IV. Ideen zu einer Reinen Phänomenologie und Phänomenologischen Philosophie. Zweites Buch: Phänomenologische Untersuchungen zur Konstitution*. Ed. Marly Biemel. Martinus Nijhoff. The Hague.
- Husserl, E. (1969). *Husserliana X. Zur Phänomenologie des inneren Zeitbewusstseins (1893-1917)*. Ed. Rudolf Boehm. Martinus Nijhoff. The Hague.
- Husserl, E. (1966). *Husserliana XI. Analysen zur passiven Synthese. Aus Vorlesungs- und Forschungsmanuskripten 1918-1926*. Martinus Nijhoff. The Hague.
- Husserl, E. (1976). *Die Krisis der europäischen Wissenschaften und die transzendente Phänomenologie: Eine Einleitung in die phänomenologische Philosophie*. Ed. Walter Biemel. Martinus Nijhoff. The Hague.
- Husserl, E. (1977). *Husserliana III-I. Ideen zu einer Reinen Phänomenologie und Phänomenologischen Philosophie: Erstes Buch: Allgemeine Einführung in die Reine Phänomenologie*. Springer. Netherlands.
- Krömmer, R. (2007). *Tool and Object: A History and Philosophy of Category Theory*. Birkhäuser. Basel, Boston, Berlin.
- Lavendhomme, R. (2001). *Lieux du sujet. Psychanalyse et mathématique*. Seuil. Paris.
- Lorenz, K. (1941). *Kant's Lehre vom Apriorischen im Lichte gegenwärtiger Biologie*. *Blätter für Deutsche Philosophie* 15, pp.94-125.
- Magnavacca, S. (2005). *Léxico técnico de filosofía medieval*. UBA, Miño y Dávila Editores. Buenos Aires.
- Marquis, J-P-. (2009). *From a Geometrical Point of View. A Study of the History and Philosophy of Category Theory*. Springer.
- Ortiz Hill, C. (2002). Tackling three of Frege's problems: Edmund Husserl on sets and manifolds. *Axiomathes*, Vol. 13, nr. 1, March 2002, pp 79–104.
- Peruzzi, A. (1989). Towards a real phenomenology of logic. *Husserl Studies* vol 6, pp. 1-24. Kluwer Academic Publishers. The Netherlands.

Peruzzi, A. (2006). The meaning of category theory for 21st century philosophy. *Axiomathes* 16, pp. 425–460. Springer.

Plotnisky, A. (2012). Experimenting with ontologies: sets, spaces, and topoi with Badiou and Grothendieck. *Environment and Planning D: Society and Space*, vol 30, pp. 351-368.

Romero Contreras, A. (2016). Analogy and mapping: philosophy, mathematics and space. *Studia Metodologiczne*. Nr. 37. Pp. 97-120.

Romero Contreras, A. (2019). *Die Gegenwart anders denken: Zeit, Raum und Logik nach dem Ende der Philosophie*. J.B. Metzler.

Rosado Haddock, G. (2017). Husserl and Riemann. In: Centrone, S. *Essays on Husserl's Logic and Philosophy of Mathematics*. Springer.

Tieszen, R. (2005). Free Variation and the Intuition of Geometric Essences: Some Reflections on Phenomenology and Modern Geometry. *Philosophy and Phenomenological Research*, Vol. 70, Nr. 1 Jan. 2005, pp. 153-173.

Thom, R. (1975). *Structural Stability and Morphogenesis*. W.A. Benjamin. Massachussets.

Zalamea, F. (2009). *Filosofía sintética de las matemáticas contemporáneas*. Editorial Universidad Nacional de Colombia. Bogotá.