

Computational Logic

Volume 1:

Classical Deductive Computing with
Classical Logic

Luis M. Augusto

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Contents

Preface to the first edition	xiii
Preface to the second edition	xix
I Introduction	1
0.1 Symbolic computation and classical computing	3
0.2 Logic: Formal, symbolic, deductive, and classical	5
0.3 Computational logic and its subfields	8
0.4 Classical deductive computing and its assumptions	10
II Mathematical foundations	15
1 Mathematical notions	17
1.1 Basic notions	17
1.1.1 Sets, relations, functions, and operations	17
1.1.2 Binary relations and ordered sets	25
1.2 Discrete structures	30
1.2.1 Algebras and models	30
1.2.2 Lattices	34
1.2.3 Graphs and trees	42
1.3 Mathematical induction	46
III Classical computing	49
2 Fundamentals of classical computing	51
2.1 Formal languages and grammars	52
2.1.1 Regular languages	60
2.1.2 Context-free languages	64
2.1.3 Recursively enumerable languages	76
2.1.4 The Chomsky hierarchy (I)	78
2.2 Models of computation	81
2.2.1 Finite-state machines	81

Contents

- 2.2.2 Pushdown automata 100
- 2.2.3 Turing machines 121
- 2.2.4 The Chomsky hierarchy (II) 135
- 2.3 Computability and complexity 136
 - 2.3.1 The decision problem and Turing-decidability . . . 136
 - 2.3.2 Undecidable problems and Turing-reducibility . . . 140
 - 2.3.3 The Chomsky hierarchy (III) 147
 - 2.3.4 Computational complexity 148
 - 2.3.5 The Chomsky hierarchy (IV) 164

IV Classical deduction and classical logic 167

3 Preliminaries: Formal logic, deduction, and deductive computation 169

- 3.1 Logical form I: Logical languages 170
 - 3.1.1 Alphabets, expressions, and formulae logical 170
 - 3.1.2 Orders 174
 - 3.1.3 Formalization 179
- 3.2 Logical form II: Argument form 187
- 3.3 Logical meaning: Valuations and interpretations 194
- 3.4 Logical systems, logics, and logical theories 206
 - 3.4.1 Logical consequence, inference, and deduction . . . 206
 - 3.4.2 Syntactical consequence and proof theory 213
 - 3.4.3 Semantical consequence and model theory 218
 - 3.4.4 Adequateness of a deductive system 223
 - 3.4.5 Logical theories 228
- 3.5 Deductive computation 230
 - 3.5.1 Logical problems and computational solutions . . . 230
 - 3.5.2 Taming FOL undecidability 233
 - 3.5.2.1 Finite satisfiability and ground extensions 233
 - 3.5.2.2 Finite models and prefix classes 238
 - 3.5.3 The complexity of logical problems 240

4 The system CL and the logic CL 247

- 4.1 The language of classical logic 247
 - 4.1.1 The language L1 247
 - 4.1.2 Substitutions and unification for L1 249
- 4.2 Classical logical consequence 255
 - 4.2.1 Classical \heartsuit -consequences 255
 - 4.2.1.1 Classical syntactical \heartsuit -consequences . . . 256
 - 4.2.1.2 Classical semantical \heartsuit -consequences . . . 258

4.2.2	Classical \blacklozenge -consequences	260
4.3	The logic of CL	262
4.4	Classical FO theories and the adequateness of CFOL	265
4.5	The extension $CL^=$: CL with equality	273
5	Classical proofs	279
5.1	The axiom system \mathcal{L}	280
5.2	The natural deduction calculus \mathcal{NK}	283
5.3	The sequent calculus \mathcal{LK}	288
6	Classical models	295
6.1	Tarskian semantics	295
6.2	Herbrand semantics	299
6.3	Algebraic semantics: Boolean algebras	306
V	Classical deductive computing with classical logic	315
7	Classical logic and deductive computation	317
7.1	The computational problem of classical satisfiability, or SAT	318
7.2	Computerizing CFOL	325
7.2.1	Literals and clauses	326
7.2.2	Negation normal form	327
7.2.3	Prenex normal form	327
7.2.4	Skolem normal form	328
7.2.5	Conjunctive and disjunctive normal forms	329
7.3	Computing the SAT	335
7.3.1	The different forms of the SAT	335
7.3.2	The SAT and unsatisfiability I: The DPLL procedure and model finding	337
7.3.3	The SAT and unsatisfiability II: Herbrand's theorem and refutation	340
8	Automated theorem proving	347
8.1	Resolution	348
8.1.1	The resolution principle for propositional logic	348
8.1.2	The resolution principle for FOL	354
8.1.3	Completeness of the resolution principle	362
8.1.4	Resolution refinements	364
8.1.4.1	A-ordering	365
8.1.4.2	Hyper-resolution and semantic resolution	369
8.1.5	Paramodulation	377

Contents

8.2	Analytic tableaux	383
8.2.1	Analytic tableaux as a propositional calculus . . .	383
8.2.2	Analytic tableaux as a FO predicate calculus . . .	391
8.2.2.1	FOL tableaux without unification	393
8.2.2.2	FOL tableaux with unification	396
9	Programming	399
9.1	Logic programming as deductive programming	400
9.1.1	Query systems and programming systems	400
9.1.2	LP programs and their meaning	405
9.1.3	Resolution and LP computations	414
9.1.4	Negation as failure	425
9.2	Declarative + procedural interpretation: Prolog	433
9.2.1	Prolog and Prolog	433
9.2.2	Logic + control: ! and fail	440
9.2.3	Negation in Prolog: The predicate not	446
9.3	Purely declarative interpretation: Datalog	451
9.3.1	Relational languages and databases	452
9.3.2	Deductive databases and Datalog	455
9.3.3	Semantics for Datalog DDBs	462
9.3.3.1	Herbrand semantics	462
9.3.3.2	Fixed-point semantics	468
9.3.4	A proof system for Datalog definite programs: SLD resolution	472
9.3.5	Datalog with negation: Datalog [¬]	480
	Bibliography	493
	Bibliographical references	495
	Index	505

List of Figures

1.1.1	A partially ordered set.	26
1.1.2	Hasse diagram of a poset.	29
1.2.1	Join table of 2^A	36
1.2.2	Meet table of 2^A	37
1.2.3	The lattice $(\mathcal{S}, \cup, \cap)$	37
1.2.4	The non-distributive lattices \mathcal{L}_1 and \mathcal{L}_2	38
1.2.5	A simple graph with five vertices and seven edges.	43
2.1.1	Derivation tree of the string $w = acbabc \in L(G)$ with the corresponding partial derivation trees.	68
2.1.2	Two leftmost derivations of the string $a + a * a$	69
2.1.3	Parse tree of an unambiguously derived string.	71
2.1.4	Parse trees for productions (1) $S \rightarrow a$ and (2) $S \rightarrow AB$	72
2.1.5	Parse tree for $z = uv^iwx^iy$	73
2.1.6	An algorithm based on the Chomsky hierarchy for deciding on the class of a language.	80
2.2.1	State diagrams of FSRs.	84
2.2.2	A FSR with two accepting states and one rejecting state.	85
2.2.3	A NDFSR accepting the language $L = \{001\}^* \{0, 010\}^*$	87
2.2.4	Equivalent NDFSR (1) and FSR (2).	92
2.2.5	Schematic diagrams for finite automata accepting (1) $L_1 \cup L_2$, (2) L_1L_2 , and (3) $(L_1)^*$	94
2.2.6	A finite automaton M for the pumping lemma.	96
2.2.7	Moore (1) and Mealy (2) machines.	98
2.2.8	A PDA M accepting the language $L(M) = \{a^m b^m \mid m \geq 0\}$	103
2.2.9	Proving the equivalence of $L(M) = N(M)$	106
2.2.10	NDFSR recognizing the viable prefixes for the CFG of Balanced Parentheses.	115
2.2.11	A Turing machine computing the function $f(m, n) = m + n$	124
2.2.12	The encodings $\langle M_T \rangle$ and $\langle M_T, z \rangle$	127
2.2.13	A Turing machine that computes the function $f(n, m) = 2n + 3m$	129
2.2.14	Program for a Turing machine computing the function $f(n, m) = 2n + 3m$	130

List of Figures

2.2.15	A combination of Turing machines.	133
2.2.16	A Turing machine.	134
2.3.1	A combination of Turing machines.	146
2.3.2	The Chomsky hierarchy and beyond: Decidable, Turing-recognizable, and not-Turing-recognizable languages.	147
2.3.3	The hierarchy of complexity classes with corresponding tractability status.	157
2.3.4	Typical structure of NP -completeness proofs by polynomial-time reductions.	161
3.1.1	Formalizations for English by means of the language of classical propositional logic.	184
3.1.2	Formalizations for English by means of the language of classical FO logic.	185
3.2.1	Some classical formally correct arguments.	192
3.3.1	Truth table for the connective \rightarrow in the 3-valued logics \mathbb{L}_3 , \mathbb{K}_3^W , and Rn_3	199
3.4.1	Adequateness of a deductive system $\mathbb{L} = (\mathbb{L}, \Vdash)$	226
4.1.1	Unifying the pair $\langle P(a, x, h(g(z))), P(z, h(y), h(y)) \rangle$	253
5.1.1	Proof of $\vdash_{\mathcal{L}} \phi \rightarrow \phi$	281
5.1.2	Proof of $\{\phi, \forall x (\phi) \rightarrow \chi\} \vdash_{\mathcal{L}q} \forall x (\chi)$	281
5.2.1	Proof of $\vdash_{\mathcal{NK}} ((A \rightarrow B) \wedge (A \rightarrow C)) \rightarrow (A \rightarrow (B \wedge C))$	286
5.2.2	Proof of an argument in (extended) \mathcal{NK}	287
5.2.3	A FO \mathcal{NK} proof.	287
5.3.1	Proof in \mathcal{LK} of a FO validity.	292
5.3.2	Proof in \mathcal{LK} of axiom $\mathcal{L}2$ of the axiom system \mathcal{L}	293
7.1.1	A tableau for the Turing machine M	323
7.2.1	Tseitin transformations for the connectives of \mathbb{L}	333
7.3.1	A DPLL proof procedure.	339
7.3.2	Closed semantic tree of $C = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3, \mathcal{C}_4, \mathcal{C}_5\}$ in Example 7.3.3.	343
7.3.3	A closed semantic tree.	344
8.1.1	A refutation tree.	350
8.1.2	A propositional argument as input in Prover9-Mace4.	350
8.1.3	Output by Prover9: A valid propositional argument.	351
8.1.4	Output by Prover 9: A valid formula.	353
8.1.5	Output by Mace4: A counter-model.	353
8.1.6	A refutation-failure tree.	355
8.1.7	Input in Prover9-Mace4: A FO theory.	356

List of Figures

8.1.8	Output by Prover9.	357
8.1.9	Output by Prover9.	358
8.1.10	Schubert’s steamroller in natural language.	359
8.1.11	Schubert’s steamroller in FOL.	360
8.1.12	Proof of Schubert’s steamroller by Prover9.	361
8.1.13	Hyper-resolution of $\Xi = (\mathcal{C}_3; \mathcal{C}_1, \mathcal{C}_2)$	370
8.1.14	Theory of distributive lattices and commutativity of meet: Input in Prover9-Mace4.	373
8.1.15	Proof by Prover9 of the commutativity of meet in a dis- tributive lattice.	374
8.1.16	A linear-resolution refutation.	375
8.1.17	Theory of commutative groups: Input in Prover9-Mace4.	380
8.1.18	Output by Prover9.	381
8.2.1	Analytic tableaux expansion rules: $\alpha\beta$ -classification.	386
8.2.2	A propositional tableau proof.	388
8.2.3	Analytic tableaux expansion rules: $\gamma\delta$ -classification.	392
8.2.4	A FO tableau proof without unification.	395
8.2.5	A FO tableau with unification.	398
9.1.1	The abstract interpreter Ψ with input (II, G) operating with ground reductions.	415
9.1.2	A LI-resolution proof on a LP program.	417
9.1.3	A LI-resolution proof tree.	418
9.1.4	A SLD-resolution proof.	420
9.1.5	A complete SLD-proof tree for a Prolog program.	421
9.1.6	SWI-Prolog answering a query and outputting traces for some “true” instantiations.	423
9.1.7	SWI-Prolog traces of a “true” and a “false” instantiation.	424
9.2.1	A SLD-proof tree for a Prolog program with !.	442
9.3.1	Table for <i>BIRD (SPECIES, NAME)</i>	453
9.3.2	The EDB <i>Avian_Center_EDB</i>	459
9.3.3	An instance of the Datalog database <i>Avian_Center_DDB</i> with respect to the program <i>Avian_Sick_Prog</i>	465
9.3.4	$Cn(Avian_Sick_Prog \cup E_{Avian_Center_DDB})$	466
9.3.5	A Datalog proof tree.	474
9.3.6	Datalog definite program <i>Avian_center_Quarantine</i>	476
9.3.7	A SLD-resolution proof of a Datalog query.	477
9.3.8	Dependency graph $\vec{\mathfrak{G}}_{\Pi_1^\neg}$ of the Datalog $^\neg$ program Π_1^\neg	483
9.3.9	Dependency graph of a non-stratifiable program.	487

Preface to the first edition

It is often the case that computer science is considered merely a branch of mathematics. This (still) often motivates the belief that logic is required for computer science just because it is required for mathematics, namely for proofs. However, logic in computing goes well beyond the context of mathematical proof, being present today in fields such as artificial intelligence and cognitive science, and having significant engineering and industrial applications. This impressive plethora of computational applications of *logic* could not be possible without a large variety of *logics*, which for our purposes can be elegantly—i.e. by means of the English connector *and*—segregated in two major classes: *classical logic(s)* and *non-classical logics*.

Yet another, but perhaps not so elegant, segregation must be contemplated when speaking of computing today: *classical computing* or *non-classical computing*. While in the latter kind one can include a large variety of computation models and computers (e.g., quantum computers, artificial neural networks, evolutionary computing), we shall consider *classical computing* to be the processing of information carried out by the von Neumann, or industrial-scale digital computer, which has as a major theoretical foundation the Turing computing paradigm. This paradigm, concretized in the Turing machine, sees computation as a spatial-temporal discrete business over symbols that can best be carried out in binary code. While this paradigm does not take into account the resources available for computation, the von Neumann computer is in fact constrained by physical—i.e. spatial and temporal—resources, which means that classical computing has more or less clearly established limitations.

When logic, whether classical or non-classical, is applied in computing, either classical or non-classical, we speak of *computational logic*. This is an important label in at least two senses. Firstly, it captures the fact that there is a subfield of formal logic that can be applied in a computational setting. This subfield might be obtained by imposing restrictions (for example, on the sets of operators), but also by extensions or just plain variations. Secondly, it helps us to distinguish clearly between computation carried out with a *logical language* from computation carried out with *other* formal languages. In effect, while the latter

Preface to the first edition

typically is concerned with preserving the legality of symbol strings (legal strings are processed into further legal strings), the former often aims at *truth-preservation*. Say that we have a theory and wish to know whether some assertion follows logically from it, i.e. belongs to it, or is true in it. The deduction theorem allows us to express this *logical following* in a single symbol string, known as a logical formula, and our question is notoriously best concretized in the *validity* and *satisfiability problems*, which ask whether a logical formula is always true, or is true in some interpretation, respectively. When these problems—in particular the latter—are posed in a computational context, we accordingly speak of *deductive computation*. When the computational solution is to be found by means of classical computing, we then speak of *classical deductive computing*.

In this book we elaborate on *classical deductive computing with classical logic*, and we do so without a specific regard to the field of application. Our foci are first and foremost two main subjects in which classical deductive computing with classical logic has a prominent role: *automated theorem proving* and *logic programming*.

This is thus a book on *applied logic*. Furthermore, this is a book on *applied mathematical logic*. We take here the label *mathematical logic* as synonymous with *formal logic*, and this in a very narrow sense: formal logic is logic whose foundations lie in mathematical objects and structures. Although these mathematical foundations may be inconspicuous at the object-language level, at the metalanguage level they do become more conspicuous or even explicit. Interestingly enough—though not surprising anymore—the mathematical structures and objects usually required in mathematical logic are precisely those needed for classical deductive computing; we talk here of lattices, graphs, trees, etc., all known as *discrete structures and objects*. This accounts for a whole chapter (Chapter 1) dedicated to the topics of discrete mathematics required for a satisfactory grasping of the material in this book. More specifically, we restrictively provide the mathematical notions that are foundational for both the theory of classical computing and classical deduction. Chapter 1 constitutes Part II of this volume, Part I being the Introduction.

Were this book on formal logic alone, there would be no need for a chapter on the *theory of computing*. Although logical languages are first and foremost formal languages, outside a computational context no issues of computability or complexity arise—certainly not in the usual treatment of logic for philosophy courses, but not even in pure mathematical logic textbooks. These issues arise when we need to compute with logical languages (e.g., Turing-completeness of programming lan-

guages). Because these issues arise here, we need to approach Turing machines, which, in turn, require the fundamentals of formal languages and models of computation, in order to be satisfactorily understood. We thus provide the basics of the *general* theory of classical computing, which includes the study of formal languages and grammars, models of computation, and computability theory. As a matter of fact, we provide more than the basics, doing so in the belief that such knowledge often comes in handy for anyone interested in computational logic. This material constitutes Chapter 2, which is Part III.

This book is one—the first—of two volumes addressing the topic of classical deductive computing. In it we focus on computing with classical logic. Although new technologies have opened a path that led to a proliferation of *new* logics, the so-called non-classical logics, classical logic remains as the standard logical system which the other, newer, systems extend or from which they diverge. This would be reason enough to justify this volume, but the fact is that, despite the many technological advances witnessed in the last decades, classical logic is still the logical system of choice for many technological applications requiring what in this book we call deductive computation.

Although the literature on classical logic is prolific, with many good introductions to the subject, with self-containment in view we provide a whole chapter (Chapter 4) on *classical logic*. This follows a comprehensive discussion on *formal logic*, *deduction*, and *deductive computation* carried out in Chapter 3, in which such fundamental notions as logical language, from the viewpoints of both form and meaning, and logical consequence, in relation to inference and deductive systems, as well as to computation, are thoroughly discussed.

The decision problem in computational logic is overwhelmingly tackled by checking for (un)satisfiability, namely by means of the so-called SAT testers or solvers. However, we thought that a working knowledge of *classical validity testing methods* is also required. These—the *classical calculi*—we present in Chapter 5, which is followed, in Chapter 6, by the different *semantics* that provide a foundation for *meaning in classical logic*.

Chapters 3 to 6, constituting Part IV of this book, comprise our discussion of classical deduction and classical logic.

In Part V, we begin by elaborating on the (*classical*) *satisfiability problem*, already introduced in Chapter 2, and by providing the means to computerize classical logic with a view to finding computational solutions to this problem. This *satisfiability testing* is extensively discussed in the remaining Sections of this Chapter 7. We then proceed with extensive treatments of the aforementioned main fields of computational

Preface to the first edition

logic, to wit, automated theorem proving (Chapter 8) and logic programming (Chapter 9). With respect to the former, we give an equal weight to resolution and analytic tableaux. This is uncommon, as the resolution calculus has all but obliterated the analytic tableaux calculus in the context of automated theorem proving, but we think this obliteration is not justified and hope to contribute to the reassessment of the pay-offs of further automating the analytic tableaux calculus. Precisely due to this imbalance our treatment of this calculus is not as comprehensive as our elaboration on resolution. As far as logic programming is concerned, we naturally focus on Prolog, as this is the major (family of) language(s) in this programming paradigm. It is our belief that by mastering the essential aspects of Prolog related to its deductive capabilities, as well as the general theory of logic programming, the reader will be well equipped to tackle most tasks involving this programming paradigm, as well as other (sub-)languages thereof, such as Datalog and Answer Set Programming.

We restrict our elaboration on classical computing to first-order predicate logic, which is known to be adequate (i.e. sound and complete) and as such provides us with a reliable means for classical deductive computation. This by no means entails that we disfavor higher-order logics, but we leave their inclusion in this text to possible future editions thereof.

As said above, this is the first of two volumes. Born in the late 1960s / early 1970s, computational logic has quickly grown to have many sub-fields or subjects; many, indeed (see Introduction). Clearly, this proliferation cannot be covered by a single volume, and we decided to divide the material we find essential in two volumes, the main segregation between both being that we dedicate this (first) volume to computing with classical logic, and we shall elaborate on computation with non-classical logics in a second volume. This segregation is justified not only by the fact that classical and non-classical logics have very different computational assumptions and applications, but also by the sheer quantity of topics that need to be addressed; a single book would certainly be too voluminous and readers may be interested in only one of these, classical or non-classical logics.

An advantage of this project over other works in the field is the breadth of its covering: the reader has in it far more content on computational logic than is usually the case in a single monograph or textbook. This, like any advantage, comes at a price, though: depth had to be relinquished. This is, however, remediated by bibliographical references to works of a more limited breadth but with greater depth of treatment. Moreover, this work contains a large selection of exercises on all the approached topics. Having in mind both that most specialized mono-

graphs and handbooks lack any exercises and the large variety of topics here approached, this is indeed yet another advantage, at least for the reader of a more practical persuasion. In our selection of exercises we included novel material (e.g., theorems not given in the main text), so that the reader is expected also to approach problems in computational logic in a creative way. Exercises asking the reader to reflect on some statements or passages, as well as to engage in research, are also included. These latter exercises are meant to complement the main text with some topics that, while not being secondary, would require some extended discussion, making of this a much larger volume.

Some final remarks: Some of the material in this volume draws on two books of ours also published in College Publications, to wit, Augusto (2020b, c). This material either is as was first published, or has been submitted to some, often substantial, revisions and extensions. As was or revised/extended, it is mostly to be found in Chapters 1, 7, and 8, as well as in all Chapters of Part IV, though not in all Sections thereof. Chapters 2 and 9, as well as many Sections in Part IV (e.g., Sections 3.1-3), are completely novel, drawing only from folklore or from works by other authors. These are orthodoxly cited and indicated in the bibliographical references, but not always did we see it necessary to do so, especially with respect to material that has to some extent already acquired the character of mathematical or logical folklore.

Being a book on computational logic, this is, as said,—also—a book on mathematical logic. This explains the usual distinction in the main text of statements into definitions (abbreviated **Def.**), propositions (**Prop.**), and the odd undistinguished paragraph that for ends of internal reference is referred to as “§”; these are all given a number indicating the Section (two digits separated by a dot) and the order in the Section. For example, **2.1.3 (Def.)** indicates Definition 3 in Section 2.1. Theorems, as well as their companion lemmas and corollaries, are numbered in the same way but separately from the other numbered statements, and the same holds for examples. Exercises are numbered according to not only Section, but also Subsection.

It is usual to provide the reader with a schematic guide for the reading of a book in the fields that are our foci. With this in mind, but not wishing to direct the reader more than the Table of Contents already is expected to do, we think that in order for the lay reader to have a *minimal* satisfactory grasping of classical deductive computing with classical logic the following topics are essential: The system of classical logic CL and the logic **CL** (Chapter 4), Herbrand semantics (concentrated in Sections 6.2 and 7.3.3), and Sections 7.1-2 for the satisfiability problem and for the necessary means to make logical formulae of CL

Preface to the first edition

amenable to computation. These are *sine-qua-non* requirements for a good understanding of automated theorem proving (Chapter 8) or logic programming (Chapter 9), or both. The novice reader wishing to gain a full grasp of our main topic cannot eschew the reading of the whole volume. It should be remarked, however, that some Chapters are self-standing in the sense that they can be used independently from the rest of the volume. This is particularly true of Chapter 2, which is largely conceived as a condensed treatment—with the usual selection of exercises—of the theory of classical computing, and thus can be of use for readers whose interest might fall exclusively on this topic.

For reasons to do with time, we do not include solutions to any of the exercises in this edition, but sooner or later they are expected to be provided, either online or in later editions. Readers wishing to contribute with original solutions to problems other than the most basic ones (e.g., proofs of theorems) are welcome to contact me for this end.

My thanks go to Dov M. Gabbay for including this work in this excellent series of College Publications, and to Jane Spurr for her usual impeccable assistance in the publication process.

Madrid, June 2018

Luis M. S. Augusto

Preface to the second edition

The first edition of the present work was rather hastily completed for many reasons. This hastiness contributed to addenda and errata lists longer than I feel comfortable with, as well as to the omission of some contents that I consider important in a comprehensive introduction to the large field of classical deductive computing with classical logic. Thus, this second edition improves on the first by both eliminating (hopefully most) addenda and errata, and including the mentioned contents. These are largely constituted by Datalog, on which I elaborate at length in a wholly new chapter (Chapter 9.3) for mainly two reasons: Firstly, Datalog has an intrinsic interest from the viewpoint of databases, thus expanding on the applications of logic programming; secondly, it provides an important illustration of the equation $Algorithm = Logic$ in computational logic, to be contrasted with the case of Prolog, which concretizes the equation $Algorithm = Logic + Control$. On a more personal level, Datalog is a highly rewarding topic to research into; more specifically, how such a frugal logical language as Datalog can call for impressively complex formal semantics promises to keep researchers busy for a long time to come.

A few more exercises, in particular exercises aiming at connecting Part III and Parts IV-V, were added in this edition. Further minor improvements were made by redrawing some of the figures and by making minor changes to the main text.

Madrid, January 2020

Luis M. S. Augusto

Note to the second printing

The long Covid-19 lockdown in Madrid allowed for a thorough reviewing of this second edition. Thus, although it was published in January, by early September a printing with some revisions is now made available. This corrects identified mistakes and omissions, has improved figures and a more uniform notation, and introduces a few notions that were either missing or not adequately defined (e.g., *trivial quantification*, *free for*).

Preface to the second edition

Most of the latter changes were made in Sections 3.1.1-2 and 4.1.2. The Turing machine in Figure 2.2.16 was replaced by a more complex one. Finally, this year I succeeded in completing second editions of three of my books, to wit, Augusto (2020a-c), and the bibliographical references were updated accordingly.

Madrid, September 2020

Luis M. S. Augusto

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Index

Index

A

Abstract interpreter, 411
Adequateness of a logical system, 225
Adequateness of a program, 411
Adequateness of a query system, 402
Algorithm, 11
Algorithm, Analytic tableaux, 383
Algorithm, CYK, 165
Algorithm, DPLL, 339
Algorithm, Robinson's, 251
Algorithm, Tseitin transformation, 332
Analytic tableaux, 383
A-ordering, 365
Argument, 188
Assumption, 281
Assumption, Closed-world (CWA), 427
Assumption, Complete database (CDB), 427
Assumption, Completion, 454
Assumption, Domain-closure, 454
Assumption, Unique-name, 454
Automated theorem proving (ATP), 347
Automaton, Finite, 86
Automaton, Linear-bounded (LBA), 131
Automaton, Pushdown (PDA), 101
Axiom, 214

Axiom system, 280
Axiom, Logical, 228
Axiom, Non-logical or proper, 228
Axioms, Blum, 150
Axioms, Particularization, 454

B

Backtracking, 419
Backus-Naur form, 176
Big-O notation, 152
Bivalence, 7
Boolean algebra, 31
Boolean expression, 310
Boolean function, 197

C

Chomsky hierarchy, 79
Chomsky hierarchy, Extended, 136
Church-Turing Thesis, 138
Clark completion, 428
Clark formula, 428
Clause, 326
Clause, Definite, 326
Clause, Dual-Horn, 326
Clause, General, 430
Clause, Horn, 326
Closure operation, 211
Closure system, 208
Closure, Existential, 177
Closure, Universal, 177
Compactness, 211
Compactness of propositional logic, 341

Index

- Completeness, 224
- Completeness theorem, 267
- Complexity classes, 153
- Complexity, Combined, 243
- Complexity, Computational, 153
- Complexity, Data, 243
- Complexity, Expression, 243
- Complexity, Space, 150
- Complexity, Time, 151
- Computation, 3
- Computation (for a machine), 82
- Computation, Deductive, 13
- Computation, Symbolic, 3
- Computation, Truth-preserving, 13
- Computational yield, 402
- Computing, Assumptions of classical, 12
- Computing, Classical, 10
- Configuration, 82
- Consequence operation, 207
- Consequence operator, Immediate, 468
- Consequence relation, 207
- Consistency, 216
- Constructive dilemma (CD), 192
- Contingency, 220
- Contradiction, 220
- Contraposition, Law of, 262
- Cook-Karp Thesis, 156
- Cook-Levin Theorem, 160
- Counter-model, 219
- Counter-proof, 215
- Cut operator, 440
- D**
- Database, Datalog, 458
- Database, Deductive (DDB), 457
- Database, Disjunctive deductive (DDDB), 472
- Database, Extended disjunctive deductive (EDDDB), 462
- Database, Extensional (EDB), 454
- Database, Indefinite deductive (ID-DDB), 462
- Database, Intensional (IDB), 455
- Database, Relational, 452
- Database, Temporal deductive, 462
- Datalog \neg , Semi-positive, 481
- De Morgan's laws (DM), 262
- Decision procedure, 232
- Deduction theorem (DT), 223
- Deduction, Computational, 232
- Deduction, Resolution, 349
- Deduction-Detachment theorem (DDT), 227
- Deductive system, 210
- Denotation, 296
- Derivability, 214
- Derivation (in a grammar), 57
- Destructive dilemma (DD), 192
- Determinacy (of a programming system), 401
- Determinism (of a programming system), 401
- Diagonalization method, 21
- Distributive laws, 331
- Domain of discourse, 200
- DPDA (Deterministic pushdown automaton), 109
- DPLL procedure, 337
- E**
- Equality, 273
- Equality substitution, 377
- Equisatisfiability, 329
- Evaluation (Datalog), 478
- Evaluation, Bottom-up Datalog, 478
- Evaluation, Top-down Datalog, 478
- Ex contradictione quodlibet (ECQ), 192

- Ex falso quodlibet (EFQ), 256
 Excluded middle, Principle of (PEM), 257
 Existential distribution, 178
 Explosion, Principle of, 196
 Extensionality, Principle of, 197
- F**
 Fact (in LP), 406
 Fail operator, 443
 Finite satisfiability, 234
 Finite transducer, 96
 Finite-model property (FMP), 239
 Finite-state machine, 98
 Finite-state recognizer (FSR), 81
 Finite-state recognizer, Nondeterministic (NDFSR), 86
 Fixed point, 469
 Fixed point, Least, 469
 Free for, 249
 Function (symbol), 174
 Function, Extended transition, 82
 Function, Transition, 81
 Functional completeness, 198
- G**
 Generalization rule (GEN), 281
 Goal (in LP), 406
 Goal clause, 416
 Goal clause, Empty, 416
 Grammar, Ambiguous, 67
 Grammar, Context-free (CFG); Type-2, 64
 Grammar, Context-sensitive (CSG); Type-1, 65
 Grammar, Formal, 52
 Grammar, LR(k), 111
 Grammar, Regular; Type-3, 63
 Grammar, Unrestricted (UG); Type-0, 76
 Graph, Dependency, 482
 Ground expression, 171
 Ground extension, 235
 Ground instance, 249
 Ground substitution, 249
- H**
 Herbrand base, 301
 Herbrand instance (H-instance), 301
 Herbrand interpretation (H-interpretation), 301
 Herbrand model (H-model), 302
 Herbrand model, Least, 425
 Herbrand model, Minimal, 425
 Herbrand satisfiability (H-satisfiability), 302
 Herbrand universe, 300
 Herbrand's Theorem, 341
 Hilbert's Tenth Problem, 146
 Hintikka set, 390
 Hintikka's Lemma, 390
 Hypothetical syllogism (HS), 192
- I**
 Identity of indiscernibles (IdI), 273
 Identity, Law of, 262
 Induction, Mathematical, 46
 Induction, Structural, 46
 Inference, 210
 Inference operation, 210
 Inference relation, 210
 Inference rule, 213
 Inference system, 210
 Instance, Database, 463
 Interpretation, 200
 Interpretation of a LP program, Declarative, 406
 Interpretation of a LP program, Procedural, 406
 Invalidity, 219
- K**
 Kleene's Least Fixed-Point Theorem,

Index

- 472
- Kleene's Theorem for regular languages, 93
- Knaster-Tarski Theorem, 472
- L**
- Language, Context-free (CFL), 64
- Language, Context-sensitive (CSL), 65
- Language, Decidable, 140
- Language, First-order (FO), 175
- Language, Formal, 52
- Language, Logical, 170
- Language, Object, 169
- Language, Propositional, 175
- Language, Recursive, 135
- Language, Recursively enumerable (REL), 77
- Language, Regular, 61
- Language, Relational, 452
- Leibniz's law (LL), 273
- Lifting lemma, 362
- Lindenbaum's Theorem, 229
- Lindenbaum-Tarski algebra, 307
- Logic (of a logical system), The, 217, 221
- Logic programming (LP), 399
- Logic, Classical, 7
- Logic, Classical first-order (CFOL), 248
- Logic, Classical propositional (CPL), 248
- Logic, Computational, 8
- Logic, Deductive, 6
- Logic, Formal, 5
- Logic, Informal, 5
- Logic, Mathematical, 4
- Logic, Truth-preserving, 6
- Logical consequence, 206
- Logical equivalence, 199, 202
- Logical system, 207
- Logics, Non-classical, 7
- Löwenheim-Skolem Theorem, 240
- M**
- Matching, 473
- Mealy machine, 96
- Meaning, 194
- Meaning of a program, 411
- Meaning of a program, Intended, 412
- Meaning, Principle of compositionality of, 197
- Metalanguage, 169
- Meta-variable, 434
- Model, 219
- Model, Computer, 82
- Model, Herbrand (H-model), 302
- Model, Herbrand least, 425
- Model, Supported, 472
- Modus ponens (MP), 192
- Modus ponens, Universal (UMP), 411
- Modus tollendo ponens (TP), 192
- Modus tollens (MT), 192
- Monotonicity, 210
- Moore machine, 96
- Myhill-Nerode Theorem, 93
- N**
- Natural deduction calculus, 283
- Negation by failure (NBF), 427
- Negation distribution, 178
- Negation law, Double (DN), 257
- Negation, Cut-failure, 446
- Non-contradiction, Principle of (PNC), 257
- Non-monotonicity, 427
- Normal form, Chomsky, 64
- Normal form, Conjunctive (CNF), 329
- Normal form, Disjunctive (DNF), 330

- Normal form, Greibach, 107
 Normal form, Negation (NNF), 327
 Normal form, Prenex (PNF), 327
 Normal form, Skolem (SNF), 329
- O**
 Ogden's Lemma, 74
 One-literal rule, 348
- P**
 $P =? NP$, 155
 Parameter, 283
 Paramodulation, 377
 Paramodulation, Ordered, 379
 Paramodulation, Simultaneous, 379
 Post's Correspondence Problem, 146
 Predicate (symbol), 174
 Predicate, Built-in, 434
 Prefix classes, 239
 Problem for 2-CNF formulae, The satisfiability (2-SAT), 335
 Problem for 3-CNF formulae, The satisfiability (3-SAT), 335
 Problem for DNF formulae, The satisfiability (DNF-SAT), 336
 Problem for dual-Horn formulae, The satisfiability (DUAL-HORN-SAT), 337
 Problem for Horn formulae, The satisfiability (HORN-SAT), 335
 Problem for k-CNF formulae, The satisfiability (k-SAT), 335
 Problem for quantified Boolean formulae, The satisfiability (QBF-SAT), 336
 Problem, Computational, 149
 Problem, Decision, 138
 Problem, Function, 149
 Problem, Hilbert's Tenth, 146
 Problem, Logical (LOGP), 230
 Problem, The Acceptance (ACPT), 141
 Problem, The Boolean satisfiability (SAT), 319
 Problem, The Busy Beaver, 146
 Problem, The Circuit Satisfiability (CIRCUIT-SAT), 159
 Problem, The Clique (CLIQUE), 160
 Problem, The Graph Colorability, 159
 Problem, The Graph Isomorphism, 160
 Problem, The Halting (HALT), 141
 Problem, The Hamiltonian Cycle (HAM-CYCLE), 160
 Problem, The Hamiltonian Path (HAMPATH), 149
 Problem, The maximum satisfiability (MAX-SAT), 337
 Problem, The Null-Value, 462
 Problem, The Relative Primes, 158
 Problem, The satisfiability (SAT), 318
 Problem, The Shortest Path, 158
 Problem, The State-Entry (STENTRY), 143
 Problem, The Subgraph Isomorphism, 159
 Problem, The Subset-Sum (SUBSET-SUM), 160
 Problem, The Traveling Salesman (TSP), 160
 Problem, The validity (VAL), 230
 Problem, The Vertex Cover (VERTEX-COVER), 159
 Production rule, 56
 Program clause, 415

Index

- Program, Datalog, 458
- Program, General, 431
- Program, Logic, 408
- Program, Prolog, 408
- Programming system, 401
- Prolog, Pure, 405
- Prolog, Real, 433
- Proof, 214
- Proof calculus, 214
- Proof system, 214
- Provability, 214
- Pumping lemma for CFLs, 70
- Pumping lemma for regular languages, 62, 95

- Q**
- Quantification, 177
- Quantification, Trivial, 177
- Quantifier (symbol), 175
- Quantifier axioms, 281
- Quantifier duality, 202
- Quantifier reversal, 178
- Query, 400
- Query system, 400
- Query, Meta-safe, 449
- Query, Restricted Prolog, 449

- R**
- Recursion, 425
- Reducibility, 142
- Reducibility, Polynomial-time, 158
- Reductio ad absurdum (RA), 262
- Reduction (in LP), 414
- Reduction, Ground, 414
- Reduction, LR(k)-grammar, 111
- Refutation, 215
- Refutation completeness, 416
- Reply, Conjunctive, 404
- Reply, Consequentially strongest correct, 403
- Reply, Most general, 404
- Reply, Provably correct, 401

- Representation theorem, 308
- Resolution principle for FOL, 354
- Resolution principle for propositional logic, 348
- Resolution refinement, 364
- Resolution with rule NF, SLD (SLDNF), 431
- Resolution, Binary, 354
- Resolution, Hyper-, 369
- Resolution, LD, 376
- Resolution, LI, 375
- Resolution, Linear, 375
- Resolution, Macro-, 369
- Resolution, RUE, 382
- Resolution, Semantic, 370
- Resolution, SLD, 376
- Resolution, Unit-resulting, 356
- Rice's Theorem, 146
- Rule (in LP), 406

- S**
- Satisfiability, 218
- Savitch's Theorem, 155
- Schema (of a Datalog program), 460
- Schema, Extensional, 460
- Schema, Intensional, 460
- Search, Breadth-first, 422
- Search, Depth-first, 419
- Semantical correlate, 197
- Semantics, 219
- Semantics, 3-valued, 492
- Semantics, Fixed-point, 468
- Semantics, Inflationary, 492
- Semantics, Least-Herbrand-model, 470
- Semantics, Perfect-model, 492
- Semantics, Stratified, 481
- Semantics, Well-founded, 492
- Semantics, Stable-model, 492
- Semi-decidability, 140
- Sentential form, 57

Sequent calculus, 288
Skolem constant, 329
Skolem function, 329
Soundness, 224
State diagram, 83
Statement (in LP), 406
Stratification, 484
Substitution, 249
Substitution principle (SubP), 273
Substitution rule (SUB), 215
Syntax, 53, 169
Syntax, Ambivalent, 407

T

Tableau proof, 383
Tarski-style conditions, 255
Tautology, 220
Theorem, 214
Theory, 228
Theory, Scapegoat, 267
Trace, 414
Tractability, 156
Transition relation, 86
Transition table, 85
Tree, Derivation, 66
Tree, Formula, 180
Tree, Parse, 66
Tree, Proof, 415
Tree, Refutation, 349
Tree, Semantic, 342
Tree, SLD-resolution, 419
Truth function, 195
Truth table, 195
Truth value, 195
Truth-functionality, 7
Truth-preservation, 258
Turing machine, 121
Turing machine, Non-deterministic,
125
Turing machine, Total, 135
Turing machine, Universal, 126
Turing paradigm, 12

Turing-completeness, 10
Turing-decidability, 138
Turing-recognizability, 147
Turing-reducibility, 143

U

Ultrafilter theorem, 312
Unicity of decomposition, 172
Unification, 250
Unification problem, 251
Unifier, Most general (MGU), 250
Unit deletion, 356

V

Validity, 219
Valuation, 195