Logical Consequences

Theory and Applications:
An Introduction

Second Edition

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Preface to the 1st edition

Wishing to anchor my research more and more in logic, pure or applied (where *or* is to be read in the logical, inclusive, sense), I aimed to increase substantially my knowledge of (classes of) logics. For both the pure and applied aspects I had in mind, tackling the central notion of logical consequence appeared as the best way to achieve my objective.

My work in this topic benefited from a short stay in early 2015 as a visiting researcher at the University of Barcelona, where, at the libraries of the Faculties of Mathematics and of Philosophy, I was able to collect much of the material that allowed me to take the turnstiles by the horns.

Melvin Fitting, Peter Schotch, and Yde Venema read parts of the manuscript and their observations helped me to improve it considerably. My sincere thanks to them.

I wish also to express my thanks to Dov M. Gabbay for including this book in this excellent series of College Publications, and to Jane Spurr for impeccable assistance during the publication process.

Madrid, February 2017

Luis M. S. Augusto

Preface to the 2nd edition

The present second edition includes some minor and a few not so minor changes with respect to the first edition. I begin with the minor changes: Besides correcting identified addenda and errata, the present second edition has improved figures and a more uniform notation; the main text was revised in some places aiming first and foremost at greater clarity.

Now, the not so minor changes:

With the aim of promoting a better understanding of the mathematical interpretation of logical consequence, order relations and associated structures were further elaborated on, namely up to the notions of ultrafilter and superideal for lattices (Section 1.2), and there is now an entirely novel Section (1.4) on closure and kernel operators.

Another important innovation is the Appendix on first-order logic, as the 2017 edition took it–unwittingly, but nonetheless wrongly–for granted that readers were well acquainted with the first-order language of (classical) logic. Hence the decision to provide the basics of first-order quantified formulae, namely with a view to exploring the (limitations in) expressive power of first-order logic via its standard language. This is done in particular by exploring the axiomatization of set theory known as ZFC, per se and in its relations to classical first-order logic. Nevertheless, this book was not written with the wholly uninitiated in mind, and only minimal discussions and examples are given of basic logic topics, reminders rather than pedagogical elaborations. For this reason, I decided not to include the material on first-order formulae and axiomatizations in the main text, opting for an Appendix instead.

Yet another novelty is the Index, which was wholly revised and is now very comprehensive.

Renewed thanks to College Publications are in order, especially for the readiness to publish the present 2nd edition.

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This Index is a hybrid of an *Index rerum* and an *Index nominum*. With respect to the former, pages for concepts are given only for their definitional occurrence (unless there are definitional variations). Frequently recurrent abbreviations (e.g., MP) are also listed as entries. As for the latter, pages for occurrences of names of authors of historical significance are exhaustively given.

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