

# Languages, Machines, and Classical Computation

Luis M. Augusto

© Individual author and College Publications 2019. All rights reserved.

ISBN 978-1-84890-300-5

College Publications  
Scientific Director: Dov Gabbay  
Managing Director: Jane Spurr

<http://www.collegepublications.co.uk>

Cover produced by Laraine Welch

---

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form, or by any means, electronic, mechanical, photocopying, recording or otherwise without prior permission, in writing, from the publisher.

# Contents

Preface	xii
<b>I Introduction</b>	<b>1</b>
<b>1 Classical computation: Turing, von Neumann, and Chomsky</b>	<b>2</b>
1.1 Computers, information, and computations . . . . .	2
1.2 Computational problems, algorithms, and decisions . . . . .	3
1.3 The Turing-von Neumann paradigm . . . . .	4
1.4 Models of classical computation: Automata . . . . .	8
1.5 The Chomsky hierarchy . . . . .	9
<b>II Preliminaries and notation</b>	<b>14</b>
<b>2 Mathematical notions</b>	<b>15</b>
2.1 Basic notions . . . . .	15
2.1.1 Sets, relations, functions, and operations . . . . .	16
2.1.2 Binary relations and ordered sets . . . . .	24
2.2 Discrete structures . . . . .	31
2.2.1 Algebraic structures and operations . . . . .	32
2.2.1.1 Algebras and morphisms . . . . .	32
2.2.1.2 Boolean algebras . . . . .	33
2.2.2 Graphs and trees . . . . .	38
2.3 Proof techniques . . . . .	43
2.3.1 Mathematical and structural induction . . . . .	44
2.3.2 Proof by contradiction . . . . .	45
<b>III Languages, machines, and classical computation</b>	<b>48</b>
<b>3 Formal grammars and languages</b>	<b>49</b>
3.1 Basic notions . . . . .	50
3.1.1 Strings and operations on strings . . . . .	50

## Contents

3.1.2	Formal languages and operations thereon . . . . .	52
3.1.3	Formal grammars . . . . .	55
3.1.3.1	Central notions . . . . .	55
3.1.3.2	Rules, symbols, and grammar cleaning . . . . .	57
3.2	Regular languages . . . . .	65
3.2.1	Regular expressions . . . . .	65
3.2.2	Regular grammars . . . . .	70
3.2.3	Properties of regular languages . . . . .	76
3.2.3.1	Pumping lemma for regular languages . . . . .	76
3.2.3.2	Algebra and linear equations for regular languages . . . . .	77
3.2.3.3	Closure properties of the regular languages . . . . .	79
3.3	Context-free languages . . . . .	84
3.3.1	Context-free grammars . . . . .	84
3.3.1.1	Context-free vs. context-sensitive grammars . . . . .	84
3.3.1.2	Normal forms for CFGs I: Chomsky normal form . . . . .	86
3.3.1.3	Normal forms for CFGs II: Greibach normal form . . . . .	90
3.3.1.4	Derivation, or parse, trees . . . . .	95
3.3.1.5	Ambiguity and inherent ambiguity . . . . .	96
3.3.2	Context-free languages . . . . .	101
3.3.2.1	Pumping lemma for CFLs and Ogden's lemma . . . . .	101
3.3.2.2	Further properties of CFLs . . . . .	104
3.4	Recursively enumerable languages . . . . .	112
3.5	The Chomsky hierarchy (I) . . . . .	118
<b>4</b>	<b>Models of computation</b> . . . . .	<b>121</b>
4.1	Finite-state machines . . . . .	121
4.1.1	Finite automata . . . . .	122
4.1.1.1	Basic aspects of finite automata . . . . .	123
4.1.1.2	Characteristic equations . . . . .	129
4.1.1.3	The pumping lemma for regular languages . . . . .	131
4.1.1.4	The Myhill-Nerode theorem and FA minimization . . . . .	132
4.1.1.5	Deterministic and non-deterministic FAs . . . . .	138
4.1.1.6	Kleene's theorem and the properties of $REG$ . . . . .	145
4.1.2	Finite transducers . . . . .	149
4.1.2.1	Moore and Mealy machines . . . . .	149

*Contents*

4.1.2.2	Equivalence of finite transducers . . . . .	154
4.1.2.3	Minimizing finite transducers . . . . .	156
4.1.2.4	Conversion of transducers into acceptors	161
4.2	Pushdown automata . . . . .	170
4.2.1	Basic aspects of PDAs . . . . .	170
4.2.2	Two acceptance modes by PDAs: Final state and empty stack . . . . .	174
4.2.3	Equivalence between CFLs and PDAs . . . . .	175
4.2.4	CFLs accepted by deterministic PDAs . . . . .	180
4.2.4.1	Deterministic PDAs . . . . .	180
4.2.4.2	LR( $k$ ) grammars . . . . .	182
4.3	Turing machines . . . . .	196
4.3.1	Basic aspects of Turing machines . . . . .	196
4.3.2	Turing machines computing functions . . . . .	199
4.3.3	Turing machines accepting languages . . . . .	201
4.3.3.1	Turing machines and unrestricted gram- mars . . . . .	201
4.3.3.2	Linear-bounded automata: Special Tur- ing machines for CSGs . . . . .	205
4.3.4	The universal Turing machine . . . . .	206
4.4	The Chomsky hierarchy (II) . . . . .	212
<b>5</b>	<b>Computability and complexity</b>	<b>216</b>
5.1	The decision problem and Turing-decidability . . . . .	216
5.2	Undecidable problems and Turing-reducibility . . . . .	219
5.3	The Chomsky hierarchy (III) . . . . .	225
5.4	Computational complexity . . . . .	227
5.4.1	Computational problems . . . . .	227
5.4.2	The Blum axioms and complexity measures . . . . .	228
5.4.3	Complexity classes . . . . .	232
5.4.4	The Cook-Levin theorem and polynomial-time re- ducibility . . . . .	237
5.5	The Chomsky hierarchy (IV) . . . . .	249
	<b>Bibliography</b>	<b>251</b>
	<b>Bibliographical references</b>	<b>252</b>
	<b>Index</b>	<b>256</b>

## List of Figures

1.5.1	The basic postulate of the Chomsky hierarchy. . . . .	11
1.5.2	Two derivation trees. . . . .	13
2.1.1	A partially ordered set. . . . .	27
2.1.2	Hasse diagram of a poset. . . . .	31
2.2.1	A simple graph with five vertices and seven edges. . . . .	40
3.2.1	A labeled digraph $\vec{\mathfrak{G}}(r)$ for a regular expression $r$ . . . . .	71
3.2.2	A labeled digraph $\vec{\mathfrak{G}}(G)$ corresponding to a left-linear grammar $G$ . . . . .	74
3.2.3	The digraph $\vec{\mathfrak{G}}'(G')$ obtained from $\vec{\mathfrak{G}}(G)$ . . . . .	75
3.3.1	Derivation tree of the string $w = acbabc \in L(G)$ with the corresponding partial derivation trees. . . . .	97
3.3.2	Two leftmost derivations of the string $a + a * a$ . . . . .	98
3.3.3	Parse tree of an unambiguously derived string. . . . .	100
3.3.4	Parse trees for productions (1) $S \rightarrow a$ and (2) $S \rightarrow AB$ . . . . .	102
3.3.5	Parse tree for $z = uv^iwx^iy$ . . . . .	103
3.4.1	A derivation graph of the string $bab$ generated by a UG. . . . .	115
4.1.1	Computer model of a FA. . . . .	123
4.1.2	State diagrams of FAs. . . . .	126
4.1.3	A FA with two accepting states and one rejecting state. . . . .	126
4.1.4	A FA for the regular language $L = \{c, ba\}^* \{ac, aab^*\}$ . . . . .	127
4.1.5	A finite automaton $M$ for the pumping lemma. . . . .	132
4.1.6	A FA (1) and its minimal equivalent FA (2). . . . .	137
4.1.7	A N DFA for the language $L = \{001\}^* \{0, 010\}^*$ . . . . .	139
4.1.8	Equivalent N DFAs with and without $\epsilon$ -transitions. . . . .	142
4.1.9	Equivalent N DFA (1) and FA (2). . . . .	146
4.1.10	Schematic diagrams for FAs accepting (i) $L_1 \cup L_2$ , (ii) $L_1L_2$ , and (iii) $(L_1)^*$ . . . . .	148
4.1.11	A FA accepting $L = L_1 \cup L_2$ . . . . .	148
4.1.12	Moore (1) and Mealy (2) machines. . . . .	151
4.1.13	A Mealy machine (1) and its equivalent Moore machine (2). . . . .	157
4.1.14	A Mealy machine (1) and its minimal equivalent (2). . . . .	162
4.1.15	A Moore machine converted into a FA. . . . .	163
4.1.16	Deterministic finite automata. . . . .	166

*List of Figures*

4.1.17	Mealy machines. . . . .	168
4.1.18	A barcode. . . . .	169
4.2.1	Computer model for a PDA. . . . .	171
4.2.2	A PDA $M$ accepting the language $L(M) = \{a^m b^m \mid m \geq 0\}$ . 173	
4.2.3	Proving the equivalence of $L(M) = N(M)$ . . . . .	176
4.2.4	NFA recognizing the viable prefixes for the CFG of Balanced Parentheses. . . . .	187
4.2.5	Pushdown automata. . . . .	192
4.2.6	A PDA accepting $L(M) = \{u \in \Sigma^* \mid u = ww^R\}$ . . . . .	193
4.3.1	Computer model for a Turing machine. . . . .	197
4.3.2	A Turing machine that computes the function $f(m, n) = m + n$ for $m, n \in \mathbb{Z}^+$ . . . . .	200
4.3.3	Turing machine $M_T$ that computes the function $f(m, n) = 2m + 3n$ for $m, n \in \mathbb{Z}^+$ . . . . .	202
4.3.4	Program for Turing machine $M_T$ that computes the function $f(m, n) = 2m + 3n$ for $m, n \in \mathbb{Z}^+$ . . . . .	203
4.3.5	The encodings $\langle M_T \rangle$ and $\langle M_T, z \rangle$ . . . . .	208
4.3.6	A combination of Turing machines. . . . .	210
4.3.7	A Turing machine. . . . .	211
5.2.1	A combination of Turing machines. . . . .	224
5.3.1	The Chomsky hierarchy and beyond: Decidable, Turing-recognizable, and not-Turing-recognizable languages. . . . .	226
5.4.1	The hierarchy of complexity classes with corresponding tractability status. . . . .	236
5.4.2	A tableau for the Turing machine $M$ . . . . .	242
5.4.3	Typical structure of <b>NP</b> -completeness proofs by polynomial-time reductions. . . . .	245

# List of Tables

- 3.5.1 The Chomsky hierarchy. . . . . 119
- 4.4.1 The extended Chomsky hierarchy: Grammars, languages,  
and associated computer models. . . . . 214
- 5.3.1 Decidability (“Yes”) and undecidability (“No”) of some prop-  
erties of interest for the Chomsky hierarchy. . . . . 226
- 5.4.1 Rates of growth of some standard functions. . . . . 233

## List of Algorithms

3.1	Grammar cleaning . . . . .	59
3.2	Left-/Right-linear grammar to right-/left-linear grammar	75
3.3	Chomsky-normal-form Transformation . . . . .	88
3.4	Greibach-normal-form Transformation . . . . .	91
3.5	Language class by grammar type . . . . .	120
4.1	Deterministic FA minimization . . . . .	136
4.2	Subset-construction algorithm . . . . .	144
4.3	Partition refinement for the states of a Mealy machine . .	159
4.4	Mealy machine minimization . . . . .	160

# Preface

Teachers tend to be picky with the material they use in teaching contexts. This may be for personality reasons, but the variety of contexts and students also plays a role in this pickiness. Be it as it may, it often is the case that students end up with teaching material in many formats and from many different sources, creating often a lack of uniformity, both in notation and terminology. Because I am picky for all the reasons above, I typically feel that my teaching task is substantially facilitated and optimized when I have gone to the great lengths of putting all the material for a particular academic subject together in a single manual or textbook. This guarantees not only conceptual and notational uniformity, but also a selection of approaches that I feel work well, or better, for particular topics or problems.

This book is not about discovering the wheel; that is, possibly no novel contents are to be found in it. The objective when writing it was that of “putting together” a textbook on the classical theory of computing. If there is any novel aspect in this textbook, it may well be the fact that I insist on preceding the terms “(theory of) computation” and “(theory of) computing” with the adjective “classical” to collect under the same label the Chomsky hierarchy and the Turing-von Neumann paradigm of computing. The former comprises three closely associated central topics, to wit, formal grammars, formal languages, and models of computation (a.k.a machines, or automata), and the latter gives to these, namely via the Turing machine, measures of the spatial and temporal costs of computation. I say that this collection constitutes *(the)* classical (theory of) computation, because many, often newer, other forms of computing have emerged or become (more) popular since the Turing “revolution,” many of which today may be said to constitute *the* non-classical (theory of) computation. This is, for the initiated, more immediately the field of quantum computing, but other forms of computation such as artificial neural networks and evolutionary computing may be seen as also non-classical versions of computing.

It is arguably possible to produce a textbook on formal languages, grammars, and automata with no emphasis on computing, let alone with any specific computational concerns. One such approach might be with linguists in mind, though contemporary linguistics is not averse to

## Preface

computation. On the extreme pole of this position, formal grammars, languages, and automata are often reduced to *the* theory of computation, namely as it serves the theoretical foundations of the digital computer. Without taking a reductive view, I discuss formal languages and grammars from the viewpoint of computation, and consider the associated automata as models thereof. This said, readers with other foci will find that the computational perspective taken here does not hinder—and may even facilitate—their particular interests and concerns.

The backbone of this book is undoubtedly the Chomsky hierarchy. Although much computing has run in the digital computer since N. Chomsky first conceived it, it still works well for combining the mostly linguistic approach with the computational one. In particular, it keeps reminding us that we are linguistic beings to the point that one of our most interesting creations—the digital computer—is language-based through and through, a feature well-patent in the famous Turing Test, a “test” conceived by the creator of the Turing machine to distinguish a human computer from a non-human one. Indeed, it seems to have been the rationale in Turing (1950) that language is sufficient to distinguish the human from the non-human computer or reasoner. More than anything, it might have been this insistence on the verbal behavior of computers that motivated the can-of-worms idea of AI (artificial intelligence) as ultimately aiming at human-like machines, at least from the viewpoint of intelligence, if not of emotion.

There is no way to go around this and it requires emphasis: (classical) computing is a mathematical subject. Although the presence of automata, of which the most famous is the Turing machine, lends it a flavor of engineering, these are not physical machines nor can they be; they are mathematical objects. To be sure, the digital computer is based on the Turing machine, but this has a feature—an infinite tape—that makes of the former a mere approximation of the latter. The mathematical nature of this subject accounts for the clearly mathematical approach in this book: I distinguish statements into definitions and propositions, and provide proofs (or sketches thereof) to further distinguished—if not distinct—statements, to wit, theorems and their companion lemmas and corollaries. The numbering of such statements finds its utility in internal referencing, if it gives a more high-brow quality to the main text. I reserve the status of theoremhood for statements of higher importance than propositions, but the reader is free to consider (most) propositions in this text as *de-facto* theorems; the fact that proofs are provided (or left as exercises) for propositions supports this view.

This mathematical nature of the subject also justifies the large selection of exercises here provided. Indeed, only few students are gifted

## Preface

with mathematical skills that free them from the arduous and time-consuming practice of doing exercises. On the other hand, some may find this a pleasant activity. Between these fall most mortals, one should think. But the selection of exercises in this book was also guided by the belief that one should be confronted with novel material and problems, in order to develop research, as well as creative, skills.

Still with regard to the mathematical nature of this text, there are throughout it a few algorithms for the computation of specific functions (e.g., computing the Chomsky normal form of a given grammar). I chose not to stick to a single pseudo-code or to a single algorithm format in the belief that different algorithms can be better grasped in distinct ways. Yet another advantage of this might be the familiarity with diverse pseudo-codes and algorithm formats. Importantly, too, no programming language or software plays any role whatsoever in this book. This is so deliberately to keep the subject matter as general as possible, untied to specific implementations or applications.

As said above, the aim for this book is not (re)inventing the wheel. Although classical computing and its theory are in a current state of development, with many a problem as focus of research—notably so the  $\mathbf{P}=?\mathbf{NP}$  problem—, the subject of the theory of classical computing has attained a certain fixed form that is historically justified. In the second half of last century, when this subject emerged, an abundance of textbooks and monographs were published, and a few of these established themselves as standard references in the field. As such, it is only natural that in pedagogical pursuits one should resort to them as sources. This I do with two such classics in particular, to wit, Davis & Weyuker (1983) and Hopcroft & Ullman (1979), the latter of which has evolved into the more undergraduate-friendly Hopcroft, Motwani, & Ullman (2013). A further source is Du & Ko (2001), a thoroughly mathematical approach. Readers can greatly benefit from a direct use of all these referenced works. Texts and manuals on this subject matter directed at undergraduate audiences abound, with many a good one to further assist readers in their academic pursuits. Referencing them all is of course impossible, but interested readers know where to find them. More specific, often more advanced, literature is cited throughout this text in the appropriate places; in particular, I cite the works in which important results (e.g., theorems) were first published.

Lastly, this textbook is a further elaboration on what was originally a chapter in a book of mine first published by College Publications, to wit, Augusto (2018). In this book, a chapter on the theory of computing appeared to be relevant, because issues such as Turing-completeness of logic programming and the complexity of the satisfiability problem

## *Preface*

(a.k.a SAT) required a minimal grasp of, among other topics, the Turing machine. Having resorted to this chapter to teach topics in automata, formal languages, and the classical theory of computation, and having obtained satisfactory results, I decided to expand it to what is now the present textbook. The main guideline for this expansion was the inclusion of topics that were left out in the mentioned chapter for spatial and temporal reasons, but which are essential for a fuller treatment of this subject. Some of these new topics—e.g., characteristic equations of finite automata, grammar cleaning algorithm—may appear quite inessential from an Anglo-Saxon perspective, but my individual work with Spanish students preparing themselves to take exams on the above-mentioned topics made me realize the need to be as encompassing and comprehensive as possible, namely with the large diversity of readers of this subject in mind.

I wish to thank Dov M. Gabbay, the scientific director of College Publications, and Ian Mackie, the editor for the Texts in Computing series, for publishing this book. My thanks go also to Jane Spurr, the managing director, for a smooth publication process.

Madrid, February 2019

Luis M. S. Augusto