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Research Article

A Decision-Making Approach Incorporating TODIM Method and Sine Entropy in q-Rung Picture Fuzzy Set Setting

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In this study, we propose a new approach based on fuzzy TODIM (Portuguese acronym for interactive and multicriteria decision-making) for decision-making problems in uncertain environments. Our method incorporates group utility and individual regret, which are often ignored in traditional multicriteria decision-making (MCDM) methods. To enhance the analysis and application of fuzzy sets in decision-making processes, we introduce novel entropy and distance measures for q-rung picture fuzzy sets. These measures include an entropy measure based on the sine function and a distance measure derived from the Jensen-Shannon divergence. In our methodology, incorporating the sine function into the entropy measure stands out as a distinctive decision, grounded in a profound understanding of the inherent characteristics of fuzzy sets. Utilizing the sine function proves especially advantageous when handling fuzzy sets that exhibit cyclical variations or fluctuations in their membership degrees. We effectively weight the criteria for an improved evaluation by using this new entropy measure. The introduced distance measure finds application in the TODIM approach, allowing the execution of TODIM method steps within a fuzzy environment until the determination of one alternative's dominance over another—an advancement beyond traditional approaches. We apply our enhanced fuzzy TODIM method to a real-life construction project management problem from the literature and compare the results with those in the literature and obtained from other MCDM methods. Our proposed measures are robust, as demonstrated by the sensitivity analysis that varied the weights of group utility and individual regret, with the results visualized in a 3D sensitivity plot. The findings demonstrate the superiority of our method in providing a more comprehensive evaluation of alternatives, making it a useful tool for decision-makers facing complex and uncertain decision-making problems.

1. Introduction

Fuzzy set theory, introduced by Zadeh [1], expands upon classical set theory's characteristic function with a membership function taking values in the closed interval [0,1]. Atanassov [2] later extended this concept to intuitionistic fuzzy sets (IFSs), which incorporate both membership and nonmembership functions. However, fuzzy sets and IFSs are sometimes insufficient for addressing real-life problems. Yager [3] introduced Pythagorean fuzzy sets (PyFSs) to overcome these limitations, characterized by membership and nonmembership functions with the sum of their squares between 0 and 1. Yager [4] also proposed q-rung orthopair fuzzy sets (q-ROFs) for better modeling real-life applications, characterized by membership and nonmembership functions with the q^{th} power between 0 and 1. q-ROFs are generalizations of IFSs and PyFSs, and some main topological properties of q-ROFs were investigated by Türkarslan et al. in [5].

Cường [6] introduced the notion of picture fuzzy sets (PFSs), characterized by membership, neutral, and nonmembership functions with their sum between 0 and 1. PFSs expand upon fuzzy sets and IFSs. Kutlu Gündoğdu and Kahraman [7] and Ashraf et al. [8] later proposed spherical fuzzy sets (SFSs) by generalizing PFSs and PyFSs. Li et al. [9] proposed the notion of q-rung picture fuzzy set (q-RPFS) in 2018, expanding upon PFs, q-ROFSs, and SFSs. All these fuzzy sets provide suitable tools for addressing unexpected

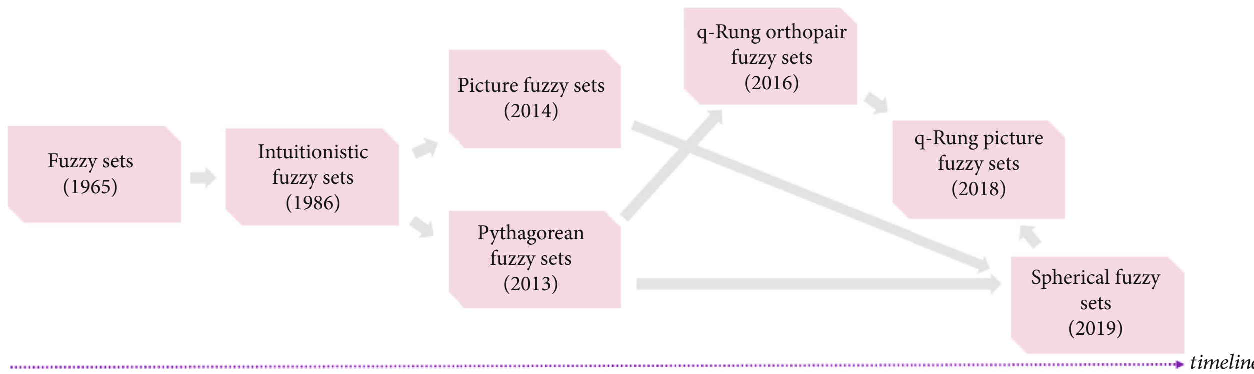

Figure 1: Development network of fuzzy set theory.

situations and uncertainties in the input data required for decision-making problems. Several researchers employed these sets in multicriteria decision-making (MCDM). For instance, Seikh and Mandal [10] introduced innovative operational laws and corresponding aggregation operators for PFSs. They also studied Archimedean aggregation operators for q-ROFSs, exploring their applications in the site selection for software operating units [11]. Ünver et al. [12] employed SFSs in addressing pattern recognition problems. Garg [13] introduced certain aggregation operators for PFSs and applied them in the context of MCDM. Özçelik [14] conducted an examination of the performances of MCDM methods and an optimization model in solving multiattribute shortest path problems under a fuzzy environment. Beg et al. [15] used q-RPFS in MCDM problems by defining some aggregation operators. Akram et al. [16] focused on MCDM with q-rung picture fuzzy information. Pinar and Boran [17] proposed a novel distance measure on q-RPFSs and its application to decision-making and classification problems. Akram et al. [18] conducted a hybrid decision-making analysis using complex q-rung picture fuzzy Einstein averaging operators. In Figure 1, the evolutionary progression of the fuzzy set theory up to q-RPFSs is depicted.

Entropy, an important measurement method for uncertainty or information, was introduced by Shannon [19]. Zadeh [20] transformed Shannon's entropy into fuzzy entropy, measuring fuzziness in a fuzzy set. De Luca and Termini [21] developed a new entropy based on Shannon's entropy, which was later extended to IFSs by Hung and Yang [22]. Arya and Kumar [23] proposed a picture fuzzy entropy. Mahnaz et al. [24] also introduced a distance-based entropy measure for q-ROPFSs, also known as t-spherical fuzzy sets [25], where the monotonicity condition of the entropy is not solely dependent on the fuzziness of the fuzzy sets. Furthermore, a distance measure, a widely used measurement form, specifies the distance between two entities and is utilized in numerous decision-making problems such as TODIM (Portuguese acronym for interactive and multicriteria decision-making) method (see, e.g., [26–28]). Distance measures are also applied in medical diagnosis, pattern recognition, classification, and MCDM. For example, Rani and Garg [29] studied distance measures for complex IFSs and their applications in MCDM. Ünver and Aydoğan [30] proposed distance measures for application in MCDM problems. Son [31] explored a generalized picture distance measure and its applications in clustering. Khan et al. [32] presented biparametric distance and similarity measures for PFSs and their applications in medical diagnosis. Jiang et al. [33] introduced a novel distance measure between IFSs based on transformed isosceles triangles, studying its applications in pattern recognition.

The TODIM method, introduced by Gomes and Lima [34] and incorporating group utility and individual regret—factors frequently overlooked in traditional MCDM methods—stands out as a widely employed discrete MCDM approach applicable to both quantitative and qualitative criteria. In this paper, we present an extended TODIM method that addresses the limitations of traditional MCDM methods by incorporating these crucial factors within the q-rung picture fuzzy environment. The distinctive choice of this method over others is grounded in its ability to comprehensively address the complexities of decision-making scenarios, considering not only the quantitative aspects but also the qualitative dimensions often crucial in real-world decision problems. By explicitly integrating group utility and individual regret, TODIM offers a more nuanced and realistic representation of decision scenarios, providing a robust framework for decision-makers to navigate uncertainties and varied criteria effectively. This adaptability and inclusiveness make TODIM a pragmatic choice, aligning with the diverse nature of decision environments encountered in practice. TODIM method determines loss and gain situations according to all criteria and uses pairwise comparisons to eliminate discrepancies [34]. It has been extensively applied to solve MCDM problems in various domains, including engineering, business, and environmental management. Recently, a number of fuzzy TODIM method have emerged in scholarly publications, which utilize fuzzy sets to address the challenges of uncertainty and vagueness in decision-making. For instance, Ju et al. [35] presented an enhanced version of TODIM method within the context of q-rung picture fuzzy framework, employing the Minkowski distance. Herrera-Viedma and Cabrerizo [36] developed a new fuzzy TODIM method to address MCDM problems with unknown attribute weights. Konwar and Debnath [37] explored continuity and the Banach contraction principle in intuitionistic fuzzy n-normed linear spaces. Lourenzutti and

Krohling [38] studied TODIM in an intuitionistic fuzzy and random environment. Ren et al. [39] proposed the Pythagorean fuzzy TODIM approach for MCDM. Wei [40] applied the TODIM method to picture fuzzy MCDM.

The present paper introduces an extended TODIM method that addresses the limitations of traditional MCDM methods within the q-rung picture fuzzy framework. The choice of q-RPFSs as the foundation for our methodology is a strategic one, driven by several compelling reasons that underscore their suitability for addressing the complexities of decision-making under uncertainty. Firstly, q-RPFSs generalize existing fuzzy set models, offering a versatile and unified framework that can seamlessly integrate various fuzzy set paradigms. Their incorporation allows us to effectively represent uncertainties and vagueness in decision-making scenarios, making them well suited for real-world applications where input data may be inherently uncertain. Moreover, the inclusion of a neutral degree for an element in a q-RPFS enhances the model's capability to handle uncertainty, providing a more nuanced representation. This is particularly advantageous in decision-making scenarios where a balanced and neutral perspective is essential. By choosing q-RPFSs, we are aligning our methodology with a proven and effective paradigm that extends beyond traditional fuzzy set models.

In the present work, we propose new entropy and distance measures for q-RPFSs, utilizing the sine function and Jensen-Shannon divergence, respectively. The choice of using the sine function instead of the absolute value function to define an entropy measure is motivated by the desire to capture the specific characteristics and properties of the fuzzy sets. While both functions can be used to quantify the spread or dispersion of values within a set, the sine function offers certain advantages. Cui and Ye [41] employed the sine function to define entropy for simplified neutrosophic sets (SNSs), applying it in the context of MCDM. Garg [42] utilized the sine function to establish operational laws for PyFSs. Ashraf et al. [43] explored fuzzy decision support modeling for Internet finance soft power evaluation, employing sine trigonometric Pythagorean fuzzy information. Türkarslan et al. [44] applied the sine function to define cross-entropy for consistency fuzzy sets (CFSs). In this paper, the sine function exhibits a periodic nature, oscillating between zero and one, which allows it to capture the cyclical patterns or fluctuations that may exist in the membership degrees. This can be particularly useful in scenarios where the data exhibits periodic or repetitive variations [42]. Furthermore, the absolute value function only considers the magnitude of the values and disregards any directional information or patterns. It treats positive and negative deviations from a central point equally, without taking into account any potential asymmetry or specific distribution characteristics. Taking into account these advanced measures, our proposed method offers a resilient approach to weighing criteria and delivers improved evaluation capabilities.

In this manuscript, we also perform sensitivity analysis by altering the weights assigned to group utility and individual regret, along with adjusting the parameters employed in the extended TODIM method. We demonstrate the practical utility of the proposed method by applying it to a real-life problem from the literature [15] in construction project management and compare the results with those obtained using other MCDM methods. The findings show that the proposed method outperforms the other methods in terms of sensitivity analysis and providing a more comprehensive evaluation of alternatives, highlighting the importance of considering group utility and individual regret in MCDM problems.

The following advantages stem from our research and the development of these pioneering information measures:

(1) This paper introduces new entropy and distance measures for q-RPFSs, expanding the existing repertoire of measurement techniques and providing alternative methods for assessing uncertainty and relationships in complex decision-making problems

(2) The proposed information measures are expected to improve the effectiveness of MCDM processes, offering better decision support by accounting for a wider range of uncertainty and vagueness inherent in real-world problems

(3) The introduced entropy measure plays a crucial role in the enhanced TODIM method by effectively assigning weights to criteria. Notably, in contrast to the classical TODIM approach, this weighting process takes place within the fuzzy environment

(4) The distance measure, derived from the Jensen-Shannon divergence, possesses the capability to gauge the similarity between objects while overcoming certain limitations of other divergence measures. In contrast to the Kullback-Leibler divergence, which lacks symmetry, the Jensen-Shannon divergence symmetrically quantifies the dissimilarity between two objects. Consequently, incorporating the proposed distance measure in the TODIM method enhances the robustness of the approach

(5) This research demonstrates the practical application of the proposed entropy and distance measures by integrating them into the well-established extended TODIM method. This integration showcases how the new measures can be used in real-life decision-making scenarios

(6) q-RPFSs extend the scope of several established fuzzy set models, facilitating the smooth integration of diverse fuzzy set paradigms. Notably, the inclusion of the neutral degree for an element in a fuzzy set enhances the treatment of uncertainty, contributing to the improved handling of fuzzy set uncertainties

(7) By comparing the results obtained using the new measures in an extended TODIM method to those from existing literature, the paper highlights the effectiveness and potential advantages of using the proposed entropy and distance measures in MCDM problems

The rest of the paper is organized as follows: In Section 2, we give a comprehensive exploration of the foundational concepts underpinning our research. We revisit key theoretical underpinnings of fuzzy set theory, emphasizing its evolution from classical set theory to more advanced forms such as q-ROFSs, PFSs, SFSs, and q-RPFSs. The section is aimed at establishing a robust theoretical foundation for the subsequent discussions on decision-making methodologies. We also give a short information about fuzzy entropy measures. Section 3 is dedicated to introducing novel information measures tailored specifically for q-RPFSs. Here, we define and elaborate on innovative entropy and distance measures, incorporating the sine function and Jensen-Shannon divergence, respectively. These measures are designed to enhance the evaluation capabilities of decision-making processes, offering a unique perspective on the weighing of criteria in complex decision scenarios. In Section 4, we present our extended TODIM method, addressing the limitations observed in traditional MCDM methods by incorporating group utility and individual regret. This section provides a step-by-step elucidation of the methodological enhancements, emphasizing the rationale behind the choice of the sine function in defining entropy measures. Furthermore, we apply the extended TODIM method to a practical, real-life construction project management problem sourced from the literature. We provide a comparative analysis of the solution using another distance-based MCDM method and conduct a sensitivity analysis. Finally, in Section 5, we conclude the paper.

2. Preliminaries

In this section, we provide a brief overview of some fundamental concepts in fuzzy set theory. Throughout this paper, we operate under the assumption that $X=\{x_1,\cdots,x_n\}$ represents a finite universal set and assume $q\geq 1$.

Definition 1 (see [4]). A q -ROFS A within X can be represented as

$$A=\{(x,\gamma_A(x),\zeta_A(x)):x\in X\}, \tag{1}$$

where $\gamma_A:X\longrightarrow[0,1]$ and $\zeta_A:X\longrightarrow[0,1]$ are functions satisfying the condition $\gamma_A^q(x)+\zeta_A^q(x)\leq 1$ for any $x\in X$.

Definition 2 (see [6]). A PFS A within X can be represented as

$$A=\{(x,\gamma_A(x),\eta_A(x),\zeta_A(x)):x\in X\}, \tag{2}$$

where $\gamma_A:X\longrightarrow[0,1]$, $\eta_A:X\longrightarrow[0,1]$, and $\zeta_A:X\longrightarrow[0,1]$ are functions satisfying the condition $\gamma_A(x)+\eta_A(x)+\zeta_A(x)\leq 1$ for any $x\in X$. The functions γ_A, η_A, and ζ_A are referred to as the membership function, neutral function, and nonmembership function of A, respectively.

Definition 3 (see [7, 8]). A SFS A within X can be represented as

$$A=\{(x,\gamma_A(x),\eta_A(x),\zeta_A(x)):x\in X\}, \tag{3}$$

where $\gamma_A:X\longrightarrow[0,1]$, $\eta_A:X\longrightarrow[0,1]$, and $\zeta_A:X\longrightarrow[0,1]$ are functions satisfying the condition $\gamma_A^2(x)+\eta_A^2(x)+\zeta_A^2(x)\leq 1$ for any $x\in X$. The functions γ_A, η_A, and ζ_A are referred to as the membership function, neutral function, and nonmembership function of A, respectively.

To briefly revisit the concept, the Hessian matrix of a function $f(x_1,\cdots,x_n)$ with multiple variables is given by

$$\mathbb{H}(f)=\begin{bmatrix} \dfrac{\partial^2 f}{\partial x_1^2} & \dfrac{\partial^2 f}{\partial x_1\partial x_2} & \cdots & \dfrac{\partial^2 f}{\partial x_1\partial x_n} \\ \dfrac{\partial^2 f}{\partial x_2\partial x_1} & \dfrac{\partial^2 f}{\partial x_2^2} & \cdots & \dfrac{\partial^2 f}{\partial x_2\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial^2 f}{\partial x_n\partial x_1} & \cdots & \cdots & \dfrac{\partial^2 f}{\partial x_n^2} \end{bmatrix}. \tag{4}$$

If $\mathbb{H}(f)$ is positive definite, then the function f is strictly convex. If $\mathbb{H}(f)$ is negative definite, then the function f is strictly concave at a point in its domain.

2.1. q-Rung Picture Fuzzy Sets. We revisit the definition of q-RPFS, which will be utilized to establish a knowledge measure for q-RPFSs.

Definition 4 (see [15]). A q-RPFS A in X is represented as follows:

$$A=\{(x,\gamma_A(x),\eta_A(x),\zeta_A(x)):x\in X\}. \tag{5}$$

Here, $\gamma_A:X\longrightarrow[0,1]$, $\eta_A:X\longrightarrow[0,1]$, and $\zeta_A:X\longrightarrow[0,1]$ are functions satisfying $\gamma_A^q(x)+\eta_A^q(x)+\zeta_A^q(x)\leq 1$ for all $x\in X$. The functions γ_A, η_A, and ζ_A are referred to as the membership function, neutral function, and nonmembership function of A, respectively. We denote the triplet of nonnegative numbers $\alpha=\langle\gamma,\eta,\zeta\rangle$ as a q-rung picture fuzzy value (q-RPFV) if $\gamma^q+\eta^q+\zeta^q\leq 1$.

It is important to note that if $\eta_A(x)=0$ for every $x\in X$, then set A becomes a q-ROFS. For $q=1$ and $q=2$, set A transforms into a PFS and a SFS, respectively.

We now review some set operations for q-RPFSs.

Definition 5 (see [15]). Let $A=\{\gamma_A(x),\eta_A(x),\zeta_A(x)|x\in X\}$ and $B=\{\gamma_B(x),\eta_B(x),\zeta_B(x)|x\in X\}$ be two PFNs.

(i) $A\cup B=\{x,\max(\gamma_A(x),\gamma_B(x)),\max(\eta_A(x),\eta_B(x)),\min(\zeta_A(x),\zeta_B(x))x,\ \max(\gamma_A(x),\gamma_B(x)),\max(\eta_A(x),\eta_B(x)),\min(\zeta_A(x),\zeta_B(x))|x\in X\}$

(ii) $A \cap B = \{(x, \min(\gamma_A(x), \gamma_B(x)), \min(\eta_A(x), \eta_B(x)), \max(\zeta_A(x), \zeta_B(x)) | x \in X\}$

(iii) The complement A^c of A is given with $A^c = (\zeta_A(x), \eta_A(x), \gamma_A(x)) | x \in X$

Now, we revisit a score function associated with the q-rung picture score function.

Definition 6 (see [15]). Let $A = \{(\gamma_A(x), \eta_A(x), \zeta_A(x)) | x \in X\}$ and $B = \{(\gamma_B(x), \eta_B(x), \zeta_B(x)) | x \in X\}$ be two PFNs. The score function $s(A)$ is defined as

$$s(A) = \frac{1}{3}\left[1 + \gamma^q + \eta^q - \zeta^q\right]. \tag{6}$$

The accuracy function $h(A)$ is defined as

$$h(A) = \frac{1}{2}\left(1 + \max\{\gamma^q, \eta^q\} - \zeta^q\right). \tag{7}$$

3. q-Rung Picture Fuzzy Information Measures

This section introduces new information measures for q-RPFSs, which include measures of entropy and distance. In the literature, several measures have been proposed to capture the uncertainty and vagueness inherent in fuzzy sets, with the entropy measure being particularly significant [19, 21–23, 45]. Entropy measures the uncertainty or randomness of a fuzzy set and has found various applications in the decision-making, pattern recognition, and classification. Therefore, it is essential to define a proper entropy measure for q-RPFSs to understand the degree of uncertainty or vagueness in the data.

3.1. A New q-Rung Picture Fuzzy Entropy Measure. Arya and Kumar [23] introduced an entropy measure for PFSs. Here, we represent the conditions for the entropy measures applicable to q-RPFSs.

Definition 7. Let q -RPFS(X) be the set of all q-RPFSs on X. A q-rung picture fuzzy entropy measure is a fuction $\mathbb{E} : q - RPFS(X) \longrightarrow [0, 1]$ satisfying the following properties:

E1. If A is a crisp set, then $\mathbb{E}(A) = 0$.

E2. $\mathbb{E}(A) = 1$ if and only if $A = \{(x, (1/2)^{1/q}, (1/2)^{1/q}, (1/2)^{1/q}) : x \in X\}$.

E3. $\mathbb{E}(A^c) = \mathbb{E}(A)$.

E4. $\mathbb{E}(A) \le \mathbb{E}(B)$ if A is less fuzzier than B; that is, $\gamma_A^q(x) \le \gamma_B^q(x)$, $\eta_A^q(x) \le \eta_B^q(x)$, and $\zeta_A^q(x) \le \zeta_B^q(x)$ if $\max\{\gamma_B^q(x), \eta_B^q(x), \zeta_B^q(x)\} \le 1/2$ or $\gamma_A^q(x) \ge \gamma_B^q(x)$, $\eta_A^q(x) \ge \eta_B^q(x)$, and $\zeta_A^q(x) \ge \zeta_B^q(x)$ if $\min\{\gamma_B(x), \eta_B(x), \zeta_B(x)\} \ge 1/2$.

Inspired by [41], we introduce a novel fuzzy entropy measure for q-RPFSs. Let A be a q-RPFS in X. We define the function $\mathbb{E} : q\text{-RPFS}(X) \longrightarrow [0, 1]$ as follows:

$$\mathbb{E}(A) = \frac{1}{3n}\sum_{i=1}^{n}\left[\sin\left(\gamma_A^q(x_i)\pi\right) + \sin\left(\eta_A^q(x_i)\pi\right) + \sin\left(\zeta_A^q(x_i)\pi\right)\right], \tag{8}$$

where q-RPFS(X) is the set of all q-RPFS on X.

Theorem 8. *The measure $\mathbb{E}(A)$ is a q-rung picture fuzzy entropy measure.*

Proof. E1. If A is a crisp set, then we get

$$\gamma_A(x_i) = 1, \eta_A(x_i) = 0, \zeta_A(x_i) = 0, \tag{9}$$

or

$$\gamma_A(x_i) = 0, \eta_A(x_i) = 0, \zeta_A(x_i) = 1. \tag{10}$$

Thus, we obtain

$$\mathbb{E}(A) = \frac{1}{3n}\sum_{i=1}^{n}\left[\sin\left(\gamma_A^q(x_i)\pi\right) + \sin\left(\eta_A^q(x_i)\pi\right) + \sin\left(\zeta_A^q(x_i)\pi\right)\right] = 0. \tag{11}$$

E2. If $A = \{(x, (1/2)^{1/q}, (1/2)^{1/q}, (1/2)^{1/q}) : x \in X\}$, then we obtain

$$\begin{aligned}\mathbb{E}(A)(x_i) &= \frac{1}{3n}\sum_{i=1}^{n}\left[\sin\left(\gamma_A^q(x_i).\pi\right) + \sin\left(\eta_A^q(x_i).\pi\right) + \sin\left(\zeta_A^q(x_i).\pi\right)\right] \\ &= \frac{1}{3n}\sum_{i=1}^{n} 3\sin\frac{\pi}{2} = 1.\end{aligned} \tag{12}$$

To prove the sufficiency, consider the function f defined by $f(\gamma, \eta, \zeta) = \sin(\gamma^q\pi) + \sin(\eta^q\pi) + \sin(\zeta^q\pi)$ for $\gamma, \eta, \zeta \in [0, 1]$. Thus, we have

$$\frac{\partial f(\gamma)}{\partial \gamma} = \pi q \gamma^{q-1}\cos(\gamma^q\pi). \tag{13}$$

To find the critical points, we need to solve

$$\pi q \gamma^{q-1}\cos(\gamma^q\pi) = 0. \tag{14}$$

Then, we have $\gamma^q\pi = (\pi/2) + k\pi$ for any integer k. So, we obtain $\gamma^q = (1/2) + k$. Since $\gamma \in (0, 1]$, we get $k = 0$ and so $\gamma^q = 1/2$. Similarly, we have $\eta^q, \zeta^q = 1/2$. So, the point $P = ((1/2)^{1/q}, (1/2)^{1/q}, (1/2)^{1/q})$ is a critical point. Zero is not considered as it makes the function zero so it cannot make the function maximum. On the other hand, we have

$$K(\gamma) = \frac{\partial^2 f(\gamma)}{\partial \gamma^2} = -\pi q \gamma^{q-2}(\cos\pi\gamma^q - q\cos\pi\gamma^q + \pi q \gamma^q \sin\pi\gamma^q) \tag{15}$$

and get the following Hessian matrix:

$$\mathbb{H}(\gamma, \eta, \zeta) = \begin{bmatrix} K(\gamma) & 0 & 0 \\ 0 & K(\eta) & 0 \\ 0 & 0 & K(\zeta) \end{bmatrix}. \tag{16}$$

Therefore, we have

$$\begin{aligned}\mathbb{H}(P) &= \begin{bmatrix} -\frac{1}{2^{(2q-2)/q}}\pi^2 q^2 & 0 & 0 \\ 0 & -\frac{1}{2^{(2q-2)/q}}\pi^2 q^2 & 0 \\ 0 & 0 & -\frac{1}{2^{(2q-2)/q}}\pi^2 q^2 \end{bmatrix} \\ &= \frac{1}{2^{(2q-2)/q}}\pi^2 q^2 \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}.\end{aligned} \tag{17}$$

Since the Hessian matrix $\mathbb{H}(P)$ is negatively defined, then f takes its maximum value at point P.

E3. The proof is trivial.

E4. Let max $\{\gamma_B^q(x_i), \eta_B^q(x_i), \zeta_B^q(x_i)\} \leq 1/2$. In this case, if A is less fuzzier than B, we have $\gamma_A^q(x) \leq \gamma_B^q(x) \leq 1/2$, $\eta_A^q(x) \leq \eta_B^q(x) \leq 1/2$, and $\zeta_A^q(x) \leq \zeta_B^q(x) \leq 1/2$. Thus, we get $\pi\gamma_A^q(x) \leq \pi\gamma_B^q(x) \leq \pi/2$, $\pi\eta_A^q(x) \leq \pi\eta_B^q(x) \leq \pi/2$, and $\pi\zeta_A^q(x) \leq \pi\zeta_B^q(x) \leq \pi/2$. Since the sine function is increasing over $[0, \pi/2]$, the proof is completed. The proof for the other case can be established similarly. □

Remark 9. Cui and Ye [41] provided a comparable entropy for SNSs, while Türkarslan et al. [44] applied a similar concept to define a cross-entropy for CFSs. The sine function introduces a distinctive feature to the entropy measure by exhibiting a periodic nature. Unlike the absolute value function, which remains constant in its behavior, the sine function oscillates between zero and one in a periodic fashion. This periodicity enables the entropy measure to capture cyclical patterns and fluctuations that may exist within the membership degrees of fuzzy sets. In decision-making scenarios, where data might exhibit periodic or repetitive variations, the sine-based entropy provides a more nuanced and accurate representation of the underlying dynamics [42]. Moreover, the sine function introduces a directional component that is absent in the absolute value function. While the absolute value function treats positive and negative deviations from a central point equally, the sine function captures both magnitude and direction. This is particularly important in scenarios where asymmetry or specific distribution characteristics play a role in defining the fuzziness of the fuzzy sets. The sine-based entropy, therefore, offers a more nuanced and context-specific assessment of the spread or dispersion of values within a set.

3.2. A New q-Rung Picture Distance Measure. In this subsection, we discuss the concept of divergence measure and its importance in measuring the difference between two q-RPFSs. We also introduce a distance measure for q-RPFSs based on the Jensen-Shannon divergence. The reason for defining a distance measure for q-RPFSs based on the Jensen-Shannon divergence is that the Jensen-Shannon divergence has several desirable properties, such as being symmetric, nonnegative, and satisfying the triangle inequality. These properties make it a suitable choice for defining a distance measure between q-RPFSs.

Definition 10 (see [46]). Let X be a discrete random variable, and let P_1 and P_2 be two probability distributions for X. The KL divergence is defined as follows:

$$\mathrm{KL}(P_1, P_2) = \sum_{x \in X} P_1(x) \log \frac{P_1(x)}{P_2(x)}. \tag{18}$$

It is worth noting that $\mathrm{KL}(P_1, P_2)$ is nonnegative, additive, and nonsymmetric.

Now, we define a new distance measure for q-RPFSs. This new measure can provide more accurate and meaningful results in various applications where q-RPFSs are used. This measure allows for a more accurate and meaningful comparison between q-RPFSs, especially in cases where there are overlaps or uncertainties in the membership degrees of the elements. In addition, the proposed distance measure considers the underlying uncertainty and vagueness in the data, which is a common feature in real-world decision-making problems.

Definition 11. Let α and β be two q-RPFVs where

$$\begin{aligned}\alpha &= (\gamma_\alpha, \eta_\alpha, \zeta_\alpha). \\ B &= \left(\gamma_\beta, \eta_\beta, \zeta_\beta\right).\end{aligned} \tag{19}$$

A new q-rung picture fuzzy distance measure is defined as

$$\mathbb{D}(\alpha, \beta) = \frac{1}{8 \log 2}\left(\sum_{J \in \mathcal{S}} J_\alpha^q \log \frac{2J_\alpha^q}{J_\alpha^q + J_\beta^q} + \sum_{J \in \mathcal{S}} J_\beta^q \log \frac{2J_\beta^q}{J_\alpha^q + J_\beta^q}\right), \tag{20}$$

where $\mathcal{S} = \{\gamma, \eta, \zeta, \pi\}$ and $\pi_A(x) = \sqrt[q]{1 - \gamma_A^q(x) - \eta_A^q(x) - \zeta_A^q(x)}$.

Theorem 12. *Let α, β, and τ be three q-RPFSs in X; then we have the following:*

D1. $\mathbb{D}(\alpha, \beta) = 0$ if $\alpha = \beta$.

D2. $\mathbb{D}(\alpha, \beta) = \mathbb{D}(\beta, \alpha)$.

D3. $\mathbb{D}(\alpha, \tau) \leq \mathbb{D}(\alpha, \beta) + \mathbb{D}(\beta, \tau)$.

D4. $0 \leq \mathbb{D}(\alpha, \beta) \leq 1$.

Proof. D1-D2. The proofs are trivial.

D3. We consider three cases.

Case 1. Let $\gamma_\alpha \le \gamma_\beta \le \gamma_\tau$ or $\gamma_\tau \le \gamma_\beta \le \gamma_\alpha$. Then, we have

$$|\gamma_\alpha^q - \gamma_\tau^q| = \left|\gamma_\alpha^q - \gamma_\beta^q\right| + \left|\gamma_\beta^q - \gamma_\tau^q\right|. \tag{21}$$

Case 2. Let $\gamma_\beta \le \min\{\gamma_\alpha, \gamma_\tau\}$. Then, we have $\gamma_\alpha^q - \gamma_\beta^q \ge 0$ and $\gamma_\tau^q - \gamma_\beta^q \ge 0$. Therefore, we have

$$\begin{aligned} &\left|\gamma_\alpha^q - \gamma_\beta^q\right| + \left|\gamma_\beta^q - \gamma_\tau^q\right| - |\gamma_\alpha^q - \gamma_\tau^q| \\ &= \begin{cases} \gamma_\alpha^q - \gamma_\beta^q + \gamma_\tau^q - \gamma_\beta^q - \gamma_\alpha^q + \gamma_\tau^q, \text{ if } \gamma_\alpha \ge \gamma_\tau \\ \gamma_\alpha^q - \gamma_\beta^q + \gamma_t^q - \gamma_\beta^q + \gamma_\alpha^q - \gamma_t^q, \text{ if } \gamma_\alpha \le \gamma_\tau \end{cases} \\ &= \begin{cases} 2\left(\gamma_\tau^q - \gamma_\beta^q\right), \text{ if } \gamma_\alpha \ge \gamma_\tau \\ 2\left(\gamma_\alpha^q - \gamma_\beta^q\right), \text{ if } \gamma_\alpha \le \gamma_\tau \end{cases} = 2\left(\min\{\gamma_\alpha^q, \gamma_\tau^q\} - \gamma_\beta^q\right) \ge 0. \end{aligned} \tag{22}$$

Case 3. Let $\gamma_\beta \ge \max\{\gamma_\alpha, \gamma_\tau\}$. Then, we have $\gamma_\beta^q - \gamma_\alpha^q \ge 0$ and $\gamma_\beta^q - \gamma_\tau^q \ge 0$. Thus, we obtain

$$\begin{aligned} &\left|\gamma_\alpha^q - \gamma_\beta^q\right| + \left|\gamma_\beta^q - \gamma_\tau^q\right| - |\gamma_\alpha^q - \gamma_\tau^q| \\ &= \begin{cases} -\gamma_\alpha^q + \gamma_\beta^q - \gamma_\tau^q + \gamma_\beta^q - \gamma_\alpha^q + \gamma_\tau^q, \text{ if } \gamma_\alpha \ge \gamma_\tau \\ -\gamma_\alpha^q + \gamma_\beta^q - \gamma_\tau^q + \gamma_\beta^q + \gamma_\alpha^q - \gamma_t^q, \text{ if } \gamma_\alpha \le \gamma_\tau \end{cases} \\ &= \begin{cases} 2\left(\gamma_\beta^q - \gamma_\alpha^q\right), \text{ if } \gamma_\alpha \ge \gamma_\tau \\ 2\left(\gamma_\beta^q - \gamma_\tau^q\right), \text{ if } \gamma_\alpha \le \gamma_\tau \end{cases} = 2\left(\gamma_\beta^q - \max\{\gamma_\alpha^q, \gamma_\tau^q\}\right) \ge 0. \end{aligned} \tag{23}$$

So far any case, we have $|\gamma_\alpha^q - \gamma_\beta^q| + |\gamma_\beta^q - \gamma_\tau^q| \ge |\gamma_\alpha^q - \gamma_\tau^q|$. On the other hand with a similar conclusion, we can get $|\eta_\alpha^q - \eta_\beta^q| + |\eta_\beta^q - \eta_\tau^q| \ge |\eta_\alpha^q - \eta_\tau^q|$, $|\zeta_\alpha^q - \zeta_\beta^q| + |\zeta_\beta^q - \zeta_\tau^q| \ge |\zeta_\alpha^q - \zeta_\tau^q|$, and $|\pi_\alpha^q - \pi_\beta^q| + |\pi_\beta^q - \pi_\tau^q| \ge |\pi_\alpha^q - \pi_\tau^q|$.

D4. Let $p_1 = \gamma_\alpha^q$, $p_2 = \eta_\alpha^q$, $p_3 = \zeta_\alpha^q$, and $p_4 = \pi_\alpha^q$ and $q_1 = \gamma_\beta^q$, $q_2 = \eta_\beta^q$, $q_3 = \zeta_\beta^q$, and $q_4 = \pi_\beta^q$. It is clear that

$$\sum_{i=1}^4 p_i = \sum_{i=1}^4 q_i = 1 \tag{24}$$

that yields that

$$\begin{aligned} \sum_{J\in\mathcal{S}} J_\alpha^q \log \frac{2J_\alpha^q}{J_\alpha^q + J_\beta^q} &= \sum_{i=1}^4 p_i \log \frac{2p_i}{p_i + q_i} = \mathrm{KL}(P, Q) \ge 0, \\ \sum_{J\in\mathcal{S}} J_\beta^q \log \frac{2J_\beta^q}{J_\alpha^q + J_\beta^q} &= \sum_{i=1}^4 q_i \log \frac{2q_i}{p_i + q_i} = \mathrm{KL}(Q, P) \ge 0, \end{aligned} \tag{25}$$

where $P = \{p_1, \cdots, p_4\}$ and $Q = \{q_1, \cdots, q_4\}$. Hence, $\mathbb{D}(\alpha, \beta) = (1/8 \log 2)(\mathrm{KL}(P, Q) + \mathrm{KL}(Q, P)) \ge 0$. It is clear that $\mathbb{D}(\alpha, \beta) \le 1$. □

4. An Application on MCDM

In this section, we propose an extended TODIM method in the context of q-rung picture fuzzy sets and apply it to a previously published MCDM problem. Following the application, we perform a sensitivity analysis of the extended TODIM method.

We now discuss the steps taken in implementing an extended TODIM method, using a similar approach proposed by Arya and Kumar [23].

4.1. An Extended TODIM Method. In this subsection, we present the steps of the promised extended TODIM method, which is a widely used MCDM approach and is applicable to both quantitative and qualitative criteria. The method is based on the distance measure $\mathbb{D}(A, B)$, defined in Definition 11, and the entropy measure $\mathbb{E}(A)$, defined in Definition 7, is used to weigh the criteria. The usage of the distance measure and entropy in the extended TODIM method is crucial for the proper evaluation of the MCDM problem. The distance measure $\mathbb{D}$ enables the calculation of the dissimilarity between two q-RPFSs. On the other hand, the entropy measure $\mathbb{E}$ allows for the weighting of the criteria in a more accurate manner. By utilizing both measures in the proposed extended TODIM method approach, a more comprehensive evaluation of the criteria can be achieved, resulting in more effective decision-making.

(i) Step 1: consider $B = \{B_1, \cdots, B_m\}$ and $C = \{C_1, \cdots, C_n\}$ as the sets of alternatives and criteria, respectively

(ii) Step 2: let $D = (d_{ij}) = (\gamma_{ij}, \eta_{ij}, \zeta_{ij})$ be a q-rung picture fuzzy decision matrix with alternatives $B_1, \cdots, B_m$ and criteria $C_1, \cdots, C_n$. The matrix D is given by

$$D = \begin{bmatrix} B_1 \\ \vdots \\ B_m \end{bmatrix} = \begin{bmatrix} d_{11} & \cdots & d_{1n} \\ \vdots & \ddots & \vdots \\ d_{m1} & \cdots & d_{mn} \end{bmatrix}. \tag{26}$$

The process of determining these values has been established in [15] that introduces q-RPFSs. The selection of these values depends on the specific application domain, the expertise of the decision-makers, and the nature of the problem being addressed. The values for the membership function, nonmembership function, and neutral function are typically determined through expert knowledge, domain-specific expertise, and consultation with stakeholders or decision-makers.

(iii) Step 3: to obtain a normalized q-rung picture fuzzy decision matrix $R = (r_{ij})$ from the decision matrix $D = (d_{ij})$, we use the following transformation:

$$r_{ij} = \begin{cases} d_{ij}^c, \text{for cost criteria}, \\ d_{ij}, \text{for benefit criteria}. \end{cases} \quad (27)$$

Here, $(d_{ij})^c = (\zeta_{ij}, \eta_{ij}, \gamma_{ij})$ represents the complement of d_{ij}.

(iv) Step 4: to determine the weights of the criteria, use the equation

$$w_j = \frac{\sum_{i=1}^m \mathbb{E}(r_{ij})}{\sum_{j=1}^n \sum_{i=1}^m \mathbb{E}(r_{ij})}, \quad (28)$$

where w_j is the weight of the criterion C_j. Next, obtain the relative weight of criterion C_j with respect to the most important criterion, denoted by rw_j, as follows:

$$rw_j = \frac{w_j}{w_r}, j, r = 1, \cdots, n, \quad (29)$$

where w_r is the maximum weight among all the criteria

(v) Step 5: calculate the dominance of alternative B_i over alternative B_t under criterion C_j using the following equation:

$$\Phi_j(B_i, B_t) = \begin{cases} \sqrt{\dfrac{rw_j \mathbb{D}(r_{ij}, r_{tj})}{\sum_{j=1}^n rw_j}}, & \text{if } r_{ij} < r_{tj}, \\ 0, & \text{if } r_{ij} = r_{tj}, \\ -\dfrac{1}{\theta}\sqrt{\dfrac{\mathbb{D}(r_{ij}, r_{tj}) \sum_{j=1}^n rw_j}{rw_j}}, & \text{if } r_{ij} > r_{tj}. \end{cases} \quad (30)$$

Here, θ is the attenuation factor of the losses. The gain or loss nature of the dominance relationship is defined based on the values of r_{ij} and r_{tj}.

(vi) Step 6: construct the dominance matrix Φ_j for each criterion C_j, where each entry $\Phi_j(B_i, B_t)$ denotes the dominance of alternative B_i over B_t under criterion C_j. The matrix Φ_j has dimensions $m \times m$ and is given by

$$\Phi_j = \begin{bmatrix} 0 & \Phi_j(B_1, B_2) & \cdots & \Phi_j(B_1, B_m) \\ \Phi_j(B_2, B_1) & 0 & \cdots & \Phi_j(B_2, B_m) \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_j(B_m, B_1) & \Phi_j(B_m, B_2) & \cdots & 0 \end{bmatrix}. \quad (31)$$

(vii) Step 7: compute the overall dominance degree of each alternative B_i under the criterion C_j as the summation of its dominance over all other alternatives B_t, given by

$$\delta_j(B_i) = \sum_{t=1}^m \Phi_j(B_i, B_t), i = 1, \cdots, m. \quad (32)$$

We can represent the total dominance consequences for each alternative as a column vector, given by

$$\Psi_j = \begin{bmatrix} \sum_{t=1}^m \Phi_j(B_1, B_t) \\ \vdots \\ \sum_{t=1}^m \Phi_j(B_m, B_t) \end{bmatrix}. \quad (33)$$

By Equation (32), the overall dominance matrix δ can be obtained as

$$(\delta_{ij}) = \begin{bmatrix} \sum_{t=1}^m \Phi_1(B_1, B_t) & \sum_{t=1}^m \Phi_2(B_1, B_t) & \cdots & \sum_{t=1}^m \Phi_n(B_1, B_t) \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{t=1}^m \Phi_1(B_m, B_t) & \sum_{t=1}^m \Phi_2(B_m, B_t) & \cdots & \sum_{t=1}^m \Phi_n(B_m, B_t) \end{bmatrix}. \quad (34)$$

(viii) Step 8: compute the positive and negative ideal solutions. The positive ideal solution is denoted as

$$\delta^+ = \left(\delta_j^+ : j = 1, \cdots, n\right), \quad (35)$$

and the negative ideal solution is denoted as

$$\delta^- = \left(\delta_j^- : j = 1, \cdots, n\right), \quad (36)$$

where

$$\begin{aligned} \delta^+ &= \left(\max_{1 \le i \le m} \sum_{t=1}^m \Phi_1(B_i, B_t), \cdots, \max_{1 \le i \le m} \sum_{t=1}^m \Phi_n(B_i, B_t)\right), \\ \delta^- &= \left(\min_{1 \le i \le m} \sum_{t=1}^m \Phi_1(B_i, B_t), \cdots, \min_{1 \le i \le m} \sum_{t=1}^m \Phi_n(B_i, B_t)\right). \end{aligned} \quad (37)$$

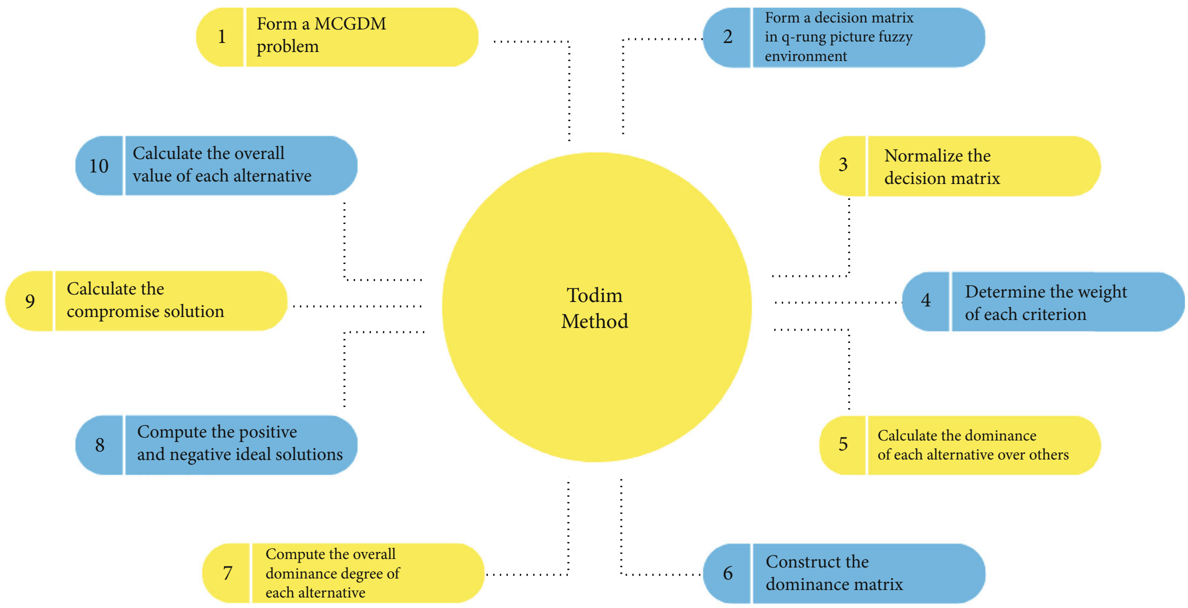

FIGURE 2: Flowchart of the extended TODIM method.

(ix) Step 9: use the maximum group utility M_i and the minimum individual regret value S_i to calculate the compromise solution as follows:

$$M_i = \sum_{j=1}^{n} w_j \frac{d\left(\delta_j^+, \delta_{ij}\right)}{d\left(\delta_j^+, \delta_j^-\right)},$$
$$S_i = \max_{1\le j\le n} w_j \frac{d\left(\delta_j^+, \delta_{ij}\right)}{d\left(\delta_j^+, \delta_j^-\right)}. \tag{38}$$

Here, $d(\delta_j^+, \delta_{ij})$ measures the distance between the positive ideal solution and the current solution δ_{ij}, while $d(\delta_j^+, \delta_j^-)$ measures the distance between the positive ideal solution and the negative ideal solution where

$$d\left(\delta_j^+, \delta_{ij}\right) = \max_{1\le j\le n} \sum_{t=1}^{m} \Phi_j(B_i, B_t) - \sum_{t=1}^{m} \Phi_j(B_i, B_t),$$
$$d\left(\delta_j^+, \delta_j^-\right) = \max_{1\le j\le n} \sum_{t=1}^{m} \Phi_j(B_i, B_t) - \min_{1\le j\le n} \sum_{t=1}^{m} \Phi_j(B_i, B_t). \tag{39}$$

(x) Step 10: calculate the overall value of Q_i as

$$Q_i = \tau \left[\frac{M_i - M^-}{M^\perp - M^-}\right] + (1-\tau)\left[\frac{S_i - S^-}{S^\perp - S^-}\right], \tag{40}$$

where $M^\perp = \max_i(M_i)$, $M^- = \min_i(M_i)$, $S^\perp = \max_i(S_i)$, and $S^- = \min_i(S_i)$. Here, the coefficient τ and $1-\tau$ represent the weights assigned to the maximum group utility (M_i) and individual regret (S_i), respectively

Steps of the extended TODIM method are visualized in Figure 2.

4.2. Numerical Example

(i) Step 1: in a recent study, Beg et al. [15] have investigated a MCDM problem using 3-RPFSs and aggregation operators. A construction supervisor plays a crucial role in overseeing construction activities at a worksite, involving tasks such as project planning, monitoring, and ensuring a safe working environment. They are responsible for managing contractors, staff, budgets, and policies and adhering to schedules. Hiring a competent construction supervisor is vital for a company's efficiency and productivity. In this scenario, a construction company is in the process of recruiting a supervisor for its ongoing projects. Three alternatives (B_1, B_2, B_3) have appeared for an interview as alternatives. The evaluation of these candidates is based on five attributes: the ability to read and understand blueprints, schematics, and construction documents (C_1); monitoring the project budget (C_2); being highly qualified (C_3); possessing communication skills (C_4); and having a reasonable salary demand (C_5)

(ii) Steps 2 and 3: we use the normalized decision matrix of [15] recalled in Table 1

(iii) Step 4: Table 2 displays the values of criteria weights calculated using (28)

Table 1: Normalized decision matrix.

	C_1	C_2	C_3	C_4	C_5
B_1	(0.5, 0.3, 0.5)	(0.7, 0.5, 0.6)	(0.4, 0.6, 0.2)	(0.6, 0.1, 0.7)	(0.7, 0.5, 0.3)
B_2	(0.6, 0.2, 0.4)	(0.4, 0.7, 0.2)	(0.8, 0.5, 0.2)	(0.6, 0.4, 0.4)	(0.7, 0.5, 0.6)
B_3	(0.7, 0.5, 0.4)	(0.3, 0.6, 0.8)	(0.7, 0.3, 0.4)	(0.9, 0.3, 0.4)	(0.6, 0.2, 0.4)

Table 2: Weights of the criteria.

$w_1 = 0.166916$	$w_2 = 0.248255$	$w_3 = 0.180575$	$w_4 = 0.188509$	$w_5 = 0.215745$

Table 3: Relative weights of the criteria.

$rw_1 = 0.672358$	$rw_2 = 1.000000$	$rw_3 = 0.479902$	$rw_4 = 0.759338$	$rw_5 = 0.869045$

As a result, Table 3 shows the relative weights of each criterion calculated using (29).

(iv) Steps 5 and 6: to determine the dominance of alternative B_i over B_t under criterion C_j, assuming $\theta = 1$, we calculate five dominance matrices Φ_1 to Φ_5 as follows:

$$\begin{aligned}
\Phi_1 &= \begin{bmatrix} 0.00000 & -0.1679 & -0.35542 \\ 0.02986 & 0.00000 & -0.32147 \\ 0.06321 & 0.05717 & 0.00000 \end{bmatrix}, \\
\Phi_2 &= \begin{bmatrix} 0.0000 & -0.46662 & 0.106671 \\ 0.123422 & 0.0000 & 0.140379 \\ -0.40329 & -0.53072 & 0.0000 \end{bmatrix}, \\
\Phi_3 &= \begin{bmatrix} 0.0000 & -0.62107 & -0.56141 \\ 0.078836 & 0.0000 & 0.047903 \\ 0.071263 & -0.37738 & 0.0000 \end{bmatrix}, \\
\Phi_4 &= \begin{bmatrix} 0.0000 & -0.38647 & -0.53887 \\ 0.077622 & 0.0000 & -0.51794 \\ 0.108232 & 0.104029 & 0.0000 \end{bmatrix}, \\
\Phi_5 &= \begin{bmatrix} 0.0000 & 0.066169 & 0.65004 \\ -0.28786 & 0.0000 & 0.090677 \\ -0.28279 & -0.39448 & 0.0000 \end{bmatrix}.
\end{aligned} \tag{41}$$

(v) Step 7: the resulting overall matrix δ is obtained as follows:

$$\delta = \begin{bmatrix} -0.52333 & -0.35994 & -1.18248 & -0.92534 & 0.131173 \\ -0.2916 & 0.263802 & 0.12674 & -0.44032 & -0.19718 \\ 0.12038 & -0.93401 & -0.30612 & 0.21226 & -0.67727 \end{bmatrix}. \tag{42}$$

(vi) Step 8: the positive and negative ideal solutions are computed as follows:

$$\begin{aligned}
\delta^+ &= (0.12038, 0.263802, 0.12674, 0.21226, 0.131173), \\
\delta^- &= (-0.52333, -0.93401, -1.18248, -0.92534, -0.67727).
\end{aligned} \tag{43}$$

(vii) Step 9: for $i = 1, 2, 3$, we compute M_i and S_i as follows:

$$\begin{aligned}
M_1 &= 0.60348, \\
M_2 &= 0.153399, \\
M_3 &= 0.503389, \\
S_1 &= 0.188509, \\
S_2 &= 0.108138, \\
S_3 &= 0.248255.
\end{aligned} \tag{44}$$

(viii) Step 10: the total value of Q_i is computed, resulting in the ranking B_3 being superior to B_1, which in turn is superior to B_2

Table 4: Q_i influence indices.

		Ranking	Ranking of [15]
B_1	0.786801	2	3
B_2	0.286801	3	2
B_3	0.888498	1	1

Table 5: Defuzzified decision matrix.

	C_1	C_2	C_3	C_4	C_5
B_1	0.3423	0.4173	0.4240	0.2913	0.4803
B_2	0.3867	0.4663	0.5430	0.4053	0.4173
B_3	0.4680	0.2437	0.4353	0.5640	0.3867

4.3. Comparative Analysis

4.3.1. Comparison with the Existing Literature. The outcomes of the current study, along with the ranking of [15], are displayed in Table 4.

The results presented in Table 4 provide insights into the influence of Q_i based on the calculated Q_i values and rankings. According to the table, B_3 holds the highest influence, as it possesses the highest Q_i value and is ranked at the top. This indicates that B_3 has the most significant impact among the evaluated factors. On the other hand, B_1 and B_2 exhibit lower influence compared to B_3, with B_1 being ranked second and B_2 ranked third. The Q_i values for B_1 and B_2 are lower in magnitude, suggesting a relatively weaker impact compared to B_3. Additionally, the rankings of [15] are provided for comparison purposes. It is noteworthy that the rankings of B_1 and B_2 differ between this study and the referenced work. This discrepancy highlights the potential variability in assessing the influence of factors, which can arise from different methodologies, contexts, or perspectives employed in the studies.

The ranking order heavily depends on the specific criteria and their relative weights assigned in each evaluation. Even a slight variation in the weights can lead to different rankings. We have demonstrated the superiority of our approach over the previous one through several key aspects. The proposed approach incorporates group utility and individual regret, which are often overlooked in traditional methods. By considering both aspects, our approach provides a more comprehensive evaluation of alternatives, capturing the collective preferences of decision-makers while addressing individual concerns.

We have introduced novel entropy and distance measures for q-RPFSs, enhancing the analysis and application of fuzzy sets in decision-making processes. These measures, based on trigonometric functions and the Jensen-Shannon divergence, offer more accurate and meaningful results in various applications. The use of trigonometric functions allows for a more flexible and nuanced representation of uncertainty in q-RPFSs. The Jensen-Shannon divergence takes into account both the overlapping and differing aspects of fuzzy sets, resulting in a more comprehensive evaluation of similarity or dissimilarity. Furthermore, our approach incorporates the TODIM method, a well-established and widely used technique for multicriteria decision-making. By integrating the strengths of TODIM method with the novel measures and the consideration of group utility and individual regret, our approach combines robustness, accuracy, and practicality.

4.3.2. Comparison with Another MCDM Method. In this subsection, we address the same problem using the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [47], another distance-based MCDM technique, and proceed to compare the outcomes. To facilitate this comparison, we perform defuzzification on the normalized decision matrix presented in Table 1 by using the score function defined in [15], as recalled in Definition 6. Thus, we obtain the decision matrix presented in Table 5.

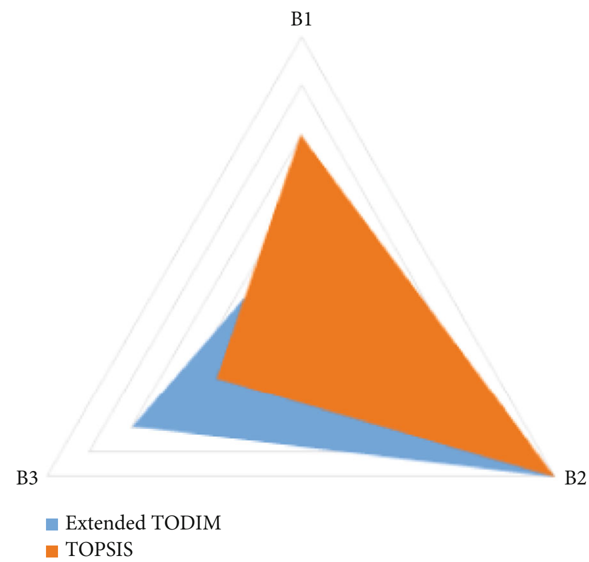

Figure 3: Comparison visualization between the extended TODIM and TOPSIS.

Upon solving the MCDM problem using TOPSIS, we identify the optimal alternative as B_2, aligning precisely with the best alternative identified by the proposed TODIM method. In this solution, the second-best alternative is B_1, while the least favorable option is B_3. The closeness coefficients provide a quantitative measure of the proximity of each alternative to the ideal solution. Higher closeness coefficients indicate greater proximity to the ideal solution. The associated closeness coefficients are 0.66, 0.46, and 0.45 for the respective alternatives.

The consistency in selecting the best alternative, B_2, by both the proposed TODIM method and TOPSIS, lends credibility to the robustness of the decision-making process. This convergence suggests that, despite the distinct methodologies employed, both approaches align in identifying B_2 as the most favorable option. The comparison between the extended TODIM and TOPSIS is visualized in Figure 3.

4.4. Sensitivity Analysis. Analyzing the sensitivity of the results to the parameters τ and θ can provide a better understanding of the robustness of the method and the relative

Table 6: Rankings for different τ values and $\theta = 1$.

τ	Ranking ($\theta = 1$)
0.5	$B_3 \succ B_1 \succ B_2$
1	$B_1 \succ B_3 \succ B_2$
1.5	$B_1 \succ B_3 \succ B_2$
2	$B_1 \succ B_3 \succ B_2$
2.5	$B_1 \succ B_3 \succ B_2$
3	$B_1 \succ B_3 \succ B_2$

importance of group utility and individual regret in the decision-making process. Varying these parameters over a range of values allows for a comparison of the rankings obtained, which can help decision-makers choose a compromise solution that is stable and satisfactory across different parameter settings. In addition, the sensitivity analysis can provide insights into the trade-off between group utility and individual regret, which can help make informed decisions. In this subsection, we present a comparison and sensitivity analysis of the results obtained with different values of τ and θ. We investigate the robustness of our proposed method to variations in these parameters and compare our results to those obtained by other methods. Specifically, we vary the values of τ and θ between 0.5 and 3 with an increment of 0.5 for each parameter.

(i) The value $\tau = 0.5$ is taken, and we get $B_3 \succ B_1 \succ B_2$. Furthermore, the ranking for different τ values is given in Table 6. Just using $\tau = 0.5$ resulted in a different ranking. The different rankings obtained for different values of τ indicate that the overall ranking of alternatives is sensitive to the weight assigned to group utility and individual regret. It is important to note that the choice of τ depends on the decision-maker's preferences and priorities. By considering different τ values, the decision-maker can have a better understanding of the impact of the weight assigned to each criterion on the overall ranking of alternatives. This sensitivity analysis can provide insights into the robustness and reliability of the decision-making process

(ii) For $\theta = 0.5$, we obtain the overall matrix δ_1, as well as the positive and negative ideal solutions, as follows:

$$\delta_1 = \begin{bmatrix} -1.04665 & -0.82656 & -2.36495 & -1.85068 & 0.131173 \\ -0.61307 & 0.263802 & 0.12674 & -0.95827 & -0.48504 \\ 0.12038 & -1.86802 & -0.18032 & 0.21226 & -1.35453 \end{bmatrix},$$
$$\delta_1^+ = (0.12038, 0.263802, 0.12674, 0.21226, 0.131173),$$
$$\delta_1^- = (-1.04665, -1.86802, -2.36495, -1.85068, -1.35453). \tag{45}$$

Table 7: Q_i influence indices for $\theta = 0.5$.

τ	Q_i values ($\theta = 0.5$)		
0.5	$Q_1 = 0.788576$	$Q_2 = 0.00000$	$Q_3 = 0.835442$
1	$Q_1 = 1.000000$	$Q_2 = 0.000000$	$Q_3 = 0.670884$
1.5	$Q_1 = 1.211424$	$Q_2 = 0.00000$	$Q_3 = 0.506326$
2	$Q_1 = 1.422848$	$Q_2 = 0.0000$	$Q_3 = 0.341768$
2.5	$Q_1 = 1.634271$	$Q_2 = 0.0000$	$Q_3 = 0.177209$
3	$Q_1 = 1.845695$	$Q_2 = 0.0000$	$Q_3 = 0.012651$

The values of M_i and S_i are calculated as follows:

$$\begin{aligned} M_1 &= 0.601539, \\ M_2 &= 0.301346, \\ M_3 &= 0.478682, \\ S_1 &= 0.188509, \\ S_2 &= 0.106962, \\ S_3 &= 0.248255, \end{aligned} \tag{46}$$

and the overall values of Q_i are computed for promised τ values, as shown in Table 7.

As a result, the rankings for various τ values are presented in Table 8.

(iii) For $\theta = 1.5$, the overall matrix δ_2 and positive and negative ideal solutions are obtained as follows:

$$\delta_2 = \begin{bmatrix} -0.34888 & -0.20441 & -0.78832 & -0.61689 & 0.131173 \\ -0.18445 & 0.263802 & 0.12674 & -0.26767 & -0.10123 \\ 0.12038 & -0.62267 & -0.18032 & 0.21226 & -0.45151 \end{bmatrix},$$
$$\delta_2^+ = (0.12038, 0.263802, 0.12674, 0.21226, 0.131173),$$
$$\delta_2^- = (-0.34888, -0.62267, -0.78832, -0.61689, -0.45151). \tag{47}$$

The values of M_i and S_i are calculated as follows:

$$\begin{aligned} M_1 &= 0.605684, \\ M_2 &= 0.30359, \\ M_3 &= 0.503978, \\ S_1 &= 0.188509, \\ S_2 &= 0.109114, \\ S_3 &= 0.248255, \end{aligned} \tag{48}$$

and thus, the overall value of Q_i are computed for different τ values and presented in Table 9. Accordingly, the rankings for different τ values are presented in Table 10.

Table 8: Rankings for different τ values and $\theta = 0.5$.

τ	Ranking ($\theta = 0.5$)
0.5	$B_3 \succ B_1 \succ B_2$
1	$B_1 \succ B_3 \succ Q_2$
1.5	$B_1 \succ B_3 \succ B_2$
2	$B_1 \succ B_3 \succ B_2$
2.5	$B_1 \succ B_3) \succ B_2$
3	$B_1 \succ B_3 \succ B_2$

Table 9: Q_i influence indices for $\theta = 1.5$.

τ	Q_i values ($\theta = 1.5$)		
0.5	$Q_1 = 0.785306$	$Q_2 = 0.00000$	$Q_3 = 0.831666$
1	$Q_1 = 1.000000$	$Q_2 = 0.00000$	$Q_3 = 0.663331$
1.5	$Q_1 = 1.214694$	$Q_2 = 0.00000$	$Q_3 = 0.494997$
2	$Q_1 = 1.429388$	$Q_2 = 0.00000$	$Q_3 = 0.326663$
2.5	$Q_1 = 1.644083$	$Q_2 = 0.00000$	$Q_3 = 0.158329$
3	$Q_1 = 1.858777$	$Q_2 = 0.00000$	$Q_3 = -0.01001$

Table 10: Rankings for different τ values and $\theta = 1.5$.

τ	Ranking ($\theta = 1.5$)
0.5	$B_3 \succ B_1 \succ B_2$
1	$B_1 \succ B_3 \succ B_2$
1.5	$B_1 \succ B_3 \succ B_2$
2	$B_1 \succ B_3 \succ B_2$
2.5	$B_1 \succ B_3 \succ B_2$
3	$B_1 \succ B_2 \succ B_3$

(iv) For $\theta = 2$, the overall matrix δ_3 and positive and negative ideal solution are obtained as follows:

$$\delta_3 = \begin{bmatrix} -0.26166 & -0.12664 & -0.59124 & -0.46267 & 0.131173 \\ -0.13087 & 0.263802 & 0.12674 & -0.18135 & -0.05325 \\ 0.12038 & -0.467 & -0.11743 & 0.21226 & -0.33863 \end{bmatrix},$$
$$\delta_3^+ = (0.12038, 0.263802, 0.12674, 0.21226, 0.131173),$$
$$\delta_3^- = (-0.26166,-0.467,-0.59124,-0.46267,-0.33863). \tag{49}$$

The values of M_i and S_i are calculated as follows:

$$\begin{aligned} M_1 &= 0.607196, \\ M_2 &= 0.304401, \\ M_3 &= 0.504516, \\ S_1 &= 0.188509, \\ S_2 &= 0.109936, \\ S_3 &= 0.248255, \end{aligned} \tag{50}$$

Table 11: Q_i influence indices for $\theta = 2$.

τ	Q_i values ($\theta = 2$)		
0.5	$Q_1 = 0.784029$	$Q_2 = 0.00000$	$Q_3 = 0.830446$
1	$Q_1 = 1.000000$	$Q_2 = 0.00000$	$Q_3 = 0.660892$
1.5	$Q_1 = 1.215971$	$Q_2 = 0.00000$	$Q_3 = 0.491338$
2	$Q_1 = 1.431941$	$Q_2 = 0.00000$	$Q_3 = 0.321784$
2.5	$Q_1 = 1.647912$	$Q_2 = 0.00000$	$Q_3 = 0.15223$
3	$Q_1 = 1.863882$	$Q_2 = 0.00000$	$Q_3 = -0.01732$

Table 12: Rankings for different τ values and $\theta = 2$.

τ	Ranking ($\theta = 2$)
0.5	$B_3 \succ B_1 \succ B_2$
1	$B_1 \succ B_3 \succ B_2$
1.5	$B_1 \succ B_3 \succ B_2$
2	$B_1 \succ B_3 \succ B_2$
2.5	$B_1 \succ B_3 \succ B_2$
3	$B_1 \succ B_2 \succ B_3$

and thus, the overall value of Q_i are computed for different τ values in Table 11.

The rankings for different values of τ are presented in Table 12.

(v) For $\theta = 2.5$, the overall matrix δ_4 and positive and negative ideal solution are obtained as follows:

$$\delta_4 = \begin{bmatrix} -0.20933 & -0.07998 & -0.47299 & -0.37014 & 0.131173 \\ -0.09873 & 0.263802 & 0.12674 & -0.12956 & -0.02447 \\ 0.12038 & -0.3736 & -0.07969 & 0.21226 & -0.27091 \end{bmatrix},$$
$$\delta_4^+ = (0.12038, 0.263802, 0.12674, 0.21226, 0.131173),$$
$$\delta_4^- = (-0.20933,-0.3736,-0.47299,-0.37014,-0.27091). \tag{51}$$

The values of M_i and S_i are calculated as follows:

$$\begin{aligned} M_1 &= 0.608457, \\ M_2 &= 0.305073, \\ M_3 &= 0.505007, \\ S_1 &= 0.188509, \\ S_2 &= 0.110922, \\ S_3 &= 0.248255, \end{aligned} \tag{52}$$

and thus, the overall value of Q_i are computed for different τ values in Table 13.

The ranking for different τ values can be seen in Table 14.

Table 13: Q_i influence indices for $\theta = 2.5$.

τ	Q_i values $(\theta = 2.5)$		
0.5	$Q_1 = 0.782479$	$Q_2 = 0.00000$	$Q_3 = 0.829507$
1	$Q_1 = 1.000000$	$Q_2 = 0.00000$	$Q_3 = 0.659014$
1.5	$Q_1 = 1.217521$	$Q_2 = 0.00000$	$Q_3 = 0.488521$
2	$Q_1 = 1.435043$	$Q_2 = 0.00000$	$Q_3 = 0.318028$
2.5	$Q_1 = 1.652564$	$Q_2 = 0.00000$	$Q_3 = 0.147535$
3	$Q_1 = 1.870085$	$Q_2 = 0.00000$	$Q_3 = -0.02296$

Table 14: Rankings for different τ values and $\theta = 2.5$.

τ	Ranking $(\theta = 2.5)$
0.5	$B_3 \succ B_1 \succ B_2$
1	$B_1 \succ B_3 \succ B_2$
1.5	$B_1 \succ B_3 \succ B_2$
2	$B_1 \succ B_3 \succ B_2$
2.5	$B_1 \succ B_3 \succ B_2$
3	$B_1 \succ B_2 \succ B_3$

(vi) For $\theta = 3$, the overall matrix δ_5 and positive and negative ideal solution are obtained as follows:

$$\delta_5 = \begin{bmatrix} -0.17444 & -0.04887 & -0.39416 & -0.30845 & 0.131173 \\ -0.07729 & 0.263802 & 0.12674 & -0.09503 & -0.000528 \\ 0.12038 & -0.31134 & -0.05453 & 0.21226 & -0.22576 \end{bmatrix},$$
$$\delta_5^+ = (0.12038, 0.263802, 0.12674, 0.21226, 0.131173),$$
$$\delta_5^- = (-0.17444, -0.31134, -0.39416, -0.30845, -0.22576). \tag{53}$$

The values of M_i and S_i are calculated as follows:

$$M_1 = 0.609525,$$
$$M_2 = 0.305637,$$
$$M_3 = 0.505459,$$
$$S_1 = 0.188509, \tag{54}$$
$$S_2 = 0.111915,$$
$$S_3 = 0.248255,$$

and thus, the overall value of Q_i are computed for different τ values in Table 15.

The rankings for different τ values can be found in Table 16.

A comprehensive examination has been conducted for each τ and θ, revealing valuable insights. The results indicate that for $\tau = 0.5$, B_3 emerges as the best alternative, while B_2 is the worst alternative. However, for $\tau = 1, 1.5, 2, 2.5, 3,$ the best alternatives are consistently identified as B_1, with the worst alternatives varying between B_2 and B_3. Comparing

Table 15: Q_i influence indices for $\theta = 3$.

τ	Q_i values $(\theta = 3)$		
0.5	$Q_1 = 0.780895$	$Q_2 = 0.00000$	$Q_3 = 0.828775$
1	$Q_1 = 1.000000$	$Q_2 = 0.00000$	$Q_3 = 0.65755$
1.5	$Q_1 = 1.219105$	$Q_2 = 0.00000$	$Q_3 = 0.486326$
2	$Q_1 = 1.438211$	$Q_2 = 0.00000$	$Q_3 = 0.315101$
2.5	$Q_1 = 1.657316$	$Q_2 = 0.00000$	$Q_3 = 0.143876$
3	$Q_1 = 1.876422$	$Q_2 = 0.00000$	$Q_3 = -0.02735$

Table 16: Rankings for different τ values and $\theta = 3$.

τ	Ranking $(\theta = 3)$
0.5	$B_3 \succ B_1 \succ B_2$
1	$B_1 \succ B_3 \succ B_2$
1.5	$B_1 \succ B_3 \succ B_2$
2	$B_1 \succ B_3 \succ B_2$
2.5	$B_1 \succ B_3 \succ B_2$
3	$B_1 \succ B_2 \succ B_3$

these results with the example in the literature [15], we observe that for $\tau = 0.5$, the best alternative remains as B_3, and the worst alternative is still B_2. However, for other values of τ, the best alternative changes to B_1. This discrepancy stems from our unique perspective and the utilization of a different method in examining the example.

To visually illustrate the outcomes of the sensitivity analysis for varying τ and θ values, we present a 3D scatter plot in Figure 4. The plot depicts the overall values of Q_i for each alternative B_i across different combinations of τ and θ. Through this plot, we can observe the impact of changes in τ and θ values on the overall ranking of the alternatives.

The findings of this study affirm the effectiveness of the proposed extended TODIM method in addressing MCDM problems in the q-RPFS environment. Additionally, the sensitivity analysis conducted by manipulating the τ and θ values reveals the potential variation in alternative rankings based on the assigned weights. Hence, careful consideration should be given to the selection of weights for group utility and individual regret to obtain the most appropriate ranking. Overall, this study underscores the significance of adopting different methods and perspectives when approaching MCDM problems in the q-RPFS environment.

4.5. Evaluation of the Application. In this section, we assess the effectiveness of applying the extended TODIM method to the recruitment process for a construction supervisor within the context of a MCDM problem, as previously delineated by Beg et al. [15].

(1) Using the proposed entropy, our approach assigns appropriate weights to the criteria based on their relative importance in the construction supervisor selection process. This weighting reflects the

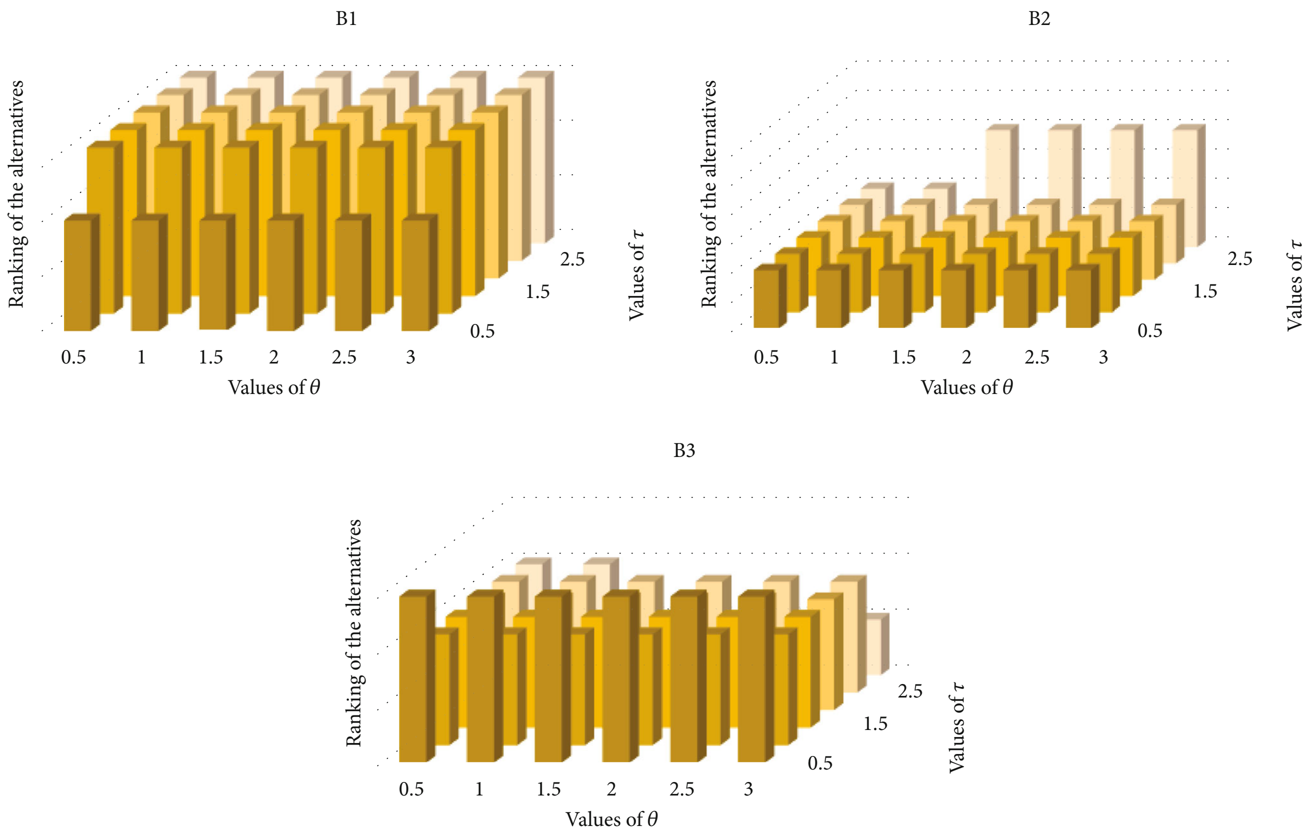

FIGURE 4: 3D sensitivity analysis plot for the extended TODIM method with varying τ and θ values.

significance of each criterion in meeting the overall objectives of the construction project management

(2) The method evaluates each candidate (B1, B2, and B3) against the established criteria (C1 to C5). The innovative entropy measures, particularly the sine function, play a crucial role in quantifying the fuzziness and uncertainties associated with each candidate's qualifications and performance in the specified criteria

(3) Unlike traditional MCDM methods, our proposed approach integrates group utility and individual regret into the decision-making process

(4) Sensitivity analysis is conducted by varying the weights assigned to group utility and individual regret, along with the parameters τ and θ used in the extended TODIM method. This analysis provides insights into the robustness of the proposed method under different scenarios, offering decision-makers a comprehensive understanding of its performance

5. Conclusion

In this paper, we proposed an extended TODIM method for MCDM problems in the q-RPFS setting. The proposed method was applied to a construction project manager selection problem from the literature, and its effectiveness was demonstrated by comparing the results with previous studies.

Through the analysis of the results, we found that the proposed method provided a flexible and efficient framework for handling MCDM problems in a q-RPFS environment. By considering the weight assigned to group utility and individual regret, the proposed method could generate different rankings for different τ values, allowing for sensitivity analysis and a more nuanced understanding of the decision problem. Furthermore, the results showed that the proposed method could yield different rankings from previous studies, indicating the importance of examining a problem from multiple viewpoints and with different methods.

In conclusion, the proposed extended TODIM method offers a promising approach for MCDM problems in q-RPFS environments and could be applied to a range of decision-making scenarios in various fields. Future research can focus on exploring additional applications of the proposed method in various domains and evaluating its effectiveness in different contexts. While the method exhibits promise in addressing the construction project management problem scrutinized in this study, its utility can be investigated in realms beyond, including finance, healthcare, and environmental management. A valuable avenue for research involves scrutinizing the sensitivity of the method to diverse parameter values, shedding light on its robustness and stability. Additionally, extending the method to tackle more intricate decision problems holds the potential to augment its practical applicability, furnishing decision-makers with a formidable and versatile tool. Moreover, our method is adaptable for application to another extension of fuzzy sets, such as quasirung fuzzy sets (see, e.g., [48, 49]). Ultimately, the method presented here can be seamlessly incorporated into fuzzy soft sets, as demonstrated in previous works such as [50–53].

Data Availability

The data used in this paper can be found in [15].

Ethical Approval

The study was conducted in accordance with ethical guidelines and does not infringe upon any existing publication ethics policies.

Conflicts of Interest

We confirm that there are no conflicts of interest regarding this manuscript.

Authors' Contributions

We hereby confirm the final authorship for this manuscript, including BA, MO, FS, and MÜ, and ensure that all authors meet the ICMJE criteria. The contributions of each author are as follows: MO and MÜ conceptualized the study and provided critical insights throughout the research process. BA conducted the data analysis and interpretation under the supervision of MO, FS, and MÜ who also contributed to the methodological design and oversight.

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