

# A Case for Higher-Order Metaphysics

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## Abstract

Higher-order logic augments first-order logic with devices that let us generalize into grammatical positions other than that of a singular term. Some recent metaphysicians have advocated for using these devices to raise and answer questions that bear on many traditional issues in philosophy. In contrast to these ‘higher-order metaphysicians’, traditional metaphysics has often focused on parallel, but importantly different, questions concerning special sorts of abstract objects: propositions, properties and relations. The answers to the higher-order and the property-theoretic questions may coincide sometimes but will often come apart. I argue that when they do, the higher-order questions are closer to the metaphysical action and so it would be better for these debates to proceed in higher-order terms.

Is the question of whether you are in pain settled by the physical facts? What about the question of whether you instantiate the property of being in pain? These are subtly different questions. When *being in pain* and *instantiating the property of being in pain* come apart, we stay closer to the important issues in the philosophy of mind when we focus on the question of whether something is in pain or not. The question of whether something instantiates the property of being in pain can be tied up in irrelevant ways with separate questions concerning the metaphysics and ontology of properties, or with the sorts of restrictions required to avoid Russell’s paradox. I use these considerations to argue that certain appeals to claims formulated in terms of properties and relations in philosophy would be better replaced with claims formulated in terms of higher-order generalizations.

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Section 1 provides a self-contained and non-technical explanation of generalization into predicate position and other non-nominal positions. Section 2 does the same for property theory. In section 3 I discuss the different ways in which higher-order generalizations and quantification over properties are analogous and disanalogous. In section 4 I argue that several debates in philosophy, when formulated in property-theoretic terms, fail to get to the heart of the matter and introduce extraneous issues. In section 5 I address a common resistance to higher-order languages. I conclude (section 6) that we would do better to replace these questions with different questions formulated in higher-order terms.

## 1 Higher-Order Generalizations

Consider a simple sentence in subject-predicate form

Socrates is wise.

We might formalize this sentence in a standard logical notation as  $Fa$ . Using the first-order quantifiers it is possible to generalize into the position that ‘Socrates’ occupies in this sentence, forming the existential and universal statements:

Something is wise.

Everything is wise.

These quantificational claims bear an intimate relationship with the sentences you get by replacing the ‘something’ or ‘everything’ in these sentences with particular names. We will call the result of performing such a replacement the quantificational claim’s *instances*:

Socrates is wise, Plato is wise, Aristotle is wise, (...and so on)

From any of the instances the first-order existential, ‘something is wise’, may be inferred, and from the first-order universal ‘everything is wise’, any instance may be inferred: given that Socrates is wise we may infer that something is wise, and given that everything is wise we may infer that Socrates is wise. Moreover, if you can prove the instances — that Socrates is wise — from assumptions that do not explicitly mention Socrates, then that reasoning would work equally well for an arbitrary individual, and we can infer the universal generalization. From the existential ‘something is wise’ you can introduce a

new name for a wise individual, say ‘Wiseman’, and infer the instance ‘Wiseman is wise’.

These inferential features pin down the logical role of the first-order quantifiers uniquely (Harris (1982)). For suppose that ‘everything’ is another quantifier subject to the same logical laws. Assuming that everything’ is wise we can infer an arbitrary instance—e.g. that Socrates is wise—by the first law applied to ‘everything’’. And since this argument proves an arbitrary instance—it could equally be applied to Plato, Aristotle, etc—we can infer that everything is wise, using the second law applied to ‘everything’’. The converse implication is identical so ‘everything is wise’ and ‘everything’ is wise’ are logically equivalent; clearly this reasoning extends to any pair of universal generalizations.

The expressions ‘everything’ and ‘something’ thus express generality with respect to name position in the sense that they bear this tight logical relationship to their instances. We will henceforth call them the ‘name quantifiers’. Although we do not any have words for them in English we might, by analogy, posit a pair of devices, the ‘predicate quantifiers’, that have the same logical relationship to predicates as the name quantifiers ‘everything’ and ‘something’ stand to names. The predicate quantifiers expresses generality in predicate position rather than name position, so the result of existentially or universally generalizing the predicate ‘is wise’ out of ‘Socrates is wise’ would have as instances the results of replacing ‘is wise’ with other predicates:

Socrates is wise, Socrates talks, Socrates is old, (...and so on)

In lieu of proper English counterparts, we will make do with ‘Socrates whatever’s’ and ‘Socrates something’s’ for the universal and existential predicate quantifiers respectively. These predicate quantifiers should bear the same logical relationship to their instances as the name quantifiers bear to their instances. From any of the instances the existential generalization may be inferred: given ‘Socrates talks’ we may infer ‘Socrates something’s’. And from the universal predicate generalization you can infer the instances: given ‘Socrates whatever’s’ we may infer ‘Socrates talks’. The remaining logical rules can be spelled out in an exactly parallel fashion; if you can prove one of the instances — ‘Socrates talks’ — from assumptions that do not contain the predicate ‘talks’, then we can prove from those assumptions ‘Socrates whatever’s’.

In standard logical notation, the result of existentially (universally) generalizing from the name  $a$  in  $Fa$  is written  $\exists x.Fx$  ( $\forall x.Fx$ ). We will represent the result of existentially (universally) generalizing from the predicate  $F$  in the sentence  $Fa$  using a similar notation:  $\exists X.Xa$  ( $\forall X.Xa$ ). This notation extends in the obvious way to other logically complex sentences. We notate

predicate generalizations using the same symbols as we use for name generalizations to emphasize the analogy in their logical role. But this is where the analogy ends: the name and predicate quantifiers are as different from one another, grammatically speaking, as names and predicates themselves are. It is sometimes tempting to pronounce  $\exists X.Xa$  as ‘Socrates instantiates some property’. But ‘some property’ is just a name quantifier restricted to the predicate of being a property. Since we have only defined the notion of an instance for the unrestricted quantifier phrase ‘something’ we might rewrite this more perspicuously as ‘something is a property and is instantiated by Socrates’. Its instances include ‘wisdom is a property and is instantiated by Socrates’, ‘Plato is a property and is instantiated by Socrates’, and so on, obtained by replacing the name quantifiers with particular names like ‘wisdom’ and ‘Plato’. But its instances *do not* include ‘Socrates is wise’. The truth of ‘Socrates is wise’ on its own does not suffice for the existence of any properties (or abstract objects whatsoever) and so does not suffice for the truth of ‘Socrates has some property’. So the name quantifier ‘some property’ does not have the right sort of logical role to be the predicate generalization.

(Word of warning: some authors prefer not to use made up English—like our ‘Socrates somethings’—to express second-order generalizations, and will look for other ways to communicate them informally. Some authors adopt a convention of using sentences like ‘Socrates has some property’ merely as a way of *pronouncing* a second-order generalization like  $\exists X.Xa$ .<sup>1</sup> But this does not mean they take these sentences to mean the same thing. The practice of using an English sentence as a way of pronouncing a sentence of a formal language does not require the former to be a synonym of the latter: it is a convention for bringing a higher-order sentence to the readers attention using a suggestive sentence of English with a different meaning.)

We have introduced devices that stand to predicates as the familiar quantifiers stand to names, but the same line of reasoning could lead one to introduce devices that generalize into any grammatical position. One could introduce quantifiers  $\exists p$  and  $\forall p$  that quantify into sentence position (letting one infer  $\exists p.\neg p$  from  $\neg(Fa)$ , for instance) operator position (letting one infer  $\exists X.X(Fa)$  from  $\neg(Fa)$ ), and so on. We may call these ‘sentence quantifiers’, ‘operator quantifiers’, and so on. Roughly speaking, ‘second-order logic’ is what one gets from first-order logic by adding variables that can take the position of a

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<sup>1</sup>Author’s who follow this practice include Williamson (2013), and Bacon (2018b) p740. Other authors instead proposed bending English somewhat to express higher-order generalizations: see for instance Prior’s ‘anywhether’ in Prior (1971) p37-39, or Rayo and Yablo (2001) §VII. Yet others avoid glossing higher-order sentences in English and get by using only the sentences of a formal higher-order language (see, for instance, Dorr (2016)).

predicate (of every arity) and adding the corresponding predicate quantifiers. ‘Higher-order logic’ does the same for every grammatical position.

It is also possible to extend other logical devices operating on names to other grammatical categories, by way of a similar sort of logical analogizing. One important example is identity.<sup>2</sup> Just as we can form an identity statement from two names, ‘Hesperus is Phosphorus’ we might introduce a binary connective,  $P \equiv Q$ , that behaves grammatically like conjunction, in the sense that it combines with two sentences to form another sentence. Like with the quantifiers, we treat it as logically analogous to the first-order identity predicate. In this case that means that it obeys the law of self identity,  $P \equiv P$ , and licenses substitution, in the sense that the analogue of Leibniz’s law holds:  $P \equiv Q \rightarrow \phi[P] \leftrightarrow \phi[Q]$ , where we are substituting sentences for sentences instead of names for names. There is no perfect gloss of it in English, but we will make do with locutions like:

For John to be a bachelor *just is* for John to be an unmarried man.

when applying the connective to the sentences ‘John is a bachelor’ and ‘John is an unmarried man’. The identity connective can be introduced by analogy with first-order identity, much as the higher-order quantifiers are. However, a connective with this logical behaviour can already be introduced through the higher-order quantifiers using operator quantification: the logical work of  $P \equiv Q$  described above can already be achieved with the formula  $\forall X(XP \leftrightarrow XQ)$  where  $X$  is an operator variable.<sup>3</sup> Any sentence,  $\phi[\dots]$ , with a gap  $\dots$  where a sentence should be, can be considered as a complex operator expression, so from this universally quantified sentence we can infer  $\phi[P] \leftrightarrow \phi[Q]$  as required.<sup>4</sup> The case of sentential identity illustrates how name identities can be generalized to sentences; we can introduce similar higher-order notions of identity for predicates, operators, and other grammatical categories by a similar process of logical analogizing.

Before continuing, it’s worth noting there are some close analogies between generalizations (both name and predicate) and infinite conjunctions

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<sup>2</sup>Advocates of this notion for doing metaphysics include Rayo (2013) and Dorr (2016).

<sup>3</sup>The reader may have observed that I have stopped short of simply treating  $\forall X(XP \leftrightarrow XQ)$  as the definition of  $P \equiv Q$ . Such an identification would take a stand on contentious higher-order identities involving the identity connective itself, such as  $(P \equiv Q) \equiv \forall X(XP \leftrightarrow XQ)$  (cf. ‘The Identity Identity’ in Bacon and Russell (2019)). On a structured conception of reality, this identification might be rejected on the grounds that there is different structure on the left and right hand side.

<sup>4</sup>This is made precise using the device of  $\lambda$ , allowing us to form an operator  $\lambda p.\phi[p]$  from an open formula  $\phi[p]$  containing a sentence variable  $p$ .

and disjunctions. Like the name generalization ‘everything is wise’, the infinite conjunction, ‘Socrates is wise and Plato is wise and Aristotle is wise and ...’, bears a close logical relationship to the first list of instances above. The conjunction entails all of the instances, and the conditions under which we can infer the universal also suffice to conclude the conjunction. Likewise, the existential ‘something is wise’ is closely related to the infinite disjunction of its instances, ‘Socrates is wise or Plato is wise or Aristotle is wise or ...’. However there are also some important disanalogies. Firstly, what instances a name generalization has depends on the language it is formulated in: in an expressively impoverished language that lacks names for some individuals the conjunction can be materially weaker than the universal.<sup>5</sup> Secondly, even in a language with names for every individual, the conjunction of instances cannot be materially weaker, but can still be modally weaker than the universal. The conjunction of instances that has a conjunct for every individual there is, ‘Socrates is wise and Plato is wise and ...’ and ‘everything is wise’ can come apart in truth value: consider a world in which all the individuals that actually exist are wise, but there are ‘new’ individuals that do not actually exist that are not wise. The same points apply just as forcefully to predicate generalizations. ‘Socrates somethings’ and the disjunction ‘Socrates is wise or Socrates talks or Socrates is old or ...’ are closely related, and we will often invoke the analogy when considering the ontological commitments of predicate generalizations, but must be distinguished for the same reasons we distinguish existential name generalizations from disjunctions.

## 2 Propositions, Properties and Relations

Metaphysics has maintained a long-standing interest in special sorts of abstract objects: propositions, properties, relations and so on. A property (say) is supposed to be an entity that represents what is common to a class of individuals sharing some trait, such as being red. An individual is red if and only if it stands in a special relationship, *instantiation*, to a further abstract object, the property of being red.

There are many questions that are intrinsic to the metaphysics of properties and relations — for instance, ‘are they located where they are instantiated?’ or ‘are they reducible to sets?’ But talk about propositions, properties and rela-

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<sup>5</sup>We have defined the notion of an instance for an existential *sentence*. Perhaps one could formulate a similar instantiation relationship at the level of reality, and introduce the relevant conjunction that way, but introducing a non-vacuous notion in the same ball-park would require some further substantive (and contentious) metaphysical assumptions.

tions have a habit of cropping up elsewhere in metaphysics. They are invoked in debates about laws and causation, the metaphysics of time and modality, the relation of the mental to the physical, the nature of moral facts, and in accounts of the structure of reality that appeal to notions like fundamentality, naturalness and ground (to name but a few).

For many (but not all) questions about properties and relations, one can formulate a structurally similar, but importantly different, question involving higher-order generalizations. It is my thesis that uses of properties and related abstract objects in metaphysics of the latter sort—that bear on other philosophical disciplines—are unnecessary. These disciplines would be better served by replacing questions formulated with properties and relations with the appropriate higher-order questions. The reason, roughly put, is that the answers to the property-theoretic questions come apart from the corresponding higher-order questions. When they do, it is the higher-order questions, in virtue of their logical relationship to their instances, that have a more direct bearing on the discipline in question — whether it be the metaphysics of modality, or laws, or morality, or what have you. I elaborate on the analogy between property theory and higher-order generalizations here. I will show how the analogy breaks down in the next section.

What ideology must one accept if we are to theorise systematically about properties? We have already mentioned one important primitive we must help ourselves to, *instantiation*:

A binary predicate  $Ixy$ , which states that  $y$  is a property and  $x$  instantiates  $y$ .

And of course, we also need:

A unary predicate  $Px$ , which states that  $x$  is a property.

In addition to the notion of instantiation and propertyhood, it is convenient (although not strictly necessary) to have a device for talking about particular properties, like *the property of being red* or *redness*, of *the property of being tall and wise*, and so on. Thus we also help ourselves to:

A term forming operation,  $\hat{x}.\phi$ , that yields a name for a property for each condition (formula),  $\phi$ : ‘the property of being  $\phi$ ’.

Not every condition  $\phi$  need correspond to a property, in which case we treat  $\hat{x}.\phi$  as referring to some default individual. (If the reader prefers they can

eliminate this device in favour of a definite description.<sup>6</sup>) I focus on properties here for simplicity of exposition, but similar primitives should exist for  $n$ -ary relations:  $I_n x_1 \dots x_n y$  means  $y$  is an  $n$ -ary relation instantiated by  $x_1 \dots x_n$ ,  $P_n x$  that  $x$  is an  $n$ -ary relation, and  $\widehat{x_1 \dots x_n} \phi$  is the name for a particular  $n$ -ary relation defined by the condition  $\phi$ . When  $n = 0$ , we say ‘ $y$  is a proposition’ and ‘ $y$  is true’ instead of ‘ $y$  is a 0-ary relation’ and ‘instantiates  $y$ ’, and we use the symbol  $\hat{\phi}$  for ‘the proposition that  $\phi$ ’.

The idea of a formal property theory goes back to Frege (1893), but finds many modern proponents in philosophy, such as George Bealer and Christopher Menzel.<sup>7</sup> Though metaphysicians may not always be as explicit about their primitives as we were above, something like the notions we introduce are implicit when they talk of individuals *having* (*possessing*, *instantiating*, etc.) properties, and in the way they refer to particular properties, such as *wisdom* (the *the property of being wise*, *being wise*, etc.).

There are some superficial similarities between higher-order questions and questions about properties and relations. For instance, from the assumption that Socrates and Plato were wise, we can infer that Socrates and Plato both something—that is,  $Fa \wedge Fb \vdash \exists X.(Xa \wedge Xb)$ . Similarly, given a suitably strong property theory, one can infer from the same assumption that Socrates and Plato instantiated a common property, namely wisdom— $Fa \wedge Fb \vdash \exists x.(Iax \wedge Ibx)$ . The first inference is a purely logical one, and is guaranteed by the way we introduced the predicate quantifiers as devices that simply express generality with respect to their instances. The second inference requires a substantive non-logical assumption about properties:

If Socrates (Plato) is wise then Socrates (Plato) instantiates wisdom.  
(i.e.  $Fa \rightarrow Ia(\hat{x}.Fx)$ )

With this assumption in place,  $Fa \wedge Fb$  entails  $Ia(\hat{x}.Fx) \wedge Ib(\hat{x}.Fx)$ , and we can infer the desired conclusion by ordinary existential generalization for name quantifiers.

Generally, there is a loose ‘correspondence’ between questions involving properties and predicate generalizations. For any statement one can formulate using predicate quantifiers, there is a ‘corresponding’ statement about

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<sup>6</sup>Although this device is not strictly necessary, it does make the exposition more convenient. We will see shortly that some conditions  $\phi$  make trouble for the naïve theory of properties. In such cases some philosophers deny that there is any such thing as ‘the property of being  $\phi$ ’, in which case we will understand ‘ $\hat{x}.\phi$ ’ as denoting some default individual—Julius Caesar, say. All my uses of  $\hat{x}.\phi$  can then be dispensed with using Russellian definite descriptions:  $\hat{x}.\phi$  should mean ‘the unique property of being  $\phi$  if there is exactly one such thing, and Julius Caesar otherwise’.

<sup>7</sup>Bealer (1982), Menzel (1986).



properties and relations; this mapping is outlined below. I should emphasize, however, that the correspondence I'm about to describe is not intended to show that the two sorts of questions are the same, or notational variants of each other. In fact, they are very different—we simply begin with some superficial similarities.

As one might expect, the correspondence is homophonic on sentences of first-order logic — those sentences that do not involve predicate generalizations:

- $(Rt_1\dots t_n)^* = Rt_1\dots t_n$  whenever  $R$  is an  $n$ -ary predicate constant and  $t_1\dots t_n$  are singular terms.
- $(\phi \wedge \psi)^* = \phi^* \wedge \psi^*$
- $(\neg\phi)^* = \neg\phi^*$
- $(\forall y.\phi)^* = \forall y.\phi^*$

Second-order languages additionally contain variables occupying predicate position, and quantifiers binding them. Here the correspondence replaces quantification into predicate position with name quantifiers quantifying over properties, and predication of a predicate variable with a first-order variable in the last argument of the instantiation relation:<sup>8</sup>

- $(Xt_1\dots t_n)^* = It_1\dots t_nx$  whenever  $X$  is an  $n$ -ary predicate variable.
- $(\forall X.\phi)^* = \forall x.(Px \rightarrow \phi^*)$

The other direction is not so clean: there are statements one can formulate about properties and relations that don't correspond naturally to any second-order statement. For instance, when a property term,  $\hat{x}.Fx$ , appears as the last argument of an instantiation sentence, such as  $Ia(\hat{x}.Fx)$  (Socrates instantiates wisdom), it may be mapped to a predication like  $Fa$  (Socrates is wise). But we have no natural options when a property term appears in any other argument place. Consider, for instance:

Wisdom is wise:  $F(\hat{x}.Fx)$

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<sup>8</sup>Here we associate to each second-order variable  $X$  a corresponding first-order variable  $x$ . To reduce unnecessary complexities, we assume the correspondence is defined on second-order formulas that do not contain the first-order variables we have associated with second-order variables. Because there are infinitely many first-order variables this does not introduce any expressive limitations.

This is presumably a false statement of property theory, but it does not correspond to a meaningful second-order statement, not even a false one. We will discuss other examples of this later. We will thus say that a property-theoretic statement of the form  $(\phi)^*$  for some second-order statement  $\phi$  corresponds to  $\phi$ , and property-theoretic statements that are not of this form do not correspond to any second-order statement.<sup>9</sup> It is possible to extend all these correspondences from generalization into predicate position to generalizations into any grammatical position.<sup>10</sup>

Notice that I have avoided calling these correspondences ‘translations’. Translations are usually thought to preserve meaning: as we will see shortly, the mappings I have described will not even preserve truth value. The mappings merely encode an (imperfect) way of analogizing statements of property theory and second-order generalizations. If other natural mappings are available then we may think of them as simply drawing a different analogy.

### 3 Property theoretic and second-order questions are not the same

The purpose of this section is to show that, despite some suggestive analogies, the second-order and property-theoretic questions have, to some extent, independent lives: both sorts of questions are meaningful, but their answers do not always coincide, and so they must be investigated separately. There are two sorts of ways the questions can come apart, which we will survey in this section.

The first way corresponds to property-theoretic questions that have no second-order analogues. Consider the following questions about properties that have captivated many philosophers interested in properties:

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<sup>9</sup>We could be a little bit more liberal if we wished and say that a statement of the property theoretic language  $\psi$  corresponds to a second order statement  $\phi$  if it is equivalent in first-order logic to  $(\phi)^*$ . As defined a property theoretic statement might correspond to several second-order statements.

<sup>10</sup>The correspondence between higher-order logic and property theory becomes more complex because we now have to consider terms of arbitrary type appearing as arguments of higher-order predicates. In this case there will be several ways of mapping higher-order claims onto property theory depending on when we choose to turn expressions of the higher-order language into singular terms using hatting and when we choose to leave them alone. As I explain below, because these mappings are not supposed to be translations but mere ways of highlighting analogies, these different mappings are all in equal standing, each providing us with a different way of analogizing higher-order questions with property-theoretic questions.

Are properties abstract or concrete?

Are properties located anywhere? Are they located where they are instantiated?

Are properties functions from worlds to extensions? Are they ordered tuples of their simple constituents? Are they not identical to sets at all, but *sui generis*?

Questions of this sort involve a name for a property, or a name quantifier ranging over properties, taking the position of a first-order predicate other than the last argument of ‘instantiates’. There simply is no coherent second-order question corresponding to ‘Is wisdom concrete or abstract?’ in the same way that ‘Is Socrates wise?’ corresponds to ‘Does Socrates instantiate wisdom?’. (The attempt at a statement ‘is wise is abstract’, and the corresponding attempt at a question, misfire as ungrammatical nonsense.) Because of the intimate relationship between a predicate generalization and its instances, quantificational claims like ‘are all properties abstract?’ equally fail to correspond to a predicate generalization. These remarks extend to the other questions listed: the predicates ‘is located’, ‘is a set’ and so on all draw distinctions between individuals—they don’t have straightforward higher-order analogues (see Jones (2018) §4.2.). Expressions that stand to predicates as predicates stand to names (i.e. predicates of predicates) are expressions that have the same type as a first-order quantifier phrase—‘three people’, ‘someone’, ‘several students’—and there is not obviously anything of that grammatical type that even bears a loose resemblance to the predicates ‘is a set’, ‘is located’, or ‘is abstract’.

The above shows that property theoretic questions are, to some extent, autonomous: if you posit any properties at all, they come with their own distinctive set of questions that have no second-order analogues. The second way the property-theoretic and higher-order questions can diverge is when our correspondence lets us draw an analogy, but their answers come apart.

Probably the most extreme way one might end up answering the questions differently is if one were to adopt a thorough-going nominalism. The nominalist denies the existence of properties altogether. Although nominalism is a negative view, it can be argued for from stronger positive theses, such as ‘everything is a material object’ given the auxiliary assumption that properties are not material objects. Thus the nominalist accepts:

Socrates is wise.

and rejects:

Socrates instantiates some property.

They accept the former based on Socrates' character, and reject the latter on the grounds that there are no properties at all. But what attitude should the nominalist have toward the second-order existential generalization:

Socrates somethings.

Of course, a nominalist could use second-order logic as another notation for first-order quantification over properties using some sort of correspondence like the one we described in section 2. But this nominalist is not addressing the question we are asking. If the nominalist is understanding the second-order quantifier in the way we introduced it here—as a device for forming generalizations across their instances—the existential generalization is an automatic consequence of any of its instances. So 'Socrates somethings' is a commitment of anyone who maintains that that 'Socrates is wise'. The nominalist thus accepts many existential predicate generalizations and rejects all of their corresponding property theoretic assertions. Arthur Prior is an example of a nominalist who was explicit about his commitment to existential predicate generalizations whilst rejecting the existence of properties.<sup>11</sup>

Now, some philosophers have taken the view that nominalists should reject second-order logic, in virtue of their rejection of properties.<sup>12</sup> But our introduction of second-order generalizations, by analogy with first-order generalizations, made no mention of properties so the nominalist's rejection of properties gives us no reason on its own to reject second-order generalizations. It is helpful to recall the analogy we drew earlier between the existential predicate generalization, 'Socrates somethings', and the infinite disjunction:

Socrates is wise or Socrates is old or Socrates talks or ...

Clearly one can believe the disjunction without believing that there are any properties. And the predicate generalization was introduced as something that bore the same logical relationship to its instances as the disjunction bears to its disjuncts. (Of course, existential quantification differs from the disjunction in *some* ways, as explained in section 1, but they do not affect the analogy.)

Now there are also less radical ways the questions could come apart. Some nominalist leaning philosophers have introduced concrete 'tropes' to do some of the work that properties are supposed to do. The trope theorist believes that every property (i.e. trope) is instantiated by one individual: there's

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<sup>11</sup>Prior (1971) chapter 2. See also Rayo and Yablo (2001).

<sup>12</sup>See Quine (1970) chapter 5.

Socrates' wisdom, and Plato's wisdom, and they are not the same. From Socrates is wise and Plato is wise we can infer that Socrates instantiates a property and Plato instantiates a property, but we cannot infer that there's some property that both Plato and Socrates instantiate. By contrast, the second-order generalization Socrates and Plato both something (i.e.  $\exists X.(Xa \wedge Xb)$ ) follows logically from this assumption, as we argued in the previous section.<sup>13</sup>

Here is another. David Lewis has identified properties with the set of individuals that instantiate that property.<sup>14</sup> Instantiation may then be simply defined as set membership. Other philosophers working in the possible worlds framework have identified properties with set-theoretic functions from possible worlds to extensions. An individual instantiates a property when it belongs to the extension obtained by applying that property to the actual world. Consider the second-order generalization:

For any  $y$ ,  $y$  somethings if and only if it is a set:  $\exists X \forall y (Xy \leftrightarrow Sy)$

this follows from the logical truth

For any  $y$ ,  $y$  is a set if and only if it is a set:  $\forall y (Sy \leftrightarrow Sy)$

The property-theoretic statement corresponding to the first claim is that there is a property instantiated by all and only the sets. For Lewis this amounts to the claim that there is a set containing all and only the sets. For the possible worlds theorist, it amounts to the claim that there is a function which maps the actual world to a set containing all and only sets. But no set contains itself, and so there is no set of all sets. So the higher-order claims and the property-theoretic ones differ.

There are thus a great many views about the metaphysics of properties in which the property-theoretic questions are, to varying extents, divorced from the higher-order questions. Indeed, that there will be some divergence somewhere is inevitable. Consider now the second-order generalization:

For any  $y$ ,  $y$  somethings if and only if  $y$  does not instantiate itself.

(That is,  $\exists X \forall y (Xy \leftrightarrow \neg Iyy)$ .)

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<sup>13</sup>For a more detailed study of the trope debate in the higher-order framework see Jones (2018).

<sup>14</sup>Lewis (1986). Properties are thus individuated extensionally for Lewis, but because a modal realist acknowledges a wider set of instances than opposing views, Lewis is spared from accepting certain obviously false identities between properties, (e.g. the property of having a heart and the property of having a kidney). Lewis later identifies properties with classes (see Lewis (1990)); a similar version of this example arises for classes.

Again, this follows from the logical truth that, for any  $y$ ,  $y$  doesn't instantiate itself if and only if it doesn't instantiate itself.<sup>15</sup> The corresponding property-theoretic statement is immediately susceptible to Russell's paradox, for it states the existence of a property,  $x$  which is instantiated by an individual  $y$  if and only if  $y$  doesn't instantiate itself (the corresponding property theoretic statement is  $\exists x(P(x) \wedge \forall y(Iyx \leftrightarrow \neg Iyy))$ ). But then  $x$  instantiates itself if and only if it doesn't. In short a statement of the form ' $\dots a$  doesn't instantiate itself  $\dots$ ' suffices for the truth of ' $\dots a$  somethings  $\dots$ ', since the former statement is an instance of the second-order existential; by contrast the corresponding inference form for properties is inconsistent.

Might one attempt to restore the parallel between higher-order and property-theoretic statements by revising our higher-order logic so that the predicate generalizations line up with the statements of the true property-theory, whatever that might be? Could a nominalist, for instance, maintain that Socrates neither instantiates any properties, nor somethings (despite being wise)? We noted above that there is something strange about combining the claims  $Fa$  and  $\neg\exists X.Xa$ , quite apart from their being logically inconsistent with the usual quantifier principles. We introduced predicate generalizations *entirely* by their logical role, as opposed to explaining them in other terms or finding English glosses (our artificial way of expressing them in English does not, of course, count). Without that logical role, what do we have left to go on when evaluating statements involving predicate generalizations, like  $\neg\exists X.Xa$ ? This is, at least, the difficulty I have with understanding the position that Socrates doesn't something (whilst still being wise). Perhaps this difficulty can be overcome. But even if it can be overcome, note that this position is motivated by an entirely different set of concerns to nominalism, or indeed the other views about the nature of properties considered above. As we noted there, nominalism can be argued for by appealing to various attractive and stronger positive theses, first-order generalizations like 'all objects are material objects', and 'no properties are material objects'. By contrast we cannot derive  $\neg\exists X.Xa$  from these claims or any first-order generalizations even assuming *classical* higher-order logic, and it is hard to see how one could motivate  $\neg\exists X.Xa$  from the same set of principles that motivate nominalism. So, even if the property-theoretic and second-order statements can be brought into alignment in a non-classical second-order logic, the agreement looks like it is an accident at best.

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<sup>15</sup>Here we are generalizing on the unary predicate 'doesn't instantiate itself', which is obtained from the grammar of English from the binary predicate 'instantiates'. In higher-order logic one would require a further device,  $\lambda$ , to obtain the unary predicate  $\lambda x.Ixx$ .

## 4 Higher-order questions are closer to the action than property-theoretic questions

Consider the notion of metaphysical necessity introduced in Kripke (1980). Whether there is a single thing satisfying everything Kripke says about metaphysical necessity is up for debate, but an important component of Kripke's conception of metaphysical necessity is that it is supposed to be 'necessity in the highest degree'. In practice this means that we can freely appeal to material conditionals like the following:

If it's metaphysically necessary that this table is mostly made of wood, then it's physically necessary that this table is mostly made of wood.

Here we invoke the idea that metaphysical necessity is as broad as physical necessity, and that physical necessity is a kind of necessity. Indeed, we can appeal to various strict conditionals as well, such as:

It's practically necessary that: if it's metaphysically necessary that this table is mostly made of wood, then it's physically necessary that this table is mostly made of wood.

Clearly the sentence 'this table is mostly made of wood' and the operators 'it's physically necessary that' and 'it's practically necessary that' are placeholders: you could replace the first with any sentence, and the second with any operators that express a kind of necessity and the resulting sentence is a consequence of the Kripkean worldview.

Kripke's ideas — both the positing of metaphysical necessity, and the thesis that it is the broadest kind of necessity — has had an immeasurable impact on philosophy over the last fifty years. Yet the mere slogan that metaphysical necessity is the broadest kind of necessity is not in itself that important without these concrete implications, such as the material and strict conditionals enumerated above. Without the implications you can't do anything with the slogan.

So how should the slogan be spelled out? We could formalize it in a first-order theory of propositions, properties, and relations, or we could use a higher-order generalization. A first pass at the former would be: for any necessity operators  $x$  and  $y$ , the proposition that *every proposition that instantiates metaphysical necessity instantiates  $y$*  instantiates  $x$ . Writing  $m$  as a singular term standing for the necessity operator and  $nec$  and  $prop$  as first-order predicates for necessities and propositions, this would become:<sup>16</sup>

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<sup>16</sup>Recall that  $\hat{\phi}$  is our notation for 'the proposition that  $\phi$ ', where we are extending our hatting notation to the empty sequence.

$$\forall xy(\text{nec}(x) \wedge \text{nec}(y) \rightarrow I(\forall z(\text{prop}(z) \rightarrow Izm \rightarrow Izy)))x)$$

To formulate the higher-order generalization we instead quantify into operator and sentence position directly, circumventing the need for problematic words like ‘instantiates’, ‘is true’ and so on, that are required to do property theory.

$$\forall XY(\text{Nec } X \wedge \text{Nec } Y \rightarrow X(\forall p(\Box p \rightarrow Yp)))$$

Here  $\Box$  is the operator ‘it’s metaphysically necessary that’,  $X$  and  $Y$  are operator variables,  $p$  a sentence variable, and  $\text{Nec}$  is now a higher-order predicate for necessities. (To state this informally would strain our conventions for glossing higher-orderese in English.)

Which of these claims fits the job description? It is clear that only the higher-order generalization lets us infer the material and strict conditionals we listed earlier, for they are consequences of the *instances* of those generalizations.<sup>17</sup> For instantiation lets us directly replace the operator variables  $X$  and  $Y$  with the operator expressions ‘it’s practically necessary that’ and ‘it’s physically necessary that’. The first-order generalization only lets us replace an individual variable with particular names; the relationship of these names to these operator expressions needs to be put in by hand by the property theorist, as a further assumption. A nominalist, for instance, cannot infer anything from a first-order claim of the form ‘all necessity operators are ...’: for the nominalist there are no such things as necessity operators, and so such universally quantified claims are vacuously true no matter what follows the quantifiers. Nonetheless, the nominalist has no reason to reject modal notions, and so should be able to meaningfully engage with Kripke’s view about metaphysical necessity, as encoded by the listed material and strict conditionals.<sup>18</sup> Each of the obstacles discussed in section 3 preventing us from moving from a first-order proposition/operator-theoretic statement to the relevant instances is one way in which the first-order claims are removed from the important metaphysical claims. If the paradoxes prevent us from moving freely between assertions about propositions instantiating necessity operators and assertions about what is necessary and contingent, then so much the worse for the proposition and operator way of talking: we shouldn’t care if the abstract object

<sup>17</sup>To make these inferences we need the auxiliary assumptions involving the higher-order predicate  $\text{Nec}$  applied to ‘it physically necessary that’ and ‘it’s practically necessary that’.

<sup>18</sup>It is not uncommon to find nominalists who also reject intensional notions (see Goodman and Quine (1947), Quine (1953)). But this admits of a sociological explanation; there is no straightforward logical connection between rejecting abstract objects, like necessity operators, and rejecting the meaningfulness of modal contexts like ‘it’s necessary that’ (see for instance Field (1984) for a nominalist happy to use modal notions).



corresponding to *this tables being made of wood* instantiates the abstract object corresponding to physical necessity unless it bears some relation to the real question, namely *whether or not it is physically necessary that the table is made of wood*.

Here is another example. Some metaphysicians, following Lewis (1983), have drawn a distinction between ‘natural’ and ‘gerrymandered’ properties, and used this to explain other important distinctions in metaphysics. For instance, Lewis defines two individuals to be intrinsic duplicates if there is a one-to-one correlation between their parts such that whenever  $x_1\dots x_n$  are correlated with  $y_1\dots y_n$ , they instantiate exactly the same perfectly natural relations.<sup>19</sup> Two mereological atoms,  $a$  and  $b$ , are thus duplicates iff:

$a$  and  $b$  instantiate the same natural properties:  $\forall x(\text{nat}(x) \rightarrow (Iax \leftrightarrow Ibx))$

Using  $\text{nat}$  as a first-order predicate applying to individuals that are properties and perfectly natural. Yet in practice the claim that  $a$  and  $b$  are duplicates immediately licenses the inference to many claims like the following:

$a$  is positively charged if and only if  $b$  is positively charged.

The nominalist ought to have a notion of being a duplicate that doesn’t require there to be any properties, and that also isn’t vacuous. It is clear that the property-theoretic statement above will not do: for it to be an adequate definition of duplication we must take a stand on the seemingly unrelated metaphysical question of nominalism. More generally, the connection between the property-theoretic claim and the instance above is tendentious and depends on what sorts of conditions correspond to properties, in accordance with your preferred response to Russell’s paradox. Other instances like:

$a$  is a set if and only if  $b$  is a set.

$a$  instantiates  $a$  if and only if  $b$  instantiates  $b$ .

should also follow from the claim that  $a$  and  $b$  are intrinsic duplicates, given the naturalness of sethood and instantiation. Observe that the latter entails  $a$  doesn’t instantiate itself iff  $b$  doesn’t instantiate itself — one of the points where a departure between higher-order property-theoretic statements is guaranteed. Of course, a predicate generalization could license these inferences. We might introduce a higher-order naturalness predicate,  $\text{Nat}$ , that combines grammatically with first-order predicates and formulate the relevant claim as follows.

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<sup>19</sup>The notion of a pair of duplicates is plays an important role in Lewis’s formulation of determinism.

$$\forall X(\text{Nat } X \rightarrow (Xa \leftrightarrow Xb))$$

Given the relevant naturalness assumptions, this generalization has the above biconditionals as instances, and so entails them in virtue of being a universal predicate generalization.

Our above examples have a common form: in cases where being  $F$  fails to correspond with instantiating the property of being  $F$ , we stay closer to the intended issues when we focus on the question of whether something is  $F$  or not. The question of whether something instantiates the property of being  $F$  can be tied up in irrelevant ways with separate questions concerning the metaphysics and ontology of properties, or the proper response to Russell's paradox. Many supervenience theses also have this form, such as the idea that the mental supervenes on the physical (the moral on the non-moral, the vague on the precise, etc). Is it possible for two people to share the same physical properties and have different mental properties? The nominalist must answer 'no' trivially, and in this sense supervenience of the mental on the physical is trivial. But they take it to be a non-trivial matter whether it's necessary that anyone with firing c-fibres is in pain, and they take many similar questions to be non-trivial; these further questions do not depend on whether there is a property of being in pain or a property of having c-fibres firing. The nominalist is, of course, the extreme case — but to the extent that Russell's paradox forces these questions to come apart, we'd do better to stick to the questions formulated in terms of higher-order generalizations.

Let me briefly turn to some examples with a different form, where I think the higher-order framework offers a distinctive advantage of perspicuity. These examples often have to do with the granularity of reality, but they do not always present themselves as such. I will illustrate it with the debate about moral naturalism. Moral naturalism is often stated in terms of propositions or facts.<sup>20</sup> When formulated this way, moral naturalism says that every moral proposition is identical to a proposition of the natural sciences: for instance a moral naturalist will believe there is a true sentence of the form 'the proposition that killing is wrong just is the proposition that the fundamental things are arranged thus and so' where 'thus and so' is to be replaced by a sentence in the language of a suitable natural science, such as physics.

When formulated in terms of facts and propositions it is all too easy to lose sight of the important issues. The word 'proposition' has multifarious uses in philosophy. Some tie them closely to thought and take them to be relatively fine-grained. Others take them to be more worldly and individuated in a coarse-grained way. The word 'fact' has more worldly connotations, but

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<sup>20</sup>See for instance §1.1 in Lutz and Lenman (2021).

without a precise account of what it is to be “worldly” fact-talk is in no better shape than talk about true propositions. It could well be that nothing satisfies all the roles associated with the word ‘proposition’ (or ‘fact’) in philosophy, but there are multiple collections of entities satisfying various fragments.<sup>21</sup>

Given all this, the question of moral naturalism bifurcates. Some take propositions to be identical to or as fine-grained as sentences.<sup>22</sup> In such a case the present formulation of moral naturalism is trivially false, for sentences belonging to the natural sciences and moral sentences are obviously distinct. Others take them to be sets of metaphysically possible worlds, in which case moral naturalism is close to trivially true (given the “least controversial thesis in metaethics” (Rosen (forthcoming) §1), namely that the moral supervenes on the natural). But fine-grained proposition theorists typically also believe in sets of possible worlds (or at the very least, equivalence classes of sentences under necessary equivalence), and coarse-grained proposition theorists still believe in sentences. So the different precisifications of moral naturalism can be made sense of no matter what we choose to apply the label ‘proposition’ to: on one of the precisifications it’s true, on the other it’s false.

But is this really all there is to the question of moral naturalism? As before, the proposition-theoretic way of formulating the question introduces noise because the relationship between the thing we are calling ‘the proposition that killing is wrong’ and killing being wrong is imperfect. This time we are arguing about what sorts of things propositions are, and losing sight of the question of what killing being wrong amounts to. The higher-order logician can formulate a thesis about what killing being wrong amounts to directly, using the binary identity connective,  $P \equiv Q$ , introduced in section 1.

For killing to be wrong *just is* for the fundamental things to be arranged thus and so.

The general thesis of moral naturalism can be formulated by generalizing into sentence position with primitive operators Moral and Natural (Moral  $P$ , for instances, stands for ‘it’s a moral matter whether  $P$ ’):<sup>23</sup>

$$\forall p(\text{Moral } p \rightarrow \exists q(\text{Natural } q \wedge p \equiv q))$$

Recall that the identity connective is introduced by its logical role, drawing on the analogy with the logic of first-order identity. This licenses many inferences

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<sup>21</sup>See, for instance, Schroeder (2013).

<sup>22</sup>Quine (1960) and Field (1978) §3, for example, take them to be identical to sentences, Soames (1987) and King (1996) take them to be as fine-grained as sentences.

<sup>23</sup>Given the standard quantificational principles and Leibniz’s law, this statement is just equivalent to the simpler claim  $\forall p(\text{Moral } p \rightarrow \text{Natural } p)$ .

that prove useful for investigating the question of moral naturalism. For instance, if it is physically contingent that the fundamental things are arranged thus and so, but not physically contingent whether killing is wrong then the above sentence identity must fail. It does *not* follow from this assumption that any two propositions are distinct without getting into side issues about what ‘propositions’ are.

Many other questions in philosophy can pick up noise from unrelated issues about propositions. Take the question of whether reality is structured. We can formulate theses in terms of propositions, properties and relations stating that they are individuated by necessary coextensiveness or individuated by their structure (call these *intensionalism* and *structuralism* respectively). Clearly we can find some entities that are structured, and call them ‘propositions’, or we can find other entities that are unstructured and call *them* ‘propositions’. But this endeavor feels pointless unless the things we are calling ‘propositions’ bear a sufficiently tight relationship with reality. Indeed, we can spell out what it means for a notion of proposition to track the structure of reality in higher-order terms, using the sentential identity connective:  $I_0(\phi) \equiv \phi$ . In plainer terms (recall that  $I_0$  is just propositional truth):<sup>24</sup>

For the proposition that killing is wrong to be true *just is* for killing to be wrong.

Provided we use the words ‘proposition’, ‘true’, ‘the proposition that’ in a way that secures this general schema, the distinctness of the proposition that  $P$  from the proposition that  $Q$  (articulated using the ordinary first-order notion of distinctness) is necessary and sufficient for the falsity of  $P \equiv Q$ , formulated using the sentential identity connective. However, the detour through propositions is unnecessary — we can reformulate moral naturalism, structuralism, intensionalism, and so on, directly in higher-order terms.<sup>25</sup>

I have focused on the case of propositions for a reason. In this case it is a substantive but consistent posit that there are propositions bearing a suitably close connection to reality — that their individuation conditions align.<sup>26</sup>

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<sup>24</sup>More generally, we have an operation that which combines with a sentence to form a name (a name for a proposition), and true that forms a sentence from a name. Our theory is that true is a left-inverse of that. In the  $\lambda$ -formalism mentioned earlier:  $\lambda p. \text{true that } p = \lambda p.p$ . This implies, for instance, that  $\forall p. \text{true that } p = p$  where  $p$  is a sentence variable. See Bacon (2018a) p47.

<sup>25</sup>See Goodman (2017), Dorr (2016), Fritz (2017) for higher-order formulations of structure and discussions thereof. Williamson (2013), Bacon and Dorr (forthcoming) for a higher-order formulation of intensionalism.

<sup>26</sup>It should be noted that while the connection for propositions consistent, it is quite

$$\hat{\phi} = \hat{\psi} \leftrightarrow \phi \equiv \psi$$

The detour through propositions is possible even if unnecessary once higher-order devices are available. In the case of properties it is not even possible to make the analogous detour. To tie the individuation conditions of properties to the higher-order individuation conditions is inconsistent. The thesis that the property of being  $F$  and the property of being  $G$  are individuated just as finely as  $F$  and  $G$  are can be formulated as follows:

$$\hat{x}.Fx = \hat{x}.Gx \leftrightarrow F \equiv G$$

where the identity on the right is a predicate identity, and on the left a first-order one. This is a version of Frege’s Basic Law V, which is famously inconsistent.<sup>27</sup> It is just a logical fact that the individuation of properties, whatever we take them to be, diverges from the individuation of reality.

Let me anticipate, and address one possible objection to my examples. Earlier we suggested that certain first-order predicates, such ‘is abstract’ and ‘is located’, have no higher-order analogues. Above we posited a higher-order analogue of our first-order predicate ‘is a necessity’, so it is natural to wonder whether this too is illegitimate. But I think we already do have the distinction, and it is arguably prior to the first-order distinction: much philosophy rests on our ability to distinguish operator expressions—ordinary expressions like ‘it must be true that’, as well as philosopher inventions like ‘it’s metaphysically necessary that’—that are used to express necessity from other kinds of operator expressions that don’t, like ‘Sally hopes that’, ‘it’s not the case that’, and so on. Furthermore, the former sorts of operator expressions are logically well-behaved—they have a ‘normal modal logic’—and one might argue that this is all there is to the distinction. In which case one could put Nec in good standing by simply *defining* it in entirely in logical terms—coding up the worldly analogue of having a normal modal logic in higher-orderese.<sup>28</sup> But even if not, Nec can at least be partially pinned down in intrinsically higher-order terms by its (higher-order) logical properties. This doesn’t seem to be possible for the notions of abstractness and locatedness, which seem, at best, to be characterized by their relationship to other individuals like regions, and other

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substantive. It is essentially a form of propositional T-schema: if we treat propositions too much like sentences, by assigning them enough structure, then it also become inconsistent because we can form propositional versions of the liar paradox. Sentence-like entities have a well-defined notion substituting a subsentence for another, which allows one to formulate a version of Gödel’s diagonal lemma.

<sup>27</sup>Russell (1904).

<sup>28</sup>Bacon (2018b), Bacon (MS) Ch.9 provide purely logical definitions of Nec in higher-order logic, and explore some of the consequences of this idea.

first-order predicates, such as causation and being extended. (For instance, one can formulate the higher-order analogue of the idea that every necessity is closed under modus ponens in entirely higher-order terms.<sup>29</sup>) A similar worry might be raised about our higher-order naturalness predicate, *Nat*. But again, the role of naturalness is one that can be spelled out as easily in higher-order terms as it can in first-order property theoretic terms, for if in the latter case it is characterized by its relationship to other properties and relations, there is a corresponding higher-order relationship we could state instead. The idea that we should treat naturalness as a higher-order predicate is a central idea in Sider (2011), and is treated more thoroughly there.<sup>30</sup> The claim that it is a moral matter whether *P*, *Moral P*, too is closely tied to deontic modals and propositional attitudes relating to action and desire, suggesting its role can be spelled in relation to other operator expressions like ‘desires that *P*’, ‘makes it true that *P*’, ‘it ought to be true that *P*’ and so on. And I think we can, in the worse case, introduce the higher-order distinction ‘*Moral P*’ and ‘*Natural P*’ by the sorts of sentences that can be used to say that *P*.

## 5 Some remarks on framework choice

I will end by briefly commenting on a common form of resistance to the very enterprise of higher-order theorizing.<sup>31</sup> It is not an objection to any particular claim *per se*—attempts to identify this resistance with disagreement tend to miss something—it is perhaps better to think of it as some considerations about ‘framework choice’ that militate against adopting higher-order languages. The worry is that once you have started talking in a higher-order language one can formulate new and ‘difficult’ questions that didn’t arise, or couldn’t be formulated, before you started theorizing in the higher-order framework, and the recognition of new seemingly intractable questions is a reason to reject a framework.

This issue is often illustrated with the following example. While the sentences of higher-order logic do not say anything about sets—they are formulated in purely logical terms without reference to distinctively mathematical words like ‘set’ or ‘member’—one can formulate purely logical theses that

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<sup>29</sup> $\forall X(\text{Nec } X \rightarrow \forall pq(X(p \rightarrow q) \rightarrow Xp \rightarrow Xq))$  captures the *K* axiom. See Bacon and Zeng (2022) for an intrinsic axiomatization of *Nec* in higher-order terms.

<sup>30</sup>Dorr and Hawthorne (2013) spell out naturalness role in some detail, and follow Sider in treating it as a family of higher-order predicates that includes the predicate of first-order predicates *Nat* appealed to above.

<sup>31</sup>My sense is this sort of resistance is common, but it has recently been articulated quite forcefully by Ted Sider (see Sider (MS of July 20, 2022)).

would imply substantive set-theoretic claims *if there were any sets*. Suppose that  $ZFC2(\text{Set}, \in)$  and  $\text{CH}(\text{Set}, \in)$  are respectively the conjunction of set-theoretic axioms (second-order ZFC) stated in terms of the primitive predicates  $\text{Set}$  and  $\in$ , and the statement of the continuum hypothesis in the same language. The continuum hypothesis is a very hard problem in set theory that we now know to be independent of all of currently accepted mathematics. The predicate generalization  $\forall F \forall R (ZFC2(F, R) \rightarrow \text{CH}(F, R))$  is a statement of pure higher-order logic (the only non-logical constants  $\text{Set}$  and  $\in$  have been replaced by variables and quantified out) which implies the continuum hypothesis given the non-logical assumption that the sets do in fact satisfy the axioms of set theory (i.e.  $ZFC2(\text{Set}, \in)$ ).<sup>32</sup> So, the worry goes, we now have to answer hard questions that we wouldn't otherwise have had to.

I will make a couple of brief remarks about this. The first concerns this particular example. Now either there are things satisfying the axioms of set theory or there are not: either  $ZFC2$  somethings ( $\exists F \exists R ZFC2(F, R)$ ) or it does not. If not, then the answer to our question is not so difficult:  $\forall F \forall R (ZFC2(F, R) \rightarrow \text{CH}(F, R))$  is vacuously true. On the other hand, if there are things satisfying the axioms of set theory—we might as well call them sets—then you already had to confront the question of the continuum hypothesis. For once you have accepted set theory, you must confront the question of the continuum hypothesis, even in first-order logic. (Note also that even those nominalists who reject set theory will devise ways of paraphrasing mathematical claims, and so have ways of raising the question of the continuum hypothesis: they might ask ‘if there *were* any sets, would the continuum hypothesis be true?’, or ‘is it true according to the fiction of mathematics that the continuum hypothesis is true?’. There are even versions of

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<sup>32</sup>This is my own version of an often raised objection, sometimes attributed to Quine. It is often raised in terms of a sentence  $\phi$  whose validity in a given set-theoretic model theory is equivalent to the continuum hypothesis. This version really misfires since there is a big difference between (i) the question of whether  $\phi$  and (ii) the question of whether  $\ulcorner \phi \urcorner$  is valid in such and such a set-theoretic model theory. For one, a negative answer to the question of whether ‘Socrates is wise’ is valid in a set-theoretic model theory will contain commitments to sets *even though* ‘Socrates is wise’ is a boring first-order claim about Socrates that has no such commitments. Second, one can theorize in higher-order languages without ever using the word ‘valid’, so commitments of statements involving the word ‘valid’ are not commitments of any statements of higher-order logic alone. Finally, the set-theoretic model theory used in these arguments is certainly not related to the intended interpretation of the higher-order formalism (which must be specified in a higher-order metalanguage), and is highly contentious even as an analysis of ‘validity’ since various extensionalist theses involving predicate and sentence identifications come out valid in it. Under the assumption of certain set-theoretic hypotheses, such as the non-existence of inaccessible cardinals, there will even be falsehoods that are valid in this sense!

the continuum hypothesis that can be formulated in the first-order language of mereology with a three-place relation between points for simulating talk of ordered pairs, by quantifying only over regions of spacetime.<sup>33</sup>)

This aside, it is clearly true that there are questions you can ask in higher-order languages that you cannot ask in first-order languages. As well as the interesting statements that bear on set theory, they include things like: ‘are there finitely many solar systems?’ or, ‘is to be wrong to be not not wrong?’ When one embraces the logical apparatus of higher-order logic one can formulate a whole load of new questions that couldn’t be formulated before.

The second remark I want to make concerns the general idea that the existence of difficult questions should play a guiding role in framework choice. First, we should note right off the bat that there is a lot of unclarity in the idea that there is a higher-order and first-order ‘framework’ or ‘worldview’ to choose between. What does it mean to ‘adopt the first-order worldview’ as opposed to the higher-order one? Unlike the dispute between the platonist and the nominalist (between  $\exists x Px$  and its negation), there does not seem to be any claim that constitutes the disagreement. They do not, for instance, disagree about whether Socrates somethings ( $\exists X Xa$ ): the first-orderist will not maintain that Socrates doesn’t something, for this is as much a higher-order statement as the claim that he does. As Wittgenstein warns us in the preface to the *Tractatus*: “in order to be able to draw a limit to thought, we should have to find both sides of the limit thinkable (i.e. we should have to be able to think what cannot be thought)” (Wittgenstein (1961) p3).

Perhaps “adopting” the first-order framework does not consist in adopting a propositional attitude toward some claim or other, but in taking some other sort of action. One could, for instance, vow to limit ones theorizing to a first-order language. It is far from clear, however, that doing this really lets you avoid the difficult higher-order questions. In some sense, restricting yourself to a first-order language prevents you from formulating these questions, but it does not mean they cannot be asked. After all, vowing to limit yourself to propositional logic, or even taking a vow of silence altogether, does not mean that first-order questions are not out there in need of answers. If there is such a

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<sup>33</sup>The pairing relation  $Pxyz$  is subject to axioms that say that it forms a one-to-one correlation between points (i.e. regions with no proper parts) and pairs of points. Letting  $x, y, z$  be variables ranging over points:  $\forall x \exists yz Pxyz, \forall yz \exists x Pxyz, (Pxyz \wedge Pxy'z') \rightarrow (y = y' \wedge z = z'), (Pxyz \wedge Px'yz) \rightarrow x = x'$ .  $Pxyz$  can be read as the point  $x$  is a representative for the pair of points  $y$  and  $z$ , and via  $P$  regions can then be thought of as coding collections of pairs, or relations. We can then proceed to state the continuum hypothesis.  $P$  may be explicitly definable from other physically acceptable primitives, such as primitive expressing metric notions.



thing as ‘the attitude of adopting an exclusively first-order worldview’ it must somehow be incompatible with that of ‘adopting the higher-order worldview’. Perhaps it consists in an additional thesis that the higher-order questions cannot even be coherently asked. But trying to draw a line in advance between what can and cannot be asked brings us back to Wittgenstein’s problem. It is one thing to do all your theorizing exclusively in first-order logic, but it is quite hard to go further and say that first-order logic is ‘the boundary of what is intelligible’, or that it ‘*exhausts* everything that can be said’ without contrasting, or otherwise talking about the things that cannot be said.

I can only think of two things this stronger attitude might amount to that do not devolve into nonsense. The first *is* a straightforward disagreement with the higher-order theorist, although it is likely too weak to properly capture Quinean resistance to higher-order theorizing. As explained above, this disagreement cannot be about any higher-order claim itself. It is rather a disagreement about a metalinguistic claim.<sup>34</sup> One might maintain that my earlier uses of the sentences ‘Socrates somethings’, ‘ $\exists X.Xa$ ’, and so on, are meaningless or nonsense. But the metalinguistic thesis is not enough: even if this particular claim about *my* uses of these sentences is right, this metalinguistic belief does not preclude one from coming to understand and ask higher-order questions by other routes, perhaps by different linguistic vehicles, or by finding a different explanation than the one provided in section 1. (We certainly do not want to say that *no* sentence with the syntax of a higher-order language is meaningful — Quine has offered first-order acceptable interpretations of these sentences in terms of sets. And we cannot simply say that no sentence can be used to make a higher-order claim about the world without conceding that there are higher-order claims about the world, even if they are inexpressible.) So it seems the metalinguistic claim is too weak to capture first-orderism. But more importantly, have we been given any reason to believe even this weak thesis? Is the fact that we could formulate new and difficult questions if higher-languages *were* meaningful a reason to reject their meaningfulness? It seems not: whether a sentence is meaningful can depend on many things—it can fail to be grammatical, some of its words may fail to make contact with reality, and so on—but the sorts of questions you could ask if the sentence *were* meaningful seems to have no bearing. Besides, there is something quite wrong about concluding a language is meaningless if the questions you encounter when you adopt it seem hard, or if reality is more complex

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<sup>34</sup>Wittgenstein, for instance, conceives of the Tractatus as drawing a limit ‘to the expression of thoughts’ as opposed to a limit on thought, before citing the problem we quoted above for the latter project.

than originally thought. It seems that to apply that sort of methodology, you have to start by asking the relevant questions too see how hard they are, at which point one has already found the questions intelligible.<sup>35</sup>

The other attitude one might take first-order worldview to involve would not be a propositional attitude at all, but rather the dispositive attitude of *not understanding*, or *finding unintelligible* the questions formulated in higher-order terms, and refusing to engage with them. At the end of the Tractatus Wittgenstein writes (speaking of the statements of the Tractatus) ‘The correct method in philosophy would really be the following: to say nothing except what can be said [...] and then, whenever someone else wanted to say something metaphysical, to demonstrate to him that he had failed to give a meaning to certain signs in his propositions’. As for myself, I find the method of introducing higher-order terms by analogy with first-order notions, explained in section 1, enough to find my way into understanding the questions. Finding something unintelligible is not a belief in any claim, but it may be a strategy for avoiding difficult questions. I can well imagine beginning inquiry with some commitments to particular claims that might guide my theorizing. However, the idea that we should avoid difficult questions by adopting the strategy of not understanding them from the outset does not seem like an appropriate attitude to towards inquiry. Indeed, the idea that having to confront difficult questions retrospectively provides a reason to adopt a strategy of not understanding them strikes me as disingenuous.

## 6 Conclusion

In the forgoing I have claimed that questions about first-order properties and the corresponding higher-order questions are both intelligible: both sorts of questions exist side by side but their answers may not be perfectly correlated.

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<sup>35</sup>One thing we can do, without taking any higher-order questions to be meaningful, is investigate the proof theoretic relationships between various higher-order sentences (where sentences here are merely uninterpreted strings). But this gets us no closer to being able to make judgments about which questions are untractable if they were meaningful. To do this we need to be able to engage in questions about which assumptions are plausible, what logical axioms are the right ones for drawing these consequences and so on. And I don’t see how to do any of this without actually asking the questions the sentences purport to express. CH, we have noted, is settled by  $\neg\exists F\exists R ZFC2(F, R)$ —does this make it tractable? To answer this we need to get into the question of whether  $\exists F\exists R ZFC2(F, R)$ . Learning mere proof theoretic relations between the string of symbols used to state CH and other uninterpreted strings gets us no closer to judgments of plausibility unless we know what those strings are saying.

My thesis is that the higher-order questions are one step closer to the action than the property-theoretic questions. When the answers to these two sorts of questions come apart, as they must at least sometimes, the answers to the property questions (at least, those that have higher-order analogues) are a few steps removed from the facts about reality. I thus recommend that philosophers would do better to proceed by replacing these question with higher-order questions that are more directly related to the subject at hand. Philosophers should avoid formulating and investigating questions that superficially involve properties and relations unless they are intrinsically about the nature of these sorts of objects.

## References

- Andrew Bacon. Radical anti-disquotationalism. *Philosophical Perspectives*, 32(1):41–107, 2018a. doi: 10.1111/phpe.12109.
- Andrew Bacon. The broadest necessity. *Journal of Philosophical Logic*, 47(5): 733–783, 2018b. doi: 10.1007/s10992-017-9447-9.
- Andrew Bacon. *A Philosophical Introduction to Higher-order Logics*. Routledge, MS.
- Andrew Bacon and Cian Dorr. Classicism. In Peter Fritz and Nicholas K. Jones, editors, *Higher-order Metaphysics*. Oxford University Press, forthcoming.
- Andrew Bacon and Jeffrey Sanford Russell. The logic of opacity. *Philosophy and Phenomenological Research*, 99(1):81–114, 2019. doi: 10.1111/phpr.12454.
- Andrew Bacon and Jin Zeng. A theory of necessities. *Journal of Philosophical Logic*, 51(1):151–199, 2022. doi: 10.1007/s10992-021-09617-5.
- George Bealer. *Quality and Concept*. Oxford, England: Oxford University Press, 1982.
- Cian Dorr. To be F is to be G. *Philosophical Perspectives*, 30(1):39–134, 2016. doi: 10.1111/phpe.12079.
- Cian Dorr and John Hawthorne. Naturalness. In Karen Bennett and Dean Zimmerman, editors, *Oxford Studies in Metaphysics: Volume 8*, page 1. Oxford University Press, 2013.

- Hartry Field. Mental representation. *Erkenntnis*, 13(July):9–61, 1978. doi: 10.1007/BF00160888.
- Hartry Field. Can we dispense with space-time? *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association*, 1984:33–90, 1984.
- Gottlob Frege. *Grundgesetze der Arithmetik: Begriffsschriftlich abgeleitet*. G. Olms, 1893.
- Peter Fritz. How fine-grained is reality? *Filosofisk Supplement*, 13(2):52–57, 2017.
- Jeremy Goodman. Reality is not structured. *Analysis*, 77(1):43–53, 2017. doi: 10.1093/analys/anw002.
- Nelson Goodman and W. V. Quine. Steps toward a constructive nominalism. *Journal of Symbolic Logic*, 13(1):49–50, 1947. doi: 10.2307/2268144.
- J. H. Harris. What’s so logical about the ‘logical’ axioms? *Studia Logica*, 41 (2-3):159–171, 1982. doi: 10.1007/BF00370342.
- Nicholas K. Jones. Nominalist realism. *Noûs*, pages 0–1, 2018. doi: 10.1111/nous.12193.
- Jeffrey King. Structured propositions and sentence structure. *Journal of Philosophical Logic*, 25(5):495–521, 1996. doi: 10.1007/BF00257383.
- Saul A. Kripke. *Naming and Necessity: Lectures Given to the Princeton University Philosophy Colloquium*. Cambridge, MA, USA: Harvard University Press, 1980.
- David Lewis. New work for a theory of universals. *Australasian Journal of Philosophy*, 61(4):343–377, 1983. doi: 10.1080/00048408312341131.
- David Lewis. *On the Plurality of Worlds*. Wiley-Blackwell, 1986.
- David Lewis. *Parts of Classes*. Blackwell, 1990.
- Matthew Lutz and James Lenman. Moral Naturalism. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Spring 2021 edition, 2021.
- Christopher Menzel. *A Complete, Type-Free "Second-Order" Logic and its Philosophical Foundations*. CSLI Publications, 1986.

- Arthur Norman Prior. *Objects of Thought*. Oxford, England: Clarendon Press, 1971.
- W. V. Quine. *Philosophy of Logic*. Harvard University Press, 1970.
- W. V. O. Quine. Three grades of modal involvment. *Proceedings of the XIth International Congress of Philosophy*, 14:65–81, 1953. doi: 10.5840/wcp11195314450.
- Willard Van Orman Quine. *Word and Object*. Cambridge, MA, USA: MIT Press, 1960.
- Agustín Rayo. *The Construction of Logical Space*. Oxford, England: Oxford University Press, 2013.
- Agustín Rayo and Stephen Yablo. Nominalism through de-nominalization. *Noûs*, 35(1):74–92, 2001. doi: 10.1111/0029-4624.00288.
- Gideon Rosen. What is normative necessity? In Mircea Dumitru, editor, *Metaphysics, Meaning, and Modality: Themes from Kit Fine*. Oxford University Press, forthcoming.
- Bertrand Russell. Letter to Frege, 12 December 1904. 1904.
- Mark Schroeder. Two roles for propositions: Cause for divorce? *Noûs*, 47(3): 409–430, 2013. doi: 10.1111/j.1468-0068.2011.00833.x.
- Ted Sider. Higher-order metametaphysics. MS of July 20, 2022.
- Theodore Sider. *Writing the Book of the World*. Oxford, England: Oxford University Press, 2011.
- Scott Soames. Direct reference, propositional attitudes, and semantic content. *Philosophical Topics*, 15(1):47–87, 1987. doi: 10.5840/philtopics198715112.
- Timothy Williamson. *Modal Logic as Metaphysics*. Oxford, England: Oxford University Press, 2013.
- Ludwig Wittgenstein. *Tractatus Logico-Philosophicus (Trans. Pears and McGuinness)*. Routledge, 1961.