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Fuzzy Networks for Modeling Shared Semantic Knowledge

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Abstract

Shared conceptualization, in the sense we take it here, is as recent a notion as the Semantic Web, but its relevance for a large variety of fields requires efficient methods of extraction and representation for both quantitative and qualitative data. This notion is particularly relevant for the investigation into, and construction of, semantic structures such as knowledge bases and taxonomies, but given the required large, often inaccurate, corpora available for search we can get only approximations. We see fuzzy description logic as an adequate medium for the representation of human semantic knowledge and propose a means to couple it with fuzzy semantic networks via the propositional Łukasiewicz fuzzy logic such that these suffice for decidability for queries over a semantic-knowledge base such as "to what degree of sharedness does it entail the instantiation $C(a)$ for some concept C" or "what are the roles R that connect the individuals a and b to degree of sharedness ε."

Keywords: Shared conceptualization; Directed graph; Description logics; Fuzzy logic; Fuzzy semantic network

1. Introduction

Semantic networks have for long been considered adequate means to represent meaning in natural languages (e.g., Richens 1956, 1958) and the way humans store meanings in their long-term memory in a structured way (e.g., Collins & Loftus 1975; Collins & Quillian 1969; Quillian 1967, 1968, 1969). With the advent of AI robotics, these networks soon appeared as important tools in providing robots with precise knowledge of meanings, in order to have them operate in the desired ways (e.g., Lehmann 1992). After the original work of L. A. Zadeh in fuzzy set theory and fuzzy logic (Zadeh 1965, 1975), the utility of this formalism for the representation of vagueness in the structure of human meanings was quickly captured in the creation of fuzzy semantic networks (e.g., Saitta 1978).

All these networks for representing human semantic knowledge neglect the crucial fact that this is a shared business, namely as far as conceptualization is concerned. In effect, concepts and relations between concepts, which constitute the basic elements of meanings for humans, find their place in a semantic network according to the degree of sharing of conceptualizations in a specific community at a specific period (see Augusto & Badie forthcoming). The importance of shared conceptualization became only recently evident, namely with the creation of the internet and more specifically of the semantic web, but it ranges over an impressively large variety of fields.[1] We elaborate on how semantic networks can represent adequately the fuzzy business of shared conceptualization.

Although fuzzy semantic networks are not recent constructs, a Google search returns a surprisingly low number of occurrences for the search "fuzzy semantic network"; 2350 to be precise, in a search in March 2021, many of which are for neural networks. A combined search for "fuzzy semantic network" and "description logic" returns a negligible result (8!) in the same date. Thus, even though there are some published articles on logic-based semantic networks (e.g., Andreasen 1997), our combination of description logic and fuzzy formalisms in an approach to semantic networks is, if not novel, certainly original. Importantly, this combination provides semantic networks with an adequate formal semantics, and their application to the analysis of shared conceptualizations connects concepts to the world rather than only to other concepts; we thus dismiss an influential critique by Johnson-Laird et al. (1984).

2. Shared Conceptualizations[2]

What is the semantic knowledge of an European with respect to breakfast as expressed in typical food items for this meal? If they are not British, it is very likely that BREAD & BUTTER will come to their minds when asked what it is they associate with this meal. They may even say outright "bread and butter," (or the equivalent expression in any other European language), but not necessarily so, as humans can represent concepts in a wholly unconscious way (e.g., Augusto 2013) and actually even if they are averbal (e.g., Lecours & Joanette 1980). Importantly, non-British Europeans who themselves do not eat bread and butter for breakfast are also likely to output BREAD & BUTTER, as they are familiar with people who eat these food items for breakfast, have often read texts or watched films in which Europeans had them, etc. This is what constitutes *shared conceptualization*, the semantic knowledge acquired either by acquaintance or by description—to adapt rather loosely an old philosophical distinction (Russell 1912)—that concepts and instantiations thereof have specific properties (e.g., "bread is a food item," "butter is yellow") and are interconnected by specific relations (e.g., "butter is spread on bread").

Despite our remark above with respect to averbal humans we shall consider that for humans exhibiting normal verbal behavior (many) concepts are associated to specific words; for instance, someone representing the concepts BREAD and BUTTER can utter or write the words "bread" and "butter." This allows for investigation on shared conceptualization to be carried out over extensive corpora available on the Internet (e.g., OWL or RDF bases). In our theoretical approach, we do not distinguish between written and audio databases.

[1] For instance, in Badie & Augusto (2022) we show its relevance for the detection of singularities in human semantic processing.

[2] We discuss shared conceptualization here only too briefly, referring the reader to Augusto & Badie (forthcoming) for an elaboration.

3. Fuzzy Semantic Networks for Shared Conceptualization

3.1. From fuzzy digraphs to fuzzy semantic networks

A directed graph (abbr.: digraph) is a triple $\mathcal{G} = (V, E, f)$ where $V = \{V_1, \ldots, V_n\}$ is a set of vertices, or nodes, $E = \{E_1, \ldots, E_k\}$ is a set of directed edges (arcs), and $f : E \rightarrow \{tail, head\}$ is a function assigning to each arch a tail and a head, called endpoints, such that arc $E_i = \overrightarrow{V_j V_l}$ connects the vertices V_j (tail) and V_l (head). For graphical convenience, we write arc E_i as the ordered pair $\langle V_j,V_l \rangle$. A *semantic network* is a labeled digraph where V is a set of concepts (e.g., FOOD, BREAKFAST, BREAD, BUTTER) and the E_i are labeled with relations r between the concepts ("is a," "is spread on," etc.), so that we have $\langle V_j,V_l \rangle_r$ denoting that the vertices $\langle V_j,V_l \rangle$ are connected by means of relation r. Furthermore, for computability reasons, we shall consider a semantic network to be a simple digraph, i.e. a graph without loops (arcs connecting a vertex to itself), despite the fact that a semantic network is often a partial order, i.e. as implementing relations r that are reflexive ("X*r*X"), anti-symmetric (if "X*r*Y" and "Y*r*X", then X = Y), and transitive (if "X*r*Y" and "Y*r*Z", then "X*r*Z").[3] This allows us to consider semantic networks from other relevant formal perspectives such as lattice theory and Galois connections, which are useful for ontologies and other formal concept analyses (see, e.g., Ganter & Wille 1999).

A *fuzzy semantic network* just is a digraph $\mathcal{G}_{fuzzy} = (V, E, f_{fuzzy})$ where $f_{fuzzy} : E \rightarrow (\{tail, head\} \times (0,1])$, i.e. f_{fuzzy} additionally assigns to every relation r a value $\varepsilon \in (0, 1]$ such that every arc of $\mathcal{G}_{fuzzy}$ is of the form $\langle V_j,V_l \rangle_{r,\varepsilon}$ denoting that the relational arc $\langle V_j,V_l \rangle_r$ is labeled with the fuzzy value ε. The interval [0, 1] of the reals has infinite cardinality, so that each arc of $\mathcal{G}_{fuzzy}$ can be labeled with any of many infinite values in the infinite interval (0, 1]. It is the latter property that makes of $\mathcal{G}$ a fuzzy labeled digraph $\mathcal{G}_{fuzzy}$.

3.2. Valuating shared conceptualizations

In a semantic network, the pair $\langle V_j,V_l \rangle_r$ typically denotes the direction (sub-)concept → (super-)concept of the relation "is a", but may also label relations such as "is adequate for," "takes place in," etc., so that virtually any relation between concepts as expressible in a natural language can be labeled by means of f in $\mathcal{G}$. In other words, a semantic network is a formally adequate means to represent diagrammatically human semantic knowledge. This is not always so, though; for instance, while most birds fly, penguins do not, so that an arc labeled "has motion type" joining the concepts BIRD (tail) and FLY (head) requires restrictions or further information. This caveat is particularly important in the case of shared conceptualizations, as these are but *approximations* in the sense that, say, BREAD and BUTTER are associated with BREAKFAST by only so many people in so many specific situations. For instance, even though one may eat bread and butter for breakfast (let us call this the *context*, or *domain of speech*), one is unlikely to associate these two breakfast food items explicitly when, say, scuba diving; on the other hand, when writing a text on scuba diving one may use the words "bread," "butter," and "breakfast," without for that combining them in the specific association "BREAD and BUTTER are BREAKFAST FOOD." Additionally, texts in which this association is explicitly made may contain alternative words or synonyms (e.g., "baguette", "margarine"), typos (e.g., "bead"), or even spelling mistakes (e.g., "buter") that will not feature in the results for the search. All this, together with many other factors, entails that any combined search for these words will return results that only in part hold

[3] For example, where *r* is the relation "is a", we have: "CAT is a CAT"; if "MARE is a FEMALE HORSE" and "FEMALE HORSE is a MARE", then MARE = FEMALE HORSE; if "APPLE is a FRUIT" and "FRUIT is a FOOD", then "APPLE is a FOOD".

as the shared conceptualization. For this reason, we see fuzzy formalisms as adequate for semantic networks for shared conceptualizations.

To obtain valuations—that can be seen as *de-facto* truth values (see Augusto 2020b)—in the interval [0, 1] we compute the frequency of a concept (i.e. its associated lexical item, or word) with respect to other concepts by means of the computation $x = n(C_1) / 100(C_2)$, where $n \leq 100$ is the number of occurrences of a concept C_1 for every 100 occurrences of some other concept of interest C_2. For instance, to obtain a fuzzy value for the shared valuation of the concept LOAF with respect to BREAD, which can be specified as the relation "LOAF is a BREAD", we carry out a search in as large a corpus as possible to determine the ratio $x = n(\text{loaf}) / 100(\text{bread})$. Yet another example: To find the fuzzy valuation for the relation "BUTTER is spread on BREAD", we compute $x = n(\text{butter + bread}) / 100(\text{spread})$. Clearly, this search can only give us approximate values, as the words in the corpus of search may not be there with these exact, explicit relations.

3.3. Description logic and knowledge bases

Description logic (DL) is widely considered as an adequate medium to represent semantic knowledge (e.g., Badie, 2018), including the terminological knowledge of individual cognitive agents (e.g., Badie, 2017, 2020a, 2020b, 2021, 2022). Instead of the *propositions* of predicate logic (PL), DL's largest complete expressions are *descriptions*. We use here the simplest decidable propositional DL, ALC (Attributive Concept Language with Complements), whose language, or description set Σ^*, is built over the Roman alphabet together with the set of logical constants $Cons = \{\neg, \sqcup, \sqcap, \sqsubseteq, \equiv, \forall, \exists\}$.[4]

Terminological axiom	Formal representation	description
concept inclusion	$C_1 \sqsubseteq C_2$	C_1 is subsumed under C_2.
role inclusion	$R_1 \sqsubseteq R_2$	R_1 is subsumed under R_2.
concept equality	$C_1 = C_2$	C_1 is equal to C_2.
role equality	$R_1 = R_2$	R_1 is equal to R_2.

Table 1. Fundamental Terminological Descriptions in DL

ALC represents semantic knowledge in terms of (possibly infinite) sets of (i) *individuals* $\{a_1, a_2, \ldots\}$ (which are equivalent to constant symbols in PL), (ii) *concepts* $\{C_1, C_2, \ldots\}$ (which are equivalent to unary predicates in PL), and (iii) *roles* $\{R_1, R_2, \ldots\}$ (which are equivalent to binary predicates in PL and can be either relations or properties). More specifically, a role expresses a relationship between individuals or it assigns a property to an individual. In DL, there are three kinds of *fundamental* symbols: (i) individuals (e.g., *pita*), (ii) atomic concepts (e.g., *Bread*), and (iii) atomic roles (e.g., *isA*). Atomic symbols are elementary descriptions from which we inductively build complex (more-specified) descriptions of the world. Typically, a DL terminological knowledge base is constituted by a TBOX, containing terminological axioms and

[4] As usually, Σ^* denotes a possibly infinite set of strings or, in logical jargon, formulae. *Cons* is actually a subset of the logical constants of ALC; on the other hand, we include in it the existential and universal quantifiers, despite the negligible use we make of them here, as ALC is a propositional DL.

definitions corresponding by-and-large to predicate formulas of the type $\forall x\ (Human(x) \rightarrow Mortal(x))$ and $\exists y\ (Fruit(y) \land Vegetable(y))$. A TBOX is structured based on terminological axioms, the most fundamental strings describing the underlying terminologies and vocabularies. Concepts and their interrelationships are, in the form of hierarchical structures, used to create a terminology in a TBOX. Table 1 presents terminological axioms in DL. Fig. 1 shows an example of a TBOX.

BAKERY ≡ BAKED_DOUGH ⊓ (SALTED ⊔ SWEET)

BREAD ⊑ BAKERY

DAIRY ≡ MILK ⊓ (SOFT_SOLID ⊔ HARD_SOLID ⊔ LIQUID)

SPREAD ≡ (DAIRY ⊔ VEGETABLE) ⊓ SOFT

BUTTER ⊑ SPREAD ⊓ ¬ VEGETABLE

MEAL ≡ ∃eats.LARGE_GROUP ⊓ (MORNING ⊔ NOON ⊔ EVENING)

BREAKFAST ⊑ MEAL ⊓ MORNING

LUNCH ⊑ MEAL ⊓ NOON

DINNER ⊑ MEAL ⊓ EVENING

Figure 1: TBox for BREAKFAST.

Semantic knowledge can be described and represented in ABOX to give us instantiations of the concepts and relations of interest, called *assertions*. DL assertional axioms (which are the most fundamental descriptions of the real world) are presented in table 2.

Assertional Axiom	Formal representation
concept assertion	$C(a)$
role assertion	$R(a, b)$

Table 2. Fundamental Assertional Descriptions in DL

$C(a)$ represents that individual a is an instance of concept C (i.e. "a is a C."; formally: $a : C$). Also, $R(a, b)$ expresses the property that a and b are related together by means of $R((a,b) : R)$. Fig. 2 shows an ABOX for the TBOX in Fig. 1.[5]

3.4. A fuzzy formal semantics for ALC

The formal semantics for the above constructs expressed in ALC is built on an *interpretation* $I = (\Delta^I, \cdot^I)$ given some non-empty set Δ^I (that is the *interpretation domain* and consists of any

[5] We abbreviate roles of the form $isA(x)$ as $A(x)$.

individual that may occur in our concept descriptions), and an *interpretation function* "$.^I$" that assigns to every individual symbol a an element $a^I \in \Delta^I$, to every atomic concept symbol A a set $A^I \in \Delta^I$, and to every atomic role symbol r a binary relation $r^I \subseteq \Delta^I \times \Delta^I$.

Bread(loaf)	*isSpreadOn(butter, bread)*
Bread(pita)	*isSpreadOn(jam, bread)*
Bread(baguette)	*isSpreadOn(margarine, bread)*
Bread(croissant)	*isEatenFor(bread, breakfast)*
Spread(butter)	*isEatenFor(butter, breakfast)*
Spread(jam)	*isEatenFor(bread, lunch)*
Spread(margarine)	*isEatenFor(bread, dinner)*
Dairy(cheese)	*isEatenFor(pita, breakfast)*
Dairy(yoghurt)	
Dairy(butter)	

Figure 2: The ABox for BREAKFAST.

Let α be an ALC (concept or role) assertion; a *fuzzy assertion* (denoted by ψ) is an expression having the form "α_ε", where the value $\varepsilon \in (0, 1]$ is computed as above.[6] An interpretation I satisfies fuzzy assertions "$(a\colon \mathrm{C})_\varepsilon$" and "$[(a, b)\colon \mathrm{R}]_\varepsilon$" iff $C^I(a^I) = \varepsilon$ and $R^I(a^I, b^I) = \varepsilon$, respectively, whenever $\varepsilon \in (0, 1]$, where by the superscript I a fuzzy valuation function $val_f : \Sigma^* \rightarrow [0, 1]$ in I is implicit. In addition, $C^I(a^I) = \varepsilon$ is defined as $a^I \in_\varepsilon C^I$, and $R^I(a^I, b^I) = \varepsilon$ is so as $(a^I, b^I) \in_\varepsilon R^I$. We say that the symbol "$\in_\varepsilon$" denotes *fuzzy membership*, because it is obtained by means of the *fuzzy membership function* $\mu : \Delta^I \rightarrow [0, 1]$ such that we have the following for the universal and existential quantifiers for $\varepsilon \in (0, 1]$:

$$(\forall R._\varepsilon C)^I = \{a \in_\varepsilon \Delta^I \mid \forall b.\ (a, b) \in_\varepsilon R^I \Rightarrow b \in_\varepsilon C^I\}, \text{ and}$$

$$(\exists R.\varepsilon)^I = \{a \in_\varepsilon \Delta^I \mid \exists b.\ (a, b) \in_\varepsilon R^I\}.$$

As above, it is by means of definitions over, or identities with, the relations and operations of set theory that ALC is provided with an adequate formal semantics. Let us represent fuzzy concept (role) subsumption by "$C_1 \preccurlyeq C_2$" ("$R_1 \preccurlyeq R_2$", respectively), and concept (and role) equivalence by "$C_1 \equiv C_2$" ("$R_1 \equiv R_2$", respectively). We have the following axioms for the fuzzy valuations based on the *fuzzy subsumption function* $\sigma : (\Delta^I \times \Delta^I) \rightarrow [0, 1]$:

i. $(C_1 \preccurlyeq_\varepsilon C_2)^I = C_1^I \subseteq_\varepsilon C_2^I$. The symbol "$\preccurlyeq_\varepsilon$" denotes here *fuzzy concept subsumption*.
ii. $(R_1 \preccurlyeq_\varepsilon R_2)^I = R_1^I \subseteq_\varepsilon R_2^I$. The symbol "$\preccurlyeq_\varepsilon$" denotes here *fuzzy role subsumption*.
iii. $(C_1 \equiv_\varepsilon C_2)^I = C_1^I =_\varepsilon C_2^I$. The symbol "$\equiv_\varepsilon$" denotes here *fuzzy concept equivalence*.
iv. $(R_1 \equiv_\varepsilon R_2)^I = R_1^I =_\varepsilon R_2^I$. The symbol "$\equiv_\varepsilon$" denotes here *fuzzy role equivalence*.

[6] Traditionally, ε denotes a value that can be as arbitrarily small as required.

Furthermore, and more briefly now for *fuzzy conjunction*, *disjunction*, and *negation*, we have the identities

v. $(C_1 \sqcap_\varepsilon C_2)^I = C_1^I \cap_\varepsilon C_2^I$.
vi. $(C_1 \sqcup_\varepsilon C_2)^I = C_1^I \cup_\varepsilon C_2^I$.
vii. $(\neg_\varepsilon A)^I = \Delta^I \setminus_\varepsilon A^I$.

3.5. Fuzzy sharedness consequence relation

Given this semantics, which provides DL with logical adequacy, i.e. both soundness and completeness (see Straccia 2005), we can obtain reliable replies to queries on the knowledge base such as "to which sharedness degree ε is a a member of C?" (*shared instantiation problem*) and "to which sharedness degree ε does C_2 subsume C_1?" (*shared subsumption problem*). But we also want to obtain replies to other queries such as "does C hold, and if so, to what sharedness degree, in the knowledge base?" (*shared concept satisfiability*), "what are the instances a of C to sharedness degree ε?" (*shared retrieval*), "what are the concepts C such that a is a shared instantiation to degree ε?" (*shared realization*), "what are the elements a and b such that they are related by means of R to sharedness degree ε?" (*shared role retrieval*), and "what are the roles R such that a and b are a shared instantiation to degree ε?" (*shared role realization*). Just as in all other logical systems, DL replies to these queries by means of its own notion of *logical consequence*. The affirmative replies to these queries are, respectively, for a KB = TBox ∪ ABox:

$$\text{KB} \vDash_\varepsilon (a{:}\, C)$$

$$\text{KB} \vDash_\varepsilon (C_1 \preccurlyeq C_2)$$

$$\text{KB} \vDash_\varepsilon C$$

$$\{a \mid \text{KB} \vDash_\varepsilon (a{:}\, C)\}$$

$$\{C \mid \text{KB} \vDash_\varepsilon (a{:}\, C)\}$$

$$\{(a,b) \mid \text{KB} \vDash_\varepsilon ((a, b){:}\, R)\}$$

$$\{R \mid \text{KB} \vDash_\varepsilon ((a, b){:}\, R)\}$$

where $\vDash_\varepsilon$ denotes the *fuzzy sharedness relation of semantic logical consequence* (*fuzzy sharedness consequence*, for short) defined below, after some preliminary definitions:[7]

Let a *fuzzy shared-conceptualization valuation* be the function $val_f : \Phi \rightarrow [0, 1]$ such that for a set of DL expressions $\Phi = \{\varphi_1, ..., \varphi_k\}$ and for some arbitrary expression φ in some interpretation I we have $val_f\ (\varphi)^I \in (0, 1]$ if and only if $\varphi^I \in_\varepsilon [\Delta^I(\Phi^I)]$, i.e. the interpreted expression φ is valuated as some ε in the interval (0, 1] according to its valued membership in the interpreted domain for Φ, denoted by "$\Delta^I(\Phi^I)$", in the same interval; otherwise, $val_f\ (\varphi)^I = 0$,

[7] Let $\Phi = \{\varphi_1, ..., \varphi_k\}$ be a set of logical formulae—a knowledge base—and φ a logical formula. The *semantic relation of logical consequence* $\vDash \subseteq 2^\Phi \times \Phi$ holds, written $\Phi \vDash \varphi$, if and only if there is an interpretation I, called a *model*, such that $val_I\ (\varphi_1 \wedge ... \wedge \varphi_k) = 1$ and $val_I\ (\varphi) = 1$. See Augusto (2020a) for details of the central notion of logical consequence. We assume the reader is familiar with the logical jargon (valuation, etc.).

whenever φ is not a member of the interpreted domain. Let further the conjunction of all the expressions in Φ be valuated as

$$(*) \qquad val_f(\varphi_1 \sqcap \ldots \sqcap \varphi_k)^I = val_f\left(\prod_{i=1}^{k} \varphi_i\right)^I = \{val_f(\varphi_i)^I\}$$

Then, given a KB = $\{\varphi_1, \ldots, \varphi_k\}$, we write KB $\vDash_\varepsilon \varphi$, and say that φ holds to degree of sharedness ε in the knowledge base KB if $val_f(\varphi)^I \geq \varepsilon$ whenever (*) = ε for $0 < \varepsilon \leq 1$.

This general definition can be specified for the cases above as follows[8]:

i. We say that a knowledge base KB entails a concept *C* to degree ε of shared conceptualization (henceforth just degree), and we write KB $\vDash_\varepsilon C$, if $val_f(C)^I \geq \varepsilon$ whenever (*) = ε for $0 < \varepsilon \leq 1$. Otherwise, we write KB $\nvDash_\varepsilon C$, or simply KB $\nvDash C$.
ii. We say that a knowledge base KB entails the subsumption $C_1 \sqsubseteq C_2$ to degree ε, and we write KB $\vDash_\varepsilon (C_1 \preccurlyeq C_2)$, if $val_f(C_1 \sqsubseteq C_2)^I \geq \varepsilon$ whenever (*) = ε for $0 < \varepsilon \leq 1$, where $val_f(C_1 \sqsubseteq C_2)^I$ is the shared valuation for the role "is a" in which C_1 and C_2 are the sub- and super-concepts, respectively. Otherwise, we write KB $\nvDash_\varepsilon (C_1 \preccurlyeq C_2)$, or simply KB $\nvDash (C_1 \sqsubseteq C_2)$.
iii. We say that a knowledge base instantiates "*y*:*C*" as "*a*:*C*" to degree ε, and we write "KB $\vDash_\varepsilon$ (*a*: *C*)", if $val_f(\varphi)^I \geq \varepsilon$ whenever (*) = ε for $0 < \varepsilon \leq 1$. Otherwise, we write "KB $\nvDash_\varepsilon$ (*a*: *C*)", or simply KB $\nvDash C(a)$.

In the following examples, we shall assume, for simplicity, that (*) for our knowledge base (Figs. 1-2) is a value ε > 0. (i) Suppose that a search for "bread" in the domain "Food and meals" resulted in 0.95. Then, we have KB $\vDash_{0.95}$ *Bread*. (ii) Suppose that for the subsumption *Breakfast* ⊑ *Meal* we have obtained the shared degree 0.89; then we have KB ⊨ (*Breakfast* $\preccurlyeq_{0.89}$ *Meal*). (iii) Let us suppose that we obtained the shared degree 0.2 for the role *isEatenFor*(*pita*, *breakfast*). Then, we have KB $\vDash_{0.2}$ ((*pita*, *breakfast*): *isEatenFor*).

Importantly, a fuzzy semantic network for shared conceptualization is *a model* of the associated knowledge base, as only concepts and relations are represented (assertions of the ABox) that actually hold (i.e. for which $\varepsilon \in (0, 1]$) as instantiations of the definitions and axioms of the TBox.[9] A node with no incoming or outgoing arcs denotes a member of the domain that, in that particular instantiation of the knowledge base, has no relations to the other nodes.

3.6. Fuzzy degrees of sharedness for negation, conjunction, and disjunction

In our fuzzy DL for shared conceptualization, we define the shared-conceptualization valuations in some interpretation *I* for the connectives ¬, ⊓, and ⊔, as follows, where φ, χ denote arbitrary DL expressions as in Table 2:

$$\left(\neg_\varepsilon(\varphi)\right)^I = 1 - val_f(\varphi)^I$$

[8] We give only a few cases: the reader can easily see how the remaining cases are computed.
[9] The fuzzy value ε can be made as close as 1 as desired (e.g., $\varepsilon \in [0.5,1]$).

$$\left(\sqcap_{\varepsilon}(\varphi,\chi)\right)^{I} = \left\{val_f(\varphi)^{I}, val_f(\chi)^{I}\right\}$$

$$(\sqcup_{\varepsilon}(\varphi,\chi))^{I} = \left\{val_f(\varphi)^{I}, val_f(\chi)^{I}\right\}$$

so that we have (*) above and also

$$\left(\left(\coprod_{i=1}^{k} \varphi_i\right)_{\varepsilon}\right)^{I} = \left\{val_f(\varphi_i)^{I}\right\}$$

These valuations, also in Straccia (1998, 2001), correspond to those of the Łukasiewicz fuzzy logic (but any t-norm fuzzy logic should be adequate, in principle; see Augusto 2020b). However, we consider that the operation of conjunction (of disjunction) in a semantic network is only defined when there is some C with *incoming* edges E_1, E_2 (*outgoing* edges E_1, E_2, respectively) labeled with fuzzy values such that we have either $E_1 \leq E_2$ or $E_2 \leq E_1$. For instance, suppose we have the fuzzy shared conceptualizations *Bread*(*loaf*) valuated as 0.95 (i.e. there is an arc from the node *Loaf* to the node *Bread* labeled "0.95") and *Bread*(*Croissant*) valuated as 0.2 (i.e. there is an arc from the node *Croissant* to the node *Bread* labeled with "0.2"). Then, by applying the above computation for ⊓, we obtain

$$val_f\left(\sqcap_{Bread}(Loaf, Croissant)\right)^{I} = \\ \left\{val_f\left(Bread(Loaf)\right)^{I}, val_f\left(Bread(Croissant)\right)^{I}\right\} = \\ \{0.95, 0.2\} = 0.2$$

In terms of shared conceptualization, we say that the shared *association* of the concepts *Loaf* and *Croissant* with respect to the concept *Bread* is of degree 0.2.[10] We proceed similarly for disjunction and speak of the shared *dissociation* of two or more concepts with respect to some other concept. As for negation, we compute, say, ¬*Bread*(*Croissant*) as

$$val_f\left(\neg_{Bread}(Croissant)\right)^{I} = \\ 1 - val_f\left(Bread(Croissant)\right)^{I} = \\ 0.8$$

and say that the degree of sharedness of the *rejection* of *Croissant* **with respect to** *Bread* is 0.8.

As the reader can easily see from looking at Fig. 3, all the above computations can be found directly from the fuzzy semantic network[11]. However, the utility of a logical language as a medium for representing knowledge, semantic or of other kind, resides in its ability *also* to find *implicit* knowledge. Here, again, a fuzzy semantic network meets this criterion: Because the elements of a fuzzy semantic network constitute a poset, we can find knowledge that is implicit in transitivity. For instance, in Fig. 3 we have $\langle Croissant, Bread\rangle_{isA,0.2}$ and $\langle Bread, Breakfast\rangle_{isEatenFor,0.89}$, so that we can extract the implicit knowledge that croissant is eaten for breakfast with a sharedness value 0.89. (Compare with $\langle Pita, Bread\rangle_{isA,0.63}$; because in this case

[10] A rather low degree, as most people do not consider a croissant to be a kind of bread.
[11] The degrees of sharedness shown are wholly hypothetical. We are here interested in the analysis of the fuzzy semantic network, and not in the veridicality of data.

we have the explicit knowledge $\langle \mathit{Pita},\mathit{Breakfast}\rangle_{\mathit{isEatenFor},0.2}$, we do not apply transitivity. Indeed, implicit knowledge should not conflict with explicit knowledge.)

Finally, information that is not to be read from the fuzzy semantic network is not assumed to be negative. For instance, according to Fig. 3 alone we cannot conclude $\mathcal{KB} \models \neg \mathit{isEatenFor}(\mathit{Baguette},\mathit{Dinner})$. In other words, we adopt here the *open-world assumption*, according to which what is not in the network is simply not known. The reply to the query $\mathcal{KB} \models^{?} \mathit{isEatenFor}(\mathit{Baguette},\mathit{Dinner})$ is simply "Unknown"; formally, $\mathcal{KB} \not\models \mathit{isEatenFor}(\mathit{Baguette},\mathit{Dinner})$.

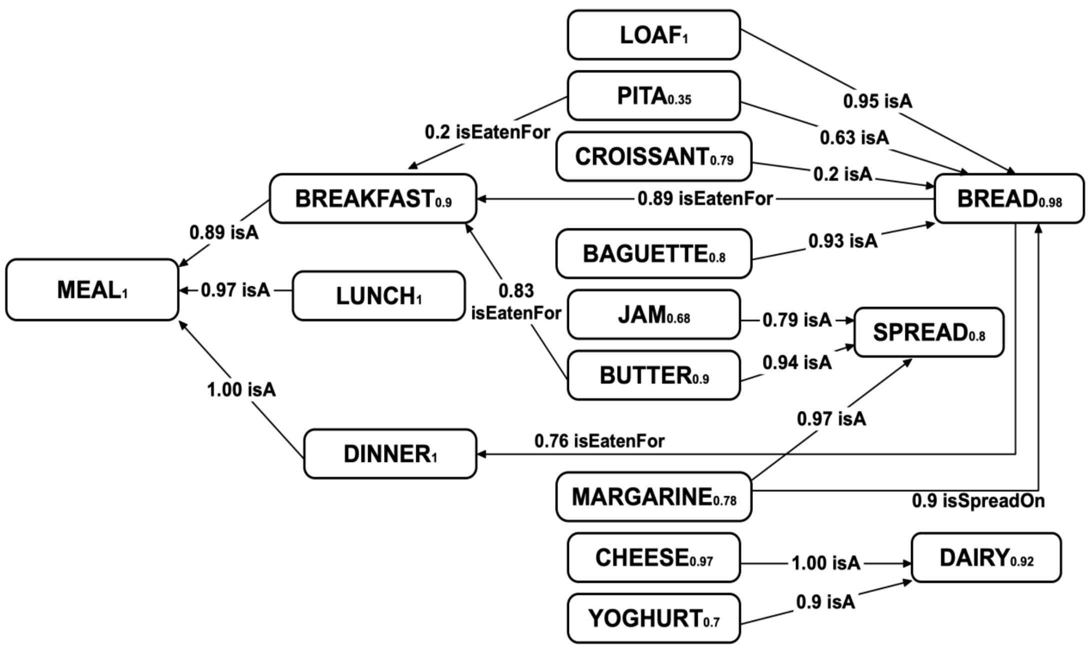

Figure 3 – Fuzzy semantic network for the ABox for BREAKFAST.

4. Applications

The advantages of having a fuzzy semantic network are many and we explore here only a few. To begin with we consider the building of representations in a continual learning setting. Foreign language teaching is a very complex process that requires the interaction of both vocabulary items with cultural aspects; besides being complex, it is also continuous, as it aims at recreating to some extent the conditions of native language learning. Consider Fig. 3 above; this semantic network can be used in at least two complementary ways: firstly, it can be used by the teachers as an aid to their teaching by providing them with the fuzzy valuations that can guide their teaching activities. For example, in classes taking place in the United Kingdom in which students learn about continental breakfast habits the teacher should emphasize the bread items with greatest fuzzy sharedness values (i.e., loaf and croissant), instead of those with very low values (pita). Additionally, teachers can use fuzzy semantic networks to check for their students' learning of food items by comparing the occurrence of specific items in their essays with those of the "canonical" semantic networks, and this at several times in the learning process.

Another way in which fuzzy semantic networks can be of import is in the clinical assessment of speech deficits. This is particularly the case in the condition known as formal thought disorder, in which irregularities at the level of the formation of individuals' semantic networks plays a central role (Augusto & Badie, 2022; Badie & Augusto, 2022). One of the main characteristics of this condition is that the semantic networks of the individuals affected by it diverge significantly from those of the majority of the members of their linguistic and cultural community as far as shared conceptualization is concerned. For instance, one finds in a semantic network of a given patient the pair ⟨Captain,Blades⟩Has, which in a typical semantic network would have a fuzzy degree of sharedness equal to zero. Of course, assessors of this condition need to consider a plethora of other symptoms, in order to be able to produce a positive diagnosis, but access to a fuzzy semantic network will be of much help to them.

5. Conclusions & Future Works

Given its increasing relevance for many domains (education, scientific taxonomies, e-commerce, etc.) efficient means to obtain quantitative and qualitative data with respect to shared conceptualization are required. This efficiency should be reflected in the fulfillment of a few criteria, such as reliability, decidability, and tractability. A quantitative approach can only provide us with approximate values, typically called "fuzzy" in the knowledge-representation literature. We propose a method to extract fuzzy values for semantic knowledge from large corpora that reliably reflect the degree to which concepts and relations between concepts are shared by a specific community or culture. The values obtained with this method are then applied in the construction of semantic networks that are in fact models of (fragments of) the associated knowledge bases formulated in ALC, the simplest description logic. ALC is well known to be decidable, so that reasoning on the knowledge bases is decidable, and in the semantic networks we couple it with the propositional Łukasiewicz fuzzy logic, which is also known to be decidable.

Obviously, the utility of semantic networks for the *direct visualization* of knowledge is limited, as they quickly become visually too complex. A semantic network is first and foremost a mathematical structure, to wit, a digraph, and as such search over it is faced with both structural and computational complexity issues. In any case, it can be implemented in an efficient algorithm, and the tractability of the semantic network search is then the tractability of the associated algorithm (see, e.g., Dehmer et al. 2019; Hunter & Kreutzer 2008). Importantly, we are interested in satisfiability alone, as validity in fuzzy logics in general is the case only for value 1, which is plausibly negligible for shared conceptualization as we discuss it here, in every model. SAT, i.e. the satisfiability problem, for the propositional Łukasiewicz fuzzy logic, is known since Mundici (1987) to be **NP**-complete.[12]

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[12] See Augusto (2020c) for the essentials of decidability and complexity theory. See Haniková (2011) for complexity of propositional fuzzy logics.

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