# Multiple-domain supervenience for non-classical mereologies

Ralf M. Bader
New York University

## 1 Introduction

The properties of complex entities generally stand in determination and dependence relations to the properties of the components from which they are made up.[1] This plausibly holds for material objects and the parts that compose them, for states of affairs and their constituents, for structural universals and the simple universals from which they are combined, for sets and their members, as well as for other complex entities, such as propositions, facts and properties. The relations that hold between these complexes and their constituents can be considered as falling under the category of parthood relations and can be characterised by means of different mereological systems, whereby many of these complex entities require non-classical mereological systems since they have much more structure than classical extensional mereology can capture.[2] The determination and dependence relations that hold between properties of complex entities and their constituents can be helpfully modelled by means of co-ordinated multiple-domain supervenience relations.[3] Making mereological supervenience claims when dealing with non-classical parthood relations requires taking into consideration ad-

[1] Which direction the determination and dependence relations go may well differ. Supervenience relations will be provided both for cases in which the direction of determination coincides with the direction of composition and for cases in which it coincides with the direction of decomposition (cf. section 4).

[2] Those who are uncomfortable with such a liberal notion of parthood can reinterpret the claims about parthood relations as claims about constitution, combination or concatenation relations. For the purposes of this paper nothing hangs on whether the different relations connecting complex entities and their constituents are all species of a common genus or whether they are fundamentally heterogeneous. All that matters is the formal features that characterise these relations and the dependence and determination relations to which they give rise. (For a pluralistic account of parthood that subsumes all these cases cf. Fine: 2010.)

[3] It is worth keeping in mind that the supervenience apparatus is limited in that it only allows us to model features concerned with property variation, leaving out important hyperintensional elements involved in determination and dependence relations that can only be captured by grounding relations.

ditional features about the ways in which entities are composed. This paper is concerned with developing multiple-domain supervenience relations for a number of non-classical parthood relations, allowing us to capture the determination and dependence relations that hold in non-classical mereological systems.

# 2 Classical mereological supervenience

## 2.1 Classical extensional mereology

Classical extensional mereology (CEM) is structurally identical to a complete Boolean algebra without the null element. It is characterised by a number of commitments that are rejected by the non-classical systems that we will be considering:

1. UNIQUENESS OF COMPOSITION: CEM is an extensional system insofar as sameness of parts implies sameness of whole, ensuring that any collection of parts can only compose a single whole that has all of them as parts.

$$\forall z(z \ll x \leftrightarrow z \ll y) \rightarrow x = y$$

2. NON-HIERARCHICAL: CEM is a 'flat' mereological system. Objects do not have any interesting mereological structure. They only differ in terms of the number and nature of parts of which they are composed, but not in the way in which they are composed. CEM does not recognise different ways in which parts can compose objects. The resulting mereological hierarchy consists of levels that are specified via the mereological complexity of objects, which is determined by the number of parts that they have.

## 2.2 Mereological supervenience

Mereological supervenience relations are concerned with determination and dependence relations amongst properties of parts and wholes. Since wholes are distinct from their parts, the domain of objects instantiating the supervening properties is distinct from the domain of objects instantiating the subvening properties, requiring us to appeal to multiple-domain supervenience relations. Supervenience relations across multiple domains are meant to track determination and dependence relations connecting members of the subvening and supervening domains.[4] In order for us to be able to formulate interesting supervenience claims, the members of these domains must be connected by some co-ordination relation (R). The co-ordination relation connects members of the subvening domain to

[4]Multiple-domain supervenience relations were first investigated by Kim in 1988 (reprinted in Kim: 1993, chapter 7). The account he put forward has important similarities to the R-related pairs strategy that we will develop, though Kim's theory is not sufficiently general to accommodate all the supervenience claims with which we will be concerned.

members of the supervening domain, allowing us to track non-holistic determination and dependency relations.[5] If it is not merely the case that the supervening domain as a whole is determined by the subvening domain as a whole, but that particular members of the latter domain determine particular members of the former domain, then the domains are co-ordinated. This connection between the domains is modelled by a co-ordination relation, whereby this relation can be very flexible. It can be a one-one, many-one, one-many, many-many or variably polyadic relation. All that is required is that the xx's that instantiate the base properties (B-properties) and the yy's that instantiate the supervening properties (A-properties) must somehow be connected.[6]

## 2.3 R-related pairs

We can appeal to this co-ordination relation to partition the members of the subvening and supervening domains into R-related ordered pairs, in such a way that the first entry of each pair is a non-empty collection taken from the subvening domain whilst the second entry is a non-empty collection taken from the supervening domain, whereby these collections are connected by the co-ordination relation. In the case of mereological supervenience, the subvening domain is the domain of parts, the supervening domain is the domain of wholes and the co-ordination relation is the composition relation. We then end up with pairs consisting of parts as well as of the wholes that they compose. That is, an R-related pair has the following structure: $\langle$xx's|y$\rangle$, where the xx's are the parts that compose y.

We can then say that two pairs $\langle$xx's|y$\rangle$ and $\langle$xx*'s|y*$\rangle$ are B-indiscernible iff there is a B-preserving mapping $\Gamma$ from the xx's onto the xx*'s, such that any x that is one of the xx's has any B-property F iff the image of x under $\Gamma$ also has F.[7] Two pairs are A-indiscernible iff y has any A-property G iff y* also has G. Given these notions of indiscernibility for pairs, we can use the standard accounts of weak and strong supervenience, i.e. A-properties supervene on B-properties iff:

WEAK-MDS: for all worlds w, and all pairs P and P* in w, if P and P* are B-indiscernible in w, then they are A-indiscernible in w.

[5] Single-domain supervenience relations can be understood as dealing with cases where the subvening and supervening domains have the same members and where the co-ordination relation is the identity relation.

[6] Every member of the subvening domain must be in the domain of R or be one of a plurality that is in the domain of R and every member of the supervening domain must be in the codomain of R or be one of a plurality that is in the codomain of R.

[7] If the set of B-properties includes irreducibly plural properties, then the notion of B-indiscernibility must be supplemented by the condition that any sub-plurality of the xx's has any plural B-property F iff the image of the sub-plurality under $\Gamma$ also has F (where the image of a sub-plurality is the plurality of the images of the members of the sub-plurality).

STRONG-MDS: for all worlds w and w*, and all pairs P in w and P* in w*, if P in w and P* in w* are B-indiscernible, then P in w and P* in w* are A-indiscernible.

This account is thus analogous to the single-domain case, with the difference that in the multiple-domain case we map R-related pairs rather than individual objects, where B/A-indiscernibility of pairs is understood in terms of the first/second members of the pairs being B/A-indiscernible. Mereological supervenience then holds if R-related pairs consisting of parts and the wholes that they compose are such that if the parts are B-indiscernible then the wholes are A-indiscernible.

## 2.4 Associated isomorphisms

The strong version of single-domain global supervenience requires that all B-preserving isomorphisms are A-preserving. This criterion does not make sense when applied to the multiple-domain case since we have different mappings for the subvening domain and for the supervening domain. As a result, we have to provide criteria as to how B-preserving mappings of the subvening domain have to be related to A-preserving mappings of the supervening domain if multiple-domain supervenience is to hold.

NC-MDS: for all worlds w and w*, if there is a B-preserving mapping of the members of the subvening domains of w and w*, then there is an A-preserving mapping of the members of the supervening domains of w and w*.

This non-co-ordinated version (NC-MDS) is too weak to be of interest. We can see this by considering the example of a Cartesian dualist who wants to claim that the mental and the physical constitute two different domains, but that the mental nonetheless supervenes on the physical. Let us consider a case in which we have two worlds each consisting of four objects {a,b,c,d} and {a′,b′,c′,d′}, where c (c′) is the soul correlated with a (a′) and d (d′) is the soul correlated with b (b′). In this case, a and a′ have B-property $F_1$, b and b′ have B-property $F_2$, c and c′ have A-property $G_1$, and d as well as d′ have A-property $G_2$. Now, if we map a onto a′ and b onto b′, then we have ensured that the mapping of members of the subvening domain is B-preserving. If we map c onto c′ and d onto d′, then we also get a mapping of the members of the supervening domain that is A-preserving. NC-MDS consequently holds. Yet, this relation would also hold if c′ were to be $G_2$ and d′ were to be $G_1$ since we could then map c onto d′ and d onto c′ to get an A-preserving mapping of the members of the supervening domain. Accordingly, we can see that it is too weak, given that a determination relation should not be compatible with a permutation of the supervening properties whilst the base properties are being held fixed.[8]

[8]This permutation problem affecting the non-co-ordinated multiple-domain supervenience relation is analogous to the kind of problem affecting weak global supervenience (cf. Bennett:

To make substantive supervenience claims, we need to connect the domains and connect the mappings. We can do this by appealing to the notion of an associated isomorphism, which uses the co-ordination relation to connect mappings of members of the different domains.

ASSOCIATED ISOMORPHISM A one-to-one mapping of members of the supervening domain $\Gamma'$ from $D_S$ onto $D_{S^*}$ counts as an associated isomorphism of a mapping of members of the subvening domain $\Gamma$ from $D_B$ onto $D_{B^*}$, if it is the case that if any collection of members $x_1 \ldots x_n$ from $D_B$ is mapped onto $x_1^* \ldots x_n^*$ from $D_{B^*}$ by $\Gamma$, then $\Gamma'$ maps the images of $x_1 \ldots x_n$ under R in $D_S$, i.e. $y_1 \ldots y_n$, onto the images of $x_1^* \ldots x_n^*$ under R in $D_{S^*}$, i.e. $y_1^* \ldots y_n^*$.

This notion allows us to specify a co-ordinated version of global multiple-domain supervenience:

C-MDS: for all worlds w and w*, every B-preserving mapping of the members of the subvening domains of w and w* is such that the associated mapping of the members of the supervening domains of w and w* is A-preserving.[9]

This co-ordinated version is sufficiently strong to capture dependence and determination relations. In particular, we can use this version to make mereological supervenience claims, i.e. every B-preserving mapping of parts is such that the associated mapping of the wholes that they compose is A-preserving.

# 3 Non-classical mereological supervenience

Non-classical mereological systems reject various features of CEM. Such systems allow us, for instance, to characterise objects that are treated as hylomorphic compounds, to make sense of coinciding objects, as well as to describe the mereological makeup of complex structured entities such as propositions and states of affairs. In order to make mereological supervenience claims within these systems, we need to modify the multiple-domain supervenience principles that we have developed so far. In particular, we need to (i) change the domains, (ii) impose restrictions on mappings, (iii) modify the co-ordination relation, and (iv) alter the things that are being mapped.

Non-classical systems that reject extensionality allow for there to be different wholes that are composed of the same parts. This leads to situations in which the R-relation will not pick out a unique image in the supervening domain, allowing

2004).

[9] This version of co-ordinated multiple-domain supervenience presupposes that there is only one associated mapping of the members of the supervening domain, which amounts to presupposing that R always picks out a unique image. This presupposition is satisfied in the case of CEM insofar as composition is unique, but as we will see below there are non-classical systems in which it fails to be satisfied.

for failures of standard mereological supervenience insofar as indiscernibility of parts need not yield indiscernibility of wholes. We will be considering five kinds of non-extensional mereologies, namely non-hierarchical mereologies that incorporate (i) order-sensitive composition relations, (ii) repetition-sensitive composition relations, and (iii) many-many composition relations, as well as hierarchical mereologies that contain (iv) composition relations that give rise to compositional structure, and (v) hylomorphic composition relations that combine form and matter.

## 3.1 Order-sensitivity

Order-sensitive composition relations allow for there to be objects that are composed of the same parts but are nonetheless distinct since they are composed of these parts in different ways. In particular, the order in which the parts compose the objects can differ. This kind of composition can be found when dealing with structured entities, such as structural universals, facts, states of affairs and propositions. For instance, the states of affairs Rab and Rba both have as their constituents particulars a and b, as well as a non-symmetric relation R, but are nonetheless distinct states of affairs that have different properties. We can model such composition relations if we expand our mereological system by including an irreflexive, asymmetric, transitive relation $>$ that imposes a strict total ordering on the parts of an object. This enables us to represent the order in which parts compose a whole. In particular, we can let $[z_1{>}z_2] \ll x$ represent that $z_1$ and $z_2$ are both parts of x, whereby they compose x in such a way that $z_1$ is prior to $z_2$ in the order of composition. Accordingly, we can allow for cases in which x and y are distinct even though they are composed of the same parts since the parts are differently ordered, i.e. both x and y are composed of $z_1$ and $z_2$ but $x \neq y$ since $[z_1{>}z_2] \ll x$ but $[z_2{>}z_1] \ll y$. We can then specify a modified order-sensitive uniqueness claim, according to which any ordered collection of parts can only compose a single whole.

$$\forall z_1 \forall z_2([z_1 > z_2] \ll x \leftrightarrow [z_1 > z_2] \ll y) \rightarrow x = y$$

R-related pairs will not contain unordered pluralities of parts but ordered pluralities, i.e. map $\langle [x_1 > x_2 \ldots > x_n] | y \rangle$ onto $\langle [x_1^* > x_2^* \ldots > x_n^*] | y^* \rangle$, whereby such pairs are B-indiscernible and have likewise ordered parts iff there is a B-preserving mapping $\Gamma$ from the xx's onto the xx*'s that is order-preserving. That is, $\Gamma$ has to be such that (i) any x that is one of the xx's has any B-property F iff the image of x under $\Gamma$ also has F, and such that (ii) any pair of parts $x_1$ and $x_2$ that belong to the xx's stands in a particular order relative to the whole composed by the xx's, i.e. $[x_1{>}x_2] \ll y$, iff the images under $\Gamma$ stand in the same order relative to the whole composed by the xx*'s, i.e. $[\Gamma(x_1){>}\Gamma(x_2)] \ll y^*$.

If we fail to impose the condition that mappings have to be order-preserving, then we can have B-preserving mappings of members of the subvenient domain

but have associated mappings of members of the supervening domain that fail to be A-preserving. For instance, we can map the constituents of states of affairs Rab and Rba onto themselves, thereby ensuring that the mapping is B-preserving yet have an associated mapping in the supervening domain that fails to be A-preserving since we can map Rab onto Rba and Rba onto Rab. Both of these wholes in the supervening domain are images of the parts under the co-ordination relation but they differ with respect to A-properties.[10] (At least this can happen if the set of A-properties includes properties with respect to which Rab and Rba differ – as long as the A-properties are restricted to only those properties that are shared by all the wholes composed of the parts in question, e.g. properties shared by Rab and Rba, then mereological supervenience will hold even given the standard supervenience relation that is not order-sensitive.)

If we impose the constraint that mappings also have to be order-preserving, then the associated mappings of B-preserving isomorphisms will be A-preserving. There will be no order-preserving mapping that is B-preserving yet has an associated mapping that fails to be A-preserving. This is because there is no order-preserving mapping from the parts of Rab onto Rba. The difference in the order of composition can thus explain the differences in properties between Rab and Rba, i.e. even though they are made up of the same parts there is no failure of supervenience since the differences between them can be explained in terms of the order in which they are composed out of the parts that they share. The difference between Rab and Rba that gives rise to their A-discernibility is the order in which the B-indiscernible parts compose these different wholes. In the one case a whole is composed out of a, b, and R by having them as parts in one order, i.e. [a>b] $\ll$ Rab, while in the other case the whole is composed by having them as parts in a different order, i.e. [b>a] $\ll$ Rba.

## 3.2 Repetition-sensitivity

Repetition-sensitive composition relations are not extensional since they allow for there to be objects that are composed of the same parts but that are nonetheless distinct since they are composed of different numbers of occurrences of these parts. Repetition-sensitive parthood relations are important when dealing with structural universals, in particular if one wants to specify ratio structures for quantities.[11] If one wants to say that the properties of a structural universal are determined by the properties of the simple universals from which it is combined, then one has to make a supervenience claim that is sensitive to repetitions.

Here it might be suggested that repetitions show up in the decomposition and that while there is a sense in which objects can be distinct even though they are made up of the same parts (namely insofar as there is some level of decomposition

[10] This ensures that only WG-MDS but not SG-MDS holds (cf. section 3.3).

[11] For an account of quantities in terms of structural universals cf. Armstrong: 1988.

at which they have the same parts), there is another sense in which these objects will be made up of different parts (namely insofar as there will be decompositions of one object that are such that the other object lacks corresponding decompositions). There are accordingly two senses of what it means for objects to be made of the same parts:

SAME PARTS 1: x and y are made of the same parts if there is some level of decomposition at which they have the same parts. Being made of the same parts in this sense is compatible with there being some level of decomposition at which the objects are made of different parts.

SAME PARTS 2: x and y are made of the same parts if every decomposition of x has a corresponding decomposition of y, i.e. if there is a bijection $\Gamma$ from decompositions of x onto decompositions of y such that any part p features in any decomposition d of x iff p features in the decomposition of y that is the image of d under $\Gamma$. Being made of the same parts in this sense is clearly not compatible with there being some level of decomposition at which the objects are made of different parts.

When one is dealing with cases in which objects are made up of the same parts only in the former but not the latter sense, then the repetition shows up in the decompositions.

> E.g., the universals *being-3-grams* and *being-2-grams* are both composed of the universal *being-1-gram*, though they are composed of different numbers of occurrences of this universal, i.e. *being-3-grams* can be decomposed into three occurrences of the universal *being-1-gram* while *being-2-grams* can be decomposed into two occurrences of the universal *being-1-gram*. Yet, there exists a decomposition of *being-3-grams* into *being-2-grams* and *being-1-gram*, though there is no analogous decomposition for the universal *being-2-grams*.

The fact that repetitions can show up in this way can be used when making supervenience claims, i.e. one can specify the mereological supervenience principle such that if x and y are made of B-indiscernible parts at all levels of decomposition, then x and y are A-indiscernible. More precisely, if the decompositions of x and y can be put into one-to-one correspondence whereby there will be B-preserving mappings between the corresponding decompositions, then x and y will be A-indiscernible.

This proposal faces the problem of providing an account as to how decomposition is to be understood. In particular, it seems that one needs to appeal to non-classical decomposition principles specified in terms of occurrences since the standard account of a complete decomposition (according to which the xx's are a collection of non-overlapping parts of y that are such that there is no z that is a part

of y but that does not overlap the xx's) is not applicable to these cases. Yet, once one needs to bring in occurrences for making sense of the decomposition principles one can also appeal to occurrences to directly specify repetition-sensitive supervenience principles.

However, it is sometimes possible to specify mereological principles indirectly by mapping objects standing in non-classical parthood relations onto suitably related entities that have a classical mereological structure. For instance, when dealing with structural universals we can appeal to the classical mereological structure of their instances to specify decomposition principles for the universals, rather than having to appeal to non-classical decomposition principles for the property parts that are the constituents of these universals. This is possible in the case of structural universals because the parts that compose the structural universal are instantiated by the parts of the objects that instantiate the structural universal.

> E.g., a structural universal F can be completely decomposed into occurrences of universals $G_1 \ldots G_n$ iff every x instantiating F has a complete decomposition into parts $p_1 \ldots p_n$, where (i) each part $p_i$ instantiates exactly one of the universals that occurs in $G_1 \ldots G_n$, and where (ii) there is a bijection $\Gamma$ from parts $p_1 \ldots p_n$ onto the occurrences of universals $G_1 \ldots G_n$ such that any part $p_i$ instantiates universal $G_i$ iff the image of $p_i$ under $\Gamma$ is an occurrence of $G_i$.

While certain forms of repetition-sensitive composition relations can be accounted for in this way, there seem to be cases where a different approach is required. In particular, if one is dealing with a restricted repetition-sensitive mereology which holds that it is not possible to fuse fusions but only to fuse simples, then there will only be one level of decomposition consisting of decompositions into occurrences of simples, ensuring that all wholes that are made of the same parts will differ only in the number of occurrences and will not differ in terms of intermediate levels of decomposition since no such levels exist.

> E.g., if we only have one simple, namely x, then the ontology will consist of: x, fu(x,x), fu(x,x,x), fu(x,x,x,x) etc. However, the ontology will not include fu(x,fu(x,x)) etc. Every whole will accordingly have a unique decomposition consisting of a number of occurrences of x.

In such cases repetitions will not show up unless one distinguishes different occurrences of the same parts. Accordingly, one cannot simply impose the condition that every decomposition must have a corresponding B-indiscernible decomposition if wholes are to be A-indiscernible. Instead, one must specify mappings that preserve occurrences.

When dealing with repetition-sensitive composition, the subvening domain

cannot be understood as a set but must instead be conceived of as a multi-set.[12] Property-preserving mappings then have to preserve occurrences of parts. There must be a bijection between the occurrences of parts whereby any occurrence of x that is one of the xx's is mapped onto an occurrence of the image of x under $\Gamma$. Put differently, for every x that is one of the xx's, each occurrence $x_i$ of x has an image, namely $x_i^*$, that is an occurrence of the image of x, namely of x*.

## 3.3 Many-many composition

Coincident objects are distinct yet made up of the same parts (at some level of decomposition). There are two ways of understanding cases of coincidence. On the one hand, coinciding objects can be conceived of as structured entities that have a formal component that distinguishes them and explains their distinctness. Such hylomorphic accounts of composition will be considered in section 3.4 below. On the other hand, they can be understood as unstructured entities that result from many-many composition relations. It is this latter account, on which coinciding objects are composed of the same parts and on which composition lacks hylomorphic or compositional structure and is neither order-sensitive nor repetition-sensitive, that is at issue here.[13]

Cases of coincidence arise if the xx's compose a plurality of yy's. These cases can be modelled either by having a unique many-many co-ordination relation or by having a non-unique many-one co-ordination relation. In the former case, the composition relation connects a plurality of parts to a plurality of composite wholes. We then have R-related pairs that are such that both members of the pairs are pluralities, i.e. $\langle$xx's|yy's$\rangle$. In the latter case, the composition relation connects a plurality of parts to individual composite objects but fails to be unique, which means that the same plurality in the subvening domain can have multiple images under R in the supervening domain. We then have a plurality of R-related pairs that are such that it is only the first member of any pair that is a plurality, but this plurality of parts can be the first member of different pairs, i.e. $\langle$xx's|$y_1\rangle \ldots \langle$xx's|$y_n\rangle$. Strong supervenience principles for coinciding objects can be devised by adopting the former option, modelling many-many composition relations by means of many-many co-ordination relations rather than un-

[12] It should be noted that only the subvening domain has to be a multi-set. As in the case of order-sensitive composition, a modified uniqueness claim holds for repetition-sensitive composition ($\ll^n$ represents the number of occurrences of a part in a whole in an analogous way that the number of occurrences of a member in a multi-set are represented by $\in^n$):

$$\forall z(z \ll^n x \leftrightarrow z \ll^n y) \rightarrow x = y$$

This uniqueness principle ensures that we only have repetitions in the subvening domain and not in the supervening domain.

[13] For a more detailed account of the considerations presented in this section, as well as for a discussion of the philosophical ramifications and underpinnings, cf. Bader: manuscript.

derstanding R as a many-one relation that fails to pick out a unique image in the supervening domain. That way one will get a unique image, namely the plurality of objects composed by the parts.[14]

R-related pairs ⟨xx's|yy's⟩ and ⟨xx*'s|yy*'s⟩ that are co-ordinated by a many-many composition relation are B-indiscernible iff there is a B-preserving mapping Γ from the xx's onto the xx*'s, such that any x that is one of the xx's has any B-property F iff the image of x under Γ also has F, while they are A-indiscernible iff there is an A-preserving mapping Γ′ from the yy's onto the yy*'s, such that any y that is one of the y's has any A-property G iff the image of y under Γ′ also has G. Given this understanding of mereological supervenience, the properties of coinciding objects will supervene on the properties of their parts. Both WEAK-MDS and STRONG-MDS will hold if a many-many co-ordination is used to pick out the members of the R-related pairs.

Analogous results can be established when appealing to associated isomorphisms to devise global multiple-domain supervenience relations. We can note that the co-ordinated global version C-MDS that was defined earlier presupposes that R picks out a unique image in the supervening domain. This presupposition is met in CEM since the commitment to uniqueness of composition that is integral to classical mereology ensures that the composition relation always picks out a unique image. Yet, when we are dealing with non-extensional mereologies, a collection of members of the subvening domain can have multiple images under R in the supervening domain. In such cases we can distinguish two kinds of global multiple-domain supervenience relations:

WG-MDS: for all worlds w and w*, every B-preserving mapping of the members of the subvening domains of w and w* has an associated A-preserving mapping of the members of the supervening domains of w and w*.

SG-MDS: for all worlds w and w*, every B-preserving mapping of the members of the subvening domains of w and w* is such that all its associated mappings of the members of the supervening domains of w and w* are A-preserving.

These co-ordinated versions differ only if the co-ordination relation fails to be unique, i.e. if $x_1 \ldots x_n$ has a plurality of images under R in the supervening domain. In such cases, a particular mapping of the subvening domain will have a plurality of associated mappings, allowing us to distinguish between a weak version of multiple-domain supervenience that requires only that one of these associated mappings be A-preserving and a strong version that requires that all of them be A-preserving.[15]

[14] When dealing with accounts of coinciding objects that allow for mutual parthood, the subvening domain, i.e. the domain of parts, should be restricted to include only parts that satisfy the weak supplementation principle. (Thanks to Gabriel Uzquiano on this point.)

[15] The distinction between the three kinds of global multiple-domain supervenience relations,

When dealing with coinciding objects, WG-MDS will hold but SG-MDS will fail if a many-one co-ordination relation is used. Yet, when appealing to a many-many co-ordination relation both WG-MDS and SG-MDS will hold.

## 3.4 Hierarchical mereologies

Wholes in CEM only differ in terms of the parts of which they are composed, but do not differ in terms of the ways in which they are composed. This means that classical mereology does not recognise any interesting mereological structure. Hierarchical mereologies, by contrast, allow for there to be different ways in which parts can compose wholes. In such cases, properties of wholes do not supervene on properties of parts considered by themselves, but on the properties of the parts together with the manner of composition i.e. the properties of wholes are determined by the properties of parts together with their mereological structure. There are two ways of conceiving of mereological structure:

1. COMPOSITIONAL STRUCTURE: wholes can have internal structure that results from the way in which they are composed out of their parts. Different wholes can be composed of the same parts due to the way that the parts are structured, due to the way in which they go together to compose the different wholes.[16] For instance, if one rejects the principle that fusing something with a fusion is identical to fusing that thing with the parts of the fusion, then one can have four different wholes composed of x, y, and z since fu(x,y,z) $\neq$ fu(x,fu(y,z)) $\neq$ fu(y,fu(x,z)) $\neq$ fu(z,fu(x,y)).[17]

2. HYLOMORPHIC STRUCTURE: hylomorphic compounds have structure that derives from their formal elements, allowing for wholes to differ in the way that form and matter are combined in composition.[18] We can represent a hylomorphic compound of material parts $x_1 \ldots x_n$ and form F as $fu(x_1 \ldots x_n|F)$. We can then have different wholes that are composed of the same material parts but have different formal components, e.g. $fu(x_1 \ldots x_n|F) \neq fu(x_1 \ldots x_n|G)$. Moreover, we can have different wholes that are composed of the same material parts and that have the same formal components but that are such that the form binds the matter in different ways, e.g.

namely non-co-ordinated (NC-MDS), weak co-ordinated (WG-MDS) and strong co-ordinated (SG-MDS), corresponds to that between weak, intermediate and strong global single-domain supervenience.

[16] It should be noted that the compositional order at issue here is distinct from the order of constituents that is at issue when dealing with order-sensitive composition. These two forms of non-extensional composition can be combined as well as kept separate.

[17] If supplementation principles are rejected as well, then one can get a significant further increase in the number of ways in which objects can be composed since in that case x $\neq$ fu(x) $\neq$ fu(fu(x)) etc.

[18] For contemporary hylomorphic accounts cf. Fine: 1999, Johnston: 2006, Koslicki: 2008.

$fu(fu(x_1 \ldots x_i|F),fu(x_j \ldots x_n|G)|H) \neq fu(fu(x_1 \ldots x_i|G),fu(x_j \ldots x_n|F)|H)$. In such a case we have distinct wholes, even though the wholes have the same material parts, namely $x_1 \ldots x_n$, as well as the same formal components, namely F, G, and H, since they differ in the ways in which form binds matter. That is, there are different ways of composing wholes out of parts resulting from different form-matter combinations.

Compositional structure can be accounted for in a relatively straightforward manner when making mereological supervenience claims. All that needs to be done is to restrict the co-ordination relation R to the relation of immediate composition, thereby ensuring that any collection of parts has a unique image under R in the supervening domain. For instance, even though fu(x,fu(y,z)) and fu(y,fu(x,z)) have the same mediate parts, namely x, y and z, they have different immediate parts, the former being immediately composed of x and fu(y,z), while the latter is immediately composed of y and fu(x,z).[19]

The strategy of restricting the co-ordination relation to the relation of immediate composition runs into problems when dealing with dense parthood orderings since such orderings allow for objects to have mediate parts without having immediate parts. In these kinds of cases additional conditions need to be imposed to ensure that compositional structure is preserved. In particular, mereological structure must be preserved at all the intermediate levels of composition.

R-RELATED PAIRS: B-indiscernible R-related pairs $\langle$xx's$|$y$\rangle$ and $\langle$xx*'s$|$y*$\rangle$ have the same compositional structure iff the B-preserving bijection $\Gamma$ from the xx's onto the xx*'s is such that any sub-plurality of the xx's has an image under R that is a part of y iff the image under $\Gamma$ of the sub-plurality has an image under R that is a part of y*.

ASSOCIATED ISOMORPHISMS: A bijection $\Gamma$ of members of the subvening domain from $D_B$ onto $D_{B^*}$ preserves compositional structure if the xx's are mapped onto the xx*'s by $\Gamma$ only if it is the case that any sub-plurality of the xx's that has an image under R that is a part of y (i.e. of the image under R of the xx's that is being mapped by the associated isomorphism $\Gamma'$) is mapped onto a sub-plurality of the xx*'s that has an image under R that is a part of y* (i.e. of the image under R of the xx*'s that is the image under $\Gamma'$ of y).

Hylomorphic structure can be accounted for in an analogous way. There are again two options for preserving hylomorphic structure. One can either restrict R to the relation of immediate composition or impose the condition that the mappings are such that intermediate structure is preserved. In either case, the additional requirement must be satisfied that formal components be preserved

[19] If we want to say that A-properties of wholes do not just supervene on B-properties of their immediate parts but also on B-properties of their mediate parts, then problems arise unless the set of B-properties is a proper or improper subset of the set of A-properties, cf. section 4.

as well. This can be done by specifying that the corresponding images under R always be images that have the same formal components. In particular, B-indiscernible pairs P and P* have to be A-indiscernible only if y and y* have the same formal component. Similarly, an associated isomorphism $\Gamma'$ preserves hylomorphic form if it maps y onto y* (where these are images under R of the pluralities mapped by the base isomorphism $\Gamma$) only if they have the same formal component. When it comes to preserving intermediate mereological structure the modified condition in both cases is that any sub-plurality of the xx's has an image under R that is a part of y and that has any formal component F iff the image under $\Gamma$ of the sub-plurality has an image under R that is a part of y* and that also has formal component F.

# 4 Priority of parts/wholes

In some cases we might want to say that certain properties of wholes, such as distributional properties, fix certain properties of parts. Formal mereological systems are neutral regarding questions of priority. They characterise parthood structure but are silent on questions of ontological priority. Accordingly the idea that wholes are prior to their parts is not in conflict with CEM. In other words, one can adopt classical mereology yet identify the direction of determination not with the direction of the composition relation but with the direction of the decomposition relation.

In cases in which the direction of determination runs from wholes to parts rather than from parts to wholes, we need to appeal to the decomposition relation rather than the composition relation to co-ordinate the domains and let the subvening domain consist of wholes and the supervening domain of parts. Since most wholes have multiple decompositions, it follows that the members of the subvening domain will have multiple images in the supervening domain. The co-ordination relation R, which in this case is the decomposition relation, will not pick out a unique collection of parts but will pick out multiple decompositions. Since there are multiple decompositions, these must be appropriately mapped if a supervenience claim is to hold.

FIRST PASS: one can restrict R such that it picks out not all decompositions but only atomic decompositions. If this is done then any whole will have a unique image under R consisting of the collection of atoms into which the whole can be decomposed. Accordingly, the mereological supervenience claim then states that if y and y* are B-indiscernible, then their atomic decompositions will be A-indiscernible, i.e. R-related pairs that are B-indiscernible will be A-indiscernible (the pairs have the form $\langle$y|xx's$\rangle$, where the xx's are the atoms into which y can be decomposed).

the xx's are an atomic decomposition of y $=_{df}$ (i) all of the xx's

> are parts of y, (ii) all of the xx's are mereological atoms, and (iii) there is no z that is a part of y that does not overlap the xx's

This account is problematic when one is dealing with gunky mereologies that allow for objects to be composed from atomless gunk. Moreover, it fails to capture the idea that properties of wholes fix properties of all of their parts and not just properties of their atomic parts. Accordingly, intermediate parts need to be included as well and R should not be restricted to pick out atomic decompositions.

SECOND PASS: one can try to take intermediate levels of the mereological hierarchy into consideration yet retain a unique image under R by restricting R to the immediate decomposition relation.

> the xx's are an immediate decomposition of y $=_{df}$ (i) all of the xx's are immediate parts of y, and (ii) there is no immediate part of y that is not one of the xx's[20]

Appealing to immediate parthood, however, does not provide us with a general solution since this is not applicable when one is concerned with dense parthood orderings. Additionally, this proposal faces problems in accounting for the idea that the A-properties of wholes fix the B-properties of all their parts and not just of their immediate parts. One might try to circumvent this problem by bringing in chains of immediate parthood relations. Yet this only works when the set of B-properties is a proper or improper subset of the set of A-properties since otherwise it will only be the case that the A-properties of the mediate parts (at one remove) of y supervene on the B-properties of the immediate parts of y and not on the B-properties of y itself. That is, unless the B-properties and not just the A-properties of immediate parts supervene on the B-properties of wholes, this way of making supervenience claims will not capture the idea that the A-properties of mediate parts also supervene on the B-properties of wholes.[21]

THIRD PASS: if y and y* are B-indiscernible, then every decomposition of y has a corresponding A-indiscernible decomposition of y*, i.e. if the xx's are a complete decomposition of y, then there exists a complete decomposition of y* into the xx*'s, such that the xx's and the xx*'s are A-indiscernible.

> the xx's are a complete decomposition of y $=_{df}$ (i) all of the xx's are parts of y, (ii) the xx's do not overlap each other, and (iii) there is no z that is a part of y that does not overlap the xx's

[20] It should be noted that the xx's will not classify as a complete decomposition of y in the traditional sense since the immediate parts of y will in most cases overlap.

[21] Analogous problems also affect the above-discussed attempts of accounting for compositional and hylomorphic structure by restricting R to the relation of immediate composition.

While an improvement over the other proposals insofar as all parts of wholes are considered, rather than just atomic or immediate parts, and insofar as there is no restriction to atomistic or non-dense mereologies, this account is nonetheless problematic since it does not preserve mereological structure. The different decompositions must be adequately connected. In particular, one needs to connect the different decompositions in such a way as to preserve mereological relations amongst them.[22]

FINAL PASS: if y and y* are B-indiscernible, then the images of y under R, i.e. the complete decompositions of y, will be A-indiscernible from the images of y* under R, i.e. the complete decompositions of y*, whereby the A-indiscernible decompositions are connected in such a way that mereological structure is preserved. This will be the case if there is a bijection $\Gamma$ from decompositions of y onto decompositions of y*, such that every decomposition of y will be A-indiscernible from its image under $\Gamma$, whereby the A-preserving mappings are co-ordinated in a structure-preserving way, i.e. any x that is one of the xx's of a decomposition of y is mapped onto x* by an A-preserving mapping $\Gamma'$ iff any image of x under R that is a member of a decomposition of y into the zz's is mapped onto the image under R of the image of x under $\Gamma'$, i.e. the image under R of x* that is a member of the zz*'s which form a decomposition of y* and are the image of the zz's under $\Gamma$.

# 5 Conclusion

Thus, we have seen that we can use co-ordinated multiple-domain supervenience relations to model determination and dependence relations between complex entities and their constituents. In particular, we developed two ways of making such supervenience claims, namely (i) by appealing to R-related pairs, and (ii) by making use of associated isomorphisms. Moreover, it was shown that suitable supervenience relations can be devised not only for classical mereological systems but also for non-classical ones, by modifying the domains and imposing various conditions on mappings that allow us to capture the additional structure of non-classical parthood relations. Additionally, we provided principles for cases in which wholes are taken to be prior to their parts that are applicable in settings in which one is dealing with dense parthood orderings and atomless gunk.[23]

[22] Failure to preserve mereological structure also undermines the attempt to appeal to total decompositions to ensure that any whole has a unique image under R in the supervening domain.

the xx's are a total decomposition of y $=_{df}$ any z is one of the xx's iff z is a part of y

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