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Towards world identification in description logics

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Abstract: Logical analysis of the applicability of nominals (which are introduced by hybrid logic) in formal descriptions of the world (within modern knowledge representation and semantics-based systems) is very important because nominals, as second sorts of propositional symbols, can support logical identification of the described world at specific [temporal and/or spacial] states. This paper will focus on answering the philosophical-logical question of 'how a fundamental world description in description logic (DL) and a nominal can be related to each other?'. Based on my assumption that nominals can support more adequate identification of the world in DL, this paper will deal with the concept of 'world identification'. Accordingly, based on a logical-terminological analysis of nominals, the paper will analyse hybridised fundamental world descriptions. The research will finally reach the idea that we can have a hybrid description logic based on the analysed concepts.

Keywords: concept, description Logic, hybridised world description, individual, nominal, world description, world identification

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1. Introduction

Hybrid logics are logics that result by adding further expressive power to ordinary modal logic, see [Braüner, 2017]. The history of hybrid logic goes back to Arthur Norman Prior's work on *hybrid tense logic* in the 1960s, see [Prior, 1967; Blackburn, 2006]. Actually the use of logical formulae as terms goes back to Prior's work. In fact, Prior's hybrid logic has focused on 'naming' worlds. The most fundamental hybrid logic is obtained by introducing *nominals* that are new kinds of propositional symbols, see [Blackburn & Seligman, 1995; Areces, 2000; Blackburn & Jørgensen, 2016b].

In the standard Kripke semantics for modal logic (see [Kripke, 1963; Menzel, 2018]), truth is relative to points in a set. Thus, a propositional symbol might have different truth-values relative to different points. Usually, these points are taken to represent possible worlds, times, epistemic states, etc., see [Braüner, 2017]. Formally speaking, any nominal (like $\mathtt{n}$) can be true at one (and only one) possible world. In fact, $\mathtt{n}$ is syntactically a marked propositional symbol that is true at one and only one state. Therefore, we can regard $\mathtt{n}$ as the addresser of a specific single state (and, correspondingly, of a time as well as of a place) that it is true at. The most significant assumption is that 'any nominal symbol can be true at exactly one state in any semantic model'. This research focuses on the logical analysis of an application of nominals in Description Logic.

Description Logics (*DLs*) are among the most widely used knowledge representation formalisms in semantics-based systems, see [Baader et al., 2007; Baader et al., 2017a; Sikos, 2017]. DLs have emerged from *semantic networks* (which are knowledge bases that represent semantic interrelationships between various concepts; see [Quillian & Minski, 1968]) and *frame-based systems* (based on which knowledge can be divided into interrelated sub-structural frames, in order to be represented; see [Minsky, 1975]). In addition, other logical representational systems based on *structural subsumption algorithms* (e.g., KRYPTON (see [Brachman et al., 1983]) and KRIS (see [Baader & Hollunder, 1991])) have constructed supportive backgrounds for DL development. Most DLs are decidable fragments of Predicate Logic (PL). More specifically, DLs are PL-based terminological systems developed out of the attempt to represent knowledge, with a formal semantics, in order to establish a common ground for human and machine interplays.

The main focus of this paper is on answering the philosophical-logical question of 'how (*i*) a world description in the standard description logic $\mathcal{ALC}$[1] and (*ii*) a hybrid logic's nominal can be related to each other?'. Based on a review of the most relevant works on the available extensions of DLs (with nominals and other hybrid operators) as well as on temporal and spacial extensions of DLs, this research focuses on the logical analysis of nominals' usability and efficacy in DL-based descriptions of the world . More specifically, the research logically-terminologically describes how we can — based on logical nominalism — provide an identification for our described world in DL. A described world in DL (equivalently: a DL *world description*) is fundamentally expressible in the form of assertional axioms. Thereby, based on logical analysis of nominals, this research

[1] $\mathcal{ALC}$ stands for Attributive Concept Language with Complements. $\mathcal{ALC}$ is the prototypical description logic.

analyses hybridised assertions in order to identify them. The very important assumption is that DL may need to identify the world (or more specifically, to make an identification of a specific world description) by addressing specific [temporal and/or spacial] states. Correspondingly, the research deals with the interrelationships between nominals and DLs' individual symbols (that are equivalent to constant symbols in PL). Consequently, the paper offers the idea that we can have a hybridised $\mathcal{ALC}$, like $\mathcal{HALC}$, that can represent *identified world descriptions.*

2. Hybrid Logic

Hybrid logic can be regarded as the hybridised version of the ordinary tense logic. *Tense logic* is a modal-logic type of approach introduced around 1960 by Arthur Prior, see [Goranko & Rumberg, 2020; Blackburn & Jørgensen, 2012; Blackburn & Jørgensen, 2016b]. In addition to the usual propositional (truth-functional) operators, the basic logical language of tense logic contains four temporal modal operators as follows:

i. P that expresses "It has at some time been the case that …"

ii. F that expresses "It will at some time be the case that …"

iii. H that expresses "It has always been the case that …"

iv. G that expresses "It will always be the case that …"

Hybrid logic interprets the phenomenon of *temporality* as an intrinsic and essential property of objects in the world, see [Blackburn, 1993; Blackburn, 2006]. Prior obtained hybrid logic by introducing 'nominals' (as the second sorts of propositional symbols). In fact, the hybrid logic which Prior used is a language built on a set of nominals as well as on a set of ordinary propositional symbols.

This research has taken into account that there is a strong logical and semantic interrelationship between (i) nominals and (ii) the concepts of *moment* (which stands for a specific state of time) and *location* (which stands for a specific state of place). More specifically, there is a correlation between the following items:

1. Nominals (that are specific kinds of propositional symbols).

2. Descriptions [of propositions].

In fact, various descriptions can be structured based on the operations that indicate, and address, specific moments and locations of the world.

In this research, nominals are regarded as logical symbols. Therefore, the state of the existence of a nominal as well as the logical validity of its relationship(s) with specific moments and locations must be taken into consideration.

3. Description Logic

Description Logics (DLs) represent knowledge in terms of *concepts*, *individuals* and *roles*, see [Baader et al., 2007; Baader et al., 2017a; Sikos, 2017].

1. A concept corresponds to a distinct [mental] entity (see [Badie, 2017a; Badie, 2017b]). Also, it can be regarded as a class of entities. Concepts and their interrelationships are — hierarchically — utilised to create *terminologies* in DL (see [Badie, 2018; Badie, 2020]). Concepts are equivalent to unary predicates in predicate logic. Atomic concepts (e.g., *Student*, *Colour*, *Company*) are the first group of atomic symbols in DL.

2. Individuals are the instances of (and, thus, are describable by) concepts. For example, the individual `john` who is a student is describable as a 'student' (and can be covered by the concept *Student*). Individuals are equivalent to constant symbols in predicate logic. Individuals (e.g., `bob`, `blue`, `google`) are the second group of atomic symbols in DL.

3. A role expresses a relationship between various individuals. Also, a role can assign a property to an individual. Thereby, roles are either relations or properties. Roles are equivalent to binary predicates in predicate logic. Atomic roles (e.g., *isA*, *produces*, *hasChild*) are the third group of atomic symbols in DL.

Note that atomic symbols (i.e. atomic concepts, atomic roles, and individuals) are the most fundamental descriptions from which we can inductively build more-specified, as well as complex, world descriptions based on logical operators. Here are some examples of the most fundamental descriptions of the world.

- Any of the individuals `ann`, `red`, and `apple` is related to itself by means of the relation of valence 0.

- The descriptions 'Fred is a student' (formally representing: *Student*(`fred`)) and 'Green is a colour' (formally: *Colour*(`green`)) are structured based on the relations of valence 1.

- The descriptions 'Tom is married to Juliana' (formally: *marriedTo*(`tom`, `juliana`)), '10 is greater than 3' (or: *greaterThan*(10, 3)), and 'Bob is the father of Alice' (or: *hasFather*(`alice`, `bob`)) are structured based on the relations of valence 2.

Regarding N_C, N_O, and N_R as the sets of atomic concepts, atomic roles, and individuals, respectively, the triple $\langle N_C, N_O, N_R \rangle$ denotes a *signature* in relevant DL-based descriptions.

The set of main logical symbols in $\mathcal{ALC}$ is: $LS = \{$conjunction ($\sqcap$), disjunction ($\sqcup$), negation ($\neg$), implication ($\rightarrow$), equivalence ($\equiv$), subsumption ($\sqsubseteq$), existential quantification ($\exists$), universal quantification ($\forall$), truth/tautology ($\top$), falsity/contradiction ($\perp$)$\}$. In addition, atomic concepts and atomic roles are represented by A and r, respectively.

Semantic Interpretations. Formal semantics of a term in DL is interpretable based on concepts, roles and individuals (which are non-logical symbols in logical descriptions). Actually, non-logical symbols do not independently have any logical consequence in a formal description. Therefore, we need to utilise a semantic interpretation in order to deal with the semantics of a DL-based term (which is structured based on those non-logical symbols). A semantic interpretation (or: '$\mathcal{I}$') consists of the following ingredients:

1. An *interpretation domain* (or: 'Δ'). Δ is a non-empty set and consists of any individual which may occur in our descriptions.

2. An *interpretation function* (in the form of '$.^{\mathcal{I}}$'). This function assigns every individual symbol (like a) to an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$. Also, it assigns to every atomic concept A, a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, and to every atomic role r, a binary relation $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$.

Note that the interpretation domain of a concept, as well as of a role, can — after being interpreted by the interpretation function — become transformed into the elements of the set $\mathcal{V} = \{0, 1\}$ in order to express the semantic concepts of 'truth' and 'falsity'. Actually, becoming transformed into '0' expresses '[being] false' and becoming transformed into '1' expresses '[being] true'. More specifically:

1. Let some individual a be an instance of some interpreted concept C. Therefore, a will be transformed into 1.

2. Let some individual b be one of the instances of some interpreted role R. Then, b will be transformed into 1.

3. Let some individual c be not an instance of some interpreted concept C. So, c will be transformed into 0.

4. Let some individual d be not one of the instances of some interpreted role R. Thereby, d will be transformed into 0.

Table 1 presents the syntax and semantics of concept constructors in $\mathcal{ALC}$ (and over LS). Also, Table 2 reports terminological and assertional axioms in DL (and over LS). In these tables, C and D stand for two concepts, and R and S stand for two roles.

Note that a semantic interpretation is called a *model* for a DL-based description if it can satisfy all the terminological and assertional axioms based on which that description has been expressed.

Table 1. $\mathcal{ALC}$ Syntax and Semantics

Syntax	**Semantics**
A	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
r	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
$\top$	$\Delta^{\mathcal{I}}$
$\bot$	$\emptyset$
$C \sqcap D$	$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \wedge D^{\mathcal{I}}$
$C \sqcup D$	$(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \vee D^{\mathcal{I}}$
$\neg C$	$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
$\exists r.C$	$\{a \mid \exists b.(a,b) \in r^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\}$
$\forall r.C$	$\{a \mid \forall b.(a,b) \in r^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\}$

Table 2. Terminological and Assertional Axioms in DL

Name	**Syntax**	**Semantics**
concept subsumption axiom	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
role subsumption axiom	$R \sqsubseteq S$	$R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$
concept equality axiom	$C \equiv D$	$C^{\mathcal{I}} = D^{\mathcal{I}}$
role equality axiom	$R \equiv S$	$R^{\mathcal{I}} = S^{\mathcal{I}}$
concept assertion	$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$
role assertion	$R(a,b)$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$

4. Literature Review

4.1. Extensions of DLs with Nominals and other Hybrid Operators

There have been strong works on the extensions of DLs with nominals as well as with other hybrid operators. This section reviews the most important ones.

Since expressive role constructors are important in many applications but can be computationally problematical, [Horrocks et al., 2000] presents an algorithm that decides satisfiability of the DL $\mathcal{ALC}$ extended with transitive and inverse roles, role hierarchies, and qualifying number restrictions. [Areces, 2000] explores and exploits the logical connections (i.e. similarities and differences) between DL and hybrid logic. [Horrocks & Sattler, 2001] presents sound and complete reasoning services for the DL $\mathcal{SHOQ}(\mathcal{D})$. $\mathcal{SHOQ}(\mathcal{D})$ is an expressive DL equipped with named individuals and concrete datatypes which has almost exactly the same expressive power as web ontology languages. [Lutz et al., 2005] works on DLs with key constraints that allow the expression of statements like 'US citizens are uniquely identified by their social security number'. Based on this idea, the authors introduce a number of natural description logics and perform a detailed analysis of the decidability and of computational complexity. [Horrocks et al., 2006] describes an extension of the description logic underlying OWL-DL (see [OWL, 2012]), $\mathcal{SHOIN}$, with all expressive means that were suggested to authors by ontology developers as useful additions to OWL-DL, and which, additionally, do not affect its decidability and practicability. The resulting logic is called $\mathcal{SROIQ}$ that includes familiar features from hybrid logic. Regarding [Horrocks et al., 2007] (based on the work of [Horrocks et al., 2006]), in order to support extensionally defined classes, $\mathcal{SHOIN}$ includes nominals (in the form of classes whose extension is a singleton set[2]). This is actually an important feature for a logic which is designed for being used in ontology language applications, because extensionally defined classes are very common and applicable in ontologies (see [Guarino, 1998]). Later, [Krötzsch et al., 2011] proposes an extension of $\mathcal{SROIQ}$ with nominal schemas which can be used like variable nominal concepts within axioms. This feature supports the authors to express arbitrary DL-safe rules in DL syntax. Later on, [Gorin & Schröder, 2012] deals with the concept of self-reference that has been recognised as a useful feature in DL but is also known to cause substantial problems with decidability. Finally, [Tobies, 2000] studies the complexity of reasoning with cardinality restrictions and nominals in expressive DLs.

4.2. Temporal and Spacial Extensions of DLs

Temporal (and to a lesser extent also spatial) extensions of DLs have been studied extensively. Here are the most significant related works.

[Schild, 1993] shows how to add full first-order temporal expressiveness to terminological logics. It analyses that this feature can be achieved by embedding point-based tense operators in propositionally closed concept languages like $\mathcal{ALC}$. [Artale & Franconi, 2000] offers a survey of temporal extensions

[2]Singleton is a set that contains exactly one element.

of DLs. In the survey, the computational properties of various families of temporal description logics are pointed out. [Artale & Franconi, 2000] emphasises that the advantages of using temporal DLs are their high expressivity combined with desirable computational properties, such as decidability, soundness, and completeness of deduction procedures. [Baader et al., 2003] addresses the extensions of DLs concerning concrete domain constraints; modal, epistemic, and temporal operators; probabilities and fuzzy logic; and defaults. [Lutz et al., 2008] surveys temporal DLs which are designed based on standard temporal logics. In particular, the authors concentrate on the computational complexity of the satisfiability problem and algorithms for deciding it. [Artale et al., 2014] designs suitable temporal DLs for reasoning about temporal conceptual data models and also investigates their computational complexity. [Baader et al., 2017b] focuses the combination of DLs with metric temporal logics over the natural numbers by introducing interval-rigid names. This allows to state that elements in the extension of certain names stay in this extension for at least some specified amount of time. Finally, [Bourgaux et al., 2019] addresses the problem of handling inconsistent data in a temporal version of ontology-mediated query answering (based on the combination of conjunctive queries with operators of propositional linear temporal logic). Subsequently, the authors work on temporal knowledge bases.

5. Logical Analysis of Nominals in DL

According to [Baader et al., 2017c], we may want to use individual names inside concepts. For example, we are going to define the class *BookOfJohn* as those books which are written by John. Hence, we can offer the following concept definition:

$$\mathit{BookOfJohn} \ \equiv\ \mathit{Book} \sqcap \exists \mathit{writes}^{-}.\mathit{John}$$

In this concept definition, writes^{-} is an inverse role[3] (of the concept *John*) which is formalised in order to relate us to the role *havingBook* (of the concept *John*) and in fact, to the specified concept *BookOfJohn*. However, the problem is that this concept definition would not work for the following two reasons:

1. John cannot be both an individual and a concept name.

2. If we were to allow John to be in the place of a concept, we would need to say what this means for John's interpretation. In fact, based on every interpretation $\mathcal{I}$, $\mathit{John}^{\mathcal{I}}$ would be an element of the interpretation domain, but concepts are interpreted as sets of elements.

[3]The *inverse role*, or R^{-}, is constructed based on the role constructor '$^{-}$' and is represented by $\mathcal{I}$. Therefore, the description logic that can model inverse roles is called $\mathcal{ALCI}$.

To enable the use of individual names in concepts and avoid the mentioned problems, nominals have been introduced. The fact that a DL provides nominals is normally indicated by $\mathcal{O}$. According to [Baader et al., 2017c], the description logic $\mathcal{ALCO}$ is obtained from $\mathcal{ALC}$ by allowing nominals as additional concepts. Considering the individual a, for an interpretation $\mathcal{I}$ in $\mathcal{ALCO}$, the mapping $\cdot^{\mathcal{I}}$ is extended as $(\{a\})^{\mathcal{I}} = \{a^{\mathcal{I}}\}$. Consequently, by utilising the interpretation $\mathcal{I}$, it is possible to redefine the concept *BookOfJohn* as:

$$\mathit{BookOfJohn} \equiv \mathit{Book} \sqcap \exists \mathit{writes}^{-}.\{\texttt{john}\}$$

In fact, by putting curly brackets around the individual name `john`, we have transformed `john` into a concept. Taking into account such a transformation, I need to offer the following definitions:

Definition – Identical Concept Constructors. An *identical concept constructor* (*IDCC*) is defined in order to turn an individual symbol into a concept. *IDCC* is formally represented by '{ }'. Any *IDCC* concentrates on a specific individual symbol and makes an identifier for it.

Definition – Identifier. An *identifier* is a name that labels the *identity* of a unique individual symbol.

It can be interpreted that any *IDCC* is a kind of role that (i) expresses the concept of *becoming* and (ii) makes an interrelationship between an individual and a concept. Regarding the latter, by relating a concept to a unique individual symbol, an *IDCC* assigns an identity to that individual.

Let me be more specific on the 'concept of an individual'. Actually, we may interpret that the existence of 'the concept of some [specific] individual' expresses the fact that 'there is, surely, one single object/thing in its own scope in our world'. Obviously, such a property is other than those forming usual concepts (e.g., *Book*, *Person*). In fact, when we [operate and] identify a concept constructor in a certain world, then a secondary concept (like $\mathcal{C}$) will arise. Accordingly, a single individual (like c) will be included in $\mathcal{C}$ [in its specific world]. It can be interpreted that $\mathcal{C}$ is empty (and meaningless) in other worlds. Therefore, $\mathcal{C}$ is a concept of different sort. In fact, properties of all individuals change from world to world.

6. First-Order Interpretations and Specific States of the World

As pointed out above, based on the standard Kripke semantics for modal logic, truth is relative to points in a set. Thus, a propositional symbol might have different truth-values relative to different points [Kripke, 1963]. It shall be taken into account that the relationship(s) between first-order interpretations and Kripke models (or more specifically, the states of a Kripke frame that can be regarded as various states of the world) is establishable as follows:

1. The states of the world are the elements of the domain of a first-order interpretation (like Δ that is a non-empty set).

2. The propositions (which can be either true or false at any state of the world) are regarded as [the outcomes of] the interpretations of unary predicates over the [interpretation] domain. Assessed by DL, this means that a proposition is described based on the interpretation of concept(s) over the interpretation domain in a DL world description.

3. The accessibility relations, which relate various states of the world together, are seen as [the outcomes of] the interpretations of binary predicates over the [interpretation] domain. Assessed by DL, the relations between various states of the world are seen as the interpretation(s) of role(s) over the interpretation domain in a DL world description.

7. World Descriptions at Specific States of the World

Accept that it is raining in Copenhagen at 17:19 on Thursday 21 September 2017. Here the nominal $\mathtt{n}$ stands for (and is identical to) the proposition 'It is in Copenhagen at 17:19 on Thursday 21 September 2017'. Then, $\mathtt{n}$ addresses a specific state (of the world) which the proposition 'It is raining in Copenhagen at 17:19 on Thursday 21 September 2017' (call this proposition 'A') is certainly true at. As pointed out above, $\{\mathtt{n}\}$ represents a concept. Regarding Δ as the domain of our interpretation, semantically we have: $(\{\mathtt{n}\})^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$. Equivalently: $\{\mathtt{n}^{\mathcal{I}}\} \subseteq \Delta^{\mathcal{I}}$.

As pointed out above, by means of the interpretation function '$.^{\mathcal{I}}$', our interpreted domain of individuals (or: $\Delta^{\mathcal{I}}$) can become transformed into the elements of the set $\mathcal{V} = \{0, 1\}$. Taking into account $\{\mathtt{n}^{\mathcal{I}}\} \subseteq \Delta^{\mathcal{I}}$, we can understand that $\{\mathtt{n}^{\mathcal{I}}\}$ has either the truth value '0' (i.e. is false) or the truth value '1' (i.e. is true).

Now suppose that the world description *Raining*($\mathtt{sky}$) expresses the proposition 'The sky is raining'. Since A is true at the state at which $\mathtt{n}$ is expressed, thus *Raining*($\mathtt{sky}$) is true at the same state and, in fact, over $\mathtt{n}$. Formally speaking, $\mathtt{sky}^{\mathcal{I}} \in Raining^{\mathcal{I}}$, when/where $\{\mathtt{n}^{\mathcal{I}}\} \subseteq \Delta^{\mathcal{I}}$. It is interpretable that $\{\mathtt{n}^{\mathcal{I}}\}$ – in order to express a truth (about A) in a model – provides a semantic reference for (i) $\{\mathtt{sky}^{\mathcal{I}}\}$ (based on the conceptualised and interpreted *Raining*) and, correspondingly, for (i) $Raining^{\mathcal{I}}(\mathtt{sky}^{\mathcal{I}})$.

At this point it shall be emphasised that the proposition A does not express a truth about itself, but about the individual $\mathtt{sky}$ at a specific state of the world.

Subsequently, *Raining*($\texttt{sky}$) is interpreted true based on the interpretation of the singleton $\{\texttt{sky}\}$ (which is, in fact, a concept) over $\texttt{n}$ in a semantic model. It shall be taken into account that such a truth about the individual $\texttt{sky}$ (which is an instance of $\{\texttt{sky}\}$) is certainly not peculiar to the state of $\texttt{n}$ (see [Øhrstrøm, 1996]). However, the remarkable logical assumption is that $\texttt{n}$ has — based on the fact that 'we are having rain' (or equivalently, 'the rain has been experienced') — provided an adequate identification for the description 'the sky is raining'.[4]

8. World Identification in Description Logic

8.1. Identification of Concept Assertions

The concept assertion $C(a)$ is made up of the concept C and the individual a. More specifically, $C(a)$ is formally-logically structured based on the collection of the concepts C and $\{a\}$.

Definition – Hybridised Concept Assertion. The formula $@_{\texttt{n}}C(a)$ represents a *hybridised concept assertion*, when/where $\texttt{n}$ is a nominal and $C(a)$ stands for a concept assertion. It is interpreted that $@_{\texttt{n}}C(a)$ expresses the co-existence of $C(a)$ and $\texttt{n}$.

Definition – Coexistence. The semantic operation '$=^{\mathcal{E}}$' between two interpreted concepts (as well as interpreted [concept] descriptions) expresses their *coexistence*. Hence, $C^{\mathcal{I}} =^{\mathcal{E}} D^{\mathcal{I}}$ means that C and D are interpreted to have co-existence. In other words, it is interpreted that C and D do exist, and be valid, together (at/in the same time and/or location).

Taking into consideration $@_{\texttt{n}}C(a)$, we can conclude that there is a co-existence relationship between 'the collection of the concepts C and $\{a\}$' and 'the concept $\{\texttt{n}\}$'. Semantically: $((\{a\})^{\mathcal{I}} \wedge C^{\mathcal{I}}) =^{\mathcal{E}} (\{\texttt{n}\})^{\mathcal{I}}$. Consequently: $(\{a^{\mathcal{I}}\} \wedge C^{\mathcal{I}}) =^{\mathcal{E}} \{\texttt{n}^{\mathcal{I}}\}$. In addition, since $\{\texttt{n}^{\mathcal{I}}\}$ is either true or false (and can

[4] In [semantics-based] information systems, ontologies which are fomal-logical descriptions of concepts as well as of concepts' [intra-/inter]relationships, can be applied in order to offer a specified shared conceptualization of various concepts over a specific domain of discourse. Obviously, in applied ontologies it is easy to interpret and understand that some individual a has a unique identification number. But what about my offered example in this section? A possible solution can be to divide our temporal space into various countable [semantic knowledge] boxes. For instance, in our example, 'Thursday 21 September 2017' can be divided into 1440 knowledge boxes (any of which would be a knowledge base for one specific minute in 'Thursday 21 September 2017'). Subsequently, we can have 1440 knowledge boxes which will contain specific 'individuals (as instances of concepts/roles)', 'identified individuals', and 'arised concepts'.

become transformed into either '0' or '1'), it is interpretable that $(\{a^{\mathcal{I}}\} \wedge C^{\mathcal{I}})$ can also be transformed into either '0' or '1'.

Note that the existence of the hybridised concept assertion $@_{\mathtt{n}}C(a)$ indicates the co-existence of '$(\{a^{\mathcal{I}}\} \wedge C^{\mathcal{I}})$' and '$\{\mathtt{n}^{\mathcal{I}}\}$'. According to $(\{a^{\mathcal{I}}\} \wedge C^{\mathcal{I}}) =^{\mathcal{E}} \{\mathtt{n}^{\mathcal{I}}\}$ it is interpretable that $\{\mathtt{n}^{\mathcal{I}}\}$ provides a semantic reference for $\{a^{\mathcal{I}}\}$ (in conjunction with the conceptualised and interpreted C). This means that $\{\mathtt{n}\}$ (which is the identifier of $\mathtt{n}$) acts as the identifier of the individual a, when/where a is interpreted to be existed with C.

Proposition. Regarding $@_{\mathtt{n}}C(a)$, the nominal $\mathtt{n}$ identifies a specific state (of time and/or place) at which $C(a)$ is certainly true. In fact, any identified concept assertion (in correspondence with a nominal) has a correlation with an identified individual. This is how an interpreted individual (that is a constant and non-logical symbol) and an interpreted nominal (that is a propositional symbol) are semantically tied together.

Example. Accept that Bob is a student in London. I will address the proposition 'Bob is a student in London' by B. The nominal $\mathtt{n}$ stands for 'It is in London'. Then, the world description $\mathit{Student}(\mathtt{bob})$ is true at the point at which $\mathtt{n}$ becomes expressed. Semantically, the conjunction of the propositions 'Bob is a student' and 'It is in London' is subsumed under the concept of truth. Considering the hybridised concept assertion $@_{\mathtt{n}}\mathit{Student}(\mathtt{bob})$, the description $\mathit{Student}(\mathtt{bob})$ can, by being transformed into $\mathtt{n}$, express a truth about 'being student by Bob'. Regarding the hybridised concept assertion $@_{\mathtt{n}}\mathit{Student}(\mathtt{bob})$, semantically we have: $(\{\mathtt{bob}^{\mathcal{I}}\} \wedge \mathit{Student}^{\mathcal{I}}) =^{\mathcal{E}} \{\mathtt{n}^{\mathcal{I}}\}$. This means that $\{\mathtt{n}^{\mathcal{I}}\}$ provides a semantic reference for $\{\mathtt{bob}^{\mathcal{I}}\}$ (in conjunction with the conceptualised and interpreted *Student*) $(*)$.

In this example, the concept $\{\mathtt{n}\}$ has — by addressing 'in London' — become subsumed under the concept of *Location*. By defining $\mathcal{L}$ as the logical concept '*Location*' in our formalism, we have: $\{\mathtt{n}^{\mathcal{I}}\} \subseteq \mathcal{L}^{\mathcal{I}}$ $(**)$.

According to $(*)$ and $(**)$, we can conclude that: $(\{\mathtt{bob}^{\mathcal{I}}\} \wedge \mathit{Student}^{\mathcal{I}}) \subseteq \mathcal{L}^{\mathcal{I}}$. In fact, regarding the interpretation of the hybridised description of 'Bob is a student', we can conclude that 'being student by Bob is identifiable in the location 'London'' if and only if: (i) being student is interpretable and meaningful in London, (ii) the individual Bob is recognisable as a concept in London, and (iii) Bob can be subsumed under the concept *Student* in London.

It shall be summarised that the concept $\{\mathtt{n}\}$ has provided a semantic reference for:

1. the concept of 'being student',

2. the identity of the individual Bob and its validity and recognisability as a concept, and

3. the logical interrelationships between (1) and (2).

8.2. Identification of Role Assertions

The role assertion $R(a, b)$ is structured based on the combination of (i) the individuals a and b and (ii) the role R (that has related a and b together). More specifically, the existence of $R(a, b)$ indicates the co-existence of the role R and the concepts $\{a\}$ and $\{b\}$.

Definition – Hybridised Role Assertion. The formula $@_{\mathtt{n}}R(a, b)$ represents a hybridised version of the role assertion $R(a, b)$, when/where $\mathtt{n}$ is a nominal. Actually, $@_{\mathtt{n}}R(a, b)$ expresses the co-existence of $R(a, b)$ and $\mathtt{n}$.

Taking into consideration $@_{\mathtt{n}}R(a, b)$ we can conclude that there is a co-existence relationship between 'the collection of R, $\{a\}$, $\{b\}$' and '$\{\mathtt{n}\}$'. Semantically: $((\{a\})^{\mathcal{I}} \wedge (\{b\})^{\mathcal{I}} \wedge R^{\mathcal{I}}) =^{\mathcal{E}} (\{\mathtt{n}\})^{\mathcal{I}}$. Consequently: $(\{a^{\mathcal{I}}\} \wedge \{b^{\mathcal{I}}\} \wedge R^{\mathcal{I}}) =^{\mathcal{E}} \{\mathtt{n}^{\mathcal{I}}\}$. Also, since $\{\mathtt{n}^{\mathcal{I}}\}$ is either true or false (and can become transformed into either '0' or '1'), we can interpret that $\{a^{\mathcal{I}}\} \wedge \{b^{\mathcal{I}}\} \wedge R^{\mathcal{I}}$ can also be transformed into either '0' or '1'.

Here the existence of the hybridised role assertion $@_{\mathtt{n}}R(a, b)$ indicates the co-existence of $\{a^{\mathcal{I}}\} \wedge \{b^{\mathcal{I}}\} \wedge R^{\mathcal{I}}$ and $\{\mathtt{n}^{\mathcal{I}}\}$. Regarding $(\{a^{\mathcal{I}}\} \wedge \{b^{\mathcal{I}}\} \wedge R^{\mathcal{I}}) =^{\mathcal{E}} \{\mathtt{n}^{\mathcal{I}}\}$, we can conclude that $\{\mathtt{n}^{\mathcal{I}}\}$ provides a semantic reference for $\{a^{\mathcal{I}}\}$ and for $\{b^{\mathcal{I}}\}$ (in conjunction with the conceptualised and interpreted R). This means that $\{\mathtt{n}\}$ (that is the identifier of $\mathtt{n}$) acts as the identifier of the individuals a and b, when/where a and b are interpreted to be existed with R.

Proposition. According to $@_{\mathtt{n}}R(a, b)$, the nominal $\mathtt{n}$ identifies a specific state (of time and/or place) at which $R(a, b)$ is certainly true. In fact, any identified role assertion (in correspondence with a nominal) has a correlation with two identified individuals. This is how two interpreted individuals (that are constant and non-logical symbols) and an interpreted nominal (that is a propositional symbol) are semantically tied together.

Example. Accept that Mary and David are hugging each other at 16:07 on Thursday 28 June 2018. I will address the proposition 'Mary and David are hugging each other at 16:07 on Thursday 28 June 2018' by H. The nominal $\mathtt{p}$ stands for 'It is at 16:07 on Thursday 28 June 2018'. Hence the world description *isHugging*($\mathtt{mary}, \mathtt{david}$) (that expresses the proposition 'Mary and David are hugging each other') is true at the state at which the nominal $\mathtt{p}$ has been expressed. In fact, the conjunction of the propositions 'Mary and David are hugging each other' and 'It is at 16:07 on Thursday 28 June 2018' is subsumed under the concept of truth. The existence of the hybridised role assertion $@_{\mathtt{p}}\mathit{isHugging}(\mathtt{mary},\mathtt{david})$ indicates the co-existence of $\{\mathtt{mary}^{\mathcal{I}}\} \wedge \{\mathtt{david}^{\mathcal{I}}\} \wedge \mathit{isHugging}^{\mathcal{I}}$ and $\{\mathtt{p}^{\mathcal{I}}\}$. Subsequently, based on $(\{\mathtt{mary}^{\mathcal{I}}\} \wedge \{\mathtt{david}^{\mathcal{I}}\} \wedge \mathit{isHugging}^{\mathcal{I}}) =^{\mathcal{E}} \{\mathtt{p}^{\mathcal{I}}\}$, we can interpret that $\{\mathtt{p}^{\mathcal{I}}\}$ provides a semantic reference for $\{\mathtt{mary}^{\mathcal{I}}\}$ and for $\{\mathtt{david}^{\mathcal{I}}\}$ (in conjunction with the conceptualised and interpreted *isHugging*) $(*)$.

In this example, the concept $\{\mathtt{p}\}$ has — by addressing 'at 16:07 on Thursday 28 June 2018' — become subsumed under the concept of *Moment*. Let us represent the logical concept of '*Moment*' by $\mathcal{M}$. It can be interpreted that: $\{\mathtt{p}^{\mathcal{I}}\} \subseteq \mathcal{M}^{\mathcal{I}}$ $(**)$.

Taking into account $(*)$ and $(**)$, we can conclude that: $(\{\mathtt{mary}^{\mathcal{I}}\} \wedge \{\mathtt{david}^{\mathcal{I}}\} \wedge \mathit{isHugging}^{\mathcal{I}}) \subseteq \mathcal{M}^{\mathcal{I}}$. According to the interpretation of the hybridised description of 'Mary and David are hugging each other', we can conclude that 'hugging David and Mary (by each other) is identifiable at the moment '16:07 on Thursday 28 June 2018'' if and only if: (*i*) the individual Mary is recognisable as a concept at the same moment, (*ii*) the individual David is recognisable as a concept at the same moment, (*iii*) 'is hugging' is an interpretable and meaningful role at the same moment, and (*iv*) Mary and David are related together by means of 'is hugging' at the same moment.

It shall be summarised that the concept $\{\mathtt{p}\}$ has provided a semantic reference for:

1. the role 'is hugging',
2. the identities of 'Mary' and 'David' and their validity (and recognisability) as concepts, and
3. the logical interrelationships between (1) and (2).

9. A Hybridised Description Logic

This research has shown that we can identify DL world descriptions (which are primarily in the forms of concept assertions and role assertions) at specific states (of the world). *World identification* is a remarkable application of nominals in description logics. Taking into account this application of nominals, we can reach the idea that there is a need for another version of a hybridised description logic $\mathcal{ALC}$ (or $\mathcal{HALC}$).

In order to syntactically model $\mathcal{HALC}$, I add the logical symbol $\mathtt{n}$ (which stands for nominals) to the usual syntax of $\mathcal{ALC}$. I also add the logical symbols $\mathcal{L}$ and $\mathcal{M}$ in order to represent the logical concepts of *Location* and *Moment*, respectively. Table 3 presents the syntax and semantics of concept constructors in $\mathcal{HALC}$.

Correspondingly, table 4 presents terminological and assertional axioms in $\mathcal{HALC}$. According to table 4, 'location subsumption axiom' and 'moment subsumption axiom' are added to the usual terminological axioms in $\mathcal{ALC}$. In addition, 'hybridised concept assertion' and 'hybridised role assertion' are added to the standard assertional axioms.

Table 3. $\mathcal{HALC}$ Syntax and Semantics

Syntax	**Semantics**
A	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
r	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
$\mathtt{n}$	$\{\mathtt{n}^{\mathcal{I}}\} \subseteq \Delta^{\mathcal{I}}$
$\mathcal{M}$	$\mathcal{M}^{\mathcal{I}}$ (i.e. when ...)
$\mathcal{L}$	$\mathcal{L}^{\mathcal{I}}$ (i.e. where ...)
$\top$	$\Delta^{\mathcal{I}}$
$\perp$	$\emptyset$
$C \sqcap D$	$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \wedge D^{\mathcal{I}}$
$C \sqcup D$	$(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \vee D^{\mathcal{I}}$
$\neg C$	$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
$\exists r.C$	$\{a \mid \exists b.(a,b) \in r^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\}$
$\forall r.C$	$\{a \mid \forall b.(a,b) \in r^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\}$

10. Concluding Remarks

Nominals are second sorts of propositional symbols and are introduced by hybrid logic. This research is relied on the assumption that we can utilise nominals as logical symbols in order to identify DL world descriptions. Actually I have believed that nominals support logical identification of the described world at specific states within DL world descriptions.

The research has taken into account that there is a strong logical-terminological and semantic interrelationship between (i) nominals and (ii) the concepts of 'moment' and 'location'. Relying on such a logical relationship, it has been assumed that the state of the existence of a nominal as well as the logical validity of its relationship(s) with specific moments and locations (within

Table 4. Terminological and Assertional Axioms in $\mathcal{HALC}$

Name	**Syntax**	**Semantics**
concept subsumption axiom	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
role subsumption axiom	$R \sqsubseteq S$	$R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$
concept equality axiom	$C \equiv D$	$C^{\mathcal{I}} = D^{\mathcal{I}}$
role equality axiom	$R \equiv S$	$R^{\mathcal{I}} = S^{\mathcal{I}}$
location subsumption axiom	$\{\mathtt{n}\} \sqsubseteq \mathcal{L}$	$(\{\mathtt{n}\})^{\mathcal{I}} = \{\mathtt{n}^{\mathcal{I}}\} \subseteq \mathcal{L}^{\mathcal{I}}$
moment subsumption axiom	$\{\mathtt{n}\} \sqsubseteq \mathcal{M}$	$(\{\mathtt{n}\})^{\mathcal{I}} = \{\mathtt{n}^{\mathcal{I}}\} \subseteq \mathcal{M}^{\mathcal{I}}$
concept assertion	$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$
role assertion	$R(a,b)$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$
hybridised concept assertion	$@_{\mathtt{n}}C(a)$	$(\{a^{\mathcal{I}}\} \ \wedge \ C^{\mathcal{I}}) =^{\mathcal{E}} \{\mathtt{n}^{\mathcal{I}}\}$
hybridised role assertion	$@_{\mathtt{n}}R(a,b)$	$(\{a^{\mathcal{I}}\} \ \wedge \ \{b^{\mathcal{I}}\} \ \wedge \ R^{\mathcal{I}}) =^{\mathcal{E}} \{\mathtt{n}^{\mathcal{I}}\}$

world descriptions) can be expressed. Accordingly, identical concept constructors (*IDCC*s) are conceptually defined in order to relate an identifying concept, like $\{a\}$, to an individual symbol (like a). More specifically, an *IDCC* — by relating a concept to a unique individual symbol — assigns an identity to that individual. Similarly, some nominal n (that stands for the propositions 'It is in/at/on somewhere specific' or/and 'It is in/at/on sometime specific'), is considered as an individual symbol. In fact, $\mathtt{n}$ makes a specific identity for the propositions 'It is in/at/on somewhere specific' and 'It is in/at/on sometime specific'. Then the identifying concept $\{\mathtt{n}\}$ is similarly defined (as the product of the transformation of $\mathtt{n}$ by means of *IDCC*).

Relying on the logical interconnections between 'nominals', 'individuals', '*IDCC*' and 'identifying concepts', I have defined the hybridised concept assertion $@_{\mathtt{n}}C(a)$ and the hybridised role assertion $@_{\mathtt{n}}R(a,b)$. According to the existence of the hybridised concept assertion $@_{\mathtt{n}}C(a)$, it is concluded that the concept $\{\mathtt{n}\}$ acts as the identifier of the individual a (when/where a has been classified under the conceptualised concept C). So, $\mathtt{n}$ identifies a specific state (of time and/or place) at which $C(a)$ is certainly true. In fact, any identified concept assertion (that has a co-existence with a nominal) has a correlation with an identified individual. Moreover, regarding the existence of the hybridised role assertion $@_{\mathtt{n}}R(a,b)$, it is concluded that the concept $\{\mathtt{n}\}$ works as the identifier of the individuals a and b (when/where a and b have been related to each other by means of the relation R). Therefore, $\mathtt{n}$ identifies a specific state (of time and/or place) at which $R(a,b)$ is certainly true. This means that any identified role assertion (that has a co-existence with a nominal) has a correlation with two identified individuals. Consequently, based on the concepts of 'hybridised concept assertion' and 'hybridised role assertion', I have concluded that the identifying concept $\{\mathtt{n}\}$ provides a semantic reference for:

1. the most central concept in a hybridised world description (which can fundamentally be described in the form of either a hybridised concept assertion or a hybridised role assertion),
2. the identities of the individual(s) (within either hybridised concept assertion or hybridised role assertion) as well as their validity (and recognisability) as concepts, and
3. the logical interrelationships between (1) and (2).

Finally, relying on the outcomes of the research, the paper has dealt with the idea that we can have a hybridised DL, like $\mathcal{HALC}$, which can be a supportive formal-logical system for representing and analysing identified world descriptions.

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