On Spacetime Functionalism

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Abstract

Eleanor Knox has argued that our concept of spacetime applies to whichever structure plays a certain functional role in the laws (the role of determining local inertial structure). I raise two complications for this approach. First, our spacetime concept seems to have the structure of a cluster concept, which means that Knox's inertial criteria for spacetime cannot succeed with complete generality. Second, the notion of metaphysical fundamentality may feature in the spacetime concept, in which case spacetime functionalism may be uninformative in the absence of answers to fundamental metaphysical questions like the substantivalist/relationist debate.

1 Introduction

In the course of work on conjectured theories of quantum gravity, physicists have begun to take extremely seriously the possibility that spacetime is not among the fundamental structures of our universe (Huggett and Wüthrich, 2013). This has prompted philosophers of science to pose the question of what it would take for a non-fundamental or emergent structure to be spatiotemporal. Independently, Harvey Brown (2005) has argued that one cannot simply posit that a certain structure, or type of structure, is spatiotemporal. Rather, a structure may only count as spatiotemporal[1] in virtue of playing the right sort of role in the laws of a theory.

Inspired by Brown's arguments and aiming to solve the problem of spacetime emergence (among other problems, including the question of how to identify spacetime structure in

[1] Or more precisely, to borrow a term of art from Brown, as having *chronogeometric significance,* i.e. as being measured directly by physical rods and clocks.

Newtonian mechanics), Eleanor Knox has recently advanced a functionalist picture of spacetime. On a functionalist picture, whether an entity (a structure, object or property–from now on I will just say "structure") counts as spatiotemporal is determined by its functional role. The functional role of a physical structure is its role in the physical laws, which often boils down to its implications about the motion of material objects.

Beyond putting forward this overall functionalist framework, Knox advocates a particular version of it, which I'll call *inertial functionalism*. She suggests that the functional role of spacetime is to determine the structure of local inertial frames in a theory. (A local inertial frame is a coordinate system over a small region of a manifold in which the dynamical laws–the laws of motion, or more generally of time evolution–take on a uniquely "simple form" (Knox, forthcoming, 12); a more precise definition will follow in the next section.)

I find the overall framework of spacetime functionalism attractive. Many geometrical structures are used in physics–state space, momentum space, configuration space. Not all of these deserve the name 'spacetime' (Brown, 2005, Section 8.2). Nor is it plausible that the difference between spacetime and these other geometries is a primitive one. It must have something to do with each geometric structure's role in the laws. As Knox rhetorically asks, "If our conceptual grasp on spacetime is not exhausted by the role it plays in our theory, what might the extra ingredient be?" (Knox, forthcoming, 9-10) I can't think of anything.

So I heartily agree with Knox's basic picture that spacetime is defined by the role it plays in theories (or laws). But *inertial* functionalism cannot be the whole story, at least not if the goal is an analysis or explication of our spacetime concept. Inertial functionalism cannot demarcate spacetime from a theory's other geometric structures without taking as "input" a prior demarcation of which coordinate systems used to represent the theory are candidate (inertial and non-inertial) frames. But this demarcation cannot be made without a prior determination of which geometric structures in the theory are apt to play the spacetime role. Moreover, there are clear examples of theories (so-called *topological quantum field theories*) which lack the sort of inertial structure that Knox's account requires, but which include structures that are obviously playing the spacetime role in a meaningful sense. Further, certain other theories (those with forces that violate parity and/or time-reversal invariance) include spacetime structures which will not count as spatiotemporal on Knox's account, because they do no work toward determining inertial structure.

On a more programmatic level, inertial functionalism proceeds from the working assumption that precise necessary and sufficient conditions for the "spacetime role" can be spelled out. But this strikes me as false, because our spacetime concept has the structure of a *cluster concept*. Rather than possessing a single set of necessary and sufficient conditions, cluster concepts can be satisfied in a variety of different ways by different entities falling under them. Having four legs is neither necessary nor sufficient for a being to fall under our concept of cat, for example–but it helps a being to count as a cat if it has four legs. The same goes for inertial structure and our concept of spacetime, I will argue.

Once this cluster concept view of spacetime is appreciated, it is apparent that a vast number of different theoretical properties and relations can help determine whether a given structure is spatiotemporal. Among the properties that help determine this, I will argue, is the property of physical fundamentality. And if this is correct, the question of substantivalism–of whether the "spacetime" structure posited by a theory is part of its fundamental ontology–is prior to the question of what counts as spacetime.

It is my understanding, from previous conversation and correspondence with Knox, that she does not intend to advance her inertial functionalism as a set of necessary and sufficient conditions that any structure whatsoever must meet in order to count as spatiotemporal. Rather, she intends her inertial criteria to be necessary and sufficient conditions for a structure to count as spacetime in a broad class of the theories most commonly discussed in foundational work (mainly Newtonian and relativistic theories). This is not always clear in written statements of her view, in which she sometimes suggests that the view is an analysis of our concept of spacetime (Knox, forthcoming, 2) and that her view entails that (for example) there is no spacetime in Aristotelian cosmology because of its impoverished inertial structure (Knox, forthcoming, 13).

Despite these passages, I take it that Knox is ultimately amenable to my conclusion that her inertial criteria do not provide general necessary and sufficient conditions for a structure to count as spatiotemporal. She has also expressed sympathy for my claim that spacetime is a cluster concept. So in these respects, the present essay should not be seen as a rejection of Knox's own position, but rather as an elaboration and filling-in of her views, together with a correction of some places in which she speaks somewhat loosely. That said, Knox would not agree with some of my particular claims about which criteria fit into the spacetime concept, and in particular she would disagree with my conclusion that the question of substantivalism is prior to the question of which structures should count as spacetime.

And once we appreciate the problem with requiring a privileged class of coordinate systems (frames) as input, and the priority of the substantivalism question (as well as the examples of parity and time-reversal asymmetry), it will become clear that Knox's inertial functionalism is not a satisfactory account of the spacetime concept even as it applies to the Newtonian and relativistic theories she has in mind. So what follows is not merely an elaboration of Knox, but also an objection to some features of her approach.

2 Knox's inertial functionalism

"I propose," Knox writes, "that the spacetime role is played by whatever defines a structure of local inertial frames." (Knox, forthcoming, 10) Where there is a unique minimal (i.e., simplest) structure required to define the inertial frames, Knox identifies that structure with spacetime.[2] For Knox, inertial frames are coordinate systems which locally (to the neighborhood of a point) satisfy the following criteria:

1. Inertial frames are frames with respect to which force free bodies move with constant velocities.
2. The laws of physics take the same form (a particularly simple one) in all inertial frames.
3. All bodies and physical laws pick out the same equivalence class of inertial frames (universality). (Knox, 2013, 348)

(Note that in pure field theories without particles or extended bodies, the first criterion will not apply and the question of which frames are inertial will boil down to the other two criteria.) In a theory like special relativity, with flat spacetime and no gravity, the inertial frames can be defined globally, but in theories with dynamical spacetime like general relativity we can at best make do with local inertial frames meeting the criteria above. These will be the frames in which the affine connection coefficients (Christoffel symbols) vanish in the infinitesimal vicinity of the location we're interested in studying (hence the

[2] Knox does not address the question of what we should say when there are multiple, inequivalent but equally simple structures which are all sufficient to define inertial frames; the question of what she should say in such a case is beyond the scope of this paper, so I will assume a unique such structure exists in the specific cases discussed.

qualification "*local* inertial frames").[3] Thus identifying the inertial frames, as Knox uses the term, requires fixing the affine (straight line) structure of a spacetime. Knox further stipulates that at least a full specification of timelike geodesics (possible inertial trajectories for matter particles) will be included in the structure that defines, in Knox's sense, the local inertial frames (Knox, 2013, 349). This is sufficient to fix the projective and conformal structure of spacetime within the framework of general relativity (Weyl, 1922, 313-314).[4]

One might worry that this leaves out a lot of what we ordinarily think of as the building blocks of spacetime. What about distances, spans of time and intervals–metric structure, in other words? What about topological structure? Fortunately this is not a problem, because the conformal and projective structure fixed by the set of timelike geodesics uniquely determines the metric of any solution in general relativity up to a global scale factor (Weyl, 1922, 313-314).[5] Since a global scale factor is not required to determine relative distances, Knox dismisses it as unphysical.[6]

Moreover, this view seems to succeed in recovering the two symmetry principles proposed by Earman (1989, 46). Earman asserts that we ought to expect the spacetime symmetries of a theory (the transformations leaving its absolute spacetime structure invariant) to match the dynamical symmetries of that theory (the transformations preserving its laws of time evolution[7]). Earman codifies this matching requirement in two symmetry principles:

> **SP1** Any [external] dynamical symmetry of [a theory] T is a spacetime symmetry of T.[8]

[3]In general relativistic field theories exhibiting so-called minimal coupling, such frames will not always exist even local to a point (see Read *et al.*, preprint). Although this is a serious problem for the scope of Knox's inertial functionalism, it is far enough removed from the considerations I wish to raise here that I will set it aside.

[4]Knox cites Ehlers *et al.* (2012) here.

[5]More may need to be said to recover non-inertial structure in other theories aside from general relativity–and indeed, more may need to be said about what licenses the assumption on Knox's part that all of this is happening within the specific theoretical framework of general relativity rather than some more general one–but rather than pursue this point here, let us move on.

[6]This is not obviously correct, though, since (for example) in a spatially closed universe the scale factor is required to determine the total volume of space, which certainly seems like a physically significant quantity.

[7]The sense in which the dynamical symmetries preserve the laws may differ from theory to theory; typically this means something like leaving the Hamiltonian and/or the Lagrangian unchanged.

[8]As Knox notes, this should only go for external symmetries–we do not expect internal symmetries such as gauge transformations to correspond to symmetries of spacetime(Knox, forthcoming, 11). Unfortunately, this introduces an ambiguity in Earman's principles, since as Knox notes, there is no accepted spacetime-independent definition of which symmetries count as "external." But Knox treats this as a peripheral

SP2 Any spacetime symmetry of T is a dynamical symmetry of T.

Knox, following Myrvold (forthcoming) (and unlike Earman himself) takes SP1 and SP2 to be analytic truths about the relationship between spacetime symmetries and dynamical symmetries (Knox, forthcoming, 11). Whenever local inertial frames meeting Knox's criteria exist, the strong equivalence principle will apply, ensuring that any local symmetries of the metric (i.e., isometries, when they exist) will be local dynamical symmetries as well (Knox, forthcoming, 12).

Citing these achievements, Knox suggests that her inertial functionalism provides everything we could want from an analysis of our spacetime concept (at least as it applies in Newtonian and relativistic theories).[9] One might object that general relativity is strictly speaking a theory with no preferred reference frames.[10] But there is far from universal agreement that this is the right way to understand general relativity. If the existence of inertial frames in general relativity is the most controversial assumption presumed by inertial functionalism, Knox's view would seem to be in good shape.

However, this is not Knox's only contestable assumption. As we will see, her inertial functionalism rests on the implausible assumption that we can identify coordinate frames prior to demarcating spacetime structure, and gives the wrong verdict about which structures to interpret as spatiotemporal when applied to several example theories–including parity and time-reversal violating theories that appear to accurately describe the actual laws of our world.

3 Which coordinate systems are frames?

We can think of Knox's inertial functionalism as an input-output machine: it takes as input a theory together with its reference frames (i.e., coordinate systems), determines which frames are the inertial frames, and then on that basis determines which of the theory's geometric structures constitute spacetime. Although Knox does not focus on this fact, the identification of certain coordinate systems definable on the theory's geometric structures as "frames" is a

complication; therefore, so will I.

[9]To reiterate, she has suggested in discussion that these theories are her view's intended domain.

[10]Knox (2013, 348) notes that Erik Curiel and James Weatherall have objected to her view in conversation along these lines.

key part of this input. The inertial functionalist is presumably not saying that in a quantum theory (for example), coordinate systems on the Hilbert space of states are candidates for counting as inertial frames.

Recognizing this key assumption brings us to the first major problem with inertial functionalism. Prior to determining which structures are spatiotemporal (which is supposed to be the task of her theory), what right does Knox have to assume that coordinate systems on Hilbert space are *not* frames? Why not suppose that the inertial frames are the Hilbert space coordinate systems in which the laws take on a particularly simple form, and conclude that spacetime is given by geometric structures on Hilbert space?

The point will be easier to illustrate with a classical example. Consider classical electrodynamics (or any other gauge theory) in its fiber bundle formulation (Healey, 2007, 7-14). The geometric structure used to formulate the theory consists of a *base space* (Minkowski spacetime in the case of special relativistic electrodynamics) which is connected to a *fiber* representing the gauge field by a projection map, generating a larger space called the *total space.* On the usual interpretation of the theory, the base space represents spacetime and the total space represents spacetime together with the state of the field in a single combined structure.

Suppose we hand an inertial functionalist the fiber bundle version of special relativistic electrodynamics and ask her to determine the theory's spacetime structure. Will she give the canonical answer, that the spacetime of the theory is Minkowski spacetime? It depends on which coordinate systems we identify as the theory's reference frames!

If we stipulate that the frames are given by four-dimensional coordinate systems on the base space, we will get the usual answer: the inertial frames are the frames on the base space (Minkowski spacetime) in which the theory takes a uniquely simple form. These will be the rest frames of special relativity, and hence the inertial functionalist will answer that the Minkowksi metric gives the structure of spacetime. But there is nothing to stop us from beginning with a different stipulation: that the "reference frames" are higher-dimensional coordinate systems on the total space. In this case, the inertial functionalist would find the coordinate systems on the total space in which the laws take on their simplest form, and answer that the structure of spacetime is given by some geometric structure on the total space.

Thus depending on our choice of which coordinate systems are the frames, Knox's account gives wildly different answers about the spacetime structure of the same theory. And any basis for deciding between coordinates on the base space and coordinates on the total space would seem to rest on assumptions about which of these two structures is more apt to represent spacetime–which is the question Knox's view was supposed to answer.

This illustrates a very serious general problem for inertial functionalism. To return to the metaphor of the input-output machine, what we need from a workable definition of spacetime is a machine that takes a physical theory's models (and no further stipulations or structures) as input, and as output tells us which of the theory's structures are spatiotemporal. Because inertial functionalism requires both a theory *and* a specification of the reference frames as input, it cannot give an unconditional answer about the spacetime structure of a theory. It can only provide a conditional answer, of the following sort: "*If* the reference frames are coordinate systems on the base space, *then* spacetime is Minkowski spacetime." But this is not sufficient to do the work Knox set out to do.

By itself, this problem is severe enough to sink inertial functionalism as an analysis of our spacetime concept. But further problems arise when the inertial functionalist account is applied to some specific cases.

4 Counterexamples to inertial functionalism

I will now present some examples which show (first) that Knox's inertial functionalism does not universally succeed as an analysis of our spacetime concept, and (second) that her inertial functionalism does not always give the right verdict in its intended domain of Newtonian and relativistic theories either. The first claim I think should be quite uncontroversial after a bit of thought; the second strikes me as more controversial but still clearly true, once the examples I have in mind are taken into account. I will begin with a simple but rather fantastical toy example, and then move on to some more physically interesting ones.

Consider a hypothetical universe consisting of a neo-Newtonian geometry with finite spatial volume, containing particles. These particles do not, however, move in accord with any forces, nor do they undergo continuous motion. Instead, they teleport from place to place: at any given instant of time, each particle has an equal chance to be located at any position in space (and an equal chance to be located in either of two regions of equal volume, etc.).

These particles follow very different laws from the physical objects we're familiar with. But I think we can recognize the structure they exist in–the structure that determines the probability that a particle is in any given region at a given time–as spatiotemporal. Yet no definition of inertial motion can be applied to these particles. The laws of this model do not include forces, since acceleration is an undefined quantity for the teleporting particles, so we cannot define inertial frames as frames in which force-free particles move at a constant velocity. The teleporting law that governs their changes of state can be stated with equal simplicity in any frame, since it appeals only to the notion of volume, which is invariant across frames.

This example may seem alien and silly, and so it is–but I don't think it's *too* silly. That is, the example is not so alien that it falls completely outside our concept of spacetime. One might object that the toy theory is empirically meaningless because it cannot describe observers (or composite objects either), but it shares this feature with Newtonian gravitation, which cannot describe stable bulk matter, at least not unless other forces are included. Or one might object that the toy theory does not describe anything we can recognize as matter, since the "particles" move discontinuously and are not subject to any fundamental forces. But again, it shares this feature with a serious theory, namely the "flash" formulation of GRW quantum mechanics (Tumulka, 2009). So I see little reason to conclude that this toy theory is so strange that our ordinary concept of spacetime must break down if applied to it.

Regardless, my case against inertial functionalism does not stand or fall with this single toy example. There are other more realistic counterexamples as well.

First, consider topological quantum field theories (TQFTs). These are quantum field theories with no local degrees of freedom. The observables (physically significant quantities) of these theories are all topological invariants–quantities invariant under spacetime transformations that don't change topology. Important examples include Chern-Simons theory (Witten, 1988) and quantized general relativity in three spacetime dimensions (Baez, 2001). Both these theories see heavy use in foundational work on quantum gravity, including applications to string theory, normally as toy models used to illuminate certain features of physically realistic theories.

Consider three-dimensional quantum gravity. It is well-known that in three spacetime dimensions or fewer, general relativity becomes trivial locally, in that the curvature of the metric at a point is uniquely determined by the matter content (stress-energy tensor) at that point (Carlip, 1995, 447-451).[11] Thus there are no additional degrees of freedom within the gravitational field. Therefore in three-dimensional vacuum general relativity ("pure gravity" theory, in which the stress-energy is zero in all states) there are no local degrees of freedom whatsoever.

That said, classical vacuum general relativity is not quite trivial globally, because multiple different topologies are possible. This remains true in the quantum version of three-dimensional vacuum general relativity. Thus it provides a rare and significant example of an indisputably *background-independent* quantum theory–a quantum theory that isn't set within a fixed spacetime.

The important thing about TQFTs for our purposes is that they describe spacetime without ascribing to it any inertial or metric structure. If one likes, one can sometimes define a metric or affine connection in a TQFT, but this amounts to imposing a sort of unphysical choice of gauge. The physical observables of these quantum theories are quantities like the length of the shortest geodesic with certain connectedness properties like wrapping around the radius of a closed space some number of times (Baez, 2001, 189). (Note that which geodesic fulfills these conditions–and indeed, which sets of points form geodesics–need not be specified in defining such observables. It may seem peculiar that metrical quantities like length could count as topological invariants, but this is possible in a quantum theory in which states are superpositions of different metrics.[12]) The local value of the metric, on the other hand, is not an observable in any TQFT. To portray these theories as describing local

[11] This holds if we understand 'three-dimensional general relativity' to mean the theory governed by the Einstein field equations in three dimensions. Interesting questions arise as to whether this is the correct way to identify general relativity in lower dimensions, given the vastly different qualitative behavior of the resulting theory (Fletcher *et al.*, forthcoming). This is an interesting question in the semantics of scientific theories, but it is not necessary to answer it for present purposes, since the important question for present purposes is only whether the lower-dimensional Einstein field theory describes spacetime–which Fletcher *et al.* agree that it does.

[12] The probability for a given value of the geodesic's length is given by integrating the amplitude over all metrics in the superposition assigning that value to the length (Barrett, 2003). These probabilities, and hence the observable's expectation value, are topological invariants, even though the individual metrics in the superposition are not.

An analogy that may help: In ordinary quantum mechanics, the singlet state $1/\sqrt{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$, and the observables defined on it, are permutation invariant even though the two terms in the superposition are individually not permutation invariant. Similarly, the state in three-dimensional quantum gravity, and the observables defined on it, are topologically invariant even though the state is a superposition of metrics which are not individually topologically invariant.

inertial structure would therefore be a serious distortion.

To say that they describe *spacetime* structure, on the other hand, is no distortion at all. The theory I've just described is a quantum gravity theory whose observables are quantities like the length of certain geodesics and whose classical limit is a spacetime theory, namely general relativity. Some candidate quantum gravity theories describe spacetime as emergent, of course, but that is clearly not the case here, since the observables are recognizably spatiotemporal. Thus it is possible for spacetime structure to exist in the absence of inertial structure, contrary to the implications of inertial functionalism.

Of course, TQFTs exist fairly far outside Knox's intended domain of familiar Newtonian and relativistic physics. So we have not yet ruled out the possibility that inertial functionalism suffices for this class of theories. To show that it does not suffice even within that limited domain will be the task of my next example.

On Knox's view, the only *spatiotemporal* structures are those that must be fixed in order to determine the class of inertial frames. Two structures that do not factor into the determination of inertial frames are the orientation of time (if the laws are not preserved by a transformation exchanging past and future) and the orientation of parity or handedness (if the laws are not preserved by a mirror-reflecting transformation that exchanges left and right). A parity transformation, which mirror-reflects everything in spacetime across some spatial plane, induces no change in which trajectories count as inertial. Neither does a time-reversal transformation which exchanges past and future.

Since they leave inertial structure invariant, Knox's inertial functionalism would predict that parity and time-reversal must always be symmetries of spacetime. But this is not so. In the Standard Model of particle physics, the weak interaction violates parity and is also thought to violate time-reversal invariance (because it violates CP and the combination of CP with time-reversal must be a symmetry according to the CPT theorem).

This means that in the spacetime where weak interactions take place, there must be spatiotemporal structures that determine a preferred direction in time and a preferred parity orientation. In their discussions of parity violation and substantivalism, Huggett (2000) and Pooley (2003) present an example of what such a structure would have to look like. To make sense of an absolute handedness, Huggett writes, "requires the use of an orientation, and this structure must be a property of or inhere in something distinct from bodies. *The only plausible candidate for the role of supporting the nonrelational structures is the spacetime* *manifold*" (Huggett, 2000, 236; emphasis in original).

This preferred orientation structure (or structures; there may be more than one) takes the form of a handed field which associates a preferred parity orientation with each point in spacetime in a continuous way (so that nearby points agree locally on which orientation is preferred; this entails that all points will agree globally in spacetimes where global orientation is a well-defined notion). As Huggett suggests in the quote above, it is not plausible to interpret this orientation field as anything but a piece of spacetime structure. The orientation field is located everywhere, carries no energy or momentum, and possesses the same state in all solutions of the theory–all features that fit much more naturally with our pre-existing concept of spacetime than with our concept of matter.

Yet the inertial functionalist cannot consistently acknowledge that the orientation field is a spatiotemporal structure. It plays no role in the definition of inertial frames, since it does not couple to the gravitational field and only affects weak interactions like the decay of certain elementary particles. Inverting the orientation field has no effect on affine structure. The same is equally true of the temporal orientation required to make sense of weak CP violation. Inverting the arrow of time does not change which trajectories are inertial. So a time orientation cannot count as spatiotemporal structure either according to the inertial functionalist.

So far I've painted a rather simple picture of the conflict between time and parity orientations and Knox's inertial functionalism. But the problem is complicated somewhat by Knox's non-standard definition of an inertial frame. Recall from Section 2 that she provides three criteria:

1. Inertial frames are frames with respect to which force free bodies move with constant velocities.
2. The laws of physics take the same form (a particularly simple one) in all inertial frames.
3. All bodies and physical laws pick out the same equivalence class of inertial frames (universality). (Knox, 2013, 348)

In this discussion of parity and time orientations, I have pointed out that these orientations are never necessary to define which frames meet Knox's first criterion. Adding a parity or time orientation to a geometry will never change which trajectories are inertial, and hence will never change which coordinate systems classify force-free objects as moving with constant velocity.

One might still hope to apply Knox's second criterion, however, and claim that the temporal or parity orientation helps determine the class of frames in which the laws are simplest. If (e.g.) inverting parity introduced additional terms into a theory's Lagrangian, that would make the equations of motion "more complicated" in some frames than in others. But in general, this does not happen in parity- or time-reversal-violating theories. Instead the only differences will appear in the form of the terms (e.g., the introduction of a minus sign). To give the most relevant concrete example, in the Lagrangian for the electroweak theory, written by the usual convention in a "right-handed" coordinate system (one in which cross products are computed using the right-hand rule), only the left-handed components of the spinor fields appear in the weak couplings. But of course if one inverts parity by changing to a left-handed coordinate system, the spinor fields do not simply disappear from those terms in the Lagrangian altogether. Instead, their *right*-handed components will appear in the parity-inverted Lagrangian's weak couplings.

So Knox cannot claim that a parity orientation helps determine which frames permit the laws to take their simplest form. That said, there is one way for Knox's second criterion to "notice" a time or parity orientation (a rather sneaky way, if you ask me). But applying her second criterion in the required way brings it into conflict with the third criterion, at least in the rather important case of the Standard Model of particle physics.

Let me explain. Knox's second criterion does not only say that the laws take on a particularly simple form in all inertial frames. It says that the laws take on the *same* particularly simple form in all inertial frames!And arguably it is false in a parity-violating theory (such as the electroweak theory) that the laws take on the same form in right- and left-handed coordinate systems (even though the form they take in right-handed coordinates is just as simple as the form they take in left-handed coordinates).[13] So taking criterion 2 literally, in a parity-violating theory, either the right-handed inertial coordinate systems (but not the left-handed ones) are inertial frames, or all the left-handed coordinate systems

[13] I say this is "arguable" because we are verging into delicate territory about what it is for the laws to take "the same form" in different frames/coordinates. For example, suppose one writes down relativistic laws with a metric signature $(+---)$, then writes down the same laws except with metric signature $(-+++)$. Has one changed the "form" of the laws? (Of course this is not a change in coordinates, but it is similar in that it is a conventional change in the geometric representation of the laws.)

are inertial frames. Thus a parity orientation does help determine the class of inertial frames after all.

It is not entirely clear that criterion 2 provides a satisfactory inertial functionalist treatment of the parity case, though. To begin with, if we conclude from the above reasoning that either the right- or left-handed frames are the inertial frames (but not both), it still seems mistaken to say that a parity orientation is doing the work of determining which frames are inertial. The orientation does not, after all, tell us whether the inertial frames are the right-handed ones or the left-handed ones. One can certainly employ the orientation to set a convention according to which the (e.g.) right-handed frames are inertial and the left-handed frames are non-inertial. But the existence of the orientation does not entail that the right-handed frames are inertial, since we could just as easily have used the same geometric structure to set the opposite convention. It is certainly not determining that the right-handed frames are inertial in the same sense that the Minkowski metric determines that the non-accelerating frames are inertial in the sense of Knox's criterion 1 (since the fact that force-free bodies move with constant velocity in these frames can be proven given the metric). Indeed, it is not obvious that any geometric structure could do the work of specifying which handedness the inertial frames possess (as opposed to specifying that they all have the same handedness). So it may be that no geometric structure could fully fulfill Knox's "spacetime role" in a parity-violating theory.

Rather than pursue this issue further, let me turn to the issue of conflict with Knox's criterion 3, which she calls universality. This stipulates that all laws must pick out the same frames as inertial. As we've just seen, taken literally, Knox's criterion 2 implies that a parity-violating law will not pick out all of the left- and right-handed frames as inertial. But in a theory like the Standard Model that combines a parity-violating law with other parity-invariant force laws, both classes of frames will count as inertial according to the parity-invariant laws. (In the case of the Standard Model, the strong force will pick out both right- and left-handed frames as inertial, but as we've seen the electroweak force will disagree.) So universality cannot obtain in any theory that combines parity-violating forces with parity-invariant forces. In such a theory, Knox's three criteria are inconsistent and there are no inertial frames, as she defines them. This is clearly an unsatisfactory result.

I've acknowledged that the overall approach of spacetime functionalism is highly attractive, but we've seen that Knox's own functionalist account cannot succeed in capturing our spacetime concept as it applies to theories like TQFTs and parity-violating interactions. But these same examples also show that a wide variety of very different structures seem to meet our existing definition of spacetime. So what hope is there for maintaining that we possess a single, unified concept of spacetime at all?

There is hope, I claim, if we understand our spacetime concept as a cluster concept.

5 The cluster concept view

A cluster concept is a concept with multiple criteria of application, none of which are necessary conditions for a thing to fall under the concept. Consider species concepts, like the concept of a cat.[14] A being's having four legs is in a sense a criterion for it to count as a cat, but there are many cats with fewer than four legs. Having four legs is neither necessary nor sufficient for cat-hood, but a being's having four legs is (so to speak) a "point in favor" of its being counting as a cat. Any being that earns enough "cat points" by satisfying enough such criteria to a sufficient degree (having two ears, chasing mice, making me sneeze, etc.) will fall under our cat concept.

In his account of our concept of art as a cluster concept, Berys Gaut provides a nice description of the logical structure of such concepts:

> There are several criteria for a concept. How is the notion of their *counting toward* the application of a concept to be understood? First, if all the properties are instantiated, then the object falls under the concept: that is, they are jointly sufficient for the application of the concept. More strongly, the cluster account also claims that if fewer than all the criteria are instantiated, this [may be] sufficient for the application of the concept. Second, there are no properties that are individually necessary conditions for the object to fall under the concept: that is, there is no property which all objects falling under the concept must possess. These conditions together entail that though there are sufficient conditions for the application of a cluster concept, there are no *individually necessary and* jointly sufficient conditions. Third, though there are no *individually* necessary conditions for the application of such a concept, there are *disjunctively* necessary conditions:

[14] Not the biological concept of a cat as a natural kind whose evolutionary descent or genetic structure is essential to it, but rather the folk concept of a cat on which (e.g.) the Cheshire cat is a cat.

> that is, it must be true that some of the criteria apply if an object falls under the concept. This clause is required, for otherwise we will merely have shown that there are sufficient conditions for a concept to obtain, rather than showing it to be a cluster concept. (Gaut, 2000, 26-27)

We can readily see how this applies to the cat example. If an entity meets none of the criteria that we ordinarily take to characterize cats, it is obviously not a cat. But any one of these properties may be absent in any particular cat, since not all cats have ears, four legs, and so on. Being a cat is a matter is a matter of satisfying enough of the criteria to a sufficient degree.

Rather than showing that inertial structure is a necessary and sufficient condition for counting as spacetime, I think Knox has shown that inertial structure is one criterion (in the sense explained above by Gaut) for counting as spacetime. The fact that the Minkowski metric in special relativity determines the structure of inertial frames is certainly a point in favor of interpreting that metric as a spatiotemporal structure. But the fact that the preferred orientation in a parity-violating theory does not help determine inertial structure is not automatic proof that the preferred orientation isn't a piece of spacetime structure. There are other criteria for our spacetime concept besides the determination of inertial frames.

Consider the reasons I adduced earlier in favor of counting the preferred orientation of a parity-violating theory as spacetime structure: it is located everywhere, carries no energy or momentum, and possesses the same state in all solutions of the theory. The last three of these criteria are not met by spacetime in general relativity. But that does not mean they aren't criteria for our spacetime concept, on the cluster concept view. On the cluster concept view, one would expect most theories to involve a picture of spacetime that fails some of the criteria for the concept–just as most real-life cats depart in several ways from the Form of the Ideal Cat.

It may be that since the advent of general relativity, we have revised our spacetime concept to reject one or more of these criteria, so that they no longer count at all toward our spacetime concept. Conceptual change tends to accompany scientific revolution, as Kuhn has taught us. So perhaps (for example) we no longer consider a structure to be a better candidate for spatiotemporal structure if its state can vary between solutions. But it seems plausible at least that this criterion has been replaced with a new one: it still seems to me to be a criterion for meeting our spacetime concept that a structure is non-dynamical with respect to non-gravitational interactions.

It's worth noting at this stage that two variants of the cluster concept picture of spacetime are possible. On what might be called the "straightforward" variant, any collection of structures that meets the criteria to a sufficient degree will count as a spacetime. This implies that the same universe (or the same state of a theory) can contain multiple distinct spacetimes, just as our world contains multiple cats as long as more than one entity satisfies enough of the criteria for the cat concept. For example, consider a theory set on Minkowski spacetime with a preferred temporal orientation. Then on this straightforward version of the cluster view, it will surely turn out that there are two sufficiently good realizers for our spacetime concept: the Minkowski geometry without the temporal orientation, and the Minkowski geometry plus the temporal orientation.

This would seem to make the straightforward variant pretty implausible as an analysis of our concept of spacetime, since we would ordinarily say in such an example that only the full geometric structure including the temporal orientation constitutes spacetime. This leads us to a more plausible version of the cluster view, the "best realizer" variant. On this version, only the (ideally unique[15]) structure that best satisfies the criteria of the cluster concept counts as spacetime, even in cases where other structures also meet the criteria to a sufficient degree that they would count as spacetime if they existed alone. Because it seems to handle examples like the temporal orientation case better than the straightforward version, I will assume the best realizer version of the cluster concept view for the remainder of this essay.

Obviously the question of which criteria count toward the spacetime concept will be a complicated one on the cluster concept view. I won't make any attempt to give an exhaustive list of candidates here, but the following are examples of criteria which are logically independent of Knox's inertial criteria and which seem to also count toward a structure's satisfying our spacetime concept:

- The structure is non-dynamical, at least with respect to non-gravitational interactions.[16]

[15]In cases where there is a tie, it remains an option to treat the different equally-good realizers as distinct coexisting spacetimes, as the straightforward variant would have it. But plausibly the criteria are many and fine-grained enough to prevent ties except in unusual examples.

[16]That is to say, the state of the structure does not differ between different states of the theory; or if it

- The structure is (in some sense) located everywhere in all states of the theory.
- The structure does not carry energy or momentum.
- "Vacuum" solutions exist which describe the (putatively) spatiotemporal structure in the absence of other structures.
- There are no *other* structures in the theory which can exist without the (putative) spacetime structure. That is, there are no solutions of the theory in which the spacetime structure does not exist but some other structure does exist.
- The structure grounds or explains a family of modal facts about which states are *geometrically possible*, where geometric possibility does not reduce to physical possibility (Belot, 2013, 50-51).
- It is a (higher-order) law of nature that the geometric symmetries of the structure are dynamical symmetries of the theory (Skow, 2006; Janssen, 2009).
- Forces propagate across the spatial distances defined by the metric characterizing the structure (so that long-range forces like electromagnetism fall off proportionately to the inverse square of this distance, and so on).

Again, this is not meant to be an exhaustive list. Rather it is meant to illustrate that a vast number of different criteria could plausibly figure into our ascription of the name 'spacetime' to a given theoretical structure, depending on the details of the laws that define that structure. And indeed, Knox's own criterion,

- The structure determines the difference between inertial and non-inertial frames of reference,

belongs high on this list, perhaps even at the top. She has certainly shown that it's a very important criterion. My only disagreement is with her claim that it is the sole criterion.

Is the cluster concept view itself a form of spacetime functionalism, as I suggested earlier? The answer depends on whether the criteria counting toward the concept are all "functional"

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does differ, its state is fully determined by a "mass-energy" source like the mass in Newtonian gravity or the stress-energy in general relativity.

criteria. Knox comes closest to defining function when she suggests that the functional role of spacetime structure is "the role it plays in our theory." (Knox, forthcoming, 10) Ultimately her point is that it cannot be a simple matter of stipulation, or a metaphysical primitive, which structures in a theory are spatiotemporal. Rather, structures must count as spatiotemporal (or not) in virtue of the roles ascribed to them by the theory. I take it that the role a structure plays in a theory is a matter of the law-governed relationships it bears to other structures in the theory. The criteria given above certainly meet that description; each criterion spells out a certain role that a geometric structure might play in a physical theory–that is, a certain way that a theory's laws might relate the structure to other entities and/or properties described by the theory.

This may strike the reader as an overly weak or inclusive definition of functional role. Although I take it to be Knox's definition as well, I have no quarrel with readers who would insist that in order to speak of the "function" of some structure within a theory, that structure must "do something," rather than simply be ascribed some property or relation by the theory's laws. On a more restrictive definition of this sort, the cluster concept view may not count as a functionalist view. But this is a purely terminological question about how to categorize the view. Ultimately, my substantive claim here is that the cluster concept view is correct. I would also suggest that it ought to count as a functionalist view, since it denies that spacetime has a primitive essence or that it can be stipulated, independently of a theory's laws, which of its structures are spatiotemporal. But if the reader comes away convinced of the view's correctness, but unconvinced that it is a form of functionalism, I will declare my mission accomplished.

6 Fundamentality and functionalism

I'd now like to advance what I think will be a more controversial criterion that I claim counts toward our spacetime concept. A structure is more apt to be called 'spacetime,' I claim, the more physically fundamental it is.

I don't intend this proposal to stand or fall with any particular account of fundamentality. As a rough guide, structure A is ordinarily more fundamental than structure B if A features in the basic laws of a more fundamental theory than those that B appears in.[17] So for

[17] As another rough guide, T is a more fundamental theory than T' if T' reduces to T in some sense.

example, the metric of general relativity is more fundamental than the metric of Newtonian gravity theory, and quantum fields are probably more fundamental than particles. As another rough guide (often related to the first), A is more fundamental than B if B is composite or emergent and A is not. So quarks are more fundamental than protons, and protons are more fundamental than hydrogen atoms.

The arrival on the scene of proposed quantum gravity theories where spacetime emerges from a more fundamental non-spatiotemporal substrate has sometimes been taken to imply that there is no connection between fundamentality and our spacetime concept. There had better not be any such connection, the thought goes, because we exist in spacetime and yet there's a good chance that spacetime is emergent in our world.

But on the cluster concept view of spacetime, this inference rests on a confusion. Fundamentality can be a criterion for the spacetime concept even if the actual spacetime we live in fails that criterion. This is especially true given that fundamentality is a graded notion, and the emergence of spacetime is only proof that it cannot be *perfectly* fundamental. The spacetime we live in might still be highly fundamental, so that the fundamentality criterion still provides part of the explanation for why the spacetime we live in falls under our spacetime concept.

On reflection, there are clear reasons why fundamentality ought to count as a criterion for our spacetime concept. To begin with, the referents of our words (and so too the extensions of our concepts) are probably determined in part by the fundamentality of objects and properties in the world. In particular, Lewis (1984) has argued persuasively that, for any descriptivist theory of meaning for words or thoughts to succeed, it must privilege interpretations of our language and concepts that refer to more fundamental properties over ones that refer to less fundamental properties.[18] (This makes fundamentality all the more important in physics, because the only serious competitor for descriptivist theories is the causal theory of reference, and causality is arguably not a well-defined concept in many areas of physics

[18] As Lewis points out, Putnam's model theoretic argument (Putnam, 1977) implies that there is no way to distinguish an intended model for a theory by adding posits to the theory itself (by adding "more theory"). Thus fundamental structures in the domain of the theory, which are more apt subjects of reference for our theoretical terms than non-fundamental structures, are necessary to avoid the unacceptable result that all possible theories (or perhaps all possible empirically adequate theories) are true. Adapting a classic example: On this view, it is the relative fundamentality of green as compared with grue that explains why our word 'green' refers to the former property and not the latter, despite the fact that its use is ambiguous between those two interpretations.

(Norton, 2003).) So other things being equal, it is likely that the referent of our theoretical term 'spacetime' will be a highly fundamental entity rather than a less-fundamental one.

Second, the notion that fundamentality is a criterion for the spacetime concept fits well with both substantivalist and relationist conceptions of spacetime. Substantivalists about general relativity, for example, tend to hold that spacetime is among the most fundamental structures described by the theory. Indeed, North (forthcoming) has recently argued that substantivalism should be defined as the claim that spacetime is at least as fundamental as material objects. (This is my preferred definition of substantivalism, and I will assume it for the remainder of this essay.) On the other hand, spacetime is obviously not a fundamental substance according to relationists.[19] But it is typical for relationists to hold that spacetime constituted by, or reducible to, a system of highly fundamental relational properties (that is to say, the spatiotemporal relations are among the most fundamental properties described by spacetime theories even according to typical relationists).[20] It is hardly common for relationists to suggest that spatiotemporal relations are reducible to something more fundamental within the domain of general relativity, for example. So all parties to the substantivalist/relationist dispute would seem to agree that spacetime is highly fundamental, if not perfectly fundamental.

Finally, the inclusion of fundamentality as a criterion in our spacetime cluster concept allows us to explain why we are tempted by what Knox calls the "container metaphor." There is obviously something intuitive about this metaphor, which has often been used by substantivalists to describe and motivate their view. The idea is that spacetime is a sort of container for matter, with material objects as its contents.

Knox considers an objection to her inertial functionalism arising from this picture: there is no guarantee that the structure determining inertial trajectories is the same structure in which matter is "contained," and so there is no guarantee that inertial functionalism gives the right answer about what should count as spacetime. Knox responds that this criticism

[19] I take relationism to be the denial of substantivalism–that is, following North, the claim that spacetime is less fundamental than material objects.

[20] As North puts it: "The relationalist says that material bodies, and certain of their properties and relations, are fundamental, and a worldï¿½s spatiotemporal structure holds in virtue of them. [...] So, for example, the fact that a world has a Euclidean spatial structure is grounded in, holds in virtue of, the fact that its particles are, and can be, arranged in various ways, with various distance relations between them." (North, forthcoming, 13) On this sort of picture, the distance relations are treated as fundamental properties even though spatial structure itself is not a fundamental substance.

is akin to (but much less plausible than) qualia-based objections to functionalism about the mind: "Just as the believer in Zombies, or the proposer of homunculus-head thinks that states could fill the functional role of mental states, but nonetheless be missing something, the proponent of the container thinks that spacetime functionalism fails to capture the essential nature of spacetime." (Knox, in progress, 16) But as Knox responds, unlike in the case of conscious experience, we cannot claim to have any direct access to the essential "container" nature of spacetime. So there is ultimately no rigorous sense to be made of the container metaphor.

This is a damning objection insofar as the container metaphor is meant to rest on a brute intuition about spacetime having a container-y nature which is not describable except via the metaphor. But it is not obvious that the metaphor's defenders can't spell out what they mean in more literal terms. For example, in his discussion of the nature of space in non-relativistic quantum mechanics, David Albert draws a distinction between "the space of possible *interactive distances*" (which I take to be the entity Knox aims to define using inertial functionalism[21]) and the "*stage* on which whatever theory we happen to be entertaining at the moment depicts the world as *unfolding*." (Albert, 1996, 282) So far all we've been given is a metaphor, but Albert continues: "a space (that is) in which a specification of the local conditions at every address at some particular time (but not at any proper *subset* of them) amounts to a complete specification of the physical situation of the *world*, on that theory, at that time." (Albert, 1996, 282-283; all italics are of course Albert's)

This is a literal, not metaphorical, definition of the sense in which Albert thinks of space(time) as a container. Indeed, it is a *functional* definition in the same sense as Knox's inertial definition of spacetime–it specifies a role in the laws that spacetime (*qua* stage/container) must fill. A container spacetime must be a geometry such that the description of its state at every location provides complete and non-redundant information about the state of the physical world.

There remains a small puzzle, insofar as theories may admit more than one geometric description that satisfies Albert's criteria. For example, a description of a classical mechanical state in phase space may provide all the same information as its description in ordinary

[21]Note that this means Knox's approach and Albert's may turn out to be compatible in a sense, since there is nothing in Albert's work to rule out the possibility that Knox's inertial functionalism is the correct way to identify the space of interactive distances.

three-dimensional space. But phase space and three-dimensional space can't both be the stage on which the classical mechanical world unfolds. I take Albert's suggestion here to be that the space in which the world "unfolds" is the *most fundamental* geometric structure meeting his criteria–that is, the most fundamental geometry whose state at all its locations provides a non-redundant description of the world. Earlier he writes: "the three-dimensional multiple-particle language of our everyday lives supervenes on the exact and complete and fundamental language of the world, which is the language of *wave functions* (and whatever else) in *configuration space*." (Albert, 1996, 279) A different geometric formalism might provide exactly the same information about the physical state, but on Albert's view, it would do so in less fundamental terms than the configuration space formalism (perhaps because it should properly be understood as reducible to the configuration space description) and thus it would not represent the space in which the quantum world unfolds.

Albert concludes that our spatial concept is really two separate concepts–interactive distance structure and stage on which the world unfolds–and that these two concepts are coextensional in classical physics but not (on his view) in quantum physics. There is certainly something to this claim, and in particular, we can always ask whether a theory posits a *fundamental background* (a perfectly fundamental geometric structure meeting Albert's criteria) which is distinct from the space(time) of interactive distances, or from Knox's inertial structure. But there is also a risk of multiplying concepts indefinitely, if we treat each logically independent feature we ordinarily take spacetime to have as a separate concept of a sense in which a structure can be spacetime. To the contrary, it seems to me that we do have an overarching concept of spacetime as the sort of entity that ideally ought to meet many different criteria–to repeat myself, a cluster concept.

The real lesson of Albert's work on configuration space is that fundamentality is one of the criteria for our spacetime concept. In some theories, there may be no fundamental background geometry, or there may be a fundamental background which fails too many other criteria to count as spatiotemporal. But the more fundamental a background geometric structure is, the more apt it is to satisfying our spacetime concept, other things being equal.

If we accept that fundamentality is one of the criteria for our spacetime concept, further lessons follow about the interesting and much-discussed case of spacetime in Newtonian mechanics and gravitation theory.

This is a debate that lends itself to a pernicious equivocation if one is not careful. The central question is usually phrased in roughly this way: What is "the spacetime setting for Newtonian physics"? (Wallace, forthcoming) The question is ambiguous between two possible readings:

A Assuming we know the laws of Newtonian mechanics are true, what should we conclude about the structure of spacetime (on the basis, perhaps, of principles like inference to the best explanation)?

B What sort of (total) spacetime structure is *entailed* by the proposition that the laws of Newtonian mechanics are true?

Clearly an answer to question B will be less modest than an answer to question A. Indeed, it seems natural to suppose that no particular spacetime structure is entailed by Newton's laws of motion, because these laws by themselves are compatible with a variety of different spacetime theories: the separate space and time that Newton himself posited, neo-Newtonian spacetime, or the dynamically curved spacetime of Newton-Cartan theory.

But if one assumes inertial functionalism, a definite answer to question B comes easily.[22] For on inertial functionalism, there is a unique minimal structure necessary to represent the theory's inertial structure–or rather its lack thereof, since such theories can make do with a geometry Saunders (2013) calls *Newton-Huygens spacetime* (more commonly called Maxwell spacetime), which lacks a global distinction between inertial and non-inertial trajectories. Subsystems in such a theory will exhibit local inertial structure, though, leading Wallace (forthcoming) to suggest (following the argument of Knox (2014)) that Newton-Cartan theory is the effective spacetime structure at the level of such subsystems (again, assuming inertial functionalism).[23]

The details of this debate about inertial structure are quite beyond the scope of this essay, but I have a single comment to add. Suppose one accepts the arguments of this paper, and in particular the arguments of this section. Then first, rejecting inertial functionalism, one ought to include that question B is misguided and question A is the interesting interpretive question in the vicinity. Second, determining the inertial structure of Newtonian mechanics,

[22]Again, assuming for the sake of argument (out of charity to Knox) that a unique simplest structure determining local inertial frames exists.

[23]For further complications of this debate, see Weatherall (forthcoming, 2016), Dewar (forthcoming) and Teh (forthcoming).

as previous parties to the debate have sought to do, will not automatically answer the question of what should count as the *spacetime* structure of the theory.

Consider the competition between Newton-Huygens and Newton-Cartan spacetime. Which counts as "spacetime"? Both are sufficient to represent local inertial structure in Newtonian gravity theory. Newton-Huygens spacetime does so somewhat more parsimoniously, assuming for the sake of argument that Wallace and Saunders are correct. But in terms of fundamentality, we have two possibilities: either Newton-Huygens geometry is more fundamental than Newton-Cartan geometry, or vice versa. Suppose that Newton-Cartan geometry is more fundamental, and thus that the geometric structures of Newton-Huygens spacetime exist only as derivative structures which are properly understood as reducible to the Newton-Cartan structures.[24]

Then, if fundamentality is a criterion for the spacetime concept, it makes sense to suggest that the tie here should be broken in favor of the more fundamental theory rather than the more parsimonious one. In other words, it may be correct to conclude: If Newton-Cartan structure is more fundamental than Newton-Huygens structure, the spacetime of Newtonian gravity is Newton-Cartan; but if Newton-Huygens structure is more fundamental, the spacetime of Newtonian gravity is Newton-Huygens. It is especially easy to imagine fundamentality breaking the tie between these two candidate geometries because they are so similar; as shown by Wallace, they share the same structure at the level of isolated subsystems.

Suppose we accept that fundamentality might be sufficient to break the tie between different candidate spacetime geometries which disagree about inertial structure. Then the question of what counts as spacetime may not be answerable without doing some metaphysics–we may need to determine which geometric structures are fundamental in order to answer it. Suppose fundamentality is indeed required to break the tie between competing spacetime pictures of Newtonian mechanics. If fundamentality is epistemically accessible to us (perhaps because more explanatory, simpler or more parsimonious structures should be assumed to be fundamental) then the question about spacetime structure may be answered, though perhaps not with great confidence. If we have no way to determine which structures in a given theory should be assumed to be fundamental, on the other hand, the question of what

[24]Following North, I take this possibility to be equivalent to substantivalism about the spacetime of Newton-Cartan theory.

is the right spacetime structure for Newtonian mechanics will likewise be unanswerable.

7 Conclusions

I am a spacetime functionalist, and if I'd grasped what it meant to be a spacetime functionalist prior to encountering Knox's work, I would always have identified as one. So I'm thankful to Knox for presenting an illuminating framework which I take to be the only reasonable framework for analyzing our spacetime concept. But I don't believe the correct analysis of that concept will take a simple, easily-stated form. We use the word 'spacetime,' fruitfully and aptly, to describe a wide variety of different theoretical structures defined within very different theories. Some of these structures have nothing to do with inertia.

It seems to me that only the cluster concept view is flexible enough to hold any promise as an accurate analysis of our spacetime concept. And even if a different approach ultimately proves better able to analyze that concept, that approach will not bear much resemblance to Knox's inertial functionalism.

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