On Spacetime Functionalism

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Abstract

Eleanor Knox has argued that our concept of spacetime applies to whichever structure plays a certain functional role in the laws (the role of determining local inertial structure). I raise two complications for this approach. First, our spacetime concept seems to have the structure of a cluster concept, which means that Knox's inertial criteria for spacetime cannot succeed with complete generality. Second, the notion of metaphysical fundamentality may feature in the spacetime concept, in which case spacetime functionalism may be uninformative in the absence of answers to fundamental metaphysical questions like the substantivalist/relationist debate.

1 Introduction

In the course of work on conjectured theories of quantum gravity, physicists have begun to take extremely seriously the possibility that spacetime is not among the fundamental structures of our universe (Huggett and Wüthrich, 2013). This has prompted philosophers of science to pose the question of what it would take for a non-fundamental or emergent structure to be spatiotemporal. Independently, Harvey Brown (2005) has argued that one cannot simply posit that a certain structure, or type of structure, is spatiotemporal. Rather, a structure may only count as spatiotemporal[1] in virtue of playing the right sort of role in the laws of a theory.

Inspired by Brown's arguments and aiming to solve the problem of spacetime emergence (among other problems, including the question of how to identify spacetime structure in

[1]Or more precisely, to borrow a term of art from Brown, as having *chronogeometric significance,* i.e. as being measured directly by physical rods and clocks.

Newtonian mechanics), Eleanor Knox has recently advanced a functionalist picture of spacetime. On a functionalist picture, whether an entity (a structure, object or property–from now on I will just say "structure") counts as spatiotemporal is determined by its functional role. The functional role of a physical structure is its role in the physical laws, which often boils down to its implications about the motion of material objects.

Beyond putting forward this overall functionalist framework, Knox advocates a particular version of it, which I'll call *inertial functionalism*. She suggests that the functional role of spacetime is to determine the structure of local inertial frames in a theory. (A local inertial frame is a coordinate system over a small region of a manifold in which the dynamical laws–the laws of motion, or more generally of time evolution–take on a uniquely "simple form" (Knox, forthcoming, 12); a more precise definition will follow in the next section.)

I find the overall framework of spacetime functionalism attractive. Many geometrical structures are used in physics–state space, momentum space, configuration space. Not all of these deserve the name 'spacetime' (Brown, 2005, Section 8.2). Nor is it plausible that the difference between spacetime and these other geometries is a primitive one. It must have something to do with each geometric structure's role in the laws. As Knox rhetorically asks, "If our conceptual grasp on spacetime is not exhausted by the role it plays in our theory, what might the extra ingredient be?" (Knox, forthcoming, 9-10) I can't think of anything.

So I heartily agree with Knox's basic picture that spacetime is defined by the role it plays in theories (or laws). But *inertial* functionalism cannot be the whole story, at least not if the goal is an analysis or explication of our spacetime concept. For there are clear examples of theories (so-called *topological quantum field theories*) which lack the sort of inertial structure that Knox's account requires, but which include structures that are obviously playing the spacetime role in a meaningful sense. Further, certain other theories (those with forces that violate parity and/or time-reversal invariance) include spacetime structures which will not count as spatiotemporal on Knox's account, because they do no work toward determining inertial structure.

On a more programmatic level, inertial functionalism proceeds from the working assumption that precise necessary and sufficient conditions for the "spacetime role" can be spelled out. But this strikes me as false, because our spacetime concept has the structure of a *cluster concept*. Rather than possessing a single set of necessary and sufficient conditions, cluster concepts can be satisfied in a variety of different ways by different entities falling under them.

Having four legs is neither necessary nor sufficient for a being to fall under our concept of cat, for example–but it helps a being to count as a cat if it has four legs. The same goes for inertial structure and our concept of spacetime, I will argue.

Once this cluster concept view of spacetime is appreciated, it is apparent that a vast number of different theoretical properties and relations can help determine whether a given structure is spatiotemporal. Among the properties that help determine this, I will argue, is the property of physical fundamentality. And if this is correct, the question of substantivalism–of whether the "spacetime" structure posited by a theory is part of its fundamental ontology–is prior to the question of what counts as spacetime.

It is my understanding, from previous conversation and correspondence with Knox, that she does not intend to advance her inertial functionalism as a set of necessary and sufficient conditions that any structure whatsoever must meet in order to count as spatiotemporal. Rather, she intends her inertial criteria to be necessary and sufficient conditions for a structure to count as spacetime in a broad class of the theories most commonly discussed in foundational work (mainly Newtonian and relativistic theories). This is not always clear in written statements of her view, in which she sometimes suggests that the view is an analysis of our concept of spacetime (Knox, forthcoming, 2) and that her view entails that (for example) there is no spacetime in Aristotelian cosmology because of its impoverished inertial structure (Knox, forthcoming, 13).

Despite these passages, I take it that Knox is ultimately amenable to my conclusion that her inertial criteria do not provide general necessary and sufficient conditions for a structure to count as spatiotemporal. She has also expressed sympathy for my claim that spacetime is a cluster concept. So in these respects, the present essay should not be seen as a rejection of Knox's own position, but rather as an elaboration and filling-in of her views, together with a correction of some places in which she speaks somewhat loosely. That said, Knox would not agree with some of my particular claims about which criteria fit into the spacetime concept, and in particular she would disagree with my conclusion that the question of substantivalism is prior to the question of which structures should count as spacetime. And once we appreciate the priority of the substantivalism question (as well as the examples of parity and time-reversal asymmetry) it will become clear that Knox's inertial functionalism is not always a satisfactory account of the spacetime concept even as it applies to the Newtonian and relativistic theories she has in mind. So what follows is not merely an elaboration of Knox, but also an objection to some features of her approach.

2 Knox's inertial functionalism

"I propose," Knox writes, "that the spacetime role is played by whatever defines a structure of local inertial frames."(Knox, forthcoming, 10) For Knox, inertial frames are coordinate systems which locally (to the neighborhood of a point) satisfy the following criteria:

1. Inertial frames are frames with respect to which force free bodies move with constant velocities.
2. The laws of physics take the same form (a particularly simple one) in all inertial frames.
3. All bodies and physical laws pick out the same equivalence class of inertial frames (universality). (Knox, 2013, 348)

In a theory like special relativity, with flat spacetime and no gravity, the inertial frames can be defined globally, but in theories with dynamical spacetime like general relativity we can at best make do with local inertial frames meeting the criteria above. These will be the frames in which the affine connection coefficients (Christoffel symbols) vanish in the infinitesimal vicinity of the location we're interested in studying (hence the qualification "*local* inertial frames").[2] Thus the structure of inertial frames, as Knox uses the term, is specified by giving the affine (straight line) structure of a spacetime. This means that at least a full specification of timelike geodesics (possible inertial trajectories for matter particles) will be included in the structure that defines, in Knox's sense, the local inertial frames (Knox, 2013, 349).

One might worry that this leaves out a lot of what we ordinarily think of as the building blocks of spacetime. What about distances, spans of time and intervals–metric structure, in other words? What about topological structure? Fortunately this is not a problem, because specifying the timelike geodesics uniquely determines the metric of any solution in general

[2] In general relativistic field theories exhibiting so-called minimal coupling, such frames will not always exist even local to a point (see Read *et al.*, preprint). Although this is a serious problem for the scope of Knox's inertial functionalism, it is far enough removed from the considerations I wish to raise here that I will set it aside.

relativity (up to a global scale factor, which is only needed to specify information which is unobservable and arguably unphysical).[3]

Moreover, this view seems to succeed in recovering the two symmetry principles proposed by Earman (1989, 46):

> **SP1** Any [external] dynamical symmetry of [a theory] T is a spacetime symmetry of T.
>
> **SP2** Any spacetime symmetry of T is a dynamical symmetry of T.

which Knox, following Myrvold (forthcoming) (and unlike Earman himself) takes to be analytic truths about the relationship between spacetime symmetries and dynamical symmetries (Knox, forthcoming, 11). Whenever local inertial frames meeting Knox's criteria exist, the strong uncertainty principle will apply, ensuring that the local symmetries of the metric will be local dynamical symmetries as well.[4]

Citing these achievements, Knox suggests that her inertial functionalism provides everything we could want from an analysis of our spacetime concept (at least as it applies in Newtonian and relativistic theories).[5] One might object that general relativity is strictly speaking a theory with no preferred reference frames.[6] But there is far from universal agreement that this is the right way to understand general relativity. If the existence of inertial frames in general relativity is the most controversial assumption presumed by inertial functionalism, Knox's view would seem to be in good shape.

However, this is not Knox's only contestable assumption. As we will see in the next section, her inertial functionalism gives the wrong verdict about which structures to interpret as spatiotemporal when applied to several example theories–including parity and time-reversal violating theories that appear to accurately describe the actual laws of our world.

[3]More may need to be said to recover non-affine structure in other theories aside from general relativity, but rather than pursue this point here, let us move on.

[4]To foreshadow one of the counterexamples I raise in Section 3 below: this will only entail SP1 and SP2 when the symmetries of spacetime are all and only local symmetries of the metric. But this condition fails when spacetime has discrete asymmetries such as time- or parity-reversal asymmetry.

[5]To reiterate, she has suggested in discussion that these theories are her view's intended domain.

[6]Knox (2013, 348) notes that Erik Curiel and James Weatherall have objected to her view in conversation along these lines.

3 Counterexamples to inertial functionalism

I will now present some examples which show (first) that Knox's inertial functionalism does not universally succeed as an analysis of our spacetime concept, and (second) that her inertial functionalism does not always give the right verdict in its intended domain of Newtonian and relativistic theories either. The first claim I think should be quite uncontroversial after a bit of thought; the second strikes me as more controversial but still clearly true, once the examples I have in mind are taken into account. I will begin with a simple but rather fantastical toy example, and then move on to some more physically interesting ones.

Consider a hypothetical universe consisting of particles with distance relations (invariant, as in Newtonian mechanics) between them. These particles do not, however, move in accord with any forces, nor do they undergo continuous motion. Instead, they teleport from place to place: at any given instant of time, each particle has an equal chance to be located at any position in space (and an equal chance to be located in either of two regions of equal volume, etc.).

These particles follow very different laws from the physical objects we're familiar with. But I think we can recognize the structure they exist in–the structure that determines the distance relations and time intervals between them–as spatiotemporal. Yet no definition of inertial motion can be applied to these particles. They feel no forces, so we cannot define inertial frames as frames in which force-free particles move at a constant velocity. The teleporting law that governs their changes of state can be stated with equal simplicity in any frame, since it appeals only to the notion of volume, which is invariant across frames.

This example may seem alien and silly, and so it is–but I don't think it's *too* silly. That is, the example is not so alien that it falls completely outside our concept of spacetime. One might object that the toy theory is empirically meaningless because it cannot describe observers (or composite objects either), but it shares this feature with Newtonian gravitation, which cannot describe stable bulk matter, at least not unless other forces are included. Or one might object that the toy theory does not describe anything we can recognize as matter, since the "particles" move discontinuously and are not subject to any fundamental forces. But again, it shares this feature with a serious theory, namely the "flash" formulation of GRW quantum mechanics (Tumulka, 2009). So I see little reason to conclude that this toy theory is so strange that our ordinary concept of spacetime must break down if applied to it.

Regardless, my case against inertial functionalism does not stand or fall with this single toy example. There are other more realistic counterexamples as well.

First, consider topological quantum field theories (TQFTs). These are quantum field theories with no local degrees of freedom. The observables (physically significant quantities) of these theories are all topological invariants–quantities invariant under spacetime transformations that don't change topology. Important examples include Chern-Simons theory (Witten, 1988) and quantized general relativity in three spacetime dimensions (Baez, 2001). Both these theories see heavy use in foundational work on quantum gravity, including applications to string theory.

Consider three-dimensional quantum gravity. It is well-known that in three spacetime dimensions or fewer, general relativity becomes trivial locally, in that the curvature of the metric at a point is uniquely determined by the matter content at that point. Thus there are no degrees of freedom within the gravitational field.

That said, the classical theory is not quite trivial globally, because multiple different topologies are possible. This remains true in the quantum version of three-dimensional gravity, which is able to represent transitions from one spatial topology to another (Baez, 2001, 184-187). Thus it provides a rare and significant example of an indisputably *background-independent* quantum theory–a quantum theory that isn't set within a fixed spacetime.

The important thing about TQFTs for our purposes is that they describe spacetime without ascribing to it any inertial or metric structure. If one likes, one can define a metric or affine connection in a TQFT like three-dimensional quantum gravity, but this amounts to imposing a sort of unphysical choice of gauge. The physical observables of these quantum theories are quantities like the length of the shortest geodesic with certain connectedness properties like wrapping around the radius of a closed space some number of times (Baez, 2001, 189).[7] To portray these theories as describing inertial structure would be a serious distortion.

To say that they describe *spacetime* structure, on the other hand, is no distortion at all. The theory I've just described is a quantum gravity theory whose observables are quantities like the length of certain geodesics and whose classical limit is a spacetime theory, namely

[7]Note that which geodesic fulfills these conditions–and indeed, which sets of points form geodesics–need not be specified in defining such observables.

general relativity. Some candidate quantum gravity theories describe spacetime as emergent, of course, but that is clearly not the case here, since the observables are recognizably spatiotemporal. Thus it is possible for spacetime structure to exist in the absence of inertial structure, contrary to the implications of inertial functionalism.

Of course, TQFTs exist fairly far outside Knox's intended domain of familiar Newtonian and relativistic physics. So we have not yet ruled out the possibility that inertial functionalism suffices for this class of theories. To show that it does not suffice even within that limited domain will be the task of my next example.

On Knox's view, the only *spatiotemporal* structures are those that must be fixed in order to determine the class of inertial frames. Two structures that do not factor into the determination of inertial frames are the orientation of time (if the laws can tell the difference between the past and future) and the orientation of parity or handedness (if the laws can tell the difference between right- and left-handed objects). A parity transformation, which mirror-reflects everything in spacetime across some spatial axis, induces no change in which trajectories count as inertial. Neither does a time-reversal transformation which exchanges past and future.[8]

Since they leave inertial structure invariant, Knox's inertial functionalism would predict that parity and time-reversal must always be symmetries of spacetime. But this is not so. In the Standard Model of particle physics, the weak interaction violates parity and is also thought to violate time-reversal invariance (because it violates CP and the combination of CP with time-reversal must be a symmetry according to the CPT theorem).

This means that in the spacetime where weak interactions take place, there must be spatiotemporal structures that determine a preferred direction in time and a preferred parity orientation. In their discussions of parity violation and substantivalism, Huggett (2000) and Pooley (2003) present an example of what such a structure would have to look like. To make sense of an absolute handedness, Huggett writes, "requires the use of an orientation, and this structure must be a property of or inhere in something distinct from bodies. *The only plausible candidate for the role of supporting the nonrelational structures is the spacetime*

[8]One might still hope to apply Knox's broader definition, and claim that the temporal or parity orientation helps determine the class of frames in which the laws are simplest, if (e.g.) inverting parity introduced additional terms into a theory's Lagrangian, making the equations of motion "more complicated" in some frames than in others. But in general, this does not happen in parity- or time-reversal-violating theories. Instead the only differences appear in the form of the terms (e.g., the introduction of a minus sign).

manifold" (Huggett, 2000, 236; emphasis in original).

This preferred orientation structure takes the form of a handed field which associates a preferred parity orientation with each point in spacetime in a continuous way (so that nearby points agree locally on which orientation is preferred; this entails that all points will agree globally in spacetimes where global orientation is a well-defined notion). As Huggett suggests in the quote above, it is not plausible to interpret this orientation field as anything but a piece of spacetime structure. The orientation field is non-dynamical, located everywhere, carries no energy or momentum, and possesses the same state in all solutions of the theory– all features that fit much more naturally with our pre-existing concept of spacetime than with our concept of matter.

Yet the inertial functionalist cannot consistently acknowledge that the orientation field is a spatiotemporal structure. It plays no role in the definition of inertial frames, since it does not couple to the gravitational field and only affects weak interactions like the decay of certain elementary particles. Inverting the orientation field has no effect on affine structure. The same is equally true of the temporal orientation required to make sense of weak CP violation. Inverting the arrow of time does not change which trajectories are inertial. So a time orientation cannot count as spatiotemporal structure either according to the inertial functionalist.

There is no question, I think, that this consequence of inertial functionalism is unacceptable. I can imagine no clearer case of a structure that should count as *part of spacetime* than a preferred direction of time or a preferred handedness.

I've acknowledged that the overall approach of spacetime functionalism is highly attractive, but we've seen that Knox's own functionalist account cannot succeed in capturing our spacetime concept as it applies to theories like TQFTs and parity-violating interactions. But these same examples also show that a wide variety of very different structures seem to meet our existing definition of spacetime. So what hope is there for maintaining that we possess a single, unified concept of spacetime at all?

There is hope, I claim, if we understand our spacetime concept as a cluster concept.

4 The cluster concept view

A cluster concept is a concept with multiple criteria of application, none of which are necessary conditions for a thing to fall under the concept. Consider species concepts, like the concept of a cat. A being's having four legs is in a sense a criterion for it to count as a cat, but there are many cats with fewer than four legs. Having four legs is neither necessary nor sufficient for cat-hood, but a being's having four legs is (so to speak) a "point in favor" of its being counting as a cat. Any being that earns enough "cat points" by satisfying enough such criteria to a sufficient degree (having two ears, chasing mice, making me sneeze, etc.) will fall under our cat concept.

In his account of our concept of art as a cluster concept, Berys Gaut provides a nice description of the logical structure of such concepts:

> There are several criteria for a concept. How is the notion of their *counting toward* the application of a concept to be understood? First, if all the properties are instantiated, then the object falls under the concept: that is, they are jointly sufficient for the application of the concept. More strongly, the cluster account also claims that if fewer than all the criteria are instantiated, this is sufficient for the application of the concept. Second, there are no properties that are individually necessary conditions for the object to fall under the concept: that is, there is no property which all objects falling under the concept must possess. These conditions together entail that though there are sufficient conditions for the application of a cluster concept, there are no *individually necessary and* jointly sufficient conditions. Third, though there are no *individually* necessary conditions for the application of such a concept, there are *disjunctively* necessary conditions: that is, it must be true that some of the criteria apply if an object falls under the concept. This clause is required, for otherwise we will merely have shown that there are sufficient conditions for a concept to obtain, rather than showing it to be a cluster concept. (Gaut, 2000, 26-27)

We can readily see how this applies to the cat example. If an entity meets none of the criteria that we ordinarily take to characterize cats, it is obviously not a cat. But any one of these properties may be absent in any particular cat, since not all cats have ears, four legs, and so on. Being a cat is a matter is a matter of satisfying enough of the criteria to a sufficient degree.

Rather than showing that inertial structure is a necessary and sufficient condition for counting as spacetime, I think Knox has shown that inertial structure is one criterion (in the sense explained above by Gaut) for counting as spacetime. The fact that the Minkowski metric in special relativity determines the structure of inertial frames is certainly a point in favor of interpreting that metric as spacetime. But the fact that the preferred orientation in a parity-violating theory does not help determine inertial structure is not automatic proof that the preferred orientation isn't a piece of spacetime structure. There are other criteria for our spacetime concept besides the determination of inertial frames.

Consider the reasons I adduced earlier in favor of counting the preferred orientation of a parity-violating theory as spacetime structure: it is located everywhere, non-dynamical, carries no energy or momentum, and possesses the same state in all solutions of the theory. The last three of these criteria are not met by spacetime in general relativity. But that does not mean they aren't criteria for our spacetime concept, on the cluster concept view. On the cluster concept view, one would expect most theories to involve a picture of spacetime that fails some of the criteria for the concept–just as most real-life cats depart in several ways from the Form of the Ideal Cat.

It may be that since the advent of general relativity, we have revised our spacetime concept to reject one or more of these criteria, so that they no longer count at all toward our spacetime concept. Conceptual change tends to accompany scientific revolution, as Kuhn has taught us. So perhaps (for example) we no longer consider a structure to be a better candidate for spatiotemporal structure if it is non-dynamical. But it seems plausible at least that this criterion has been replaced with a new one: it still seems to me to be a criterion for meeting our spacetime concept that a structure is non-dynamical with respect to non-gravitational interactions.

Obviously the question of which criteria count toward the spacetime concept will be a complicated one on the cluster concept view. I won't make any attempt to give an exhaustive list of candidates here, but the following are examples of criteria which are logically independent of Knox's inertial criteria and which seem to also count toward a structure's satisfying our spacetime concept:

- The structure is non-dynamical, at least with respect to non-gravitational interactions.

- The structure is (in some sense) located everywhere in all states of the theory.
- The structure does not carry energy or momentum.
- "Vacuum" solutions exist which describe the (putatively) spatiotemporal structure in the absence of other structures.
- There are no *other* structures in the theory which can exist without the (putative) spacetime structure.
- The structure grounds or explains a family of modal facts about which states are *geometrically possible*, where geometric possibility does not reduce to physical possibility (Belot, 2013, 50-51).
- It is a (higher-order) law of nature that the geometric symmetries of the structure are dynamical symmetries of the theory (Skow, 2006; Janssen, 2009).
- Forces propagate across the spatial distances defined by the metric characterizing the structure (so that long-range forces like electromagnetism fall off proportionately to the inverse square of this distance, and so on).

Again, this is not meant to be an exhaustive list. Rather it is meant to illustrate that a vast number of different criteria could plausibly figure into our ascription of the name 'spacetime' to a given theoretical structure, depending on the details of the laws that define that structure. And indeed, Knox's own criterion,

- The structure determines the difference between inertial and non-inertial frames of reference,

belongs high on this list, perhaps even at the top. She has certainly shown that it's a very important criterion. My only disagreement is with her claim that it is the sole criterion.

5 Fundamentality and functionalism

I'd now like to advance what I think will be a more controversial criterion that I claim counts toward our spacetime concept. A structure is more apt to be called 'spacetime,' I claim, the more physically fundamental it is.

I don't intend this proposal to stand or fall with any particular account of fundamentality. As a rough guide, structure A is more fundamental than structure B if A features in the basic laws of a more fundamental theory. So for example, the metric of general relativity is more fundamental than the metric of Newtonian gravity theory, and quantum fields are probably more fundamental than particles. As another rough guide (often related to the first), A is more fundamental than B if B is composite or emergent and A is not. So quarks are more fundamental than protons, and energy is more fundamental than temperature.

The arrival on the scene of proposed quantum gravity theories where spacetime emerges from a more fundamental non-spatiotemporal substrate has sometimes been taken to imply that there is no connection between fundamentality and our spacetime concept. There had better not be any such connection, the thought goes, because we exist in spacetime and yet there's a good chance that spacetime is emergent in our world.

But on the cluster concept view of spacetime, this inference rests on a confusion. Fundamentality can be a criterion for the spacetime concept even if the actual spacetime we live in fails that criterion. This is especially true given that fundamentality is a graded notion, and the emergence of spacetime is only proof that it cannot be *perfectly* fundamental. The spacetime we live in might still be highly fundamental, so that the fundamentality criterion still provides part of the explanation for why the spacetime we live in falls under our spacetime concept.

On reflection, there are clear reasons why fundamentality ought to count as a criterion for our spacetime concept. To begin with, the referents of our words (and so too the extensions of our concepts) are probably determined in part by the fundamentality of objects and properties in the world. In particular, Lewis (1984) has argued persuasively that, for any descriptivist theory of meaning for words or thoughts to succeed, it must privilege interpretations of our language and concepts that refer to more fundamental properties over ones that refer to less fundamental properties.

Second, the notion that fundamentality is a criterion for the spacetime concept fits well with both substantivalist and relationist conceptions of spacetime. Substantivalists about general relativity, for example, tend to hold that spacetime is among the most fundamental structures described by the theory. Indeed, North (forthcoming) has recently argued that substantivalism should be defined as the claim that spacetime is at least as fundamental as material objects. (This is my preferred definition of substantivalism, and I will assume it for the remainder of this essay.) On the other hand, spacetime is obviously not a fundamental substance according to relationists.[9] But it is typical for relationists to hold that spacetime is constituted by a system of highly fundamental relational properties. It is hardly common for relationists to suggest that spatiotemporal relations are reducible to something more fundamental within the domain of general relativity, for example. So all parties to the substantivalist/relationist dispute would seem to agree that spacetime is highly fundamental, if not perfectly fundamental.

Finally, the inclusion of fundamentality as a criterion in our spacetime cluster concept allows us to explain why we are tempted by what Knox calls the "container metaphor." There is obviously something intuitive about this metaphor, which has often been used by substantivalists to describe and motivate their view. The idea is that spacetime is a sort of container for matter, with material objects as its contents.

Knox considers an objection to her inertial functionalism arising from this picture: there is no guarantee that the structure determining inertial trajectories is the same structure in which matter is "contained," and so there is no guarantee that inertial functionalism gives the right answer about what should count as spacetime. Knox responds that this criticism is akin to (but much less plausible than) qualia-based objections to functionalism about the mind: "Just as the believer in Zombies, or the proposer of homunculus-head thinks that states could fill the functional role of mental states, but nonetheless be missing something, the proponent of the container thinks that spacetime functionalism fails to capture the essential nature of spacetime." (Knox, in progress, 16) But as Knox responds, unlike in the case of conscious experience, we cannot claim to have any direct access to the essential "container" nature of spacetime. So there is ultimately no rigorous sense to be made of the container metaphor.

This is a damning objection insofar as the container metaphor is meant to rest on a brute intuition about spacetime having a container-y nature which is not describable except via the metaphor. But it is not obvious that the metaphor's defenders can't spell out what they mean in more literal terms. For example, in his discussion of the nature of space in non-relativistic quantum mechanics, David Albert draws a distinction between "the space of possible *interactive distances*" (which I take to be the entity Knox aims to define using

[9]I take relationism to be the denial of substantivalism–that is, following North, the claim that spacetime is less fundamental than matter.

inertial functionalism) and the "*stage* on which whatever theory we happen to be entertaining at the moment depicts the world as *unfolding*." (Albert, 1996, 282) So far all we've been given is a metaphor, but Albert continues: "a space (that is) in which a specification of the local conditions at every address at some particular time (but not at any proper *subset* of them) amounts to a complete specification of the physical situation of the *world*, on that theory, at that time." (Albert, 1996, 282-283; all italics are of course Albert's)

This is a literal, not metaphorical, definition of the sense in which Albert thinks of space(time) as a container. Indeed, it is a *functional* definition in the same sense as Knox's inertial definition of spacetime–it specifies a role in the laws that spacetime (*qua* arena/container) must fill. There remains a small puzzle, insofar as theories may admit more than one geometric description that satisfies Albert's criteria. But I take Albert's suggestion to be that the space in which the world "unfolds" is the *most fundamental* geometric structure meeting his criteria. Earlier he writes: "the three-dimensional multiple-particle language of our everyday lives supervenes on the exact and complete and fundamental language of the world, which is the language of *wave functions* (and whatever else) in *configuration space*." (Albert, 1996, 279) A different geometric formalism might provide the same information about the physical state, but on Albert's view, it would do so in less fundamental terms than the configuration space formalism, and thus it would not represent the space in which the quantum world unfolds.

Albert concludes that our spatial concept is really two separate concepts–interactive distance structure and stage on which the world unfolds–and that these two concepts are coextensional in classical physics but not (on his view) in quantum physics. There is certainly something to this claim, and in particular, we can always ask whether a theory posits a *fundamental background* (a fundamental geometric structure meeting Albert's criteria) which is distinct from the space(time) of interactive distances, or from Knox's inertial structure. But there is also a risk of multiplying concepts indefinitely, if we treat each logically independent feature we ordinarily take spacetime to have as a separate concept of a sense in which a structure can be spacetime. To the contrary, it seems to me that we do have an overarching concept of spacetime as the sort of entity that ideally ought to meet many different criteria–to repeat myself, a cluster concept.

The real lesson of Albert's work on configuration space is that fundamentality is one of the criteria for our spacetime concept. In some theories, there may be no fundamental background geometry, or there may be a fundamental background which fails too many other criteria to count as spatiotemporal. But the more fundamental a background geometric structure is, the more apt it is to satisfying our spacetime concept, other things being equal.

If we accept that fundamentality is one of the criteria for our spacetime concept, further lessons follow about the interesting and much-discussed case of spacetime in Newtonian mechanics.

This is a debate that lends itself to a pernicious equivocation if one is not careful. The central question is usually phrased in roughly this way: What is "the spacetime setting for Newtonian physics"? (Wallace, forthcoming) The question is ambiguous between two possible readings:

A Assuming we know the laws of Newtonian mechanics are true, what should we conclude about the structure of spacetime (on the basis, perhaps, of principles like inference to the best explanation)?

B What sort of spacetime structure is *entailed* by the proposition that the laws of Newtonian mechanics are true?

Clearly an answer to question B will be less modest than an answer to question A. Indeed, it seems natural to suppose that no particular spacetime structure is entailed by Newton's laws of motion, because these laws by themselves are compatible with a variety of different spacetime theories: the separate space and time that Newton himself posited, neo-Newtonian spacetime, or the dynamically curved spacetime of Newton-Cartan theory.

But if one assumes inertial functionalism, a definite answer to question B comes easily. For on inertial functionalism, there is a unique minimal structure necessary to represent the theory's inertial structure–or rather its lack thereof, since such theories can make do with a geometry Saunders (2013) calls *Newton-Huygens spacetime*, which lacks a global distinction between inertial and non-inertial trajectories. Subsystems in such a theory will exhibit local inertial structure, though, leading Wallace (forthcoming) to suggest (following the argument of Knox (2014)) that Newton-Cartan theory is the effective spacetime structure at the level of such subsystems (again, assuming inertial functionalism).[10]

[10] For further complications of this debate, see Weatherall (forthcoming, 2016), Dewar (forthcoming) and Teh (forthcoming).

The details of this debate about inertial structure are quite beyond the scope of this essay, but I have a single comment to add. Suppose one accepts the arguments of this paper, and in particular the arguments of this section. Then determining the inertial structure of Newtonian mechanics, as previous parties to the debate have sought to do, will not automatically answer the question of what should count as the *spacetime* structure of the theory. For example, it may turn out that Newton-Huygens structure is sufficient to describe motion in Newtonian mechanics, but that the fundamental geometric structure of the theory should be understood to be the dynamically curved geometry of Newton-Cartan theory. (Following North, I take this possibility to be equivalent to substantivalism about the spacetime of Newton-Cartan theory.) It is especially easy to imagine fundamentality breaking the tie between these two candidate geometries because they are so similar; as shown by Wallace, they share the same structure at the level of isolated subsystems.

Suppose we accept that fundamentality might be sufficient to break the tie between different candidate spacetime geometries which disagree about inertial structure. Then the question of what counts as spacetime cannot be answered without doing some metaphysics–we need to determine which geometric structures are fundamental in order to answer it. If fundamentality is epistemically accessible to us (perhaps because more explanatory, simpler or more parsimonious structures should be assumed to be fundamental) then the question about spacetime structure may be answered, though perhaps not with great confidence. If we have no way to determine which structures in a given theory should be assumed to be fundamental, on the other hand, the question of what is the right spacetime structure for Newtonian mechanics is likewise unanswerable.

6 Conclusions

I am a spacetime functionalist, and if I'd grasped what it meant to be a spacetime functionalist prior to encountering Knox's work, I would always have identified as one. So I'm thankful to Knox for presenting an illuminating framework which I take to be the only reasonable framework for analyzing our spacetime concept. But I don't believe the correct analysis of that concept will take a simple, easily-stated form. We use the word 'spacetime,' fruitfully and aptly, to describe a wide variety of different theoretical structures defined within very different theories. Some of these structures have nothing to do with inertia.

It seems to me that only the cluster concept view is flexible enough to hold any promise as an accurate analysis of our spacetime concept. And even if a different approach ultimately proves better able to analyze that concept, that approach will not bear much resemblance to Knox's inertial functionalism.

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