**The Case for Mach’s Neutral Monism[[1]](#endnote-1)**

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**Introduction**

For many years, the received view of the Viennese physicist and philosopher Ernst Mach (1838-1916) has been that he was a phenomenalist, who only believed in given human sensations and who held that everything else was unverifiable metaphysical nonsense. This view comported well with Mach’s historical influence on members of the Vienna Circle and seemed to explain both his negative skepticism about atoms and his positive skepticism of Newton’s absolute space which was so influential on Einstein. In recent years, however, this phenomenalistic view of Mach’s work has given way to a more realistic and nuanced “neutral monist” view, far more in line with his contemporary reception by Paul Carus, Hans Kleinpeter, William James, Bertrand Russell and American Realists, such as Ralph Barton Perry and leading to Herbert Feigl and Wilfrid Sellars and beyond to the contemporary neutral monist movement. There are now two main traditions in the literature, one tying Mach to positivism and the other to neutral monism. In this paper, I will defend the neutral monist tradition and show that it is actually a form of scientific realism, not positivism.

I start with a characterization of what I believe to be some tenets of neutral monism in general, many of which were shared by James and Russell, both deeply influenced by Mach. I will then go point by point and find evidence for these views in Mach’s texts (including his notebooks and other documents). Seeing Mach as a kind of realist also casts much light on his scientific views and corrects a number of historical misconceptions regarding both atomism and Mach’s philosophy of space and time. Finally, I will discuss Mach’s place in the neutral monist movement of James, Russell and the American Realists, and the revival of these views in the recent philosophy of mind.

**What is Neutral Monism?**

1. **The elements**. Neutral monism holds that objects and the human mind are both concatenations of neutral elements (e) neither exclusively mental nor physical in nature. These elements are transient event particulars that never recur. They are bound up in functions with one another such that the same element (e/s) may partake of both mental variations (memory, association) and physical variations (connections to other objects, physical laws). Objects and egos are functional complexes of elements not substrata or substances prior to events.
2. **Unobserved elements**. In addition to elements which obey both mental and physical variations (e/s), there are also elements that make up the rest of the physical world, other minds, animal sensations, etc. These are found by causal continuity with our experienced sensations, by extending the functions we find in perception to complete unobserved objects “in thought” and by adding extra (*hinzugedachte*) elements. We know these elements only through their causal functional relations but we do not experience their quality directly, so we can say little concrete about them, barring future experiments, or what Mach called “an extension of biology.”
3. **Functions**. Elements are not atomistic or logically independent simples, as in the case of Hume’s impressions or Russell’s simples from his logical atomism period. They are always bound up in functions that supervene on their causal powers to affect each other. Polyadic functional relationships from complex to complex are also possible. The division between mental and physical functional relationships is provisional only, due to our incomplete knowledge and will eventually be overcome as mental variations are traced to the physical states of the brain. There will then be one set of elements and one set of natural variations in a future state of the science. Functions are naturalistic, not merely logical or combinatorial possibilities without a physical grounding in the qualities of natural events.
4. **Forceful Qualities**. Elements are events that express qualities, or powers with causal force to affect other events. They are not inert sense data or logically independent simples. Nor are their qualities mere “epiphenomenal qualia” with no causal efficacy. Even physical events possess qualities of their own, whether or not these look anything like the manifested qualities of our sensations.[[2]](#endnote-2) The qualities of elements, are occurrent and particular, not universals.
5. **The ego**. Consciousness is a set of complex functional relations, not a single unitary phenomenon. The unity of consciousness is an illusion. Sensations and elements always occur in functional complexes supervening on their causal relations to each other, and one of these complexes is the conscious mind, but the elements do not *acquire* their quality, or existence, from being beheld by the mind. It is rather the reverse: the mind acquires its features through the complex functional relation of its contents in perception and also in judgments.[[3]](#endnote-3)
6. **Matter**. Physical objects are also functional constructions out of events and are not absolutely permanent. Underlying the notion of the physical object however is the much more solid permanence of functions and conserved systems of elements. Conservation first establishes the possibility of identity of systems through time and over spatial transformations in place, rotations and velocity boosts. It is also the basis for identifying properties preserved through transformations like energy, momentum, angular momentum. These are the only true universals. Other semi-permanent things (species, organisms, tables, chairs) are quasi “objects” only as long as their semi-permanence and ultimate composition out of elements is recognized. So-called properties established by similarity relations[[4]](#endnote-4), or by correlation with logical subject terms, predicates, or sentences, are merely linguistic or psychological in nature.
7. **Naturalized Epistemology-Direct Realism**. Elements are not a “given” basis for a foundational construction of knowledge such as in the *Aufbau* or Russell’s early theory of knowledge by acquaintance. Rather elements are provisionally the smallest divisions of experience we are capable of making at the present time. Analysis of the elements may, and should, continue indefinitely. Mental events have no intrinsic intentional power to represent other objects. Where no causal relations can be established to objects, mental events do not refer to anything beyond themselves. Where causal relations can be established, mental events may lead to knowledge or error along the same causal links and even using the same methods of inquiry. There are no established canons of inductive or even hypothetical deductive or any other scientific methods guaranteed to lead to knowledge. Where the object can be connected up with mental events, our knowledge of the object is *directly realistic* because the whole object will then include events that are part of it, both physically and also as part of the knower. The only methods for attaining knowledge are fallible scientific methods of tracing causal chains in the whole set of naturalistic functions, *not* knowledge via propositions and logical relations of truth and falsity to linguistically delineated facts.

Not all neutral monists would subscribe to all of these points (see for example Stubenberg 2016). In fact, James and Russell dissented on points 3 and 7 respectively (see Banks 2014 and 2017). So far as I know, Mach is the only philosopher who would match up point by point to this ideal list. I will now seek to prove this reading.

1. **Mach’s Neutral Sensation-Elements**

In the *Analysis of Sensations*, Mach expresses the neutrality of the elements this way:

A color is a physical object as soon as we consider its dependence, for instance upon its luminous source, upon other colors, upon temperatures, upon spaces and so forth. When we consider however its dependence upon the retina, it is a psychological object, a sensation. Not the subject matter but the direction of our investigation is different in the two domains. (Mach 1959 p. 18 CF. Mach 1905/1976 pp. 15-16).[[5]](#endnote-5)

He also says that the term ‘sensation’ is only relative to the set of mental variations. Otherwise the same colors, sounds, etc. are to be considered physical elements:

In what follows, wherever the reader finds the terms “sensation” “sensation complex” used alongside of or instead of the expressions “element” “complex of elements” it must be borne in mind that it is only in the connexion and relation in question, only in their functional dependence that the elements are sensations. In another functional relation they are at the same time physical objects. We only use the additional term “sensation” to describe the elements, because most people are much more familiar with the elements in question as sensations (colors, sounds, pressures, spaces, times), while according to the popular conception it is particles of mass that are considered as physical elements, to which the elements, in the sense used here are attached a “properties” or “effects.” (Mach 1959, p. 16).

It is clear from the last quote that elements are not objects but events. It is also clear from the early pages of the *Analysis* that elements are not types or universals but event particulars that do not recur (see Mach 1959, pp. 2-7, 29, 331).

As many authors have observed, a color patch does not seem like a physical object, like a particle or a field; it seems more like a sensation only—a “secondary quality” in the mind with no connection to the real properties of objects such as the reflected wavelengths of light. In boldly calling sensory qualities physical events, some like V.I. Lenin sensed a word game.

Mach and Avenarius secretly smuggle in materialism by means of the word “element,” which supposedly frees their theory of the “one-sidedness” of subjective idealism, supposedly permits the assumption that the mental is dependent on the retina, nerves and so forth, and the assumption that the physical is independent of the human organism. In fact, of course, the trick with the word “element” is a wretched sophistry, for a materialist who reads Mach and Avenarius will immediately ask: what are the “elements"? It would, indeed, be childish to think that one can dispose of the fundamental philosophical trends by inventing a new word. Either the “element” is a sensation, as all empirio-criticists, Mach, Avenarius, Petzoldt, etc., maintain—in which case your philosophy, gentlemen, is idealism vainly seeking to hide the nakedness of its solipsism under the cloak of a more “objective” terminology; or the “element” is not a sensation—in which case absolutely no thought whatever is attached to the “new” term; it is merely an empty bauble. (Lenin 1908, p. 48-49)

Quine (1966, p. 667) and Thomas Nagel (at least in 2000) also attacked neutral monism as not “really neutral,” but a disguised attempt to reduce the physical to the mental (see also Stubenberg 2016 Sec 7.2 for more examples of this criticism).

Myself (Banks 2014 Chapter Five) I think it is clear that the sensation-elements like blue are assumed to be complex electrochemical brain events, and that blue is simply how the physical powers manifested in these sorts of configured events appear *to us*. From outside the brain the same event of our seeing the blue will appear to an external observer as a complex firing pattern of 10,000 complexly configured neurons. If the external observer siphons off the neural energy into 10,000 electrodes, the event of seeing blue will vanish for the internal observer which is what you would expect if they were identical in some way. If the external observer then reconfigures the 10,000 events along wires in exactly the same way with exactly the same causal powers, I would also expect the blue sensation to be recreated within the complex of powers and manifestations in the wires. See my Chapter Five for a more complete neutral monist view of sensation qualities in terms of identical powers and non-identical manifestations.) If any such explanation is workable, or even just logically coherent, I see absolutely no reason why it can be claimed a priori that sensory qualities like blue *could not possibly be* physical events, as is often done. It seems to me that *a priori* arguments or intuitions have nothing to do with the question of whether sensations are physical.

Passages about the two orders and the neutrality of the sensation-elements occur in both James’s 1904 “Does ‘Consciousness’ Exist?” the first of his radical empiricist essays and of course in Russell’s 1919 Essay “On Propositions” and The 1921 *Analysis of Mind*. Russell saw Mach’s basic insight about the physicality of sensation as a breakthrough and a new truth for breaking the stranglehold of the mind-body problem. “Mach argued that our sensations are part of the physical world and thus inaugurated the movement toward neutral monism.” (Russell 1914/1984 p. 16). Even before his own conversion to the view in 1918, Russell accepted as “a service to philosophy” Mach’s and James’ idea that “what is experienced may itself be a part of the physical world and often is so” (Ibid, p. 31) or that “constituents of the physical world can be immediately present to me” (Ibid. p 22).

This brings us to feature 2. and a fundamental question for neutral monism. Are all physical elements *also* sensations (e/s) or are there world elements (e) that are *not* interpretable as anyone’s sensations? You could say, and Mach did say sometimes, that the question is moot if minds are not fundamental anyway and if sensations are *already* physical in any desired sense of “mind external physical objects.” Since there is no distinction for him, he may have considered the question to be nonsense. A more modest reading would perhaps be that even if this is true, Mach never actually goes beyond assuming those mind-independent physical elements that *also* bear an interpretation as sensations. Indeed, this is by far the most common view of Mach’s neutral monism.

So a clearer position is still necessary on whether there *are* further elements in causal relations to the (e/s) that fill out perceived objects and objects in causal relations to us that are simply (e) and never themselves (e/s), or a denial that there are *no* such elements. Fortunately, there is evidence Mach took the further step to the pure (e) elements as well.

1. **The Existence of the World Elements[[6]](#endnote-6)**

In the *Analysis of Sensations* Mach talks about the need to add elements in thought (*hinzudenken*) to those observed in order to complete our experience of objects, or to complete “half-observed facts” (1959, p., 333; 1883/1960, p.587). This will involve adding elements not directly experienced by us, such as the elements of other human beings by analogy, animals and elements of physical objects like the backs of chairs, distant stars and so forth. Mach is clear that we do not experience the qualities of these added elements but he is also clear on their causal powers and linkage to experienced elements. They are not *mere* thought additions without some kind of causal functional efficacy backing them up. If they are linked to elements with causal powers, then they too must have causal powers, and cannot be inert thought things even though we must add them in thought to our experiences. The merely imagined back of a chair for example is not what establishes the solidity and permanence of the object of perception. Mach also says that the analogy to the sensations of another person is a “causal” not imaginary analogy:

When I speak of the sensations of another person, those sensations are not, of course, exhibited in my optical or physical space; they are mentally added and I conceive them causally, not spatially, attached to the brain observed or rather functionally presented. (Mach 1959 p. 361)

He also says that in a causal presentation one need not even distinguish between the sensations of one ego from another anyway, all sensation-elements can be represented by their causal connections to each other in a kind of map, and then one completes the causal relations on the map by adding the necessary elements required to complete the causal relations, or “partially observed facts,” even if the further assumed elements are not observable.

From the standpoint which I here take up for purposes of general orientation, I no more draw an essential distinction between my sensation and the sensations of another person than I regard red or green as belonging to an individual body. The same elements are connected at different points of attachment, namely the egos. But these points of attachment are not anything constant. They arise, they perish, and are incessantly being modified. But where there is no connexion at a given point there is no perceptible reciprocal influence. Whether it may or may not prove possible to transfer someone else’s sensation to me by means of nervous connexions, my view is not affected one way or the other. (Mach 1959, p.27)

There may be many ways to complete the map of course, all consistent with observation, but there are no in principle theoretical difficulties since all the components are of the same homogeneous kind and we are connecting like with like. We never for example assume elements in no possible causal relation to experience. This would be a dreaded extra-causal Ding an sich, permanently isolated from our experience, which he thinks science cannot tolerate.

As I showed in my 2003 book, Mach’s commitment to mind independent elements had a very long background in his early career when he was studying Herbart and Fechner and working his way towards his own position. For a time, Mach says, he was an idealist, a monadologist, (1959 p 30n) and even a panpsychist (1959, p. 362), all views he later abandoned. These early views and Mach’s development intrigued me, so I investigated many of these early writings, including the *Vorträge über Psychophysik* of 1863, where, influenced by Fechner’s panpsychism, he seems to have held that even atoms had some kind of qualitative inner nature:

We cannot attribute to atoms an outer side. If we must think anything, we must attribute to them an inner side, an inwardness analogous in some respects to our own soul. In fact, where could the soul come from in a combination of atoms in the organism if the kernel did not already lie in the individual atom? (Mach 1863, p. 364).

In an 1866 paper “Über die Entwicklung der Raumvorstellungen” (1866), Mach speculated about inner states, or qualities, of pressure making up the phenomenon of spatially extended matter, similar to the Leibnizian constructions of matter and space in Herbart’s metaphysics from forceful qualities pressing upon each other (see Banks 2003 Chapter Three). Mach concluded that space and time extension were unnecessary to physics and physics could instead confine its attention to functionally and combinatorially related “inner states” like forces or pressures instead, from which spatio-temporal phenomena would arise by construction, as in Herbart’s works.

Finally in lectures entitled “Über einige Hauptfragen der Physik” (Ernst Mach Nachlass, Deutsches Museum, 1872 NL 174/1/003) Mach put together these ideas and asked whether sensation could be considered “a general property of matter.” If so, then nothing in principle stood in the way of unifying the mental with the physical, since both would consist of the same kind of qualities in different sorts of causal relations:

Sensation is a general property of matter, more general than motion. Let us seek to set down this proposition clearly. An organism is a system of molecules. Electrical currents run into the interior and come back again into the muscles. Everything is physically explainable. But not that the person should have sensations. What we can investigate physically is always merely physical. We find no sensation. And yet the human being senses. The material flows forward and through and through him. The old departs. The new comes in. We have therefore the problem of finding something fundamentally new in the whole that is not in the parts. We escape this difficulty when we consider sensation a general property of matter….

What scientific value this assumption of a general sensation of matter has, this can only be decided by how much better we can deduce and understand physical phenomena through them. Rules for deduction of our sensations with the help of other sensations added in thought and in causal relation to them. Thus, as the most immediate goal of science the construction of the world out of sensations. The appearance of matter, in so far as it is sensation, built up similar to the way physics has built up material added in thought (the atom). (Haller and Stadler 1988, p. 173)

What seems clear is that world elements played a very significant role in the development of Mach’s neutral monism. They were not a later add-on to a phenomenalist view, as it might appear from a cursory read of the *Analysis of Sensations*. Recognizing the role qualitative elements could play in the construction of matter, as well the construction of mind, appeared to convince Mach that the elements were ideally suited to play a role *neutral* between the two and that the two categories could henceforth be abandoned in favor of the elements in one natural array capable of constructing both mind and matter. That view made its first appearance at the end of Mach’s *Principle of the Conservation of Energy* in 1872.

Mach continued to write this way even after the development of neutral monism, referring always to a “future state of science” in which the sameness in kind between sensations and physical elements would be recognized in the way he hoped by “building a tunnel” through from the one to the other. There are letters to Friedrich Adler and Gabriele Rabel (see Banks 2003 pp. 6-7) in which he says that he refused to limit himself to the observed element/sensations only, but reserved speculation on world elements for the future. Adler asked him point blank: “*Do you consider necessary the assumption of elements that belong to an object but not to a subject?* Obviously such elements are not directly given, but do you assume them hypothetically as it appears to me you do from passages cited in the *Analysis of Sensations*?” (loc. cit.) Mach wrote to Adler that indeed he “assumed analogous elements in animals, plants and inorganic bodies. This hypothesis serves only to round off the world view provisionally and in hope of the future construction of biology…Healthy biological research must teach if this hypothesis has any worth and if so, what worth. Speculation cannot manage this. Provisionally it appears to me that we completely overlook a side of our experience when we overlook this hypothesis…I have not further cultivated all of these matters, for I feared the nearness of the metaphysical abyss where there is no experiential foundation.” (loc. cit.). There are also published passages in the 1883 *Mechanics*, Mach’s 1882 lecture “On the Economical Nature of Physical Inquiry” and the 1905 *Knowledge and Error* to this effect:

Careful physical research will lead to an analysis of our sensations. We shall then discover that hunger is not so essentially different from the tendency of sulfuric acid for zinc, and our will not so different from the pressure of a stone as it now appears. We shall again feel ourselves nearer nature without its being necessary that we should resolve ourselves into a nebulous mass of molecules or make nature a haunt of hobgoblins. (Mach 1883 p. 559)

If the ego is not a monad isolated from the world but a part of it, in the midst of a cosmic stream from which it has emerged, and into which it is ready to dissolve back again, then we shall no longer be inclined to view the world as an unknowable something and we are then close enough to ourselves and in sufficient affinity to other parts of the world to hope for real knowledge. (Mach 1905 p. 361)

Clearly “real knowledge” of nature would involve some sort of actual acquaintance with mind independent natural qualities, which Mach thought might be possible through the further development of biology, perhaps by adapting the nervous system so that previously unexperienced natural qualities actually fell within our experience. In other words, the existence of the world elements is an experimental question, or so Mach hoped. It is not a question to be blocked off by philosophical positions such as phenomenalism. This is one of the errors one makes in interpreting Mach as a traditional philosopher building a closed and incorrigible system. No such thing is going on here. His terms are simply starting points for future inquiries, nothing more.

1. **Natural Functions and Forceful Qualities**

Mach was aware that elements and functions would seem to many people too flimsy a foundation to build up egos and solid objects; he wrote:

My world of elements, or sensations, strikes not only men of science but also professional philosophers as too unsubstantial. When I treat matter as a mental symbol standing for a relatively stable complex of sensational elements, this is described as a conception which does not make enough of the material world. The external world, it is felt, is not adequately expressed as a sum of sensations; in addition to the actual sensations, we ought at least to bring in Mill’s possibilities of sensations. In reply to this, I must observe that for me also the world is not a mere sum of sensations. Indeed, I speak expressly of functional relations of the elements. But this conception not only makes Mill’s “possibilities” superfluous, but replaces them with something much more solid, namely the mathematical concept of function. (Mach 1959, pp. 362-363)

It should be pointed out that Mach had held this view at least since *his Principle of the Conservation of Energy* in 1872, where he described the idea that objects should be complexes that exhibit a sturdy functional dependence of the elements f(α,β,γ…)=0. Mach also pointed out that elements are always bound up in complexes and never occur in isolation. They obey a general principle called “the reciprocal functional dependence of elements on each other,” which Mach substitutes for the principle of causality, and also for ideas of conservation, which otherwise would require a substratum of conserved substances, stuffs or fluids, all of which he rejected as “metaphysical additions to physics.” It is also clear from these and other writings that Mach thought of elements as having causal force or power to abut upon each other and generate those reciprocal variations (an idea which Mach may have retained from Herbartian psychology where qualities behaved just like forces, see Banks 2003 Chapter Three). I have also ventured the idea that the elements are like energy-potential differences equalized by forces, a view that fits in with Mach’s view of the natural processes as tending to the maximum (or minimum) possible equalizations of potential differences by the law of least action (Banks 2003, 2014).[[7]](#endnote-7) If elements are really direct qualitative manifestations of energy differences, manifested in natural events, then calling them forceful would make much sense since this is what forces are, according to physics.

The functional dependence of the elements is guaranteed because the functional behavior is grounded in the qualities and causal forces expressed by the elements themselves. This is indeed a solid foundation, more so than any impermanent object which merely seems solid to us: bodies become “embodied laws of conservation” like f(α,β,γ…)=0.

James and Russell further developed Mach’s causal-functional connections into spatio-temporal perspectival systems. The elements, which James called “pure experiences” and Russell called “event-particulars,” represented an object by breaking it up into all of its perspectival views, or causal interactions, it might have with observers and other objects. This has the extra benefit of giving the causal relations some further perspectival structure and implicitly defining the objects as the invariants of perspectival transformations from one perspective to another.

It is clear that the functions used by Mach, James and Russell to give the world structure are not mere mathematical functions or combinatorial relations or other logical devices such as the spurious quasi-analysis relations of Carnap. These are naturalistic functions grounded in the behavior of the elements and leading to natural ideas of conservation and invariance. I do not find this to be true of Carnap’s later constructions of similarity classes in the *Aufbau*. Similarity, if a relation does not supervene on causation, is not a natural relation but a purely psychological or perceptual similarity in the eye of the beholder. It is not even clear there exists in nature a real relation of similarity that is reflexive, symmetric and non-transitive. It is certainly not an equivalence relation of the kind that would indicate the presence of a real physical property over systems or events. In fact, there is a close relationship between equivalence relations and groups: for every equivalence relation producing a partition on a set there is a group of transformations that preserves the partition. For example, if we have a system of bodies classified by their momenta with respect to a reference body at rest, we can construct a partition of bodies with the same momenta. We can then introduce a group of velocity transformations on the partition and preserve the identified property of momentum through that group. Groups in turn follow naturally from the adoption of a system of perspectives (see Frank 2001, Chapter Five, and pp. 84-85). This, I believe, is how the functional relations should be parsed out in a Machian way.

1. **“*Das Ich is unrettbar*” The Self cannot be Saved**

One of Mach’s most famous and characteristic ideas is the rejection of the self or consciousness. He still retained the idea that consciousness was a kind of complex functional relation of associated sensations, images, memories, spaces, times and the like. In fact, the *Analysis of Sensations* goes a long way towards characterizing these mechanisms as a series of machine-like reflexes conditioned largely by the evolutionary history of the organism. In that sense, Mach was already a completely modern psychologist in an age when bizarre ideas still held sway over that field of study. Mach did not believe that the contents of the mind depended upon consciousness as a kind of circumambient medium. A sensation of blue retains its existence and quality even if it is not being attended to or embedded in a conscious act of observing the blue:

Consciousness is not a special mental quality or class of qualities different from physical ones; nor is it a special quality that would have to be added to physical ones in order to make the unconscious conscious. Introspection as well as observation of other living things to which we have to ascribe consciousness similar to our own shows that consciousness has its roots in reproduction and association: their wealth, ease, speed, vivacity and order determine its level. Consciousness consists not in a special quality but a special connection between qualities…A single sensation is neither conscious nor unconscious: it becomes conscious by being ranged among the experiences of the present. (Mach 1905/1976, p. 31-32).

Of course Mach’s famous and rather chilling “headless body” illustration (Mach 1959, p. 19) makes the absence of the ego abundantly clear. He was known for this idea even outside philosophy in cultural circles. This picture may be one inspiration for Robert Musil’s *Man without Qualities* (Musil wrote his doctoral dissertation on Mach). Of course, James soon followed suit in discarding consciousness in “Does ‘Consciousness’ Exist in 1904 and Russell, in 1919, was led by Mach and James’ example to abandon his theory of acquaintance which was fundamental to his theory of knowledge up to that time (Russell 1959 pp. 134-135).

1. **Space and Time**

Mach’s skepticism of the basic concepts of physics is well established in the literature. He is usually considered a relationist in the philosophy of space and time because of his famous critique of Newton’s bucket experiment in the *Science of Mechanics*. According to Mach, accelerated and rotating reference frames should be exchangeable for a rest frame if the rest of the bodies in the universe accelerate or rotate in the opposite direction. The fictitious “inertial forces” that appear to work on bodies in these resting reference frames (K) are to be replaced by gravitational forces at a distance or through a medium due to the other bodies in the universe moving around it instead (A,B,C). Einstein famously enshrined this idea as “Mach’s principle” and originally thought general relativity upheld it. However, as became clear over time (See Janssen 2014) the gravitational field has its own independent existence in general relativity and does not depend upon generating source masses as proved by the cogency of the DeSitter solution and the rejection of Einstein’s “masses at infinity.” The gravitational field is however the source for both gravitational and inertial effects, which can be transformed into each other, and so the field *can* be described as a combined inertio-gravitational field. What is hardly ever mentioned is that Mach *also* admitted the possibility of an inertio-gravitational field independent of, or collateral to, source masses, in defiance of Mach’s Principle[[8]](#endnote-8), *provided* it did not require to be embedded in an absolute space and time. That is, inertial effects could be due to accelerated motion and rotation with respect to the field but would also presumably occur if the field itself moved or rotated relative to the body also. This passage actually goes back to the 1883 first edition of the *Mechanics*:

It might be indeed that the isolated bodies A,B,C… play a collateral role in the determination of the body K and that this motion is determined by a medium in which K exists. In such a case we should have to substitute this medium for Newton’s absolute space. (Mach 1960/1883 pp. 282-283).

So far as I know, this passage has played *no role whatever* in the vast discussion of Mach’s Principle, a huge oversight, but it should be more widely known, since it shows Mach was not at all wedded to relationism, even if he expresses a strong preference for the determination of motion with other bodies as reference points (Ibid.).

I think Mach’s well known relationism (like Leibniz’s) hid a much deeper eliminationist view about space and time which takes us back to the germination of his theory of elements and functions, in specific in two papers, the aforementioned 1866 “Raumvorstellungen” paper and the 1871 paper “Über die physikalische Bedeutung der Gesetze der Symmetrie.” There, Mach indicated his desire to eliminate spatio-temporal representation from physics altogether, not just to reduce space and time to spatio-temporal relations between bodies. This too is an oversight deserving correction, especially from scientists. In the “Raumvorstellungen” paper, Mach suggested replacing forces as functions of distance by distance as a function of the intensity of forces. (Reprinted in Mach 1872/1910 pp. 88-89). Space and time were to be entirely replaced by causal functional dependencies among the elements, expressed in abstract form and not dependent on any prior extended representation. As he put it in the *Principle of the Conservation of Energy*:

I think I must add, and have already added in a previous publication, [The “Raumvorstellungen” paper, E.B.] that the express drawing of space and time into the law of causality is at least superfluous. Since we only recognize what we call space and time by certain phenomena, spatial and temporal determinations are only determinations by means of other phenomena. (Mach 1910/1872, p. 60).

In the “Symmetrie” paper, he wrote that it made no sense to force all physical phenomena into the three-dimensional space of our imagination (*Über Symmetrie*, p. 147). What if natural phenomena were discovered that had more degrees of freedom to consider? He suggested in the *Principle of the* *Conservation of Energy* that the spectral lines of different chemical elements might present such a case. He added in a footnote: “It follows from this that the dependence of natural phenomena be expressed through relations of number, not spatially or temporally.” (*Über Symmetrie* 147n).

This program in his early papers was amplified in later works. Mach came to believe that spatio-temporal representations in physics were based in anthropomorphic human visualization and psychology and that a future physics would be of his more abstract and sparse element and function ontology. His deeper theory of elements explains why he did not think relativity was the ultimate answer to the space-time problem, since it still assumed an, albeit relativized, spatio-temporal format and did nothing to reduce it to non spatio-temporal foundations. Likewise, Mach could not accept little spatio-temporally extended atoms, but rather thought his theory of elements could go deeper and explain the energy changes *within* atoms as evidenced by the spectral lines. As Paul Feyerabend pointed out in a seminal paper: “elements as envisaged by Mach are more fundamental than atoms” (Feyerabend 1984, p. 11). And as I have shown, Mach continued to hope for a constructive *elimination* of space and time from physics to the end of his career (Banks 2003 Chapter Fourteen). Like so much else about Mach, this program has received almost no attention at all, a very puzzling state of affairs. It is assumed by philosophers of science that Mach’s relationism in physics was simply a result of his phenomenalism in philosophy, thus compounding an error with another error.[[9]](#endnote-9)

1. **Knowledge and Error**

Mach may have been one of the first naturalized epistemologists, *avant la lettre*, since he believed that the only methods for reaching knowledge were scientific methods.[[10]](#endnote-10) He still held on to introspection in his psychology, of course, but supplanted it with careful measurements. Mach certainly was not a foundationalist and did not start with a set of indubitable givens from which to construct the world by comparison with Carnap and Schlick, (see Feyerabend 1970 pp. 179-181, Uebel 2007, Banks 2013). He insisted that his elements were “provisional only…the smallest divisions science is capable of making at any given time” (Mach 1905/1976 p. 12n) hence his view that the elements of spectral lines were a level of analysis *beneath* atoms, as Feyerabend pointed out. The functions grouping the elements into systems were the naturalistic functions they obeyed by virtue of their causal powers, not second-order tortured logical or linguistic constructions with little relation to reality. The Ramsay sentence, or Craig’s Theorem, for example are the farthest things possible from a Machian approach to science even though many would take these developments to be the apotheosis of the Machian view.

Mach’s theory of knowledge as represented in *Knowledge and Error* was biological and evolutionary in nature. He proposed the idea that thoughts accommodated themselves to experience and to each other in an evolutionary or adaptive process. He also saw conceptual and linguistic knowledge as resulting naturally and continuously from preconceptual habit and reflex, enhanced by social communication. In some ways, perhaps, he does not do justice to the abstract structure of logic and mathematics or the generality of abstract concepts. (I now believe this is a serious gap in his philosophy, though not one that cannot be filled, I think.) Mach did, however, recognize that hypotheses could not be arrived at by Baconian enumerative induction but involved a kind of abductive leap which could then be compared with experience. He also believed that nature exhibited patterns or “great facts” which could become accessible to human intellect, for example various partial differential equations that showed up across various domains in physics. He even hoped for a “phenomenological physics” linking all the great facts under abstract, master principles like the conservation of energy, the second law of thermodynamics, and the law of least action, all of which rested upon experience for their evidence. It is very wrong to say that Mach was skeptical of laws or abstract principles and believed only in economical lists of particulars. His view of economy is much more subtle than that (see Banks 2004).

In *Knowledge and Error*, Mach suggested that inquiry took many paths, calling his book a series of “sketches” of different techniques some of which might lead to knowledge or else error depending on the circumstances of the problem, most of which are out of our control:

Knowledge and error flow from the same mental sources, only success can tell the one from the other. A clearly recognized error, by way of correction, can benefit knowledge just as a positive piece of knowledge can (Mach 1905/1976, p. 84).

As he says pointedly, this catalogue of approaches is all we have: there are *no codified scientific methods* guaranteed to lead to truth. Notice that knowledge and error flow along the causal links between our thoughts and ideas and circumstances in which an expectation is met, or thwarted or even found to be illusory. Mach holds a causal theory of knowledge and not a propositional theory, in which a proposition has an intrinsic power to picture a state of affairs beyond itself, and to correspond truly or falsely to it. James, who also rejected any intrinsic intentionality for mental images and judgments, held a very similar view, first presented in his 1894 address “The Knowing of Things Together” later printed as “The Tigers in India” in the *Meaning of Truth*:

The pointing of our thought to the tigers is known simply and solely as a procession of mental associates and motor consequences that follow on the thought and would lead harmoniously, if followed out, into some kind of ideal context, or even into the immediate presence of the tigers. It is known as our rejection of a jaguar if that beast were shown to us as a tiger; as assent to a genuine tiger if so shown…In all of this there is no self-transcendency in our mental images taken by themselves. They are one physical fact; the tigers are another; and their pointing to the tigers is a perfectly commonplace physical relation…I hope you may agree with me now that in representative knowledge there is no special inner mystery, but only an outer chain of physical or mental intermediaries connecting thought and thing. (James 1977 pp. 155-156).

I have presented my own reconstruction of how I think James analyzes away phenomenal intentionality in my 2014 Chapter Three. Incidentally, Bertrand Russell, who also converted to neutral monism, never fully accepted this epistemology or naturalistic theory of meaning and reference. He still believed that one needed images and image propositions to “mean” non-present states of affairs, and to correspond to them truly or falsely, of which the images could not be considered direct copies or memories. There is thus a residue of propositions and propositional judgement in Russell that was never eliminated (for which see Banks 2014, Chapter Four).

I have also argued elsewhere that it is a mistake to attribute the modern observable-unobservable distinction to Mach (Banks 2013). Mach emphasized a different distinction based upon causal continuity with experience. Unobservable elements are at least causally continuous with observation and allow for analogies and even the assumption of the unobservable, if it can be shown to play a role in extending organizing experience. Mach said as much in his famous discussion with Einstein in 1907 (related in Frank 1947). What Mach rejected as “metaphysical” were:

1. an isolated Ding an sich with no possible causal connection to other experiences. He originally thought atoms were like this, forever unobservable things of thought. When he began to think of them as continuous with observation, his opinion seemed to change, although he never regarded them as ultimate. Mach probably would have been much happier with Heisenberg’s abstract matrix atom made up of energy transitions, very much like his elements in fact.

2. intuitive pictures or visualizations based on human psychology being substituted in thought where human intuition could not be expected to reach. Again, remember his remark in “Uber Symmetrie” about a natural system with more degrees of freedom than will fit into three-dimensional space. Mach wanted to purge physics of psychological visualizations, substances, extended space, time, which interfered with its content, and called this “metaphysics.”

**Conclusion: The Two Traditions of Mach Interpretation, Relation to Contemporary Philosophy of Mind**

So, to return to the beginning, there are two main strands of Mach interpretation. The first “phenomenalist” strand runs from Lenin, Schlick, Carnap and Popper, to John Blackmore and many scientific writers and historians of ideas. In fact, to put things in the strongest terms, most historians of philosophy, and the vast majority of philosophers, and scientists unfortunately, still associate Mach’s views closely with the Vienna Circle as a primitive early form of what later became 20th century logical positivism. I wish to add also that our understanding of the Vienna Circle has advanced considerably in recent years, however, and the old “received view” of logical positivism (for example in Suppe 1977) is currently under attack. Friedrich Stadler 2001 has questioned the Mach-Vienna Circle association in some detail and Thomas Uebel 2007 has set apart the *Protokollsatz* debate in the Vienna Circle from anything having to do with Mach’s elements. Other active investigations seek to link Mach to European pragmatism, or even Nietzsche’s evolutionary naturalism (see Gori 2009), rather than with positivism. I hesitate to declare this positivist strand *dead*, since so many people still believe it and articles still often appear declaring Mach a phenomenalist. It is like a myth that has become so widely believed it is not possible to change anyone’s mind who has not looked into the matter. But these are usually superficial accounts using Mach for some subsidiary purpose rather than interpreting him directly, and usually citing the same recycled quotes from the *Analysis of Sensations* or the *Mechanics* that are printed and reprinted in historical anthologies, but rarely going into details or citing the secondary literature at all.

The second “neutral monist” strand of interpretation I have been defending here runs from Mach, James and Russell to the American Realists (see Banks 2014 and 2016). Paul Carus and Hans Kleinpeter also made a special effort to insist that Machian elements were *really* physical and that the realistic side of his work had to be fully acknowledged for any understanding of his views, as Russell understood so well. All of these early writers at least seem to have grasped the realistic tenor of Mach’s neutral elements. For example, Ralph Barton Perry, one of the founding American Realists, claimed that “Mach’s book the *Analysis of Sensations* deserves to be numbered among the classics of realism” *not* positivism (Perry 1925, p. 79).

After Russell, neutral monist ideas lived on in the work of Wilfrid Sellars and Herbert Feigl, but the recent revival of neutral monist ideas in the philosophy of mind takes its cue from Russell’s *Analysis of Matter*. Recently however this tradition has been traced back further to Mach and James (for example by Stubenberg 2016). Michael Lockwood and Grover Maxwell are most directly responsible for reviving Russellian ideas in philosophy of mind. Russellian monism as it is called, was first offered as an alternative to Kripke’s modal arguments against physicalism in *Naming and Necessity* and later against David Chalmers’ “zombie” argument in *The Conscious Mind* (1996). The basic idea behind both arguments is that if pain and c-fibers are both rigid designators, identifying the same state across all possible worlds, or counterfactual situations, then the identity should be necessary but there should *also* be no conceivable way to imagine their non-identity. Another way to put it a la Chalmers is that if two rigid designators like pain and c-fibers agree in their secondary intensions, or reference, they should also agree in their primary intensions, or meanings. This is not true for say the meaning of non-rigid “watery stuff”: clear colorless liquid in lakes and ponds which might pick out H20 on earth and XYZ on Twin Earth in a counterfactual situation. By contrast, really grasping what it means to be in pain is the same thing as *being* in pain, grasping the meaning of c-fibers is just knowing what c-fibers *are*, and thus there should be an conceptual identity between the meanings too, however complicated, which someone could potentially work through concepts alone, as it were on paper even if the derivation was extremely complicated. Since this is clearly not the case for the identity between pain and c-fibers, they are not identical and physicalism is false.[[11]](#endnote-11) So runs that argument.

Roughly, Russellian monists respond to this, valid but I think unsound, argument by defending a kind of enhanced physicalism in which the outer structural aspect of physics is actually grounded, or realized, in some kind of qualitative interior or categorical properties of physical objects. Hence when you physically designate c-fibers you might fail to designate their interior aspect of pain, as Maxwell argued (1977). Russellian monists then argue that as the atoms and molecules combine physically, their qualitative interior, or categorical properties, also combine internally into minds. Sometimes these views veer off into panpsychism attributing protophenomenal qualities to all bits of matter (as we saw in the early Mach before he embraced neutral monism).

David Chalmers and Galen Strawson have both offered compositional panpsychist versions of Russellian monism. Both authors also think qualities of experience imply awareness or an ego to behold them, thus I do not hold this view to be a neutral monist view. In my 2014 book defending neutral monism I rejected *both* the pan-psychism and also the a priori compositionality of the protophenomenal qualities in matter into minds, which I believe to be the more basic error. Yet, despite these differences, the realist strand of neutral monism lives on today in a variety of forms. Who knows, just as Mach once evolved from a panpsychist to a neutral monist and discarded the ego on the way, the present movement may end up retracing his steps, and soon we might even see a revival of Mach’s work parallel to Russell’s.

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1. Much of the background material for this paper can be found in my two books on Mach (Banks 2003, 2014). On Mach and the Vienna Circle see Banks 2013. On Mach versus James on pragmatism and other differences see Banks 2016. [↑](#endnote-ref-1)
2. In panpsychism the unobserved qualities are analogized to our sensations so instead of blue we have “protoblue” inside atoms (or perhaps neurons) that combine to make blue when enough of them are combined in a certain way. See Chalmers 2002 for the introduction of protophenomenal qualities. [↑](#endnote-ref-2)
3. This is a second difference with panpsychism, which holds that qualities always require conscious egos to behold them. Chalmers and Strawson are panpsychists of this sort. It isn’t ruled out that a higher order judgment couldn’t affect a quality that is part of the complex we call the judgment. But judgments are nothing besides the complexes to which they belong. It would be wrong to think of judgment as some special mental act determining its content. It would also be wrong to think of judgment as somehow responsible for determining the quality of the content merely because it can affect that quality (which is surely the case). For example Daniel Dennett’s attempt to muddle the issue with his example of the Maxwell’s House tasters would certainly have to be rejected by the neutral monist. [↑](#endnote-ref-3)
4. As will be discussed below, similarity is not an equivalence relation. It therefore is not suitable for physical properties determining a partition of individuals that is, in turn, invariant through transformation groups. [↑](#endnote-ref-4)
5. Where exactly neutral monism originates is hard to tell. Similar passages about the two orders and the neutral stuff occurred in writings by C.S. Peirce, W.K. Clifford and Richard Avenarius. Mach’s view still has priority because it seems to have originated in writings of the 1860s and may have appeared in the first draft of the Analysis of Sensations in 1864. G.T. Fechner’s refusal to let Mach dedicate the book to him in 1864 may have delayed publication for twenty years (the Analysis of Sensations was first published in 1886 long after Mach had formulated his ideas) (see Michael Heidelberger 1993 for this interesting story). [↑](#endnote-ref-5)
6. The term world elements is not Mach’s but Friedrich Adler’s (1908). I also used it in the title of my 2003 Book *Ernst Mach’s World Elements*. [↑](#endnote-ref-6)
7. I mean that the principle of least action is an expression that the differences between the time averages of the kinetic energy *minus* the potential energy is a minimum. [↑](#endnote-ref-7)
8. Einstein described “Mach’s Principle” in a letter to Mach in 1913. Shortly thereafter is dated the preface to the *Optik* in which Mach disavows being a forerunner to the theory of relativity. According to Gereon Wolters, 1987, the preface is a forgery by Mach’s son Ludwig and Mach had accepted relativity and atomism by this time. Wolters has made a case for his view with many supporting documents. For me, however, the timing is too close between the two documents to be a coincidence (see Banks and since I do not think Mach would have accepted Mach’s Principle as the only reconstruction of his thoughts on Newton, Mach may have been trying to head off Einstein’s all too neat summary of his ideas. The forbidding tensor calculus field equations may have been another reason Mach did not want to be associated with theories he did not understand mathematically. [↑](#endnote-ref-8)
9. I have often wondered why so many misinterpretations proliferate about Mach’s works. I have been told that Mach is hard to read in the original and that most readers consult anthologies or “Cliff Notes” versions in other authors or erroneous histories of philosophy which seek to pigeonhole authors in neat historical movements. I urge authors considering writing about Mach to read his original works and also to become familiar with the scientific issues he discusses. [↑](#endnote-ref-9)
10. Although Mach was not like Peirce at all and did not canonize scientific methods as an epistemology. The difference is subtle, see my 2017. [↑](#endnote-ref-10)
11. The focus on conceivability is a bit of a red herring, since the real issue is the existence of an a priori conceptual connection between the secondary intensions. [↑](#endnote-ref-11)