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Explanatory Information in Mathematical Explanations of Physical Phenomena

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In this paper I defend an intermediate position between the 'bare mathematical results' view and the 'transmission' view of mathematical explanations of physical phenomena (MEPPs). I argue that in MEPPs, it is not enough to deduce the explanandum from the generalizations cited in the explanans. Rather, we must add information regarding why those generalizations obtain. However, I also argue that it is not necessary to provide explanatory proofs of the mathematical theorems that represent those generalizations. I illustrate this with the bridges of Königsberg case.

Scientific Explanation; Mathematical Explanation; Explanatory Proofs; Bridges of Königsberg; Asymmetry; Relevance

1. Introduction

According to Carl Hempel's Deductive-Nomological (D-N) model of scientific explanation, scientific explanations are deductive arguments that are given in order to answer a *why-question*. These arguments must include at least one law of nature that is essential to the derivation of the explanandum [Hempel 1965: 336-7]. However, for some decades now the common agreement in the scientific explanation literature has been that merely deducing the explanandum from a generalization cited in the explanans (along with some initial conditions) does not constitute an adequate explanation. First, mere deduction from a generalization fails to indicate how these initial conditions bring about the explanandum; this is called the problem of relevance. In addition, in some cases deduction can be symmetrical, so taking deduction as the explanatory relationship fails to explicate why the initial conditions explain the explanandum but not in the other way around; this is called the problem of asymmetry. For these and other reasons, Hempel's D-N model was abandoned and causal models were introduced instead. According to these models, scientific explanations work by tracking the causal history of the *explanandum*. Causal models account for explanatory asymmetries because causation is an asymmetric relation. In addition, the causal factors cited by these explanations are precisely the factors relevant to bring about the explanandum.

Now, in recent years there has been much discussion in the philosophy of science concerning noncausal scientific explanations. These explanations do involve physical objects endowed with causal powers, however, they do not work by exploiting those powers. One particular kind of noncausal explanation that has been the focus of much attention is so called Mathematical Explanations of Physical Phenomena (MEPPs). One aspect that has not been properly discussed in the literature is how MEPPs deal with the problems of asymmetry and relevance, given that they ignore causal factors. This will be the focus of my paper. I use as a case study the bridges of Königsberg.

2. The Bridges of Königsberg case

The inhabitants of XVIIIth century Königsberg wondered whether it was possible to walk over all seven bridges of the city without retracing their steps.[1] Leonard Euler's solutions to this problem

[1] This example was first brought to discussions on the applicability of mathematics in science by Christopher Pincock [2007].

not only showed *that* the trip was impossible; they also explained *why*. Euler discovered that a road system allows a continuous walk that covers all of its bridges if and only if it has either zero or two landmasses connected by an odd number of bridges. Now, philosophers disagree about the role this generalization plays in the explanation, so let us take a closer look.

Consider the following statement:

> It is impossible to walk over the seven bridges of Königsberg without retracing one's steps.

It is easy to verify that this is true. For example, we can try all the available routes over the bridges system; these are computable in polynomial time [Moore and Mertens 2011: 4]. Once we do this, we will find that none of them allows such a walk. This answer, however, does not explain why the statement is true. For all we know, the reasons why we failed to perform that walk in each case may differ, and so the fact that it is impossible to perform such a walk over the bridges may be a coincidence.

Now, consider the following situation. You open a book on graph theory, find the theorem discovered by Euler, and then provide the following argument:

Argument 1

P1) A road system allows a continuous walk that covers all its bridges if and only if it has either zero or two landmasses connected by an odd number of bridges

P2) The bridges of Königsberg system has neither zero nor two landmasses connected by an odd number of bridges

C) It is impossible to walk over the bridges of Königsberg system without retracing one's steps

Is this an adequate explanation of the impossibility of performing an Eulerian walk over the Königsberg system? There are two competing views about this issue. On the one hand, defendants of the 'bare mathematical results' view argue that, if a mathematical theorem features in the explanans, the explanation must simply show how the explanandum follows from it. On this view, Argument 1 above does constitute an adequate explanation of C. On the other hand, defendants of the 'transmission' view argue that, in order to be successful, the explanation must include an explanatory proof of the theorems it uses. According to this view, Argument 1 is not adequately explanatory because it misses an explanatory proof of P1.

In this paper, I adopt an intermediary position. I claim that Argument 1 above does not explain the impossibility because it suffers the problems of relevance and asymmetry in the same way Hempel's D-N model does. In that sense, in order to explain the explanandum it would not be enough to deduce it from P1. This does not mean, however, that we should provide an *explanatory proof* of the mathematical representation of P1, as the transmission view suggests. The ultimate goal of the mathematical operations performed at the level of the mathematical representation is to convey information about the physical structures represented by P1, regardless of whether those operations would qualify as genuine proofs (let alone explanatory proofs) of the mathematical version of P. In my view, in order to account for the explanatory power of MEPPs, we do not need to adopt a position in the philosophy of mathematics debate concerning the nature of explanatory proofs. MEPPs should meet the standard of scientific explanations, not of mathematical explanations. The plan of the paper is as follows. In section 3 I present and discuss the 'bare mathematical results' view. In section 4 I reconstruct Euler's solutions to the bridges of Königsberg case under my view and argue that my view deals with the problems of the bare mathematical results view. In section 5, I present and discuss the 'transmission' view.

3. 'Bare Mathematical Results' View

3.1. Mathematical Generalizations as Initial Conditions

According to Alan Baker, the mathematical facts used in a scientific explanation do not need to be explained. When we use a mathematical result in an explanation, we only need to know *that* such result has been proven, but the proof itself is unimportant [2012: 263]. As long as it is known that a given theorem is true, it can successfully be used in empirical applications. This is especially the case in MEPPs (what Baker calls science-driven mathematical explanations (SDME)):

> In general, all scientists need to know when they appeal to a given mathematical result in the context of a science-driven mathematical explanation is *that* this result has been proved. Nothing else about the proof matters for the purposes of the overall SDME, nor does the proof need to be included in the presentation of the SDME [Baker 2012: 263].

Baker argues that, in a scientific explanation, the mathematical facts cited in the explanans have the same status as the initial conditions presented in the situation to be explained. We do not usually need to explain the initial conditions, for example, we do not usually need to explain why there are raindrops in the air in order to explain the rainbow. In the same way, we do not need to explain the mathematical facts. Rather, what the explanation does is show how, given the initial conditions *and* the mathematical facts, the explanandum follows. Baker tracks this view back to

Hempel [Baker 2012: 263]. In the bridges case, Argument 1 would be an explanation because P1 is a mathematical fact. We do not need to explain P1, we must just use it to *deduce* C. Other versions of the 'bare mathematical results' view include Pincock's account of abstract explanations [2015][2], and Marc Lange's account of distinctive mathematical explanations [2013, 2017].

3.2. Problems with the 'Bare' Mathematical Results View

a) Relevance

One of the problems of Hempel's D-N model is that merely deducing the explanandum from facts cited in the explanans does not inform us about the features relevant to the occurrence of the explanandum. For example, as James Woodward has pointed out, if we want to explain why a given animal is black, the following argument will not be enough:

(a) All ravens are black

(b) x is a raven

(c) x is black (a+b)

This argument can be used to justify the truth of 'x is black', and it makes 'x is black' expectable given the facts cited in the explanans. But it does not identify the features of ravens relevant to their blackness, and it doesn't show how these features lead to the blackness of the animal

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[2] Although Pincock endorses the 'bare mathematical results' view [2015: 874, fn12], his account of abstract explanations is not necessarily committed to it. For example, in his treatment of the soap bubbles case, Pincock includes mathematical operations that may count as explanatory on my view [see 2015: 858 and f.f.]

[Woodward 2003: 187 and f.f.] This is even more evident in Wesley Salmon's famous Mr. Jones case:

(a) All males that take birth control pills fail to get pregnant

(b) Mr. Jones is a male that has been taking birth control pills.

(c) Mr. Jones failed to get pregnant (a+b)

For Salmon, the reason why Mr. Jones taking birth control pills should not be included in the explanation is that we do not need to mention this in the reconstruction of the causal history that lead to him not being pregnant (see for example [Salmon 1989: 50]).

Now, MEPPs are noncausal explanations, so how do they deal with the problem of relevance? Pincock [2012, 2015] holds that MEPPs work by appealing to instantiations of mathematical structures, and that only the physical features relevant to the instantiation of these mathematical structures are relevant to the explanation. Thus, in the bridges case, an explanation of the impossibility of performing the desired walk over the bridges must appeal only to those features of the system of bridges relevant to instantiate the mathematical graph, namely, number of landmasses and the connections between them [Pincock 2012: 208-10]. With this in mind, we identify P1 as the correct mathematical relationship to be used in this case, and we explain C by deducing it from P1.

However, I do not think this fully solves the problem of relevance. It is one thing to identify the aspects relevant to explaining a situation, and a different thing to correctly use those aspects in the explanation. Argument 1 only establishes that the property of allowing an Eulerian walk is related to the property of having either zero or two landmasses, and that, given the system's structure, such a walk is not possible. But although it does cite all that is relevant, it does not tell us how exactly these features bring about, or are somehow responsible, for the explanandum. The problem is that if we do not know this, we may still accidentally incorporate irrelevant aspects in the explanation. Consider the following argument:

Argument 2

P1') A road system allows a continuous walk that covers all its bridges if and only if it has either zero or two landmasses connected by an odd number of bridges, *or if the total sum of the number of bridges connecting to each landmass is not higher than 2*.[3]

P2') The Königsberg system has neither zero nor two landmasses connected by an odd number of bridges, *and the total sum of the number of bridges connecting to each landmass is higher than 2*.

C) It is impossible to walk over the Königsberg system without retracing one's steps.

This case is meant to be analogous to the Mr. Jones case. Although this argument only uses the connections and nodes, and leaves aside many irrelevant features such as areas, material composition, lengths, etc., it still incorporates irrelevant information regarding the sum of the degrees of all nodes. The 'bare mathematical results' view cannot explicate why Argument 2 is

[3] In modern graph-theoretical terms, the landmasses would be the nodes and the bridges would be the edges. The number of edges connecting to a given node is called the degree of the node. Here, what I am adding to P1 is the requirement that the sum of the degrees of all the nodes must not be higher than 2.

not an adequate explanation because Argument 2 satisfies the requirement of deducing the explanandum from some mathematical facts.

As I will argue below, Euler's solutions to the bridges case not only identify the properties relevant to bringing about the lack of an Eulerian walk, they also describe the exact way these properties bring about this. And in this description, there is no need to mention that the sum of the degrees of all nodes is higher than 2. The crucial point is that we can only learn what is and what is not relevant when we disaggregate P1 into the reasons why it obtains. Merely mentioning P1, as the 'bare mathematical results' view suggests, is not enough.

b) Asymmetry

Another famous problem of Hempel's D-N model is that it fails to account for explanatory directionalities. This is illustrated by the famous flagpole case[4]:

(a) Trigonometrical laws[5]

(b) Other physical laws (rectilinear propagation of light, opacity, etc.)

(c) Length of the shadow is L.

(d) Other initial conditions

(e) Height of the flagpole is H (a – d)

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[4] This example was informally presented to Hempel by Sylvain Bromberger (see [Bromberger 1992: 8].

[5] Note that they are not the trigonometric theorems themselves, but the properties of physical space approximately captured by them, that are intended to be explanatory.

The argument above is the 'reverse case' of a correct explanation that would appeal to features of the flagpole to explain its shadow's length (the phrase 'reverse case' is from Carl Craver and Mark Povich [2017: 33]). Common sense indicates that it should be the flagpole's height that explains the shadow's length, and not in the other way around, as the reverse case may suggest[6]. The purpose of this reverse case is to show that mere deduction from the trigonometrical theorem that relates the two legs of a right triangle is not enough to explain the length of the shadow, and that something else must be mentioned. But that something else is not captured by the D-N model.

Following Woodward [2003] and other defendants of ontic views of scientific explanation, Pincock argues that explanations must exploit objective relations of asymmetric dependence. As he puts it, 'C explains E iff E depends on C' [2015: 878]. In the case of MEPPs, Pincock argues that the explanatory directionality can be captured by appealing to the asymmetrical dependence

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[6] Against this, Bas van Fraassen [1980] famously argues that, if you want the shadow to have a specific length at a certain time of day, then that length would explain the flagpole's height [1980: 132-4]. More recently, Mary Leng argues (personal communication) that, if the question is why, *given the length of the shadow*, the height must be such and such, then we must use the shadow to explain the height. But I do not believe that these modifications actually show that a reverse flagpole case can be explanatory, which is ultimately what the problem of asymmetry is about. With respect to van Fraassen's modified version, as Philip Kitcher and Salmon [1987] pointed out, it is not the shadow, but your beliefs and desires, along with facts about the flagpole's material constitution and the rectilinear propagation of light, that explain why the flagpole was made with such a height ([1987: 316-7]; see also Salmon [1989: 144]). Similarly, in Leng's case you do not use the shadow to explain the height; rather, you use the shadow to explain a conditional fact that involves the shadow and the height. Again, this does not show that a reverse flagpole case is explanatory.

between abstract mathematical facts and their concrete instantiations [2015: 876]. In the bridges case, for example, the bridges system instantiates a graph, but not in the other way around, and this is why facts about the graph explain facts about the bridges system, but not in the other way around.

I believe, however, that this does not work in the bridges case *as I have presented it*. In my version of this case, the explanation must explain relationships between two instantiated properties (the bridges structure and the impossibility of an Eulerian walk), so the asymmetry in the instantiation relation (between the abstract property and its instantiation) is not the one we are looking for. Something more needs to be said if one instantiated property is going to be used to explain another. To see this, consider the following argument:

Argument 3

P1) A road system allows a walk that covers all its bridges if and only if it has either zero or two landmasses connected by an odd number of bridges

P2*) It is impossible to walk over the Königsberg system without retracing one's steps

C*) The Königsberg system has neither zero nor two landmasses connected by an odd number of bridges

Argument 3 is the 'reverse case' of Argument 1. Here, P2* corresponds to the conclusion of Argument 1, C* corresponds to its second premise, and P1 describes a symmetrical relationship between the bridges system having a given structure and the fact that it does not allow an Eulerian walk. However, common sense indicates that the bridges structure should explain the impossibility, and not in the other way around. The problem here is not whether the bridges system instantiates a mathematical graph. Rather, the idea is that some properties of the bridges structure explain other properties, regardless of whether the structure itself is mathematical. The 'bare mathematical results' view, therefore, cannot explicate why Argument 3 is not an explanation. As I argue in section 4, we can identify the source of explanatory power in the bridges case if we pay attention to Euler's solutions.

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Now, my rejection of the 'bare mathematical results' view does not imply that we must prove the mathematical results in the explanans. For one thing, this will render many simple cases of MEPPs unnecessarily complicated. For example, in Lange's strawberries case [2013] –where, allegedly, we explain the impossibility of sharing 23 whole strawberries evenly among three children by appealing to the mathematical fact that 23 is not divisible by 3– proving the relevant arithmetical result (for example, from Peano axioms) would not only be very long but also would make it difficult to see how this proof may track the relevant features of the explanandum[7]. Fortunately, however, we do not need to include these proofs because, in my view, in MEPPs mathematics simply tracks down the relevant physical structures responsible for the occurrence of the explanandum. In the strawberries case, 23, 3, and the division operator, capture all and only the features relevant to explaining the explanandum, ignoring things such as fruit color, children's height, etc.[8] This representation works because we are dealing with physical structures that, when

[7] Thanks to an anonymous reviewer for raising this point.

[8] In [Barrantes 2019] I call those representations that include all and only the features relevant to explaining the explanandum (once it has been described at the appropriate level) 'optimal

described at the appropriate level, can be suitably modeled by whole numbers and the division operator. This does not occur, for example, with gases (23 gallons of gas fit in several three-gallon containers). Once this representation is in place, the mathematical impossibility of dividing 23 by 3 is used to track down the relevant physical relations that explain the impossibility of accomplishing such a division of strawberries. Imagine for example that we give one strawberry to each child, and then ask them to eat their strawberry *only after* everyone else has received one, and then repeat this. There will be a point where one child will be missing a strawberry and the other two will not be allowed to eat theirs. Why is it that 23 strawberries cannot be evenly divided among three children? For the same reason that you cannot give one strawberry to two children: unlike gases, strawberries cannot occupy another space in addition to the one they currently occupy. '23 is not divisible by 3' is shorthand for the physical process of failing to establish a one-to-one correspondence between children and strawberries, in virtue of both the numbers of strawberries and children and the aforementioned empirical property of strawberries. Merely deducing the physical impossibility from the mathematical impossibility does not explain the contingent fact that this operation works for strawberries but not for other things.

4. Euler's Solutions to the Bridges Case

In his *Solutio Problematis at Geometriam Situ Pertinentis* [1736], Euler provided two solutions to the problem faced by the Königsberg inhabitants. In doing so, he not only proved that such a walk was impossible, he also explained why.

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representations.' I argue that one essential feature of MEPPs is that they use these optimal representations.

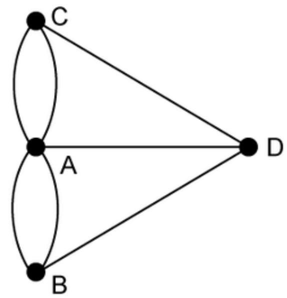

Figure. 1. Graph-Theoretical representation of the Bridges of Königsberg system

Euler's Solution 1[9]

> Define a path-sequence as the sequence of letters each particular path covers. Since there are seven bridges, a path that successfully covers all of them only once must feature exactly eight letters. However, if we count the number of times each letter will appear in the path-sequence, given the number of bridges that connect to it, we will see that B, C, and D will appear two times each, and A will appear three times (see Figure. 1). This gives us a total of nine letters, which exceeds the eight letters a successful path-sequence must have. Therefore, such a successful path is impossible.

This is an explanation because it tracks down how the relevant structural features of the bridges system bring about the lack of an Eulerian walk. From the perspective of the number of bridges, eight letters must appear in the path-sequence, but from the perspective of the number of bridges connecting to each landmass, nine letters must appear in the path-sequence. The incompatibility of these two facts renders the desired walk impossible. Moreover, the explanation

[9] What follows are my paraphrases of Euler's solutions.

shows how these two facts depend on the way bridges and landmasses are structured in the Königsberg system, and on the extremely simple fact that a bridge connects two landmasses.

Now, it may be argued that Solution 1 is not general enough. It only works for the specific system of bridges in Königsberg. Contrary to this, the objection goes, Argument 1 subsumes the impossibility under the broader generality given by the (allegedly mathematical) fact P1. I believe this objection correct. However, it only shows that we need a more general explanation, not that Argument 1 is a correct explanation of C. Euler himself thought about this problem and provided a more general solution:

Euler's Solution 2

Define a generalized road system with *o* odd landmasses, *e* even landmasses, and *n* bridges. Every odd landmass has h_i bridges connecting to it, and every even landmass has k_j bridges connecting to it. A path-sequence is a list of all the landmasses covered by a given path.

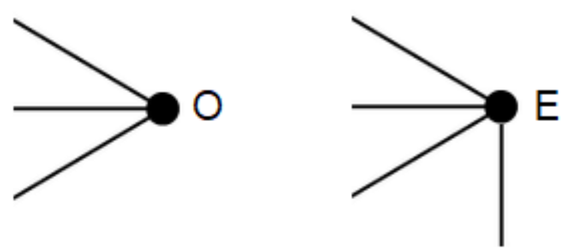

Figure. 2. Example of an odd landmass and an even landmass

1) Every time you cross a bridge, two letters feature in the path-sequence, the starting point and the ending point.

2) If a connected system has n bridges, a successful Eulerian path-sequence will have n+1 letters. (1)

3) Every odd letter will feature $\frac{h_i+1}{2}$ times in a successful Eulerian path-sequence. (1)

4) If the path starts in an even landmass, its letter will feature $\frac{k_j}{2}$ +1 times in a successful Eulerian path-sequence. (1)

5) If the path does not start in that even landmass, its letter will feature $\frac{k_j}{2}$ times in a successful Eulerian path-sequence. (1)

6) Define r as a binary variable that takes value 1 if the path starts on an even landmass and 0 if it does not.

7) The number of letters in any path-sequence is:

$$\#\text{letters} = \sum_{i=1}^{o} \frac{h_i+1}{2} + \sum_{j=1}^{e} \frac{k_j}{2} + r \qquad (3\text{-}6)$$

8) In a successful Eulerian path sequence the following holds:

$$\sum_{i=1}^{o} \frac{h_i+1}{2} + \sum_{j=1}^{e} \frac{k_j}{2} + r = n+1 \qquad (7+2)$$

9) $\sum_{i=1}^{o} h_i + \sum_{j=1}^{e} k_j + o + 2r = 2n+2$ (8)

10) Every bridge has been counted twice, so: $\sum_{i=1}^{o} h_i + \sum_{j=1}^{e} k_j = 2n$

11) $2n + o + 2r = 2n + 2$ (9+10)

12) $o + 2r = 2$ (11)

13)[10] $o_1 = 2$; $r_1 = 0$

$o_2 = 0$; $r_2 = 1$ (12+6)

14) A road system allows a walk that covers all its bridges if and only if it has either zero or two landmasses connected by an odd number of bridges. (13)

15) The bridges of Königsberg system has neither zero nor two landmasses connected by an odd number of bridges[11]

C) It is impossible to walk over the bridges of Königsberg system without retracing one's steps (14+15)

This explanation counts the number of letters in a path-sequence both from the perspective of the number of edges (2) and from the perspective of the number of connections to each node (3-6). In order to avoid the incompatibility that occurs in the Königsberg case, in a successful path-sequence the number of letters measured from these two perspectives must be the same (8).

As I mentioned, this is a non-causal explanation. Although the bridges system is a physical system composed by elements with causal powers (for example, wooden bridges), the explanation does not work in virtue of the bridges' causal powers (this has also been emphasized by Lange

[10] This should be read thus: if a connected system has two odd landmasses, a successful path is possible, and it must start in one of the odd landmasses; if a connected system does not have odd landmasses, a successful path is possible, and (evidently) must start in an even landmass.

[11] Note that 14 and 15 correspond to premises P1 and P2 in Argument 1.

[2013: 509] and Pincock [2015: 895]).[12] The causal components of the bridges system are not relevant to the explanation. But despite being noncausal, the explanation does not face the problems of relevance or asymmetry.

First, Solution 2 not only informs us about the aspects of the system that are relevant for the explanation. It also tells us how these aspects relate to each other so as to bring about the explanandum. Specifically, Solution 2 shows how the fact that a bridge connects two landmasses

[12] Lange argues that MEPPs (what he calls distinctive mathematical explanations) are non-causal because 'they do not work by supplying information about a given event's causal history or, more broadly, about the world's network of causal relations' [2017: 5]. Instead, for Lange these explanations appeal to facts that are modally stronger than ordinary causal laws [2013: 506]. Yet Lange also holds that not citing causes is not a reason for an explanation not to be causal. For example, Lange holds that Jackson and Pettit [1990]'s example of a circular peg not fitting in a square hole due to geometrical incompatibility is a causal explanation [2013: 506]. But this is confusing. After all, the geometrical incompatibility is modally stronger than the contingent fact that the peg does not fit in the hole, so it is not clear why this would not be a distinctive mathematical explanation. Lange may reply that, because these geometrical properties are 'causally relevant' (in the sense that they 'program' that there will always be a part of the peg bumping onto a part of the wall (see [Jackson and Pettit 1990: 115]), they belong to 'the network of causal relations'. But again, this programing role sounds like a constraining role. Overall, as Sorin Bangu [2017] points out, it is hard to see why for Lange some explanations are non-causal. My view of MEPPs does not have this problem. I argue that MEPPs do not give information about the explanandum's causal history, but, contrary to Lange, I believe that they must give information about the world's causal network. Otherwise, as also pointed out by Bangu, the explanation would be explaining purely mathematical facts, which would disqualify them as being genuine MEPPs (see [Bangu 2017] for details). Thanks to a reviewer for asking me to clarify this.

is responsible for both facts that an even landmass will feature in a successful path-sequence a number of times that is half the number of bridges connecting to it, plus 1, and that an odd landmass will feature in a successful path-sequence a number of times that is half the sum of the number of bridges connecting to it and 1. This is not the causal history of the explanandum, but it does explain how these relevant features of connections and landmasses bring it about, or are responsible for it.

In addition, as opposed to Argument 3, Solution 2 is explanatory because it shows how the impossibility of the desired walk depends on the structure of bridges because it ultimately depends on the fact that a bridge connects two pieces of land. This is the source of the explanatory asymmetry because the fact that a bridge connects two pieces of land does not depend on the lack of an Eulerian walk. The crucial point here is that, contrary to what the 'bare mathematical results' view suggests, we can only learn this information if we disaggregate P1 in terms of the reasons why such generalization obtains, which is precisely what Euler did in Solution 2.

Against this, a defendant of the 'bare mathematical results' view may argue that 1-13 explain why 14 holds, and 14 and 15 together explain C (Argument 1). In that sense, 1-13 would be indirectly contributing to explaining C, but this indirect contribution should not be part of the explanation of C, otherwise every assumption in an explanation would in turn have to be explained, which would make any explanation subject to infinite regress[13]. This objection presupposes a distinction between explanations of particular explananda and explanations of regularities[14], the

[13] Thanks to an anonymous reviewer for raising this point.

[14] Woodward [2003] calls this the 'two-level' approach, and rejects it on the grounds that this distinction is absent in scientific practice.

first being appropriately described by the D-N model, and the second may appeal to causes, mechanisms, or in the bridges case, structural relations. On this view, deducing C from 14 and 15 would be a first level (D-N) explanation, and deducing the regularity 14 from 1-13 would be a second level explanation.[15] However, I see two problems with this view. First, 14 and 15 *do not* explain C, because if they did, then 14 and C would also explain 15, which is not the case as I have shown in my analysis of Argument 3. In addition, although I agree that in causal explanations one does not need to explain why the initial conditions obtain (we don't need to explain why there are droplets in the air in order to explain the rainbow), this is not what is going on in the bridges case. Here, my view does not require an explanation of why the bridges system has been built in such a particular way. Rather, my point is that, *given the bridges structure*, we must show how some of its features lead to the lack of an Eulerian walk. Simply providing an argument establishing that such a walk is impossible fails to do this.

5. Do MEPPs Need Explanatory Proofs?

One of the first analyses of the structure of MEPPs was given by Mark Steiner [1978]. Based on a distinction between explanatory and non-explanatory proofs (between proofs that merely verify a theorem and proofs that, in addition, explain why the theorem obtains), Steiner holds that there are three elements involved in a MEPP: the physical explanandum P', a mathematical explanandum M', and a mathematical explanans M. Given some bridge assumptions, the physical explanandum P' is represented by the mathematical explanandum M', which in turn is explained by the mathematical explanans M. The MEPP should include these three elements, and, crucially, it

[15] One defendant of the two-level approach is Bradford Skow (see for example [2016, 2017]).

inherits its explanatory power from the explanation of M’ by M. Schematically the relation is like this:

$$\boldsymbol{M \rightarrow M' — P'}$$

Figure. 3. Schematic representation of the Transmission View (adapted from Baker [2012: 247]).

Thus, for Steiner, at the core of a MEPP there is always a ‘pure’ mathematical explanation. On this view, the special feature of MEPPs is that if ‘we remove the physics, we remain with a mathematical explanation – of a mathematical truth’ [1978: 19], as opposed to ordinary scientific explanations, where ‘after deleting the physics nothing remains’ [1978: 19]. The idea is that, by working over the mathematical explanandum, it is possible to find an explanation that also explains the physical explanandum P’. Once the explanation of M’ has been found, nothing explanatorily relevant is added when one applies the explanation to phenomenon P’.

In the same vein as Steiner’s, Mark Colyvan calls MEPPs ‘extra-mathematical explanations’, and defines them as ‘intra-mathematical explanations’ that ‘‘spill over’ into physical applications’ [2012: 90]. He provides the following condition for an explanation to be a MEPP:

> [C]onsider an explanatory proof of some mathematical theorem. If that theorem has some physical application, then the proof of the theorem might well explain what’s going on in the physical situation [2012: 90].

For Steiner and Colyvan, what distinguishes MEPPs from other scientific explanations is that, in order to work as explanations, MEPPs must include an explanatory proof of the mathematical facts used in the explanans. Baker calls this account ‘the transmission view’ [2012: 246], because the explanatory power of the explanation comes from the mathematical explanation of the mathematical fact, which is then 'transmitted' to the explanation of the physical fact. In the bridges case, for example, the fact that no one can perform an Eulerian walk over the bridges is represented as the absence of an Eulerian path over a mathematical graph. If we have a proof of why the graph does not allow such a path, this proof would also explain why the bridges system does not allow such a walk. But, crucially, not any proof will do. According to Colyvan:

> [A] proof with a brute force, combinatorial proof that there is no Eulerian cycle for the above multigraph... would indeed deliver the result, but armed only with such a proof, we would be none the wiser as to why there is no Eulerian cycle for the multigraph in question. We'd just know that all options had been tried and none of them worked [2018: 27].

We saw that for the 'bare mathematical results' view, merely deducing the conclusion C from premises P1 and P2 constitutes an explanation. The 'transmission' view says that the bridges case must include an explanatory proof of C, and that this proof is not just a deduction of C from P1 and P2, as in Argument 1. We must include an *explanatory proof* of P1 as well. For Colyvan, not every proof is explanatory:

> [D]eduction cannot be the key to mathematical explanations. If it were, all proofs would be explanatory, but this is clearly not the case. For example, the brute force, combinatorial proof… also delivers a deductive result. But… this proof is not explanatory [2018: 27].[16]

[16] In their recent book, Otávio Bueno and Stephen French also endorse the view that brute computation strategies are not explanatory [2018: 167, fn. 21].

Although focused on explicating intra-mathematical explanations, this analysis of the bridges case aligns perfectly well with my criticism of the 'bare mathematical results' view. As we saw, Argument 1 is not an explanation because it fails to capture the sense in which the bridges structure brings about the impossibility of performing the desired walk. However, I think that Steiner and Colyvan's requirement of MEPPs necessitating *explanatory proofs* is too demanding. To reiterate a point I made in section 2, once P1 is represented mathematically, the goal of the operations performed at the mathematical level is to convey information about the physical explanandum, regardless of whether those operations amount to explanatory proofs. Whether or not steps 1-13 in Solution 2 constitute an *explanatory proof* of P1 is unimportant. The crucial thing for the success of a MEPP is that the derivations performed in the mathematical representation give explanatory information about the physical structures involved. It is as if, when performing steps 1-13 in solution 2, we were moving around the physical structures that explain the impossibility of performing an Eulerian walk, tracking down the story of how these connections are arranged in a way such that they make it impossible to perform an Eulerian walk. But ultimately, mathematics can be understood as a tool to find out about these structural relationships (relationships such as 'a bridge connects two pieces of land'[17]). Whatever operations are performed at the mathematical level are not important in themselves, but only in so far they provide this information *about bridges and roads*. A proof, including an explanatory one, therefore, is not a necessary condition for a MEPP.

Now, the reader may wonder why I keep calling these cases *MEPPs*, if, as I have argued, these explanations can be understood in empirical terms. The preferred label would be *structural*

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[17] 'Two' here may stand as a shortcut for its nominalized expression in first order logic.

explanations, because these explanations work by exploiting structural features of the physical system involved, as opposed to the causal features of it. [18] However, they are called mathematical explanations in the literature, and I do not want to be too revisionary with the terminology. Besides, there is a strong sense in which we would not have been able to discover most of these explanations without using mathematics, especially since mathematics seems to be a privileged tool to explore the realms of possibility and modality. Regardless of the label, the significance of these explanations in the broader discussion on scientific explanation is that, despite being non-causal, they still depend upon objective relationships of (structural) dependence, and do not suffer the problems causal accounts of explanation were supposed to solve.[19]

The only features of the bridges structure relevant to the explanation are bridges and landmasses, and they can be perfectly represented as edges and nodes.[20] We use the graph representation for convenience, but the explanatory work is carried out by the physical structures represented by it. The graph works as a proxy to access the relevant empirical features, and every manipulation at the level of the mathematical model must convey information about how these empirical features explain the explanandum. The crucial point is that, when analyzing how MEPPs work, we should not pay too much attention to the debate in the philosophy of mathematics regarding the nature of explanatory proofs. The main focus should be whether the mathematical

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[18] Besides causal and abstract/structural, Pincock calls 'constitutive' those explanations that appeal to the 'parts of the phenomenon to be explained' [2018: 43]. He compares each of these kinds to Aristotle's efficient, formal, and material causes respectively [2018: 46, fn. 11].

[19] Thanks to an anonymous reviewer for asking me to clarify this point.

[20] As I mentioned in footnote 8 above, representations like this one are 'optimal'.

operations performed in the mathematical model convey explanatory information about the represented empirical structures. As long as they do this, it does not matter whether these operations amount to mathematical proofs.

6. Conclusion

In this paper I have defended an intermediate position between the 'bare mathematical results' view and the 'transmission' view of MEPPs. Although it is not enough to deduce the explanandum from the generalizations cited in the explanans, it is not necessary to provide explanatory proofs of the mathematical theorems that represent those generalizations.

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Figures

Figure. 1. Graph-Theoretical representation of the Bridges of Königsberg system

Figure. 2. Example of an odd landmass and an even landmass.

Figure. 3. Schematic representation of the Transmission View (adapted from Baker [2012: 247]).