PROTOCOL ANALYSIS IN CREATIVE PROBLEM-SOLVING

by

Steven James Bartlett

Visiting Scholar in Philosophy and Psychology, Willamette University
and
Senior Research Professor, Oregon State University

Webpage: http://www.willamette.edu/~sbartlet

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STEVEN BARTLETT

Protocol Analysis in Creative Problem-Solving*

Until recently, training of cognitive skills has been haphazard and unsystematic. Students in courses which seek to develop and improve problem-solving abilities have been given an almost purely vicarious experience of problem-solving: They bear passive witness to the instructor's solution of typical problems and then are expected successfully to undertake similar problems on their own. "Better" students are those able to do this with some success; "less able" students encounter difficulties in generalizing and actively extending the passive classroom experience.

There has been a rapid growth of interest in university-level training of a wide range of cognitive skills, in particular those skills needed in creative problem-solving (Hollaway, 1975) — i.e., skills essential in the solution of problems which require more than the application of an algorithm, a formula, a set of step-by-step instructions. It is in connection with such problems whose solutions do not fit the programming-of-memory mindset which traditional education has fostered that the development and improvement of student skills become more difficult, the teaching of creative problem-solving more challenging, and the tendency to divide students into "cans" and "cannots" less easy to resist.

Several approaches, none of them as yet mature, have been proposed: George Polya has advocated the teaching of problem-

solving by means of a Socratic, discovery-oriented, in-class dialogue with students.[1] When skillfully implemented by the instructor, the growth of heuristic self-consciousness in students can serve as a useful and exciting guide for the discovery of solutions to problems similar to those encountered in class. When less successful, the discovery-oriented approach can consume excessive class time, and discourage many students who feel lost and insecure in the open search for a solution.

Another approach, which can be used to complement a heuristic involvement in problem-solving, attempts to upgrade the level of reasoning of the student[2] to fit the class of problems to be studied under this method, rather than to restrict the study of such problems to students who have already shown some competence with them. In other words, either the "entry skills" of the students can dictate the material to be studied, or the study material selected can assume that certain prerequisite skills have already been developed. The latter is the developmental approach advocated by Karplus and others, influenced by the investigations of the Swiss psychologist, Jean Piaget.[3] *Learning cycles* can be designed to upgrade the reasoning level of students so that their access to the class of problems to be studied is assured.[4] Learning cycles *do* work[5] but experience indicates that reliance upon them is overly time-consuming; an instructor will be hard put to cover the material expected of his course using predominantly a Piagetian learning cycle approach.[6]

A third approach, which also can be used in combination with either of the two above, has been suggested by research in the field of artificial intelligence and information processing. In order to acquire a good idea of how a machine can be programmed to perform a certain task, a human subject is asked to report verbally what goes on in his mind as he encounters, attacks, and solves a problem successfully. The verbal report, or *protocol*, can then be analyzed, broken down into "subroutines," and then organized in a form which can serve to instruct a machine to accomplish the same and similar tasks. Protocol analysis is useful precisely because it makes entirely explicit the *process* by which the effective problem-solver bridges the gap between the initial problem and the desired final result.

An analysis of protocols can have a variety of purposes. One's interest may be in the literal analysis, or breakdown, of a protocol into subroutines in an attempt formally to represent the problem-solving process in the form of an algorithm capable of mechanically simulating human results.[7] Protocol analysis serves a different end, however, if one is interested in problems demanding "creative" solutions, i.e., problems which cannot be

solved by means of known algorithms. The purpose in this case becomes embedded in a context of human-to-human communication rather than human-to-machine programming.[8] In a word, listening and reflecting upon the protocols of effective human problem-solvers can serve to explicate and model for others how to solve different though related problems demanding different but related skills in creative problem-solving.[9] In this sense, protocol analysis is an expression of the very old concept that education essentially involves learning by example. The example provided to assist the student differs from a Polya discovery-oriented dialogue or a Karplus learning cycle primarily in that a model problem-solving process is made very explicit and then is analyzed.

It is often the case that in the solution of creative problems instructors as well as students are unable to *say* how they were able to solve a given problem. They may claim, "It suddenly occurred to me," "And then I just saw what I needed to do," etc. Such insight-based solutions, common as they are, have virtually no instructive value, and fail to help the student who is having trouble understanding, much less dealing creatively with, the material. For such students it is invaluable to step back from a model protocol, express a dissatisfaction with reports of insight, and emphasize a careful and patient understanding of the conditions posed by a problem.

Protocol analysis should be approached in the spirit of openness that characterizes all good problem-solving: If our problem is how to teach effective, creative problem-solving, the more solutions we can obtain and utilize in a complementary manner, the richer will be our communication of the experience of good problem-solving. Teaching creative problem-solving is itself an expression of creative problem-solving: There are no fixed strategies, no certain recipes.

In this sense, what I have summarized briefly above should be taken as a protocol of someone who is involved in the attempt to teach creative problem-solving. If it can serve as one model among others for those who want to work with this challenge, then it will have served some purpose. The model I would recommend is one of open pluralism:

- Do not share answers with your students: Explore together in Polya's spirit.

- Do not try to bring the mountain of your subject matter to Joe Mohammed; instead, bring Mohammed to the mountain through learning cycles.

- In this, model good problem-solving by seeking, before your students, to transform your honest, hesitant, careful, at times halting and perplexed problem-solving process, with which they can identify, into words from which they may learn. Your willingness to take this risk will contribute more than anything else to establishing an open atmosphere in which students will feel free to participate and respond to the task of creative problem-solving.

There follow two rather different kinds of problems of the sort treated in the general problem-solving course I teach. A sample protocol for the solution of each problem is given.

PROBLEM 1   A hobo can make one whole cigar from every five cigar butts that he finds. How many cigars can he make if he finds 25 cigar butts? The answer is not five.

Protocol   The problem, on the face of it, leads me immediately to an answer which the problem itself claims is incorrect: For, if the hobo can make a single whole cigar from each five cigar butts he finds, then if he has found 25 cigar butts (and I know 25 is 5 x 5), he should certainly be able to make five whole cigars. Yet, for some reason, this is not the right answer. This is very frustrating. Either my reasoning up to this point is not good, or else I must be overlooking something in my solution.

Well, let me go over my first attempt. Is my reasoning valid? If the hobo can recycle five cigar butts and use them to make a whole cigar, he should be able to do this five times if he finds a total of 25 cigar butts. O.K., I will let this reasoning stand—I can't see how it could be wrong.

Yet, the answer is not supposed to be five. I must be missing something then. Let me go back again to the original problem. The problem describes what a hobo does with cigar butts he finds. What does a hobo do with recycled cigars he makes? Well, obviously he must smoke them; at least this is probably a good assumption. What happens, then, if the hobo smokes one of his recycled cigars? He smokes it, and finally is left with a cigar butt. And being a hobo who scavenges for cigar butts, he surely will not throw the cigar butt away! This must be the key. . . .

In other words, every time the hobo makes a recycled cigar, once he smokes it, he will be left with a cigar butt. If he finds 25 cigar butts, as the problem states, of course he will be able to make five recycled cigars. But when he smokes these, he will be left with five cigar butts, one from

each recycled cigar. And the problem states that from each five butts he can make a whole cigar. So, having smoked the five recycled cigars he made from the 25 butts he found he could then make one more cigar with the cigar butts he would have left. So, in all, he could make six whole cigars from the 25 butts (5+1=6). And this must be the reason why the answer isn't five, as I first thought.

Analysis of Protocol

I first thought about the problem as stated and tried out a solution that seemed like an obvious one. But that solution was ruled out as incorrect by the problem statement itself. Of course this made me feel uncomfortable and frustrated. I could easily have given up at this point. Overcoming my frustrated sense of certainty about my first solution was perhaps the real problem I had to deal with: to go beyond my first attempt I had to question my initial reasoning and then try to see whether there might not be more to the original problem than I had first seen. Already the problem had become familiar, and this made it much harder for me to see the problem in a new light.

So, I tried to see what new information the problem statement could communicate to me. The problem had to do with a hobo and his cigar-making. Why a hobo? And why cigar-making? Presumably, because hobos need to make do with what they find and are unlikely to throw away what they can use later, including their own cigar butts. Why cigar-making? Presumably to smoke the cigars made. Two presumptions, but both very nearly "implied" by the conditions of the problem.

I then went back to my initial, verified reasoning, and tried to see what would happen if I added the two assumptions. And, very quickly, I could see that the two assumptions led me one step further, beyond an answer the problem ruled out as incorrect to an answer which was ingenious, surprising, and satisfying. The solution felt right: it displaced the feeling I had of being frustrated by my first attempt.

So my solution-finding went through several stages:
- Thinking about the conditions described by the problem.
- Trying out a solution.
- Running into a wall, made up primarily of my own pride and certainty with respect to my first attempt.
- Extending myself beyond my first attempt.
- Reinterpreting the problem.
- Reworking the problem with my reinterpretation in view.

A conversation took place between two friends, a philosopher and a mathematician, who had not seen or heard from one another in years. The mathematician, who had an exceedingly good memory, asked the philosopher how many children he had. The philosopher replied that he had three. The mathematician then asked how old the children were. His friend, who knew how much most mathematicians enjoy puzzles, said he would give him a number of clues to his children's ages. The philosopher's first clue: "The product of the children's ages is 36." The mathematician immediately replied that this was insufficient information. The philosopher's second clue: "All of the children's ages are integers; none are fractional ages, e.g., 1½ years old." Still, the mathematician could not deduce the correct answer. The philosopher's third clue: "The sum of the three children's ages is identical to the address of the house where we played chess together often, years ago." The mathematician still required more information. The philosopher then gave his fourth clue: "The oldest child looks like me." At this point, the mathematician was able to determine the ages of the three children. This is the problem: What were their ages, and what was the mathematician's reasoning?

Protocol   What information do I have? (1) There are three children. (2) The product of their ages is 36. (3) All their ages are integers. (4) The sum of their ages is identical to the address of a house where the two friends used to play chess together frequently. (5) The oldest child looks like the philosopher. Anything else? Yes, (6) the mathematician is endowed with a very fine memory. Anything more? Yes, (7) only when he possessed all of the information (1)-(5) could the mathematician deduce the children's ages.

Now I will try to work with these clues to see if I can reach a conclusion as the mathematician did. I have the date, (1)-(7) above, before me. Some of these pieces of information make sense, e.g., (1), (2), (3), and (7); but (4) and (5) don't help me very much — unlike the mathematician with his good memory (6), I don't know the address of the house where the two friends used to play chess, and I can't see how (5) helps at all.

Well, I'll try to work with the information I can understand and hope the rest falls into place gradually. . . . Let me see, there are three children, their ages when multiplied together equal 36, and none are fractions. What possible combinations of ages satisfy these conditions?

There seem to be eight combinations; I believe I have exhausted them all, since I rechecked my list several times carefully:

a. 1 x 1 x 36	e. 1 x 6 x 6
b. 1 x 2 x 18	f. 2 x 2 x 9
c. 1 x 3 x 12	g. 2 x 3 x 6
d. 1 x 4 x 9	h. 3 x 3 x 4

Now let me add up the ages in each of the above combinations to see what the sums look like:

a. 38	e. 13
b. 21	f. 13
c. 16	g. 11
d. 14	h. 10

What have I done so far? I have listed the different combinations of three ages which when multiplied together lead to a product of 36. I have included only ages that are integers. And I have added up these combinations of ages and now have eight totals, ranging from 10 to 38.

Since I know the mathematician has an exceedingly good memory, he should of course know what the address was of the house in which he and his philosopher friend played chess frequently. However, he was unable to determine which of the eight cases above was the correct one, even with this information. He needed yet another clue from the philosopher, the fourth clue. Why would he need more information as he did?

As I look over the eight totals above, I see that two of them, e. and f., are both 13. Only in these two cases would the mathematician remain uncertain, since otherwise his exceedingly good memory would enable him to identify the number identical to the address of the house where the two friends used to play chess together. Since he remained uncertain at this point, his uncertainty must have been due to this fact that the sums of both combinations e. and f. were correct, yet there was no way as yet to decide between them.

The philosopher's fourth and last clue did, however, enable the mathematician to deduce the children's ages. What was that clue again? It was, "the oldest child looks like me." What information is contained in this clue? Well first of all, that there is a child who is the eldest of the three and that this child looks like the philosopher. So we know one of the three children must be older than the other two. And this enables me, along with the mathematician, to choose between combinations e. and f., since only in combination f., in which the children's ages are 2, 2, and 9, is an *oldest* child. This, then, must be the answer and this must have been the mathematician's reasoning.

Analysis of Protocol   First, I tried to summarize what information the problem contained. Some of this information made good sense to me, some

didn't. I decided to work with what made sense and accept the feelings of uncertainty and confusion I had at the beginning. I then tried to list all the combinations of ages which might be involved in the problem. Eight combinations seemed to exhaust the possibilities; I rechecked my list several times and verified this. When I totalled these combinations I discovered that two had identical sums, and this explained the mathematician's need for an additional clue. And then I saw how the philosopher's fourth and last clue dispelled any doubt between these two combinations.

My reasoning went through several stages:

- Clarifying the information contained in the problem statement.
- Being willing to continue working in spite of a feeling of uncertainty.
- Patiently trying to list all possible cases, and rechecking these to make sure all were covered.
- Understanding *why* the mathematician was unable to solve the problem without the final clue.
- Discovering the solution to the problem in the light of the fourth clue from the philosopher.

In solving the problem, the most important stages, at which I was tempted to give up, were the second and third: when I was confronted by uncertainty and lack of clarity, and when I had to list, rather tediously and patiently, all the combinations of ages that were possible. I should try to remember, then, my need for a willingness to cope with uncertainty, and the need for slow and painstaking thought.

FOOTNOTES

[1]Polya (1941), (1945), (1950), (1954), (1962-65). For related views and additional references, see Bartlett (1977), Bruner (1961), contributions in Cohen-Feyerabend-Wartofsky (1976), Hadamard (1945), Kleinmuntz (1966), Lakatos (1976), and Skinner (1966).

[2]There are alarming indications that some 50 per cent of college freshmen are not, in Piagetian terminology, formal operational (i.e., cannot perform conceptual tasks successfully which involve, e.g., hypothetico-deductive inferences, contrary-to-fact propositions, separation of variables, holding variables constant, or proportional reasoning). Cf. Duly (1976), Karplus (1974) and (1975), Kohlberg-Gilligan (1971), Lawson-Renner (1974), Lovell (1961), Tomlinson-Keasy (1972) and (1976), and Wason (1968).

[3]See Inhelder-Piaget (1958), Kohlberg-Gilligan (1971), Lawson-Blake-Nordland (1975), Longet (1962), Lovell (1961), Palmer (1965), Piaget (1964) and (1972), Sheehan (1970), Sticht (1971), Tomlinson-Keasy (1972), and Wason (1968).

[4]For an exposition of the concept of learning cycles, see Karplus (1970-1974), (1974), (1975), and (1976), Campbell-Fuller (1976), Lawson, Nordland (1977), Lawson-Wollman (1977), and Shoemacker (1967).

Recent literature attests to a strong interest in Piaget's work as it can be applied in the form of learning cycles. For example, Athey-Rubadeau (1970),

Anderson (1976), Beard (1969), Elkind (1970), Ford-Flamer (1971), Furth (1970) and (1974), Ginsburg (1969), Gorman (1972), Lawson-Renner (1974), Renner (1975), Richmond (1971), Schwebel-Raph (1973), Tomlinson-Keasy (1976), and University of Nebraska-Lincon (1976).

[5]There is evidence that a student retains comparatively little from lectures, but is able to remember actively a much greater proportion of what he is taught when he learns via learning cycles. Cf. FIPSE (1975), Petr (1976), and University of Nebraska-Lincoln (1976).

[6]The use of learning cycles admittedly limits the amount of material that normally could be covered — albeit not very effectively (see note 5) — by means of the lecture approach. See, e.g., Duly (1976, page 58), Hazen (1976, page 107), Thornton (1976, page 48), and Tomlinson-Keasy (1976, page 7).

Learning cycles used in moderation, however, seem to constitute a promising alternative to the traditional lecture-approach. Learning cycles have been developed in a variety of fields, for example: Arons (1976), Duly (1976), Fuller-Karplus-Lawson (1977), Hazen (1976), Lawson-Renner (1975), Petr (1976), and Thornton (1976).

[7]On the use of protocol analysis in studies of information processing and artificial intelligence, see Forehand (1966), Hayes (1966), Newell-Simon (1962), (1965), and (1972), Newell-Simon-Shaw (1958), Paige (1966), and Simon (1962).

[8]On the use of protocol analysis in studies of the psychology of human problem solvers, cf. de Groot (1965) and (1966), Gagné (1959), (1965), and (1966), Hadamard (1945), Kleinmuntz (1966), Laugherty-Gregg (1962), Skinner (1966), and Simon (1973).

[9]On the use of protocol analysis in the training of cognitive skills, see Bartlett (1977) and (1977a), Bloom-Broder (1950), Marron (1965), Whimbey (1977), Whimbey-Barberena (1977), Whimbey-Ryan (1969), and Whimbey-Whimbey (1975).

REFERENCES   ANDERSON, B. *Science teaching and the development of thinking.* Göteborg, Sweden: Acta Universitatis Gothoburgensis, 1976.

ARONS, A. B. Cultivating the capacity for formal reasoning: objectives and procedures in an introductory physical science course. *American Journal of Physics*, 1976, *44, (834),* 834-838.

ATHEY, I. J. & RUBADEAU, D. O. (eds.). *Educational implications of Piaget's theory,* Waltham, MA: Xerox College Publications, 1970.

BARTLETT, S. A metatheoretical basis for interpretations of problem-solving behavior. *Methodology and Science,* 1977, (in press).

BARTLETT, S. The use of protocol analysis in philosophy. *Metaphilosophy,* 1977(a), (in press).

BEARD, R. M. *An outline of Piaget's developmental psychology for students and teachers.* NYC: Basic Books, 1969.

BLOOM, B. S. & BRODER, L. *Problem-solving processes of college students.* Chicago: University of Chicago Press, 1950.

BRUNER, J. S. The act of discovery. *Harvard Education Review,* 1961, *31,* 21-32.

CAMPBELL, T. C. & FULLER, R. G. A teacher's guide to the learning cycle: a Piagetian approach to college instruction. In *ADAPT* (monograph report). University of Nebraska-Lincoln, 1976, 9-28.

COHEN, R. S., FEYERABEND, P. K, & WARTOFSKY, M. W. (eds.). *Essays in memory of Imre Lakatos. Boston studies in the philosophy of science.* XXXIX. Dordrecht, Holland: D. Reidel, 1976.

deGROOT, A. D. *Thought and choice in chess.* The Hague: Mouton, 1965.

deGROOT, A. D. Perception and memory versus thought: some old ideas and recent findings. In Kleinmuntz, B. (ed.), *Problem solving — research, method and theory.* NYC: Wiley, 1966, 19-50.

DULY, L. C. ADAPT history: the inexpensive unification of Clio and Piaget.

In ADAPT (monograph report). University of Nebraska-Lincoln, 1976, 53-58.

ELKIND, D. *Children and adolescence: interpretative essays on Jean Piaget.* NYC: Oxford University Press, 1970.

FIPSE (Fund for the improvement of post secondary education). Program information and application procedures, Department of Health, Education, & Welfare, Document PMB No. 85-R-0287, 1975.

FORD, M. P. & FLAMER, G. B. (eds.). *Measurement and Piaget.* NYC: McGraw Hill, 1971.

FOREHAND, G. A. Constructs and strategies for problem-solving research. In Kleinmuntz, G. (ed.), *Problem solving — research, method and theory.* NYC: Wiley, 1966, 355-383.

FULLER, R. G., KARPLUS, R. & LAWSON, A. E. Can physics develop reasoning? *Physics Today,* 1977, *30 (2),* 23-28.

FURTH, J. H. *Piaget for teachers.* Englewood Cliffs, NJ: Prentice-Hall, 1970.

FURTH, H. G. & WACKS, H. *Thinking goes to school: Piaget's theory in practice.* NYC: Oxford University Press, 1974.

GAGNÉ, R. M. Problem solving and thinking. *Annual Review of Psychology,* 1959, *10,* 147-172.

GAGNÉ, R. M. The analysis of instructional objectives for the design of instruction. In Glasser, R. (ed.), *Teaching machines and programmed learning, II. Data and directions.* Washington, DC: National Education Association, 1965.

GAGNÉ, R. M. Human problem solving: internal and external events. In Kleinmuntz, B. (ed.), *Problem solving — research, method and theory.* NYC: Wiley, 1966, 128-148.

GINSBURG, H. & OPPER, S. *Piaget's theory of intellectual development.* Englewood Cliffs, NJ: Prentice-Hall, 1969.

GORMAN, R. M. *Discovering Piaget: a guide for teachers.* Columbus, OH: Merrill, 1972.

HADAMARD, J. *An essay on the psychology of invention in the mathematical field.* Princeton, NJ: Princeton University Press, 1945.

HAYES, J. R. Memory, goals and problem-solving. In Kleinmuntz, B. (ed.), *Problem solving — research, method and theory.* NYC: Wiley, 1966, 149-170.

HAZEN, M. V. H. The impact of the ADAPT program on the teaching of data processing. in *ADAPT* (monograph report). University of Nebraska-Lincoln, 1976, 103-107.

HOLLAWAY, O. *Problem solving: toward a more humanizing curriculum.* Philadelphia: Franklin Publishing Co., 1975.

INHELDER, B. & PIAGET, J. *The growth of logical thinking from childhood to adolescence.* NYC: Basic Books, 1958.

KARPLUS, E. & KARPLUS, R. Intellectual development beyond elementary school. I: Deductive logic; II: Ratio: a survey; III: Ratio: a longitudinal study; IV: Ratio: the influence of cognitive style. *School Science and Mathematics.* I: 1970, 5 *(60),* 398-406; II: 1970, 9 *(70),* 913-820; III: 1972, 8 *(72),* 735-742; IV: 1974, 7 *(74),* 476-482.

KARPLUS, R. *Science curriculum improvement study: teacher's handbook.* Berkeley, CA: Regents University of California, 1974.

KARPLUS, R. Science teaching and the development of reasoning. Presented at the National Association for Research in Science Teaching, April 23, 1976; forthcoming in *Research and Science Teaching.*

KARPLUS, R., KARPLUS, E. F., FORMISANO, M. & PAULSEN, A. C. *Proportional reasoning and control of variables in seven countries.* Berkeley, CA: Lawrence Hall of Science, 1975.

KLEINMUNTZ, B. (ed.). *Problem solving — research, method and theory.* NYC: Wiley, 1966.

KOHLBERG, I. & GILLIGAN, C. The adolescent as a philosopher. *Daedalus,*

1971, 100, 1051-1103.

LAKATOS, I. *Proofs and refutations: the logic of mathematical discovery.* Worrall, J. & Zahar, E. (eds.). NYC: Cambridge University Press, 1976.

LAWSON, A. E. & RENNER, J. W. A quantitative analysis of responses to Piagetian tasks and its implications for curriculum. *Science Education,* 1974, *58 (4),* 545-560.

LAWSON, A. E. & RENNER, J. W. Piagetian theory and biology teaching. *American Biology Teacher,* 1975, 37, *(411),* 336-343.

LAWSON, A. E. & BLAKE, A. J. D. & NORDLAND, F. H. Training effects and generalization of the ability to control variables in high school biology students. *Science Education,* 1975, 59, 387-396.

LAWSON, A. E. & NORDLAND, F. H. The factor structure of some Piagetian tasks. *Journal of Research in Science Teaching,* 1977, *13 (5),* 461.

LAWSON, A. E. & WOLLMAN, W. T. Encouraging the transition from concrete to formal cognitive functioning; an experiment. *Journal of Research in Science Teaching, 1977, 13 (5),* 413.

LAUGHERTY, K. R. & GREGG, L. W. Simulation of human problem-solving behavior. *Psychometrika, 1962, 27,* 265-282.

LONGET, F. Un essai d'application de la psychologie génétique à la psychologie differentielle. *Bulletin de l'Institute National d'Etude, 1962, 18 (3),* 153-162.

LOVELL, K. A. A follow-up study of Inhelder and Piaget's, Growth of logical thinking. *British Journal of Psychology,* 1961, *52 (143).*

MARRON, J. E. *Special test preparation, its effect on college board scores and the relationship of effected scores to subsequent college performance.* Office of Director of Admissions and Registrar, U. S. Military Academy, West Point, New York, 1965.

NEWELL, A. & SIMON, H. A. Computer simulation of human thinking and problem solving. In Greenberger, M. (ed.), *Management and the computer of the future.* NYC: Wiley, 1962.

NEWELL, A. & SIMON, H. A. Programs as theories of higher mental processes. In Stacey, R. W. & Waxman, B. (eds.), *Computers in biomedical research,* (Vol. II). NYC: Academic Press, 1965, 141-172.

NEWELL, A. & SIMON, H. A. *Human problem solving.* Englewood Cliffs, NJ: Prentice-Hall, 1972.

NEWELL, A., SIMON, H. A. & SHAW, J. C. Elements of a theory of human problem solving. *Psychological Review,* 1958, 65, 151-166.

PAIGE, J. M. & SIMON, H. A. Cognitive processes in solving algebra word problems. In Kleinmuntz, B. (ed.), *Problem solving — research, method and theory.* NYC: Wiley, 1966, 51-127.

PALMER, E. G. Accelerating the child's cognitive attainments through the inducement of cognitive conflict: an interpretation of the Piagetian position. *Journal of Research in Science Teaching,* 1965, *3,* 318-325.

PETR, J. L. Piaget and learning economics. In *ADAPT* (monograph report). University of Nebraska-Lincoln, 1976, 31-44.

PIAGET, J. Development and learning. *Journal of Research in Science Teaching, 1964, 2,* 176-186.

PIAGET, J. Intellectual evolution from adolescence to adulthood. *Human Development, 1972, 15 (1),* 1-12.

POLYA, G. Heuristic reasoning and the theory of probability. *American Mathematical Monthly,* 1941. *48.* 450-465.

POLYA, G. *How to solve it.* Princeton: Princeton University Press, 1945.

POLYA, G. Let us teach guessing. *Etudes de Philosophie des Sciences, en hommage a Ferdinand Gonseth.* Neuchatel: Editions du Griffon, 1950, 147-154.

POLYA, G. *Mathematics and plausible reasoning, vol. I: Induction and analogy*

in *mathematics; vol II: Patterns of plausible inference*. Princeton: Princeton University Press, 1954.

POLYA, G. *Mathematical discovery: on understanding, learning and teaching problem solving.* NYC: Wiley, 1962 (Vol. I), 1965 (Vol. II).

RENNER, J. W. et al. *Research, learning and teaching with the Piagetian model.* Norman, OK: University of Oklahoma Press, 1975.

RICHMOND, P. G. *An introduction to Piaget.* NYC: Basic Books, 1971.

SCHWEBEL, M. & RAPH, J. (eds.). *Piaget in the classroom.* NYC: Basic Books, 1973.

SHEEHAN, D. S. *The effectiveness of concrete and formal instructional procedures with concrete and formal operational students.* Ph.D. thesis, SUNY, Albany. Ann Arbor, MI: University Microfilms, 1970, No. 70-25, 479.

SHOEMACKER, H. A. The functional context of instruction. Professional paper 35-67, Human Research Office, George Washington University, 1967.

SIMON, H. A. An information-processing theory of intellectual development. In Kessen, W. & Kuhlman, C. (eds.), *Thought in the young child: monographs of the society for research in child development.* Yellow Springs, OH: Antioch Press, 1962, 150-161.

SIMON, H. A. & CHASE, W. G. Skill in chess. *American Scientist*, 1973, *61*, 394-403.

SKINNER, B. F. An operant analysis of problem solving. In Kleinmuntz, B. (ed.), *Problem solving — research, method and theory.* NYC: Wiley, 1966, 225-257.

STICHT, T. G. Comments on Kenneth Lovell's paper: does learning recapitulate ontogeny? In Green, D. R., Ford, M. P. & Flamer, G. B. (eds.), *Measurement and Piaget.* NYC: McGraw-Hill, 1971.

THORNTON, M. C. Piaget and mathematics students. In *ADAPT* (monograph report). University of Nebraska-Lincoln, 1976, 45-51.

TOMLINSON-KEASY, C. Formal operations in females aged 11 to 54 years of age. *Developmental Psychology*, 1972, *6, (364)*.

TOMLINSON-KEASY, C. Piaget's theory and college teaching. In *ADAPT* (monograph report). University of Nebraska-Lincoln, 1976, 3-8.

University of Nebraska-Lincoln. *ADAPT* (Accent on developing abstract processes of thought) program (monograph report). University of Nebraska-Lincoln, 1976.

WASON, P. C. Reason about a rule. *Quarterly Journal of Experimental Psychology*, 1968, *20 (273)*.

WHIMBEY, A. E. Teaching sequential thought: the cognitive-skills approach. *Phi Delta Kappan*, 1977, *December, 255-259.*

WHIMBEY, A. E. & BARBERENA, C. J. *A cognitive-skills approach to the disciplines: intelligence beyond Arthur Jensen.* CUE project technical paper #2, published by CUE (Competency-based Undergraduate Education) Project, Bowling Green State University, 1977, December.

WHIMBEY, A. E. & RYAN, S. F. Role of short-term memory and training in solving reasoning problems mentally. *Journal of Educational Psychology*, 1969, *60*, 361-364.

WHIMBEY, A. E. & WHIMBEY, L. S. *Intelligence can be taught.* NYC: Dutton, 1975.

Steven J. Bartlett, Associate Professor, Department of Philosophy.
Address: Saint Louis University, 221 North Grand Boulevard, Saint Louis, Missouri 63103.