Reflection in Apophatic Mathematics and Theology*

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Abstract

A long tradition in theology holds that *the divine* is in some sense incomprehensible, ineffable, or indescribable. This is mirrored in the set-theoretic literature by those who hold that the *universe of sets* is incomprehensible, ineffable, or indescribable. In this latter field, set theorists often study *reflection principles*; axioms that posit indescribability properties of the universe. This paper seeks to examine a *theological reflection principle*, which can be used to deliver a very rich ontology. I argue that in analogy with set-theoretic reflection principles, we should understand theological reflection via schematic commitment.

Introduction

Ineffability, indescribability, and incomprehensibility—the idea that the divine can neither be properly picked out by a concept, not described, nor known—is advocated on many conceptions of theology. A related idea is present within set theory and its philosophy—in particular one often assumes that the *universe of sets* is somehow ineffable.[1] This idea is often held to be captured by *reflection principles* which state that any property held by the universe of sets is held by some

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[1] See [Roberts, MS] for a survey.

set of a particular kind. Such principles have many 'richness' consequences for the universe of sets, implying that various kinds of 'large cardinal' exist. Less explored, though not unexamined, is the idea that there might be a kind of *theological reflection*. Such a principle says that the ineffability of the divine can be explored via reflection-like ideas—any property (or perhaps some analogue thereof) held by the divine is held by some creature[2]. Often such theological ideas are left somewhat underspecified and are *used* to motivate the standard set-theoretic reflection principles. However not much has been done in the other direction—using observations from mathematics and set theory to articulate the notion of theological reflection more fully. This paper seeks to (partly) fill this gap.

The idea of apophatic theology can be used to motivate theological reflection. But (as we'll see) there is an apparent tension here in that theological reflection itself can be viewed as a true claim about the divine. This is an instance of a common problem in apophatic theology, namely that the negative view seems self-undermining. I think that an approach to reflection principles in set theory—developed under a correspondic apophatic view in mathematics—can help to inform characterisations of theological reflection. In particular, I will argue that:

Main Claim. Taking inspiration from reflection in set theory, there is a formulation of theological reflection that is compatible with the apophatic stance. More specifically, we should view theological reflection as a *schematic* commitment.

Exactly what this means, and in particular the nature of theological reflection and schematic commitment, I hope to make clear in due course. For now, here's the plan for the article:

§1 will set up some theoretical background, including apophaticism/negative theology, and the ideas of incomprehensibility, ineffability, and indescribability. §2 will present the theological reflection principle, and discuss some of its consequences. In particular I note that the ontology thereby provided is very *rich*, but that there are possibilities for interpreting these somewhat counterintuitive ideas. §3 presents a possible problem (familiar in the literature on negative theology) that the apophatic theologian contravenes their own doctrine. I'll adapt this problem to the specific case of theological reflection. §4 goes on to present what I call "apophatic mathematics"; the view that only first-order claims about the universe of sets can be true. §5 then

[2]Here, and throughout, I will follow much of the literature in using the term 'creature' to denote non-divine entities

explains how the apophatic mathematician faces a similar problem concerning set-theoretic reflection as the apophatic theologian does concerning theological reflection. I'll explain how the apophatic mathematician can respond the this problem by using a notion of *schematic commitment*. §6 will then explain how theological reflection can similarly be understood via schematic commitment, dissolving the tension of §3. §7 provides some conclusions and open questions, in particular noting issues that need to be addressed by the proponent of theological reflection.

1 Apophatic principles

Let's first set up some claims about divine incomprehensibility, indescribability, ineffability, and apophaticism. These details will be known to specialists in the philosophy of religion, but may be less familiar to philosophers of logic and mathematics. Moreover, the terminology in this literature can be somewhat fraught, so it will be helpful to lay down what we mean by various terms. We start with:

Definition 1. (Informal) Say that an entity[3] is *incomprehensible to a community C given a class of properties P*, if members of C are unable to grasp truth about whether properties from P hold of it.[4]

We can now lay down the following principle:

Definition 2. (Informal) **Incomprehensibility.** The divine is incomprehensible to the whole of humanity for a significant class of properties P.

Of course, the rub will come in specifying for what class of properties we take the divine to be incomprehensible. We'll examine some responses to this problem in a moment, once we have some other principles on the table. For now, we should note what it means for the divine to be **Incomprehensible**—it is a claim about the extent to which properties held by the divine (a metaphysical fact) are (not) humanly graspable (an epistemic state of affairs). Closely linked is the idea of:

[3]**Note:** I use the term 'entity' in a broad sense, and do not commit to whether or not entities are individuals, since the assumption that the divine is given by an individual is highly problematic in this context.

[4]This is adapted from [Keller, 2018]. She uses the term 'weak incomprehensibility' but since the difference between strong and weak incomprehensibility won't play a role in what follows, I've suppressed it.

Definition 3. (Informal) We say that an entity is *ineffable* to a community C relative to some concepts Co, if there is no *concept* from Co that C could employ that would pick it out.

And again, we can identify:

Definition 4. (Informal) **Ineffability.** The divine is ineffable to the whole of humanity for a significant class of concepts.

Note that there is not a metaphysical component to **Ineffability** (as there was with **Incomprehensibility**). Rather **Ineffability** is a claim about what can be done with *concepts*. In this way it is a *metasemantic* claim about how our concept relate to entities in the world. A final piece of our puzzle is:

Definition 5. (Informal) An entity is *indescribable* to a community C using language $\mathscr{L}$ if C is unable to pick it out using language $\mathscr{L}$.

Definition 6. (Informal) **Indescribability.** The divine is *indescribable* to humanity using human languages.

Indescribability (unlike **Ineffability** and **Incomprehensibility**) is a *linguistic* claim, concerning how our *languages* and *language use* relates to entities.

For the sake of brevity, I'll fix the following:

Definition 7. (Informal) I'll refer to **Indescribability**, **Ineffability**, and **Incomprehensibility** as the ***3Is***.

Let's discuss the relationship between the **3Is**. First, we should note that these claims are often put forward by many theological standpoints. Ideas of ineffability appear in Hinduism [Priest, 2018, §2.3]and Buddhism [Priest, 2014, §13.9]. Within Christian thought the claim is repeatedly advanced, for example by Gregory of Nyssa, John Chryostom, Pseudo-Dionysus, and Thomas Aquinas. The literature here is *enormous*, and so I will only discuss it insofar as it relates to the issues I'll consider below.[5]

However we should also note that each is a *separate* claim, despite the fact that the **3Is** are often thrown together. An entity can be incomprehensible (given some properties), but neither ineffable nor indescribable (say because the community cannot grasp how the property corresponding to the concept picking out an entity applies to said entity). Similarly ineffability entails neither incomprehensibility nor

[5]Concise surveys are available as parts of [Keller, 2018], [Horsten, MS], and [Priest, MS], see also [Hewitt, 2020] for a recent book-length treatment.

indescribability (say because (i) the properties that apply to it are graspable, but do not correspond to any possessed concept, and (ii) there is a piece of language that describes the entity, but not through any concept possessed by the community, e.g. in a case of deference to an expert outside the community). Finally, indescribability entails neither ineffability nor incomprehensibility (say if a property is graspable and a concept picks out an entity, but the community lacks the language to express claims involving the concept).

However, the **3Is** clearly *fit well together*, even if there are not *logical entailments* between them. If an entity is incomprehensible, then it is likely it is ineffable and indescribable too (since if its properties are not graspable, then it is unlikely that we have the linguistic/conceptual resources to pick it out). If an entity is one of ineffable or indescribable, and we hold a tight metasemantic link between language and concepts, then it will likely be the other too. And we might take both ineffability and incomprehensibility as indicative that the properties of the relevant entity are not graspable.

Second, we might think that the **3Is** are *necessary* features of the divine (at least for a broad class of possible worlds). As Simon Hewitt puts it:

> With the proviso that the needed clarification will be provided, I think [**Indescribability**] is true. Nor is it true in a purely contingent way—it doesn't just so happen that God can't be described in words, but had our languages been more sophisticated or our brains bigger, things would have been different. The intended reading of [**Indescribability**] is one that could be appropriately interpreted in the familiar heuristic: at no (metaphysically) possible world can God be described in words. [Hewitt, 2020, p. 13]

It thus is not merely that if we were just cleverer and/or could speak a richer language, we could grasp properties of the divine or use concepts that picked it out. Rather, it is of the divine's nature that it is *always* incomprehensible/ineffable/indescribable, at least insofar as we are human. (Perhaps the divine is self-comprehensible or can pick out itself in some sense, but for *human* possibility, it is necessarily incomprehensible/ineffable/indescribable.)

Often the **3Is** (and their necessity) are linked to the following aspect of the divine:

Definition 8. (Informal) **Transcendence.** The divine is radically dissimilar from every other entity.

Radical dissimilarity can be characterised in terms of logical space.[6] I can say that my couch is dissimilar from my table in various respects. But they are not *radically* dissimilar in virtue of being members of many of the same contrast classes (e.g. furniture, physical objects, etc.). Radical dissimilarity on the other hand occurs when entities cannot even be put into the same contrast class or region of logical space, there is no intelligible basis on which to contrast them. **Transcendence** is sometimes seen as the *metaphysical* correlate or motivation for the *epistemic* **3Is**. If the divine is radically dissimilar from every other entity then it makes sense that its properties would not be graspable to us, or that we couldn't pick it out with concepts/language. Since we primarily come into contact with creaturely entities, and our language is designed for talking about these, there is a gap between the tools we have and the nature of the divine.[7] The idea that the **3Is** might hold links closely to the idea of:

Negative theology/Apophaticism. For some class of propositions P, we can only *deny* sentences expressing propositions from P (or perhaps propositions from P are not truth-evaluable), and we cannot *truthfully assert* any proposition from P of the divine.

The motivation from the **3Is** and **Transcendence** is as follows. Given these assumptions, it's natural to think that given a property, concept, or description of the divine, it falls short, especially since the divine does not even occupy the same region of logical space. Therefore, we cannot truthfully assert that said property, concept, or description applies.

Again, **Apophaticism** has been advanced variously throughout the literature. Famous examples include Nicolas de Cusa, Augustin, Karl Barth, Plotinus, and Pseudo-Dionysus, as well as more recent authors. Again the literature is large and others are better placed to assess it, so I'll say little about it here.[8]

Apophatic theology contrasts with *cataphatic* theology, on which we are able, for a given class of propositions, to assert them of the divine. But now a puzzle emerges. We'll term this difficulty, following

[6] This is also the move of [Hewitt, 2020] (e.g. p. 14).

[7] **Transcendence** in turn is sometimes motivated by:

Simplicity. The divine is a simple (as opposed to complex) entity (i.e. the divine is not composed from many simples).

See, for example, [Hewitt, 2020] for an argument to this effect. I won't use **Simplicity** (though it is in the background), so I relegate mention of it to this footnote.

[8] For a survey of apophatic theology, see [Hewitt, 2020] (Chapter 1) and for an introductory book-length treatment, see [Turner, 1995].

[Lebens, 2014] (who in turn is drawing on [Plantinga, 2000]), the problem of **Incoherence**. Suppose that we are committed to a strong form of the **3Is** and **Apophaticism**, on which we think that *nothing* can be truthfully asserted of the divine. But then we simply note that each of the **3Is** and **Apophaticism** *themselves* make apparent (truthful) assertions about the divine. Thus (so the charge goes) **Apophaticism** must draw on cataphatic assertions in stating the position. So, by their own lights, they contravene their own doctrine. The position is self-undermining.

Exactly how one cashes out this problem is something of a subtle issue ([Lebens, 2014] and [Hewitt, 2020, Ch. 2] provide a thorough examination) but the above informal statement will be enough for present purposes. The problem has been around pretty much as long as **Apophaticism** itself, and again there's lots one might discuss here. We'll narrow down on one response to the problem and set some others aside. Since we will want to say that properties reflect from the divine to creatures, and motivate this on the basis of the **3Is** and **Apophaticism**, I'll want to:

(1.) be able to say that the **3Is** and **Apophaticism** are *correct* in an important sense (e.g. they are not merely false but heuristically useful claims), and

(2.) be able to say that there *are* similarities between the creaturely properties we're familiar with and ones held by the divine.

This will rule out some responses to **Incoherence** as unfit for what we'll do here (though they may be useful for other purposes). One could, for example, take a dialethist approach, accept the contradiction, and adopt a paraconsistent logic for reasoning about the divine. This is the strategy of [Beall, 2021] and [Priest, MS]. Whilst interesting, I lack expertise here (and also I happen to not be a dialethist), so I'll set them to one side. A different approach is that of [Lebens, 2014] who (drawing on Wittgenstein) takes the **3Is** to be illuminating falsehoods ([Hewitt, 2020] suggests that we should take them instead as *meaningless* within Lebens' framework). This clearly contravenes (2.). An alternative is to delineate some class of properties and hold that the **3Is** and **Apophaticism** lie outside this class.[9] One such is [Hick, 1989]'s

[9]There is something of a similarity with theories of truth, and responses to the liar here. For example, on the Tarskian account, paradox is avoided by having the truth-predicate for an object language not be within that language. Of course, in that context we get a hierarchy of truth predicates, which isn't such a common response for the apophatic theologian.

distinction between formal, non-formal, negative, and positive properties. I find these distinctions hard to maintain and it's unclear to me that they can do the job one wants. I will simply refer the reader to [Lebens, 2014] for some criticism of this approach. Another is the 'fundamental properties' approach of [Jacobs, 2015], on which we can say nothing about the divine's *fundamental* intrinsic nature, but can make other claims (the **3Is** and **Apophaticism** then become non-fundamental claims). This would be fiddly to operate with regarding reflection, as we'll constantly have to assess whether our predicates are fundamental or not, so again I'll shelve it for now. Another way out is provided by [Keller, 2018], on which knowledge of the divine should be understood as 'Franciscan' (i.e. personal) rather than propositional. Again this will create difficulties for reflection, and so I set it to one side.

Perhaps the above can be modified so as to work with reflection, but I'll leave this open. For now, we'll want something satisfying both (1.) and (2.). For this I will follow approaches that hold that the divine holds properties that are *analogical* to the ordinary creaturely ones that we're familiar with. Such an idea can be linked to the work of Aquinas and Anselm (see chapter 65 of the *Monologian*), and include the apophatic approaches of Herbert McCabe and Alasdair MacIntyre, as well as the more recent Grammatical Thomism of Simon Hewitt.[10] Going through the many subtle details of these theological approaches would take us too far afield, however, what is common to all is that they hold that whilst we can't make true *univocal* predications of the divine, we can gain *partial* understanding of the divine's properties via *analogy*. Again there are various ways to do this. One can posit a realm of 'analagous' divine predicates/properties to which we don't have access. However, there are also more 'mundane' ways of understanding the distinction. For example, in [Hewitt, 2020]'s Grammatical Thomism, analogy should be understood via the extension of predicate use:

> Imagine that we know how to use a predicate F with respect to a certain class of entities. We know when to apply F to a member of the class, and we know when to withhold it. The know-how we thereby exercise may be vague or admit borderline cases. To know how to use an expression is to be initiated into a social practice, and these are often not precise affairs. Still, we know how to use F of a certain portion of reality. Now suppose we consider the members of

[10] See [McCabe, 1987], [MacIntyre, 2009], and [Hewitt, 2020].

> some second disjoint class of entities. We wonder whether we ought to say (of some or all of them) that they are F. At least some of the rules of use for F with respect to the original class are not applicable to members of the new class. It nonetheless does not follow that we must refuse to predicate F-ness of any member of the new class. It may be that some of the rules for F are applicable within the new class, as when say of human bodies and dinners alike that they are healthy, and in both cases are prepared to infer that this is something desirable. Then again it may be that entities in the new class bear some interesting relation, perhaps causal, to the F-ness of entities in the old class, such that it seems appropriate to us to apply F to at least some of these entities. Again, this is the case with predications of healthiness: we call dinners healthy because they cause healthiness in human beings...
>
> In a case where use is extended in this sort of continuous way, or where variation in use could be illustrated by the supposition that it has been, I will term the relationship between the uses (and therefore the meaning) analogous. Moreover this is all I have in mind when talking about analogy: no high metaphysics, no systematic theory of predication, merely the observation that the use of words can be extended in certain ways. [Hewitt, 2020, pp. 113–114]

So there are a great many ways that we might cash out this analogousness. I'll remain neutral on this score—I do not require much, I just need some way of there being a *resemblance* between creaturely properties and divine ones. And these approaches provide a way of getting as close to cataphatic statements as possible, whilst maintaining a degree of **Apophaticism** consistent with the **3Is**.

There is a question of whether or not they go too far, and lapse into full-blown cataphatic language. But there are scholars who hold that such use is consistent with **Apophaticism**. As MacIntyre writes:

> Theists in recognising that God exceeds the grasp of our understanding must also recognise that in trying to speak of God we are extending our use of words and the application of our concepts, so that we no longer understand what we mean when we talk about God to the same extent and in the same way that we do in our speech about finite beings [MacIntyre, 2009, p. 7]

So this talk of 'extending' is important, we simultaneously are able to make meaningful claims and have *partial* understanding, the view is apophatic in that wet are unable to make *univocal* claims about the divine.

Interestingly several mathematical logicians have found **Apophaticism** attractive. Cantor writes:

> The absolute can only be acknowledged, but never known—and not even approximately. (Cantor, in [Ewald, 1996, p. 916])

Whilst Cantor proposes **Apophaticism** here, it is a very strong version and it is unclear whether this is compatible with analogousness accounts of divine properties.[11] Gödel, however, seems more sympathetic. He writes the following in his notebooks:

> It is clear that we have no adequate concept of God but merely approximations. [Gödel, 2021, p. 252]

Gödel's talk of 'approximations' is highly suggestive of the analogousness account of divine properties. It is thus interesting to note that this kind of **Apophaticism** appears in the ideas of mathematical logicians of the early late 19th and early 20th century, as well as the more traditional sources.

Perhaps the approach sketched here dances dangerously close to cataphatic theology. However, the ability to make kinds of predications will be needed for what follows, and I hope that the examination of theological reflection will actually help elucidate the apophatic view.[12]

2 A theological reflection principle

Given the **3Is**, **Apophaticism**, and the characterisation of divine talk via analogousness, we can come to a consideration of the idea of *theological reflection*, the main focus of this paper. In this section, I'll state

[11] See [Newstead, 2009] for a discussion of Cantor and infinity regarding the divine mind.

[12] There are still lots of objections to the **3Is** and **Apophaticism** in general that I haven't considered. In particular [Plantinga, 2000] argues that there is a lack of argument for the position, it is too revisionist, and arrogant. Nothing I've said here has any bearing on those matters, and I set them to one side (though see [Lebens, 2014] for a rebuttal).

the principle, discuss its historical roots, and explain some of its consequences.

Let's start with some motivation. Given that the **3Is** hold, we know that we fail to talk about the divine *univocally*. Instead our talk should be taken *analogically*. But it certainly seems that there are people who *have something in mind* when they *aim* to talk univocally about the divine. There are plenty of folks out there who really do think that there is a being who is all powerful, where powerfulness should be understood as a property/predicate of the same logical kind as my ability to lift a certain amount of weight, but more. One way to interpret these folks is simply as confused, conflating univocal predicates/properties with their analogical counterparts (this is very much the default). Certainly, this has to be part of the story for the apophatic theologian. But perhaps we can be more charitable. Perhaps there *is* something in the world that they conflate with the divine because it *very closely resembles it*.

This view, at first blush, looks bizarre. Part of what I'll do in this section is formulate the principle and try to argue that it's perhaps not as outlandish as it seems. To get going, let's formulate the principle a little more carefully:

Definition 9. (Informal) The *Theological Reflection Principle* (or TRP) is the claim that if the divine possesses a (possibly ungraspable) property P, then there is a creature that possesses a creaturely property P' (of which P is the divine analogue).

I make no secret of the fact that I've basically just extracted this formulation from set-theoretic reflection principles. If you replace 'creature' with 'set', 'divine' with 'universe of sets', and interpret 'analogue' via quantifier restriction, you'll get a set-theoretic reflection principle. What is interesting is that (1.) there are significant similarities between this principle and some things theologians have said, and (2.) similar moves can be made in the case of theological reflection as set-theoretic reflection. The eagle-eyed reader may already see revenge problems lurking in this latter regard. One of the main points of this paper (as we'll see after this section) is that similar responses are available in each case. For now, let's examine some places where this kind of theological reflection is proposed.

[van Atten, 2009] and [Horsten, 2016] note the following passage from Odo Reginaldus, as quoted by Côté:

> How can the finite attain [knowledge of] the Infinite? On this question some said that God will show Himself to us

> in a mediated way, and that he will show Himself to us not in His essence, but in created beings. This view is receding from the aula... (Odo Reginaldus, quoted in [Côté, 2002, p. 78], [Horsten, 2016]'s translation)

van Atten claims the following regarding Reginaldus' remarks:

> From here it is only a small step to: "Suppose creature A has a perception of God. Then God is capable of making a creature B such that A's perception cannot distinguish between God and B." ([van Atten, 2009] footnote 84, p. 22):

Presuming (in line with **Apophaticism**) that we cannot distinguish a creaturely property from its divine analogue (call this idea $(*)$) van Atten's principle closely relates to the TRP. By the TRP, given a property P possessed by the divine, there is a creature C possessing some P' of which P is an analogue. By $(*)$, we can't distinguish the divine from C using P'. Assuming that this goes for all creaturely perceptions of the divine (in line with the necessity of the **3Is**), given a creature A and a 'perception' of some property of the divine, we can always generate such a C. So even if not obviously equivalent, van Atten's principle looks motivated if we assume the TRP. We'll put this to one side since for the apophatic theologian, 'perception' of the divine is a tricky issue. The TRP is a little easier to state given what we have, but it is an interesting question whether there could be a 'perceptual' version of the principle.

[Horsten, 2016, pp. 114–115] conjectures that by "view that is receding from the aula" Reginaldus is referring to Philo of Alexandria, who says in 'On Dreams':

> Thus in another place, when he had inquired whether He that is has a proper name, he came to know full well that He has no proper name, [the reference is to Exodus 6:3] and that whatever name anyone may use for Him he will use by licence of language; for it is not in the nature of Him that is to be spoken of, but simply to be. Testimony to this is afforded also by the divine response made to Moses' question whether He has a name, even "I am He that is (Exodus 3:14)." It is given in order that, since there are not in God things that man can comprehend, man may recognise His substance. To the souls indeed which are incorporeal and occupied in His worship it is likely that He should reveal himself as He is, conversing with them as friend with

> friends; but to souls which are still in the body, giving Himself the likeness of angels, not altering His own nature, for He is unchangeable, but conveying to those which receive the impression of His presence a semblance in a different form, such that they take the image to be not a copy, but that original form itself.[13]

This, Horsten has argued in recent work, can viewed as an endorsement of something like our TRP:[14]

> This is a reflection phenomenon not from the world to God (as with Augustin), but from God to the world. An 'angel' reflects the essence of God in the form of an image. But this angel-image is such a perfect copy that we cannot distinguish it from God in any way, so we humans tend to take such an 'angel' to be God himself. [Horsten, MS, p. 6]

So versions of the TRP, or perhaps TRP-like intuitions, are at play in the thought of historical figures as well as contemporary authors. But what might some *consequences* of the TRP be?

Well, it seems clear that if the divine satisfies the TRP, then the world is going to be *extremely* ontologically rich. Presumably, the divine is not finite (a core tenet of many religions). And presumably our grip on this kind of 'analogous' infinitude is sufficient to get us an infinite creature via the TRP.[15]

But standard reflection-style arguments are then available. Call our infinite creature above C_1. We can then observe (using the assumption that the divine explains the existence of every creature), argue that the the divine is analogously-infinite and can analogously-see an infinite creature (namely C_1). We then (by the TRP) get a creature C_2 that reflects this property and can see C_1. Clearly this idea iterates to get any number of infinite creatures you like.

There are some similarities to Cantor's theologico-mathematical thought here. After remarking that the Absolute can never be known (and noting a similarity between his thought and the apophaticist Nicolas de Cusa) he provides a similar kind of reflection argument:

[13] See [Horsten, 2016, pp. 114–115].

[14] [Horsten, MS] terms this 'ontological', rather than 'theological' reflection.

[15] This is a little controversial, since whether this analogue of 'infinite' is appropriately related the usual property *infinite* needs support. I think that it's natural to think that the analogue will deliver an infinite creature under the TRP, so I'm happy to simply make this as an assumption.

> For just as in numberclass (I) every finite number, however great, always has the same power of finite numbers greater than it, so every supra-finite number, however great, of any of the higher number-classes (II) or (III), etc. is followed by an aggregate of numbers and number-classes whose power is not in the slightest reduced compared to the entire absolutely infinite aggregate of numbers, starting with 1. As Albrecht von Haller says of eternity: 'I attain to the enormous number, but you, O eternity, lie always ahead of me.' The absolutely infinite sequence of numbers thus seems to me to be an appropriate symbol of the absolute; in contrast, the infinity of the first number-class (I), which has hitherto sufficed, because I consider it to be a graspable idea (not a representation ⟦Vorstellung⟧), seems to me to dwindle into nothingness by comparison. (Cantor in [Ewald, 1996, p. 916])

Although Cantor has the infinite cardinal/ordinal numbers in mind here, the intimate relationship he sees between the mathematical and theological absolute suggests a more theological reading of this idea, in line with our reasoning above.[16]

It is possible that we can extend even further. Above, we reflected *individual* properties to particular creatures. But the creatures we obtain can change as we consider different properties. What if we wanted an "angel-image" that is a "perfect copy" (as in [Horsten, MS]'s reading of Philo of Alexandria)? We would then need a creature that reflected *every* (analogous) property of the divine. We therefore define:

Definition 10. (Informal) The *Extended Theological Reflection Principle* (or ETRP) is the claim that there is a creature C, such that if the divine possesses a (possibly ungraspable) property P, then C possesses P' (of which P is the divine analogue).

This principle is stronger than the TRP due to the different scope of the quantifiers. Suppressing the talk of analogue vs. non-analogous properties: For the TRP we had that for any property P, there is some creature C possessing P, for the ETRP we have a single creature C that reflects every property P.

We can strengthen further by giving additional axioms that can be fed in to the TRP/ETRP. Note that we can view "x satisfies the TRP/ETRP" is a predicate itself (I've *used* this predicate above). We

[16] See [Hauser, 2013] for a further detailed discussion of the link between reflection, apophaticism, and the (divine) absolute in Cantor's thought.

can then ask whether there is a *property* corresponding to this predicate. Here's a way of making these claims:

Definition 11. (Informal) The *double reflection axiom* (or DRA) states that there is both a creaturely and divine property corresponding to the TRP. The *extended double reflection axiom* (or EDRA) there is both a creaturely and divine property corresponding to the ETRP.

Adding the DRA or EDRA on top of the TRP or ETRP has some interesting consequences. Under both, we obtain a creature that satisfies some creaturely version of the TRP/ETRP, simply by using the TRP/ETRP to reflect the *divine version* of the TRP/ETRP to a creature with the *creaturely version* of the ETRP/TRP. Let's see just how strong this can be by adopting the ETRP and EDRA. Since we have the ETRP and EDRA, we can have a creature C_1 that is an almost perfect copy of the divine, up to and including the ETRP itself. But then, running our familiar reflection argument, we obtain a creature C_2 that can see C_1 and also satisfies the (creaturely version of) the ETRP and EDRA, and is also an almost perfect copy of the divine. Iterating this idea, we can generate any number we like of creatures that both reflect and closely resemble the divine.

There are affinities here with views of 'angelic orders' proposed by the likes of Pseudo-Dionysus. In *De Coelesti Hierarchia* he suggests that there are three hierarchies of angels, each of which is split into three orders. The theology proposed here suggests that Pseudo-Dionysus was was on the right track but out by many infinite orders of magnitude. Under the ETRP and EDRA, Francesco Botticini's *The Assumption of the Virgin* (see Figure 1) needs to be extended, fractal-like, into an infinite reflecting tower of angels. Gustav Doré's illustration of the abode of God in Danté's *Paradiso* (see Figure 2) gets closer to the mark, but he still ran out of space and ink.

The view of the divine and the creaturely proposed by the TRP is thus *exceedingly* rich. One might, at this point (especially with regard to the comparison with angels) worry that it is just obviously wrong. All this talk of apparently rich and reflecting beings is surely too much for a respectable theology, no?

I am not sure, once one is engaging in talk of the divine and **Apophaticism**, that holding these kind of reflection 'axioms' is *substantially* worse. We are *already* engaging in philosophy on the *verge* of mysticism. Adopting theological reflection is a bit like taking one small step off the Apollo 11 lunar lander. Getting to space and landing on the moon whilst keeping the astronauts alive was the bigger step.

In any case, there is a dialectical point to be made here. Even if the view is *false*, if it can be made *coherent*, then this represents a win

Figure 1: *The Assumption of the Virgin* by Francesco Botticini

Figure 2: An illustration from Dante's *Paradiso* by Gustav Doré

for apophatic theism. If we can argue that the view is coherent *even when augmented* with ludicrously strong principles like the ETRP and EDRA, then we can have an increased confidence in the view itself. One can never make a coherent position incoherent by *taking away* assumptions.

We should, however, try to make these reflection ideas *somewhat* more palatable. One point to note is that the kind of ontological richness postulated appears in the thought of Gödel, who writes in his notebooks:

> ...in order to get to know an object a, I need to have: 1.) certain fundamental mappings of certain empirical objects (possibly beginning with God = universal set; devil = an irrefutable wrong system (or empty set); human being = ω_1; animal = ω (or correct ethical and theoretical system); time = real number). (Gödel in [Gödel, 2021, pp. 246–247])

Gödel's remarks are somewhat cryptic, and it is a little unclear what the epistemological point he is trying to uncover is. However, it is clear that he has a picture on which entities can be correlated with cardinal numbers (or perhaps ordinals), and in particular animals can be mapped to the countable, humans the least uncountable cardinal, and the divine the cardinality of the universe. An immediate question is then whether the proper class of cardinals above $\aleph_1 = \omega_1$ have corresponding entities. And a natural thought is that they can be, and these 'angels' can just be thought of as the entities that are so correlated. Assuming that this is at least on the right track, and Gödel had thoughts in line with the ontological richness yielded by the reflecting theology proposed here, we are at least in good company.[17]

However, more can be said. It is tempting, given the kinds of beings delivered by theological reflection, to think of these 'angels' as fantastical divine-like beings (with flaming swords and all the other clichés). But there is no *requirement* to think of them this way. Rather they may be conceived of as more abstract kinds of creature. For example, one could think of them as abstract possibilities for existence, that may or may not be physically instantiated. Though we think that some infinite cardinals are physically instantiated (e.g. its a popular assumption to think that spacetime is continuum-sized) we are not required to think *every* cardinality is so instantiated. Similarly with these 'angels' delivered by reflection; they could be understood as abstract

[17] For some additional remarks on Gödel's theology in relation to set theory (and, indeed, some connections to Aristotelian thought) see [Engelen, F].

entities that do not interact with the physical realm directly, and are not physically instantiated.

There is a certain affinity here with Spinozean metaphysics. Under one reading, Spinoza holds that the divine is a being with infinite attributes that exhibit an infinite diversity of modes. Any possible way of being is exemplified by these modes (to have anything less would be a limitation).[18] So one way of thinking of these 'angels' might be to view them as partial reflections of the entire structure of attributes. And to have less than what is given by the theological reflection would be a kind of limitation. (Of course, it is unclear whether the theological reflection can be squared with Spinoza's monism, but we set this issue to one side.)

These observations indicate some possible ways out of viewing theological reflection as delivering a radically implausible ontology. But before moving on, I should emphasise that these are the mere germs of a response that would need working out more fully in order to be satisfactory. Moreover, a full defence of the coherence of theological reflection should really be obtained by formalising and providing a mathematical model for the 'axioms' I've proposed, and I don't have the space to do so here. My focus here is not to provide a complete response to these thorny issues, but rather indicate some affinities between theological reflection and reflection in set theory. And I do think (as we'll see) that the relationship between the two is sufficiently close to indicate that the former is coherent.

Before we move on to this relationship, I want to identify a nice feature delivered by theological reflection concerning epistemology of the divine. A salient problem given **Apophaticism** is that we cannot get knowledge of the divine. As noted earlier, a natural way to do so is via some understanding of analogousness. In this context, we gain partial understanding of the divine's properties via creatures. As

[18]This interpretation is controversial, but can be viewed as extending his remarks about the number of attributes in *Ethics*, §VI:

> By God (Deus) I understand a being absolutely infinite, that is, a substance consisting of infinite attributes, each of which expresses eternal and infinite essence.
>
> Explanation. I say absolutely infinite, but not in its kind. For of whatever is infinite only in its kind, we can deny infinite attributes; but to the essence of what is absolutely infinite there appertains whatever expresses essence and involves no negation.

Is is an excellent question whether ideas of *reflection* also appear in Spinoza's thought.

Hewitt writes:[19]

> ...all talk of God is licensed via more direct talk about creatures, ... it arises out of radical questioning of more readily comprehensible creaturely reality... [Hewitt, 2020, p. 78]

One question, given this perspective, is to what extent we can gain partial understanding of the divine via studying creatures. Given theological reflection, we have a tremendously *positive* outlook on this question. For, any property of the divine is instantiated by some similar property in the creaturely, and we *can* have understanding of creatures. Theological reflection thus *guarantees* us the existence of the creatures needed to obtain partial understanding of the divine—we can substitute direct investigation of the divine with examining creatures (and their properties). Thus, while the theology proposed under the theological reflection, the **3Is**, and **Apophaticism** is *negative*, it is perhaps the *most positive* of such theologies. Our understanding is always partial, but can always be improved via a more expansive understanding of creaturely reality.[20,21]

3 Tensions between apophatic theology and reflection

We are now in a position where we have some characterisations of theological reflection on the table. We've seen that it whilst they ap-

[19]Here Hewitt is actually considering why questions concerning the existence of the divine are poorly posed, so is intended for a slightly different objective. However, it suffices to get the epistemic position of understanding of the divine via an understanding of creatures on the table.

[20]Of course, cataphatic theology can hold versions of theological reflection too, it is just that it is less likely to be motivated on apophatic grounds. Moreover, theological reflection seems to conflict with some other commitments that are normally accepted by cataphatic theologies. For example, it is unclear on this picture how the divine could be a *person* (in any reasonable sense of the term). I'll set this issue to one side, despite its interest.

[21]This account of the epistemology of the divine under versions of theological reflection has some further nice consequences for questions about the consistency of the divine. Here, we have set dialethism to one side, but it bears mentioning that if theological reflection could be made to work in the dialethist context, we can address many questions regarding consistency. If the divine instantiates inconsistent properties, then this inconsistency should be reflected (by theological reflection) in creaturely reality. We thus provide a bridge (using theological reflection) between the usual theological questions concerning the consistency of the divine and dialethism regarding creaturely reality: If the divine is inconsistent, then there is a creature exemplifying a similar inconsistency that we can go and *look for*.

pear to present a very strange pictures of the world, they can be made somewhat more palatable and have some attractive properties for the apophatic viewpoint. In this section, I want to raise a challenge for the advocate of the theological reflection. We'll then see in the rest of the paper how to resolve this tension via comparison with the set-theoretic case.

The core question is whether, by advocating the theological reflection, we thereby lapse into cataphatic theology. This is essentially the old problem of **Incoherence** mentioned above, but theological reflection presents an additional challenge.

The core issue is that to possess the TRP/ETRP is *itself* a property that the divine can satisfy (we seemed to trade on this when formulating the DRA/EDRA). But presumably the TRP/ETRP are understood as a *non-analogous* claim about the divine. If this is the case, however, then we seem to have contravened **Apophaticism**, we have predicated something of the divine *univocally*.

The problem is brought into sharper focus if we consider one natural way of phrasing the TRP/ETRP via *quantification*. We'll just consider the TRP, but it easily generalises to the ETRP. One natural way we might state the TRP is as follows:

Definition 12. (Informal) The *Quantificational Theological Reflection Principle* (or QTRP) holds that for *any* property P, if P is held by the divine and if P is the analogue of a creaturely property P', then there is a creature with P'.

The QTRP looks like it could be additionally problematic. The version of **Apophaticism** we're considering merely has a notion of analogous property, and a commitment that claims about the divine should be understood as partial understanding of analogous properties. But here we are quantifying over all properties, both analogous and non-analogous. How should we undertand this? Is this not an attempt to predicate something *univocally* of the divine? Have we not lapsed into cataphatic theology?

The issue is especially troubling, as it presses a kind of *revenge* incoherence problem. Note that we motivated theological reflection on the basis of the **3Is** and **Apophaticism**. Perhaps this motivation can be questioned. But if it is accepted, then *even granting* a notion of analogousness for the apophatic theologian, there is a still a question of **Incoherence** using theological reflection. If **Apophaticism** and the **3Is** motivate theological reflection, but theological reflection is incoherent with **Apophaticism** and the **3Is**, then **Apophaticism** and the **3Is** are incoherent (at least insofar as the motivation is robust). Do we have a reductio of the apophatic viewpoint?

In the rest of the paper, I will show how this problem can be overcome. We will see that there is a version of mathematics that is apophatic in its own way, and that similar problems concerning reflection occur there. We will also see, however, that the set-theoretic case is perhaps more tractable and there is a clear answer there, and the similar theological problem admits of a similar solution.

4 Apophatic mathematics

An important distinction in set theory is the distinction between *proper classes* and *sets*. A *proper class*, loosely speaking, is an extensional class that does not form a set (on pain of contradiction). Familiar examples from set theory include the Russell class (the class of all non-self-membered sets), the class of all ordinals, and the universal class. There is a rich and detailed literature on how to handle talk of classes (and whether, for instance, we can think of every class as a set in an expanded domain).

For our purposes, we will want to consider an analogy with the absolute as it appears in theology, and the absolute as it appears in set-theoretic mathematics. For this reason we will consider the following position:

Definition 13. (Informal) **Universism** is the claim that there is a unique universe of set theory that contains every possible set.

Universism contrasts with multiversism (the position that there are multiple equally legitimate universes of set theory, with no maximal such), and potentialism (the position that the universe is modally indefinite).[22] In the context of set theory, one can view **Universism** as a little like (mono)theism in theology: There is an absolute out there.

Given **Universism**, there is a question of explaining the nature of proper classes, given that they are also extensional collections. As Boolos writes:

> Wait a minute! I thought that set theory was supposed to be a theory about all, "absolutely" all, the collections that there were and that "set" was synonymous with "collection" [Boolos, 1998, p. 35]

There are some responses out to Boolos' challenge. For instance, proper classes could be thought of as given by plural reference to sets

[22] See [Barton, 2021] for a survey.

or extensions of properties/predicates.[23] One option that is *not* available to the universist, however, is to simply view proper classes as sets in some expanded domain (since our universe provides a maximal domain). A different possibility is to hold that there are *no* proper classes. This meshes well with the following position:

Definition 14. (Informal) **Apophatic Mathematics** is the view that *all* talk of proper classes is *meaningless* (or perhaps *false*, where it involves existential quantification). The only legitimate proper class talk is that which can be paraphrased by *first-order* talk about sets.

The parallel with **Apophaticism** is that there is there is class of statements that we cannot truly assert about the absolute. Perhaps there are other assertions we can make claims (e.g. **Universism** itself is a claim about the absolute), but for mathematical statements (set theory) or statements about divine properties (theology), we cannot make true assertions. Instead, all talk of the absolute can only be true insofar as they talk about sets (set theory) or the creaturely (theology). There is thus a clear parallel here between sets/creatures and the universe of sets/the divine, and how we talk about them.

This said, we *do* use proper classes in set theory (much as the apophatic theologian will often predicate things of the divine for ease of expression). So how exactly does one interpret proper class talk given **Apophatic Mathematics**? Before we get going, it will be useful to set up the following:

Definition 15. The *language of set theory*, or $\mathscr{L}_\in$, is the extension of the language of first-order logic with a single non-logical dyadic predicate symbol "$\in$" (intended to denote membership).

Now, we can begin by noting that there are various proper classes that are *defined by formulas* in $\mathscr{L}_\in$. Good examples are the formulas "$x \notin x$" (the Russell class), "x is an ordinal" (the class of all ordinals), and "$x = x$" (the universal class). So one option is just to regard *all* talk of classes this way. Given a set x, and adding a parameter $\bar{x}$ for x to $\mathscr{L}_\in$, we can interpret claims like "x is a member of the Russell class" simply as "$\bar{x} \notin \bar{x}$". Whilst the former statement seems to talk about classes, the latter is simply a sentence in $\mathscr{L}_\in$ with $\bar{x}$ added. And this technique can be used for far more complicated sentences, extending to discussion even of large cardinals. This is perhaps the traditional approach that is used in many set theory textbooks (e.g. [Kanamori, 2009]). And it provides the apophatic mathematician with a way of interpreting the

[23] See §1 of [Barton and Williams, MS] for a survey.

(very useful) proper class talk in a manner more congenial to their viewpoint.

The approach is somewhat controversial. Many find more liberal interpretations of proper class talk preferable, on both philosophical and mathematical grounds.[24] But we will set aside these tricky issues in the philosophy of set theory—**Apophatic Mathematics** is clearly a popular and viable view. What is important for us is that the apophatic mathematician may want to use versions of *set-theoretic* reflection, and we will see similar problems as in the theological case.

5 Reflection for the apophatic mathematician

The basic idea of a reflection principle is that the universe of sets is so rich that parts of the universe resemble the universe itself. This motivation is somewhat controversial, but let's just assume it for now.[25] Reflection principles are often seen as some of the most attractive principles of set theory.[26] The relationship between theological reflection and set-theoretic reflection has already been noted by [Welch and Horsten, 2016] and [Horsten, MS]. Important for us will be a specific challenge in the case of the apophatic mathematician. To see this, let's discuss a little the role of reflection in set theory.

Insofar as the most widely accepted theory of sets (Zermelo-Fraenkel set theory with the Axiom of Choice or ZFC) is part of our conception of set, so is some kind of reflection. Within ZFC (indeed ZF)[27] the universe can be stratified into levels. In particular one can define:

Definition 16. (ZFC) Let $\mathcal{P}(x)$ denote the *power set* of x (i.e. the set of all subsets of x). *The Cumulative Hierarchy of Sets* or V is defined as follows:[28]

(i) $V_0 = \emptyset$

(ii) $V_{\alpha+1} = \mathcal{P}(V_\alpha)$, where $\alpha + 1$ is a successor ordinal.

(iii) $V_\lambda = \bigcup_{\alpha<\lambda} V_\lambda$ (if λ is a limit ordinal)

[24] See here [Hamkins et al., 2012], [Barton and Williams, MS].

[25] See [Barton, 2016] and [Incurvati, 2017] for discussion of this issue.

[26] See [Tait, 2005], [Koellner, 2009], and [Welch and Horsten, 2016] among many others.

[27] From now on I'll just assume the Axiom of Choice holds, and suppress details about the exact background theory.

[28] For simplicity, I am giving the version for pure sets, if you want to include Urelemente then clause (ii) should be replaced by $V_{\alpha+1} = \mathcal{P}(V_\alpha) \cup V_\alpha$.

We can also prove:

Theorem 17. (ZFC) For every set x, there is an ordinal α such that $x \in V_\alpha$.

One question that is often asked is whether there are levels that can resemble the universe (much as we asked earlier if there are creatures that can resemble the divine). The answer is again affirmative. Start with the following:

Definition 18. (ZFC) The Lévy-Montague Reflection Principle, is the following schema of assertions (where $\phi(\vec{x})$ is a sentence in $\mathscr{L}_\in$ in the parameters $\vec{x}$):

$$\phi(\vec{x}) \rightarrow \exists\alpha\phi^{V_\alpha}$$

Where ϕ^{V_α} is the restriction of all quantifiers and parameters in ϕ to V_α.

We now note:

Theorem 19. ZFC proves every instance of the Lévy-Montague Reflection Principle.

Insofar as ZFC is justified then, so is some sort of reflection.

Interestingly, the generalisation of reflection to *higher-order logics* results in stronger principles. Introducing variables and quantifiers for (possibly proper) classes into $\mathscr{L}_\in$ yields a language we'll call $\mathscr{L}_{\in,2}$. Doing so allows us formulate theories of classes. The class theory GB (sometimes denoted NBG or VNBG) is the theory obtained by adding to all first-order axioms of ZFC second-order versions of the Axiom of Replacement and adopting a predicative comprehension axiom for classes. Predicative comprehension only asserts that there are classes corresponding to any formula where all class quantification is *bounded*. GB is consistent relative to ZFC, and so represents a very mild extension thereof. Kelley-Morse class theory KM is obtained by adding an *impredicative comprehension scheme* to GB where *arbitrary* formulas are allowed in class comprehension (sometimes a second-order version of the Axiom of Choice is included in KM too). The distinction is important for the apophatic mathematician, since given a model $M \models$ ZFC, we know that the structure $(M, \in, Def(M))$ (i.e. the structure where the classes of M are interpreted as the *definable* classes) satisfies GB. An apophatic mathematician can then just view GB as giving us a nice way of talking about the definable classes the universe, but not to be taken ontologically seriously. Not so for KM, which implies the existence of a large number of *non-first-order definable* proper classes.[29]

Now we have second-order resources on the table, we can formulate *second-order* reflection principles:

Definition 20. (GB) The *second-order reflection principle* is the following schema of assertions in $\mathscr{L}_{\in,2}$ (given a formula ϕ in the possibly second-order parameters $\vec{X}$):

$$\phi(\vec{X}) \rightarrow \exists\alpha(V_\alpha, \in, \mathcal{P}(V_\alpha)) \models \phi(\vec{X})^{V_\alpha}$$

Where $\phi(\vec{X})^{V_\alpha}$ is obtained by restricting all quantifiers and parameters in $\phi(\vec{X})$ to V_α.

Second-order reflection is known to be quite strong, implying the existence of many cardinal numbers not provable to exist in ZFC (so called "large cardinals"). And, taking some care with the parameters allowed, we can generalise reflection to higher and higher orders.[30]

There is a challenge here however for the apophatic mathematician. Suppose that they accept that the universe is very rich, and wish to advocate reflection on this basis. Second-order reflection seems to make reference to classes, formulated as it is in GB, but as noted above GB can be viewed as a convenient way for talking about first-order definable classes, if so desired. However, we can now point out:

Fact 21. Over GB, second-order reflection implies every instance of the *impredicative* comprehension scheme.[31]

[29]See here Kameryn J. Williams' thesis [Williams, 2018] for details.

[30]Reflection with arbitrary third-order parameters is inconsistent, see [Reinhardt, 1974] and [Koellner, 2009]. Restricting to only second-order parameters but arbitrary orders of logics is widely believed to be consistent, and is relatively low in the large cardinal hierarchy. We'll see some discussion of this below with *totally indescribable cardinals*.

[31]This observation was first communicated to me by Øystein Linnebo and Sam Roberts and I'm grateful for their input.

Proof. If impredicative comprehension fails, then there is an instance ϕ witnessing such a failure. There is thus a $(V_\kappa, \in, \mathcal{P}(V_\kappa))$ that violates ϕ. But this is impossible, every $(V_\kappa, \in, \mathcal{P}(V_\kappa))$ satisfies every instance of the impredicative comprehension scheme. □

This proof is highly generalisable, and applies to any sentence true over every $(V_\kappa, \in, \mathcal{P}(V_\kappa))$. For example, so long as we have ZFC in the universe, second-order reflection will reverse also to *class* choice (since we get class choice over every $(V_\kappa, \in, \mathcal{P}(V_\kappa))$ by using *set* AC in the universe).

Given this (and noting that the impredicative comprehension scheme implies the existence of non-first-order-definable classes) one *cannot*, as an apophatic mathematician, accept the second-order reflection principle about the universe.

But can the apophatic mathematician *approximate* higher-order reflection principles? There is a trick one can pull here. The apophatic mathematician may accept the *idea* of reflection as a motivating (but non-formalisable) principle, but then find formal axioms that capture *something* of the idea.

We'll approach a full response slowly. We've already seen that the Lévy-Montague reflection principle is provable in ZFC. But it's also interesting to note that it can be used to *get* richness in the universe:

Theorem 22. Let ZC - Infinity be the axioms of ZFC with the Replacement Scheme and Axiom of Infinity removed. Then if we add every instance of Lévy-Montague reflection to ZC - Infinity, we can prove Infinity and every instance of the Replacement Scheme.

I want the reader to note a couple of things about Lévy-Montague reflection at this point. The first, a theme that will recur repeatedly from now on, is that Lévy-Montague is a *scheme*, and cannot be given by a single axiom in ZFC.[32] Commitment by the apophatic mathematicians to this kind of reflection is thus *schematic*, they commit to every instance of the scheme, but *not* a *single axiom*.

Second, I want the reader to note some affinity with the TRP. Lévy-Montague reflection asserts that anything true in the universe is already true in some initial segment thereof. But this initial segment can change as we vary ϕ (much as the creature we reflected to can change under the TRP).[33] The question we must now ask ourselves is whether we can get something closer to the ETRP, where we reflect *all* sentences of the universe to a *single* initial segment.

The answer is affirmative and again uses the notion of schematic commitment. Suppose we believe that the universe is so rich that there is some V_κ that satisfies all the same sentences of $\mathscr{L}_\in$ as the absolute universe of sets. The existence of such a V_κ will not be definable in V by Tarski's theorem. However, we can once again (via the use of parameters and schematic commitment) formulate a version of the principle:

[32]If it were, ZFC would be finitely axiomatisable, but this isn't possible by Gödel's Second Incompleteness Theorem and reflection.

[33]The analogy is not perfect, since the closer analogy when talking about properties of the divine are *higher-order* versions of reflection, but we'll get to that in a second.

Definition 23. Let $\mathscr{L}_{\in,\bar{\tau}}$ be the language obtained by adding a single additional predicate $\bar{\tau}$ into $\mathscr{L}_{\in}$. Let ZFC_{τ} be the following theory in $\mathscr{L}_{\in,\bar{\tau}}$:

(i) Every axiom of ZFC (including instances of the Replacement Scheme in this new language).

(ii) The axiom "$\bar{\tau}$ is an ordinal"

(iii) The following axiom scheme:

$$\phi \leftrightarrow \phi^{V_{\bar{\tau}}}$$

for every formula ϕ of ZFC.

This move essentially copies a strategy from [Feferman, 1969] to obtain a countable transitive model of ZFC.[34] ZFC_{τ} is known to be consistent (relative to very mild theories).[35] But ZFC_{τ} is just a first-order theory using one extra set parameter, and hence is acceptable to the apophatic mathematician.

Here we have something closely resembling the ETRP, in that ZFC_{τ} provides us with a V_{τ} that reflects *every* sentential truth of V in $\mathscr{L}_{\in}$. Of course, V_{τ} is only *partial* in other respects—if we have some sentence in an expanded language with a parameter $\bar{\theta}$ for some $\theta > \tau$, then V_{τ} doesn't say anything about θ. But for truth in $\mathscr{L}_{\in}$, V_{τ} agrees with V.

Let's now move on to higher-order reflection for the apophatic mathematician. Here we'll see that a similar move can be made, and again schematic commitment is vital. In order to see the trick, we'll need the *complexity hierarchy*. For natural numbers m and n, the complexity hierarchy classifies formulas as either Σ^m_n or Π^m_n. The superscript denotes the largest order of quantifiers appearing in the formula, starting with 0 for first-order, 1 for second-order etc. The subscript denotes how many quantifier alternations there are, starting with 0 for quantifier-free formulas, Σ^m_1 denoting a single bank of m-order quantifiers, and Π^m_1 denoting a single bank of universal quantifiers, and then numbers increasing with alternations of quantifiers (so a Π^3_2 formula starts with a bank of 4th-order universal quantifers, then has a bank of 4th-order existential quantifers, followed by a quantifier-free formula). We can then define:

[34] Feferman's axioms are basically the same, with axiom (ii) replaced by "τ is countable and transitive", and axiom scheme (iii) replaced by every instance of "$\phi \leftrightarrow \phi^{\tau}$".

[35] For example, if we have a truth predicate, then one can prove that V is the union of an chain of such V_{τ}, all elementary in V. See [Marek and Mostowski, 1979, p. 475].

Definition 24. (ZFC) For $Q = \Sigma$ or $Q = \Pi$ and $m, n \in \omega$, a cardinal κ is Q^m_n-indescribable iff for any formula ϕ of Q^m_n-complexity and parameter $A \subseteq V_\kappa$ (with m-order quantifiers over any V_α interpreted as ranging over $V_{\alpha+m}$) we have:

$$(V_{\kappa+0}, ..., V_{\kappa+m}, A, \in) \models \phi \rightarrow \exists \beta < \kappa (V_{\beta+0}, ..., V_{\beta+m}, V_\beta \cap A, \in) \models \phi$$

we can generalise this to:

Definition 25. (ZFC) A cardinal κ is *totally indescribable* iff κ is Q^m_n-indescribable for both $Q = \Pi$, $Q = \Sigma$, and any $m, n \in \omega$.

What a totally indescribable cardinal provides is effectively a *set* satisfying higher-order reflection of arbitrary order with second-order parameters. But now we can define:

Definition 26. Let $\mathscr{L}_{\in,\bar{\sigma}}$ be the language obtained by adding a single additional predicate $\bar{\sigma}$ into $\mathscr{L}_\in$. Let ZFC_σ be the following theory in $\mathscr{L}_{\in,\bar{\sigma}}$:

(i) Every axiom of ZFC (including instances of the Replacement Scheme in this new language).

(ii) The axiom "$\bar{\sigma}$ is a totally indescribable cardinal"

(iii) The following axiom scheme:

$$\phi \leftrightarrow \phi^{V_{\bar{\sigma}}}$$

for every formula ϕ of ZFC.

This theory does exactly what we did before, but now σ is a totally indescribable cardinal rather than some ordinal or other. In this way ZFC_σ is something like having the ETRP with the EDRA added—V_σ resembles V for first-order truth *and* satisfies arbitrary amounts of higher-order reflection. And again, we can note that the theory ZFC_σ is a first-order theory with one parameter $\bar{\sigma}$ added for σ.

But once more, such a V_σ is only an approximation to V (in line with apophatic mathematics). Whilst V_σ and V satisfy all the same $\mathscr{L}_\in$ sentences, there are sets (and thus available set parameters) outside V_σ. In the end, V_σ is merely a *partial* approximation, even if a *very close* one. But importantly, we can learn about V via studying V_σ. And if some sentence ϕ from $\mathscr{L}_\in$ can be proved using reflection (of any order!) to hold in V_σ, then ϕ is true *simpliciter* (assuming that we think ZFC_σ

is true). Much like Philo of Alexandria's image, V_σ provides a clear picture of the absolute, even if it would be confused to say that it is identical with it.

Does this kind of idea extend yet further? Suppose now one wants to consider a set x such that $x \notin V_\sigma$. Since *every* set is in some V_α, we know that $x \in V_\gamma$ for $\gamma > \sigma$. Without loss of generality, we can make γ as large as we like. And, given our commitment to richness motivating reflection properties, we extend ZFC_σ with the following axioms:

Definition 27. Let $\mathscr{L}_{\in,\bar{\sigma},\bar{x},\bar{\gamma}}$ be the language obtained by adding two additional predicates $\bar{\gamma}$ and $\bar{x}$ into $\mathscr{L}_{\in,\bar{\sigma}}$. Let $\mathsf{ZFC}_{\sigma,\gamma,x}$ be the following theory in $\mathscr{L}_{\in,\bar{\sigma},\bar{\gamma},\bar{x}}$:

(i) Every axiom of ZFC (including instances of the Replacement Scheme in this new language).

(ii) The axiom "$\bar{\sigma}$ is a totally indescribable cardinal"

(iii) The axiom "$\bar{\gamma}$ is a totally indescribable cardinal"

(iv) The axiom "$\bar{x} \notin V_{\bar{\sigma}}$"

(v) The axiom "$\bar{x} \in V_{\bar{\gamma}}$"

(vi) The following axiom schemes:

$$\phi \leftrightarrow \phi^{V_{\bar{\sigma}}}$$

for every formula of $\mathscr{L}_{\in}$

$$\phi \leftrightarrow \phi^{V_{\bar{\gamma}}}$$

for every formula of $\mathscr{L}_{\in,\bar{x},\bar{\sigma}}$ (i.e. just taking out any formulas containing the parameter $\bar{\gamma}$).

This theory axiomatises the idea that we now have a new totally indescribable cardinal γ above another totally indescribable cardinal σ, such that V_γ contains x, and V_γ and V_σ are elementary in V for first-order truth in $\mathscr{L}_\in$. Indeed V_γ gets much more, capturing truth when we allow parameters for x and σ too. But V_γ is still only partial, and if we consider some $y \notin V_\gamma$ we may have to expand our language and axioms again to get a totally indescribable cardinal elementary in V that is also able to capture truth when we have a parameter for y.

What this shows is that there are two kinds of schematic commitment going on for the apophatic mathematician who holds that reflection cannot be truthfully stated about the universe, but nonetheless wants to capture the effect of reflection on it. The first is the use of schemes in the theories proposed. The second is more metatheoretic in nature: Such an apophatic mathematician has a commitment to accept that if you give me any set z, I will formulate a theory with a totally indescribable cardinal κ such that V_κ contains z and is $\mathscr{L}_{\in,z}$-elementary in the universe. But any such *particular* theory always manifests a *partial* rather than full understanding, in line with **Apophatic Mathematics**.[36]

6 Resolving the tensions?

The parallels between the versions of the theological reflection under **Apophaticism** and set-theoretic reflection for the apophatic mathematician are (by design) close. Moreover, we've seen that the apophatic mathematician can formulate versions of reflection that they find acceptable. But can we use the insights from the apophatic mathematician to avoid the charge of **Incoherence**?

I think we can. The core point from the case of the apophatic mathematician is to note that reflection was formulated via *schematic* commitments (both in the object language and metatheoretically). And we can use a notion of schematic commitment under **Apophaticism** too.

The charge of **Incoherence** only bites if we think that theological reflection should be formulated as a propositional claim about the divine (in particular the QTRP looked like such a formulation). But we might rather formulate theological reflection *schematically* as follows. For the TRP, instead of holding that its a proposition about the divine, we can take on a commitment such that, if we're given some creaturely property P, and think that an analogue of P holds of the divine, then we think that there's a creature with P. We treat talk of the relevant kind of P and its analogues as *schematic*, rather than *quantificational*. In the formulation theological reflection, we treat mention of a divine property P as a kind of open variable for suitable properties. And indeed, this is what should want for apophatic theology. Predicating properties of the divine is tricky business (especially under **Apophaticism**), and we should expect ourselves to be routinely able to quantify over divine properties. However *given* some property P, we can consider whether it is plausible that the analogue of that property might hold of

[36]One might wonder, can a produce such an axiomatisation but with a *proper class* of totally indescribable cardinals, each elementary in V, using a scheme? This would capture any given set in some member of the proper class. The only way I know of doing so—communicated to me by Kameryn J. Williams—adds in a class predicate $\bar{D}$ and the schema that every $\gamma \in \bar{D}$ reflects every formula. But the introduction of predicates for proper classes is unlikely to sit well with the apophatic mathematician.

the divine, though our knowledge of said property is merely partial. And if we *do* think it's plausible that some divine property holds, then the TRP says that there is a creature possessing P.

Moreover, much as we did for V_τ, there's no obstacle to extending this strategy to the ETRP, even with the EDRA added. This informal theory produces divine-like entities that seem to reflect 'every' property of the divine, including the ETRP itself. But just as with V_σ, we need not believe this via quantification over properties. We can hold that there is a creature C such that *if we are given* a property P, if we think an analogue of P holds of the divine then P holds of C. Moreover, we can hold that the creaturely version of the ETRP holds of C, but here we really can think of this via quantification if we so desire. And anything we accept about creature C (without attending to its specific creaturely-ness) we will know provides an analogical resemblance to the divine. We have *negative* theology, but the most *positive* version thereof.

As we did before with V_γ, we can extend by reflecting on the creaturely nature of C. If we introduce a name $\bar{C}$ for C, we use the schematic commitment to the ETRP again to reflect a creature C' that can see that C is a mere image of the divine. But we have to *introduce a name for* C and assume it *denotes a creature* to do so, in a more restricted language, C is indistinguishable from the divine. And again this iterates; we can reflect on the creaturely nature of C' to get a C'', and so on. We obtain arbitrarily close but partial creaturely images of the divine, without ever committing to the idea that we have some univocal language in which we can distinguish the divine from creatures.

There is a similarity here with a particular view of negative theology proposed by Merlin Carl and Rico Gutschmidt.[37] They view Nicolas de Cusa's negative theology in line with Cantor's diagonal argument. They propose that both can be understood as a kind of schematic commitment to performatively undermine a given understanding of the divine/the universe of sets, given some candidate understanding. As they write:

> As we have argued... the *via negativa* is not committed to making a theoretical statement about the incomprehensibility of the absolute, which would be self-defeating, but can be—and usually is—understood as a practice that performatively undermines our putative understanding of the absolute. We also pointed out that this practice concerns

[37] See [Carl and Gutschmidt, MS]. I am very grateful to both Dr. Carl and Dr. Gutschmidt for several helpful discussions here

> not only the theological notion of God, but also, particularly in Neoplatonism, the philosophical problem of the totality of the world. In terms of the One, Plotinus performatively undermines our putative understanding of this totality and its origin. Similarly, we just argued that the method of diagonalization can be interpreted as performatively undermining the notion of absolutely everything. Thus, we think that this method constitutes a modern version of the *via negativa*. [Carl and Gutschmidt, MS, p. 19]

The present proposal meshes very well with this approach. Theological reflection presents a schematic commitment that any creature, no matter how closely it resembles the divine, can be witnessed to be creaturely by some other creature. But this is a schematic commitment, the creature itself may even satisfy the ETRP and be indistinguishable from the divine, *until* its creaturely nature is reflected upon.

7 Conclusions and open questions

In this paper, I've proposed a particular kind of theological reflection principle. I've argued that whilst it is a somewhat strange idea, it has historical roots and can be given a more charitable interpretation. Whilst we posed some initial questions about its coherence, I argued that they can be resolved in analogy with the apophatic mathematician's attitude to set-theoretic reflection. In particular, I argued that the theological reflection should be understood through *schematic commitment* and that, given the apophatic stance, is exactly how things should be.

I want to close with two open questions. The first concerns an underlying theme of this paper:

Question. *How* similar are theological and set-theoretic reflection?

In particular, whilst we drew something of a tight analogy between the two in virtue of schematic commitment, there are differences. For the apophatic mathematician, talk of 'properties' of V can only be given in a first-order fashion. But 'first-order' here just means in terms of statements about sets, whereas the version of **Apophaticism** that makes a distinction between creaturely properties/predicates and their divine analogues, *does* have some notion of divine-but-not-creaturely property. There is also the point that reflection is a standard part of cataphatic *mathematics*, on which we do allow the use of non-first-order definable proper classes. This contrasts sharply with many cataphatic theologies, where, though TRP and ETRP can be formulated, they do not mesh well with several commitments normally associated with the view (e.g. the personhood of the divine).

The second question concerns an issue already given a cursory treatment in the text:

Question. How should we understand the *metaphysical* picture proposed under the TRP, ETRP, DRA, and EDRA? In particular, what is the metaphysics of these "divine-like" creatures?

Whilst I did make some short remarks in this direction, linking their existence to "abstract possibilities of existence" and Spinozean metaphysics, this is scarcely a full account. If we wish to really move forward using theological reflection, a better understanding is required. In line with apophaticism in both mathematics and theology, our knowledge of theological reflection remains incomplete.

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