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Structural Explanations: Impossibilities vs Failures

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The bridges of Königsberg case has been widely cited in recent philosophical discussions on scientific explanation as a potential example of a structural explanation of a physical phenomenon. However, when discussing this case, different authors have focused on two different versions, depending on what they take the explanandum to be. In one version, the explanandum is the *failure* of a given individual in performing an Eulerian walk over the bridge system. In the other version, the explanandum is the *impossibility* of performing an Eulerian walk over the bridges. The goal of this paper is to show that only the latter version amounts to a real case of a structural explanation. I will also suggest how to fix the first version, and show how my remarks apply to other purported cases of structural explanations.

Scientific Explanation; Structural explanation; Non-causal explanation; Mathematical explanation.

1. Introduction

The bridges of Königsberg case has been widely cited in recent philosophical discussions on scientific explanation as a potential example of a structural explanation of a physical phenomenon. One of the reasons this case is so appealing is that it is, so to say, 'air-tight', in the sense that, no matter what new information we may get, we can be completely sure that we have got it right: no one will ever find a continuous path that covers all the Königsberg bridges (as they were arranged in 1736) exactly once.[1]

However, when discussing the bridges case as an explanation, different authors have focused on two different versions, depending on what they take the explanandum to be. In one version, the explanandum is the *failure* of a given individual in performing an Eulerian walk over the bridge system. In the other version, the explanandum is the *impossibility* of performing an Eulerian walk over the bridges. In this paper I show that only the second version is a real case of a structural explanation[2].

The paper is structured as follows. In section 2, I introduce the distinctions between lower-order and higher-order explanations, and causal and non-causal explanations. In section 3, I introduce the bridges case as a purported example of a non-causal, higher-order explanation, which I will call structural explanation (as I explain below, I do not call it *mathematical explanation* in order to avoid the debate on whether there are genuine mathematical explanations in science, or whether physical systems can instantiate mathematical structures). Section 4 shows how this case has been described in two different ways in the relevant literature; and section 5 argues that only one of those versions is a genuine instance of a structural explanation. In section 6, I address potential objections to my view,

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[1] This is not the case with other famous examples of structural/mathematical explanations such as the Honeycomb case or the Cicada case, whose dependance on specific scientific facts opens them to the possibility of not actually being genuine structural/mathematical explanations (see Wakil and Justus 2017, and also Bueno & French 2018, 160ss).

[2] Pragmatic views of explanation, like van Fraassen (1980)'s, emphasize that different explananda call for different explanantia. More recently, Kostic (2020) and Kostic & Khalifa (2021) have highlighted the role of these pragmatic aspects in non-causal explanations, such as the ones I discuss here.

and suggest a way of reframing the first version of the explanandum. In section 7, I show how my remarks generalize to other purported cases of structural explanation.

2. Structural Explanations

After the failure of Hempel's DN model, the philosophical discussion on scientific explanation shifted focus to causal explanations. A causal explanation is one that tracks down the causal relationships that led to the occurrence of the explanandum[3]. One of the most discussed philosophical models of causal explanation is Wesley Salmon's Causal-Mechanical (CM) model, according to which to explain is to track down the spatiotemporally continuous causal processes and interactions that produced the explanandum (Salmon 1984). The CM model, however, cannot explicate why the following case is a successful scientific explanation:

> THE GLASS CONTAINER CASE: The water in a closed glass container reaches boiling temperature—the mean molecular motion is at such and such a level—and the container cracks. Why did it crack? Because, due to an energy transfer from an external source, there was an increase in the internal temperature of the container (cf. Jackson & Petit 1990, 110).

A proper CM explanation would have to appeal to the momentum of the molecules that struck some molecular bonds in the container's surface. But, obviously, it is virtually impossible to determine which molecules actually hit the surface, let alone back tracking the molecular interactions that led them to do so. This result is puzzling. If scientific explanation was in every case only a matter of tracking down spatiotemporally continuous causal processes and interactions, the cracking could not

[3] I am actually referring to the phenomenon described by the explanandum-statement, but for simplicity I will just call it 'explanandum'.

be explained. And yet, the explanation that appeals to the increase in temperature does seem to be a satisfactory one.

James Woodward's Interventionist model (Woodward 2003), another one of the most discussed philosophical accounts of causal explanation, can explicate what is going on in cases like this. On Woodward's view, to explain is to show how causal interventions in the objects or situations mentioned in the explanans would causally produce changes in the explanandum. On this view, the purpose of an explanation is to answer 'what-if-things-had-been-different' questions. In the glass container case, for example, the interventionist model admits citing higher-order properties such as the water overall temperature or average kinetic energy, the fragility of the container walls, etc. (Woodward 2003, 354ss). Since it is possible to conceive causal interventions that would change the cited properties in order to see a difference in whether or not the cracking occurs, this explanation is causal. The explanation answers what-if-things-had-been-different questions with counterfactual scenarios conceived taking into account the causal dependence between the cracking and the cited higher-order properties.

Higher-order explanations are valued, among other things, for their modal component. They show that the explanandum was somehow bound to happen, independently of the actual trajectory of the causal processes that produced it. In the glass container case, for example, even if it had not been the exact same set of water molecules that hit the container's walls, another set of molecules would have produced the cracking anyway. In that sense, higher-order explanations show that the outcome was modally stronger than what would have seemed if all we had was a lower-order explanation.

Now, there are several cases of higher-order explanations that cannot be understood causally because, even though they deal with empirical explananda (which include objects with causal powers), they do not work by appealing to the causes of those explananda: the explananda are not taken as having been produced by interactions between causal properties. But even in these cases the counterfactual condition, which is the main idea of Woodward's account, can be retained. As Woodward points out, we can keep the notion that an explanation shows how changes in the objects/structures cited in the explanans would make a difference in the explanandum even in those cases where the dependence between the explanandum and the objects/structures cited in the explanans is not causal (see., Woodward 2003, 221). In fact, many counterfactual, non-causal philosophical accounts of scientific explanation do this (e.g., Kostic 2020; Kostic & Khalifa 2021; Pincock 2015; 2018; Reutlinger 2018; Woodward 2018).

I will call these cases *structural explanations*. One of the most cited cases of this kind is the Bridges of Königsberg case. In the philosophical literature on scientific explanation, this case has been proposed as an example of a purely *mathematical* explanation of a physical phenomenon. The reason behind this is that the physical bridge structure involved can be perfectly represented using a connected graph —a mathematical object—, and the explanation works almost exclusively over the graph. This will be the main case-study in the rest of the paper. I will argue that the bridges case can be understood as a strutural explanation, *but only if one describes it using the right explanandum*. What is more, structural explanations, being higher-order explanations, are supposed to be superior to their causal-process, lower-order counterparts because they provide modal information that shows that it was necessary for the outcome to occur, regardless of the many physical details of the actual causal processes that led to it. But, I argue, this advantage would not apply to the bridges case (and similar cases) if we fail to specify the explanandum correctly.

One clarification before I begin: I want my discussion to be neutral regarding whether physical structures are genuine instantiations of mathematical structures, instead of simply being represented by them. So, I will avoid talking about mathematical explanations *per se* and refer to (the right version of) the bridges case as an instance of a *structural* explanation. Whether structural explanations are inherently mathematical, and whether this would support some form of mathematical realism will not be topics of concern[4].

3. The Bridges of Königsberg Case

Let us examie the bridges case in more detail. In a letter addressed to mathematician Giovanni Marinoni, Leonard Euler wrote:

> A problem was posed to me about an island in the city of Königsberg, surrounded by a river spanned by seven bridges, and I was asked whether someone could traverse the separate bridges in a connected walk in such a way that each bridge is crossed only once. I was informed that hitherto no-one had demonstrated the possibility of doing this, or shown that it is impossible. (Cited in Hopkins & Wilson 2004, 201).

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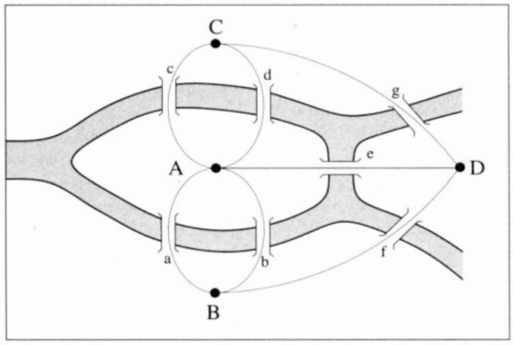
[4] In the literature on this issue, the phrase 'mathematical explanation' has been used to imply that mathematics itself is somehow responsible for explaining an empirical phenomenon, and this has been taken by many to justify commitment to mathematical realism, the view that mathematical objects or structures exist. The most discussed version of this reasoning is the enhanced indispensability argument (e.g. Baker (2005)), whose goal is to show that, if there are genuine mathematical explanations in science, then the principle of inference to the best explanation would justify both scientific and mathematical realism. Of course, simply renaming the type of explanation, like I do in this paper, is not going to completely avoid this debate. My point is simply that the issues that I discuss here are pretty much neutral with respect to this debate, in the sense that whether or not the explanatory structures appealed to by these explanations are physical (see e.g. Bueno & French 2018, p.10, fn. 32) or mathematical is irrelevant. I address this debate in Barrantes 2019). Thanks to an anonymous reviewer for encouraging me to clarify this point.

In modern terms, the problem is whether the Königsberg bridge system allows a continuous path that covers all its bridges only once (i.e., an Eulerian path).



(Baker 2013, 691)

Euler's solution led him to discover the following theorem:

> THEOREM: A system of connected roads allows a path that covers each road only once if the system has either zero or two nodes connected by an odd number of roads (cf. Euler 1986 [1736], 8).

In discovering this theorem, Euler not only showed that the trip was impossible; he also explained why. This example has been widely cited in debates regarding the applicability of mathematics in science, especially, as I mentioned, as a case of a genuine mathematical explanation of a physical phenomenon. Euler's proof of the theorem explains why it is impossible to perform an Eulerian walk over the bridge system (see Barrantes 2020 for a discussion).

When correctly described (see section 4, below), the bridges case is an example of a structural (i.e. higher-order, non-causal) explanation. If we were to focus on explaining why each individual path is not Eulerian, the fact that none is may look like a simple coincidence. But Euler's proof shows that this is not a coincidence: it is necessary that none of these paths is Eulerian because the bridge structure is such that an Eulerian path is simply impossible. The explanation is also non-causal (or at any rate it would not count as one under Salmon's CM model and Woodward's Interventionist model) because, even though it is about concrete bridges, landmasses, and paths, it does not depend on tracking down the causal factors that led to the impossibility. Moreover, it is not possible to conceive this situation in terms of possible causal interventions over the bridge system (see Woodward 2018, 128). Rather, the explanation shows that, as long as the bridge system instantiates a given structure, an Eulerian path will be impossible, regardless of the causal properties of bridges and landmasses. As I have shown in Barrantes 2020, the explanation depends on the way the bridge system is structured, as well as on the simple fact that a bridge connects two landmasses: by definition of 'bridge', there is no possible causal intervention that would make a bridge connect a different number of landmasses.

4. The Explanandum in the Bridges Case

Despite having been widely cited, the bridges case has been described in two different ways in the literature, depending on what the explanandum is supposed to be. On the one hand, the explanandum has been taken to be the failure of an individual or group of individuals to perform the desired walk (explanandum 1). On the other, it has been taken as the impossibility of performing such a walk (explanandum 2). Here are some versions of explanandum 1:

> "Why did Marta fail to walk a path through Königsberg in 1735, crossing each of its bridges exactly once…?" (Craver & Povich 2017, 33).

> "Why has no one ever succeeded (or: why did a given person on a given occasion not succeeded) in crossing all of the bridges of Königsberg exactly once…?" (Lange 2013, 488-489)

"No one has ever continuously walked over Königsberg's seven famous bridges, passing over each bridge exactly once. Why?" (Lyon 2011, 4).

"The residents of Königsberg wondered why they had failed to make a circuit of their city that involved crossing each of its seven bridges exactly once" (Pincock 2018, 45).

"[N]o one… ever succeeded in the attempt to cross all of the bridges exactly once" (Reutlinger 2018, 83).

"Why… would Immanuel Kant fail… to accomplish this task?" (Strevens 2018, 98).

And here are some versions of explanandum 2:

"[W]hy it was impossible for the citizens of Königsberg to complete a tour of all the bridges in the city while crossing each bridge exactly once" (Baker 2021, 2).

"There is no way of crossing all the bridges exactly once" (Bueno & French 2018, 216).

"[T]here is no way of traversing the seven bridges of Königsberg once and only once in a single trip, beginning and ending in the same place." (Colyvan 2018, 26).

"Königsberg cannot be toured by crossing each and every bridge only once" (Jansson & Saatsi 2019, 821).

"[W]hy it is impossible to traverse the city crossing every bridge exactly once" (Kostic & Khalifa 2021, 14145)

"Königsberg in 1735 does not permit a Eulerian walk" (Leng 2021, 10427).

"Why it was impossible to walk a certain kind of path across the bridges of Konigsberg" (Pincock 2007, 257).

"The Königsberg bridge configuration… contains no Eulerian path… it is impossible to cross each bridge exactly once" (Woodward 2018, 127-128).

Explanandum 1 is the event that either someone or everyone so far has failed in their attempt at crossing the bridges in the desired way. Explanandum 2 is the fact that there is no such a path available, or that it is impossible to cross the bridges in the desired way. I believe that only explanandum 2 requires a strutural explanation. Here is why: since explanandum 1 is about the failure(s) of an agent or group of agents in crossing the bridges, it opens the door to explanations that rely on psychological, social, or even random causes, and most likely will require mentioning this causal information (as we will see below, at least under a counterfactual account of explanation, neither the causal information nor the structural information would be sufficiently explanatory). On the contrary, explanandum 2 is about an impossibility that, although can in principle be verified empirically (according to More & Mertens (2011, 4), there are only 2^7 available routes), it stands in need of explanation, since the brute computation strategy would show *that* the trip is impossible but not *why*. Explaining why the trip is impossible requires a structural explanation; explaining why such-and-such person or group of people have failed in performing that trip, does not.

5. Individual Failures vs Impossibility

Whether or not any given road system allows an Eulerian path entails different things regarding the kind of information that we can get about the system's structure. On the one hand, the success of one person in performing the desired walk is evidence that the system is walkable in such a way, that the system has such a path available. And if you are aware of Euler's theorem, with this information you can infer without a doubt that the system in question is Eulerian, that is, that it has either zero or two landmasses connected by an odd number of bridges.

On the other hand, you cannot infer, from a failed attempt, that a given system is non-Eulerian. In fact, one failed attempt says little about the road structure, since there are many reasons why the attempt may have failed. In the bridges case, for example, it may be that the walker just got tired of walking and decided to quit even when she still had uncrossed bridges available, or that she was walking over an Eulerian system but started in the wrong path, or that she was in a non-Eulerian system, etc. This is not just a problem for individual attempts because, even if many people failed in crossing the system, we would not be able to tell whether the system is Eulerian or not because, again, the failure may be due to all of those people starting in wrong paths or getting tired and interrupting their walks, etc. These scenarios, which may seem far-fetched, become real possibilities in cases with many more bridges such as the bridges in Venice (with more than 300 bridges).

This distinction is relevant in the present discussion. When the explanandum is presented as 'why no-one succeeded', or 'why Marta failed', which are versions of explanandum 1, you leave room for explanations that rely on causal factors such as people simply not having tried all the available routes, not being systematic in their attempts, not trying very hard, leaving available bridges uncrossed, etc. The situation is analogous to the following:

> THE PEG CASE: Imagine a square peg that does not fit into a circular hole whose diametrer equals the side of the square. Why doesn't it fit?

A higher-order explanation of this would rely on the geometrical properties of the peg and the hole, and it would be superior than a lower-order explanation that appeals to, say, the actual particles of the peg that bumped onto the wall surrounding the hole. The reason is that the higher-order explanation shows why such a fitting is impossible, regarless of the particular way one tries to do it. But in order for this geometrical, higher-order explanation to work, one must specify that the phenomenon in need of explanation is the fact that it is the peg that does not fit, not that a particular person (e.g., a 1-year-old) failed to fit it, which opens the door to many other causes that may actually be completely unrelated to the shapes involved.[5] This can also be seen in this other case:

> THE LIVING ROOM CASE: Imagine a person who is locked in a very confortable living room. She does not have the key to leave, but she is not aware of not having the key. The fact is that she is so comfortable in the living room that decided to spend all the afternoon sitting on the couch, reading. Why she has not left?[6]

In this case, the question 'why she has not left?' does not have an obvious answer. We can try to appeal to her current state of mind: she was very comfortable, she found the book very interesting, she did not feel the need to leave, etc., or we can appeal to the fact that the room was locked. But neither of the two answers, by themselves, seem to explain the not leaving, at least not under counterfactual accounts of explanation, whether causal or non-causal. The reason is that they appeal to things that, even if they had been different, would not have made a difference on whether or not she left the room. But if we were interested in the question: 'why it is not possible for her to leave the room?', we would not have this problem (I will say more about this case when answering Objection

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[5] This case was originally proposed by Hilary Putnam (1975), and was also used by Jackson and Pettit (1990). Putnam's original formulation corresponds to what I take to be the correct explanandum ("the peg… does not pass through the round hole" (1975, 295)), while Jackson and Pettit's does not ("I try and fail to fit a square peg in a round hole of diameter equal to the side of the square. Why did it not go through?" (1990, 110)).

[6] This case is inspired by John Locke's famous example of a sleeping man (Locke, *Essays* II, XXI, 10 in Ariew & Watkins (2019, 380)), as well as by the Frankfurt cases (Frankfurt (1969).

3, below).

The point is that showing that something is not possible does not necessarily explain an individual failure. In the two previous cases, as well as in the bridges case, cashing out the explanandum in terms of an individual failure (failure to fitting the peg, failure to walk the bridge system, 'failure' to exit the room) leaves open the possibility that the higher-order explanation may not fully explain it or even may not be relevant at all. If we want to use these cases to illustrate how higher-order explanations work, we must focus on a different explanandum, namely, why it is not possible to fit the peg, walk the system, or leave the room. In the specific case of the bridges of Königsberg, when we ask 'why explanandum 2 occured', we are asking about something we can be sure depends on a structural feature of the bridges system, not about the contingent fact that one or several people may have actually decided to walk over the bridges, nor about the decisions the walkers may have made during their walks.

6. Objections and Replies

Objection 1

It may be argued that I am criticizing the choice of explanandum 1 on the grounds that explanandum 1 can be better explained citing causal factors, but this does not take away the fact that it can also be used to illustrate structural explanations. After all, the objection goes, the fact that the individual failure can be explained both structurally and, say, psychologically, does not prevent it from being used to correctly illustrate how structural explanations work. This would make my criticisms almost trivial: who cares if the explanandum can be explained differently, as long as there is a structural explanation available?

This objection misses the mark. My argument is not that explanandum 1 admits both causal and structural explanation. My point is, rather, that if the explanandum is poorly specified, one cannot be sure that the structural explanation actually explains it at all. When you ask 'why no-one has walked across the bridge system?', you leave the possibility open for a purely arbitrary cause. And if that was indeed the case, if, say, people in Königsberg did in fact get tired after trying out a couple of bridges, they would not have been able to cross the bridge system *regardless of whether the system was Eulerian or not*. If the explanandum is cashed out in terms of failures, it would be possible that the structural properties of the bridge system were not difference-making, and so, not relevant to the explanation.

Objection 2

Another objection can go like this: even if individual failures are explained by lower-order explanations, they are *also* explained by the structural explanation, because the structural explanation unifies all those failures under a single, higher-order, explanatory pattern. That is precisely what higher-order explanations do: they account not only for the causal processes that actually occurred, but for other causal processes that could have occurred.

It is true that structural explanations and other kinds of higher-order explanations bring information that delivers a unified understanding of the different causal processes that were actually involved (and could have been involved) in the production of the explanandum. But this would not be the case if what one wants to explain is explanandum 1. The *de facto* failure of everyone who tried to cross the seven bridges would not be unified by citing properties of the non-Eulerian system because it is possible that those failures were not in fact unified at all, as they could have been due to different reasons. An artificial unification based on a property that did not make an actual difference to the occurrence of the explanandum would not be an explanation at all.

Now, if one wants to highlight the difference between the lower-order processes and higher order structures, one can ask why *one or several paths* are not Eulerian. In this case, a structural explanation would provide a unified understanding of this, as it will show that, necessarily, no path can be Eulerian. It seems to me that asking why a given person could not perform such-and-such a walk does not add anything relevant to the discussion.

Objection 3

It could also be argued that the fact that someone did not try hard enough, fell asleep, etc., cannot cause or explain why they failed to make an Eulerian walk on a non-Eulerian system because it is not true that, had they tried harder, not fallen asleep, etc., then they would have made an Eulerian walk on a non-Eulerian system. In such a system, effort, being awake, etc. do not make an actual difference on whether or not a person succeeds in taking an Eulerian walk.

I believe these remarks are correct, but they do not work against the point that I am making. In fact, they strengthen it, as the also rule out the possibility of a structural explanation of explanandum 1. If the bridge structure actually explained the individual failure, then, counterfactually (following the reasoning of objection 3), this would mean that you could change the structure and make a difference on whether or not the failure occurs. This seems to be the idea behind these remarks by Reutlinger (2018):

> "Euler's theorem supports counterfactuals such as (i) 'if all parts of Königsberg had been connected to an *even* number of bridges, then people would *not* have failed to cross all of the bridges exatly once', and (ii) 'if exactly two parts of town were connected to an *odd* number of bridges, then people would *not* have failed to cross all of the bridges exactly once" (2018, 84).

However, if the failure was in fact caused by someone falling asleep, changing the bridge structure would not have made a difference either. Therefore, the bridge structure would not explain the failure either.[7]

Let me illustrate this point with the living room case. It is true that the counterfactual, 'had she not been so comfortable, she would have left' is false, which casts doubt on the explanatory force of her state of mind as an explanation of her not leaving. But the counterfactual, 'had the room not been locked, she would have left' is also false, so the room being locked cannot be the explanation either. None of the two explanations work by themselves, and a full account of cases like this must show exactly how this information must be integrated into a full explanation (below I show why none of the current models achieves this). But here is the crucial point: the previous remarks show that her not leaving the room is not fully explained by the door being locked, regardless of the fact that it is not fully explained by her comfort either. In the same way, explanandum 1 is not fully explained by the bridge structure, even if, arguably, it is not fully explained by psychological or contingent causes either. Explanandum 2, on the contrary, is fully explained by the bride structure, so it perfectly illustrates structural explanations.

Now, both the living room case and the bridges case are classical cases of overdetermination. In the living room case, for example, the first counterfactual does not work because of the higher-order constraint, and the second counterfactual does not work either, due to the lower-order contingency. Taken by itself, her not wanting to leave determines her staying, that is, her 'failure to leave'. Similarly, the door being locked also determines her staying because there is no way she could

[7] Reutlinger includes a footnote adding some caveats that, supposedly, would make the counterfactuals work. I address this in my response to Objection 4.

have left. Despite the fact that both the lower-order and higher-order factors together *overdetermine* the outcome, I do not think that they fully explain it. Determining and explaining are different things.

Debates revolving around overdetermination have been usually about causation (the breaking of the window is overdetermined because both Bob and Suzy's throwing of their respective rocks caused it) (see Paul & Hall, 2013, 143-161 for a comprehensive discussion). Because explanation has been traditionally taken to be causal, one may be tempted to say that, because something is overdetermined, *necessarily* there are more than one successful explanations of it. But I believe that we should resist this temptation, or at any rate, that is not how I understand determination in this paper. I take determination to be a logical relation of inference: 'given that the door is locked, it follows that she will stay'; 'given that she wants to stay, it follows that she will stay'. But if there is anything that we have learned from the asymmetry problem suffered by the DN model, it is that explanation *is not* just about logical relations of inference: the shadow's length entails the flagpole's height, but does not explain it. The lesson is that A entailing (or determining) B, does not amount to A being an explanation of B.

Coming back to our examples, in the living room case, the fact that she stayed in the room may be overdeterminated by two different factors (the higher-order one and the lower-order one), but this does not mean that there are two full explanations available. There is no contradiction in saying that higher-order properties determine, but do not explain, the explanandum. These remarks also apply to the case of a person who fails to perform an Eulerian walk over the bridges of Königsberg due to the fact that she is lazy (say). The failure is overdetermined (is entailed) both by her laziness and by the fact that the bridge system is non-Eulerian. This does not necessarily mean that it is explained by those factors.[8]

Objection 4.

It may also be argued that the philosophers who use explanandum 1 have in mind a list of caveats that would block the extraneous possibilities that I have mentioned, in such a way as to make the individual failures depend solely on the pursued paths being non-Eulerian. I am thinking of caveats such as: 'and the walker walked until the end', or: 'the walker did not fall asleep', or: 'the walker is intelligent and tried repeatedly', etc. The idea would be to point the reader into a charitative mindset: 'do not think about all the possibilities that may make the failure depend on some thing else, but rather, think of the failure as being due to the fact that the chosen path is non-Eulerian.'[9]

I understand that that may be the intention of those who use explanandum 1, but this is precisely what worries me about this methodology: why describing the explanandum in terms that introduce the possibility of psychological and other extraneous explanations, only to, immediately after, add an incomplete list of caveats aimed at removing those possibilities? Nothing is gained by opening the door to psychological considerations only to immediately point the reader to closing those

[8] My thanks to an anonymous reviewer for pressing me on this issue.

[9] Lange (2017, 131ss), and Povich (2019) address the issue of the different explananda in this and similar cases. Their discussions, however, rely on the reader assuming a list of caveats similar to the ones I mentioned. Reutlinger (2018) holds that, once we assume these caveats, the counterfactuals would work (2018, 84 fn12), but I do not think this is true. For example, repeatedly trying to cross the bridges does not guarantee success, even if the walker walks over an Eulerian system.

options. Why introducing a psychological element when everything can be captured perfectly well without appealing to the individual's intentions?

Objection 5

Finally, it may also be said that those extrenous possibilities are *automatically* ruled out by providing the higher-order explanation. It is not clear to me, however, that there is any account of explanation that would make this work. As we have seen, the higher-order explanation of the failure does not support counterafctuals, so counterfactual accounts would not justify this claim. It is not clear that a purely pragmatic view would justify using a structural explanation either. If the explanandum is 'why Marta failed', we can emphasize that it was *Marta* but not Silvia, and we would have to mention details about Marta's laziness, as opposed to Silvia's. And if we emphasize that Martha *failed*, we would still have to mention that today, as opposed to yesterday, she was particularly lazy. In both cases, the causal information needs to be mentioned and will not be automatically ruled out by citing the higher-order constrain. Finally, Lange's view doesn't work either. As Lange puts it:

> [T]he explanation would not have been distinctively mathematical if it had been that no one ever turned left rather than right after crossing a given bridge, or the bridges were made of a corrosive material, or someone was poised to shoot anyone who tried to cross a given bridge. (2013, 489).

Here, the implication is that these caveats are introduced precisely because the higher-order explanation does not *automatically* rule them out. The same can be said of Reutlinger's view. The clarificatory footnote is introduced precisely because the counterfactual explanations do not *automatically* rule out the extrenous possibilities, for the reasons discussed above.

7. Other cases

The previous remarks are not limited to the bridges case. There are other purported cases of structural explanations that may need more precision when specifying the explanandum. Consider the following example:

> THE STRAWBERRIES CASE: No one can distribute 23 strawberries exactly among 3 children, without cutting any strawberry. Why? (Lange 2013, 488).

As an explanation, this situation can be described with the explanandum: ‘a person failed to evenly distribute 23 strawberries among 3 children’ (the equivalent of explanandum 1), or with the explanandum: ‘it is impossible to evenly distributing 23 strawberries among 3 children’ (the equivalent of explanandum 2). If my previous remarks are correct, only the second version would be explained by the structural explanation. Just like in the bridges case, the impossibility of distributing the strawberries can be established by a survey of all possible distributions that verifies that none of them is even. The absence of such a distribution makes it impossible for anyone to distribute the strawberries evenly. This impossibility can be explained by a structural explanation. Whether or not a given person fails is not necessarily explained by the mathematical/structural impossibility.

Another such an example is the following:

> THE TREFOIL KNOT CASE: A trefoil knot cannot be untied. Why? (Lange 2013, 489).

A trefoil knot is a non-trivial knot. A trivial knot, also called unknot, is a closed loop that is not knot (like a circular band). A trefoil knot, being a non-trivial knot, is such that there is no way to untie it to turn it into an unknot; it is a knot that cannot be untied without being cut. Again, as an explanation, this situation can be described with the explanandum: ‘a person failed to untie a trefoil knot' (the equivalent of explanandum 1), or with the explanandum 'it is impossible to untie a trefoil knot' (the equivalent of explanandum 2). And just like in the previous cases, the impossibility of untying the knot can be established by performing all combinations of the available untying moves (called Reidemeister moves) and verifying that none of the combinations will succeed in untying the knot. It is this impossibility that would require a structural explanation. Whether a given individual fails in achieving this is not necessarily explained by the theory of knots.

8. Conclusion

Out of the two common versions of the bridges of Königsberg explanation, only the one whose explanandum is the impossibility of performing an Eulerian walk unequivocally counts as a structural explanation. As we have seen, the failure of an individual or group of individuals to perform an Eulerian walk over the bridge system does not necessarily need a structural explanation. But if we focus on the paths themselves (either why this path is not Eulerian, or why none are), then this amended version would in fact count as a structural explanation.

None of my remarks actually constitute criticisms to the accounts of higher-order, non-causal explanations discussed. My point is simply that, in order to qualify as such, the cases discussed must be cashed out in terms that appeal to the structural features involved, and not in terms that appeal to individual failures. Individual failures may be beyond the scope of these accounts.

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Declarations

Conflict of interest: The author declares that there is no conflict of interest.

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