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ORIGINAL RESEARCH

Towards an Understanding of the Principle of Variable Embodiments

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Abstract
The theory of variable embodiments has been primarily formulated to model ordinary objects as things that change their parts over time. A variable embodiment */f/* is a *sui generis* whole constructed from a principle *f*, the principle of a variable embodiment, and it is manifested at different times by different things picked out by such a principle *f*. This principle is usually clarified as a function that picks out, at any given time the variable embodiment exists, its corresponding manifestation at that time, and it fails to pick out anything at any other time. The first purpose of this article is to examine and refute three natural understandings of such a principle. Specifically, first, we argue against the view that it should be understood extensionally as either a partial function or total function. Next, we provide some motivations to reject the understanding of the principle as a function in intensional terms, where the notion of intension is analyzed in terms of a specific version of a possible world semantics. The second goal of the article is to make it plausible that the principle has an intensional character and, on the basis of the results achieved, to formulate some constraints that an account of such a principle should meet.

Keywords Object · Variable Embodiment · Function · Ontological Dependence

1 Introduction

We ordinarily hold that an ordinary object may undergo a change in its parts or qualities while remaining the same. A car changes its tires, while remaining the same. How to account for this phenomenon is a vexed question in metaphysics.[1] The theory of

[1] See Simons (1987), Sider (2001), Wasserman (2006), Hansson (2007), Hofweber (2009).

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variable embodiments, as Fine (1999, 2008) claims, has been primarily developed to account for this phenomenon. One can elucidate the notion of variable embodiment by considering a variable embodiment as an entity that is embodied, or manifested, by different things at different times. Suppose we are willing to distinguish The Mayor of The City, and Charles, who happens to be The Mayor of The City at a given time. According to the theory of variable embodiments, The Mayor of The City is embodied, or manifested, by Charles at the given time. Moreover, The Mayor of The City may be embodied by different persons, for example Tom, at different times. How is it that a variable embodiment is manifested by different entities at different times? The answer to this question calls into play the key notion of the principle of a variable embodiment. Any principle of a variable embodiment is specific to a unique variable embodiment, and Fine describes it as a function that picks out, at any given time the variable embodiment exists, its corresponding manifestation at that time, and it fails to pick out anything at any other time.[2]

Now, the theory of variable embodiments adequately accounts for the nature of objects *only if* it clarifies what the principle of a variable embodiment is. The goal of this article is to assess the plausibility of several understandings of the principle of a variable embodiment. In particular, the previous Finean gloss suggests that we investigate whether the principle of a variable embodiment is a function from times to things in the mathematical-extensional sense. Is this a possible understanding of the principle of a variable embodiment? And if not, why not?[3] First, we examine the thesis that the principle is a *partial* function from times to things in the mathematical-extensional sense, and we provide three arguments against this understanding. However, when one says "X is a partial function from set A" one may mean something like: "X is a *total* function from a proper subset of A". Thus, we also consider the idea that the principle is a *total* function from times to things, and given widely shared assumptions, we argue that this option is also not plausible. The rejection of the previous options will lead us to examine whether the principle is a function understood in intensional terms. Given such an option, we also provide some reasons to reject the understanding of the principle of a variable embodiment as a function in intensional terms, where the notion of intension is analyzed in terms of a specific conception of the possible world semantics.[4] Finally, given the previous results, we fix some *desiderata* for an account of the principle of a variable embodiment. Since the plausibility of the theory of variable embodiments depends on the clarification of the principle of a variable embodiment, and given that our results refute some of its most natural interpretations, it is urgent for the supporter of this view to provide an explanation of the nature of this principle.[5]

[2] Fine, 1999, p. 68. See, also, Fine (2022).

[3] Koslicki (2008, p. 78) does not provide any argument against the understanding of the principle of a variable embodiment as a function in the mathematical-extensional sense, and she leaves this problem as an open question.

[4] See Menzel (2016).

[5] Jacinto and Cotnoir (2019) supply a formal semantics for Fine's theory of embodiments. Since their account provides a semantic *representation* of Fine's theory of embodiments (see, p. 918), plausibly their account need not to capture the nature of the principle of a variable embodiment. However, it is relevant

The article is structured as follows. In Sect. 2, we introduce the theory of variable embodiments, and we show how it models ordinary objects as things that may change over time. In Sect. 3, we examine the idea that the principle of a variable embodiment is a partial function in mathematical-extensional sense, and we provide three arguments against this thesis. Then, we assess the thesis that this principle is a total function in the mathematical-extensional sense, and we argue that, given some widely shared assumptions, such an option is not plausible. In Sect. 4, we make it plausible that the principle has an intensional character, but we reject the idea that the principle can be interpreted through a specific version of a possible world semantic analysis of the intension of a relation. Finally, we establish some constraints on an account of the principle of a variable embodiment.

2 The Theory of Embodiments

The theory of embodiments (Fine, 1999, 2008, 2022) is called to account for how an entity is capable of having the parts it does, and the ways it has the parts it does. On the one hand, as Fine (1999) claims, a ham sandwich possesses its parts timelessly – namely, it makes no sense to ask for how long the ham sandwich possesses the slices of bread that are its parts. One the other hand, a car has its parts temporarily – it does make sense to ask for how long the tires have been part of the car. The theory of embodiments is specified in two theories. The theory of rigid embodiments that deals with things that have their parts timelessly, and the theory of variable embodiments that accounts for the variation over time of an entity. While Fine (1999, 2008) formulates his theory of embodiments primarily to account for the nature of ordinary objects, his theory of embodiments has been employed to account for other kinds of entity, such as processes, acts and actions, musical works, groups, intentional collectives as well as abstract things such as bodies of law.[6] In what follows, we only focus on the theory of variable embodiments, and on how it accounts for ordinary objects as things that may change.

2.1 The Theory of Variable Embodiments

The key notion of the theory of variable embodiments is the notion of the principle of a variable embodiment. As glossed in the introduction, this principle is a function from times to things such that, for any time t the principle f is defined at, it picks out a

that they define this principle as a function whose input domain is the set of times (see their Definition 5, p. 917). We believe that some of the results of this article are relevant also for their account.

[6] Processes are conceived as variable embodiments in Guarino (2017) and Fine (2022). This view is also examined in Baratella (2023). Relevantly, Fine (2022) assumes that processes have the character of going on or progressing over time, while events do not have such a character. Specifically, Fine (2022) adopts as his starting point Stout's clarification of the distinction between processes and events (Stout, 1997, 2016, 2018). Processes and events are conceived as rigid embodiments in Baratella (forth). Acts and actions are accounted for in terms of Fine's theory of embodiments in Fine (2022). Musical works and bodies of law are accounted for by Fine's theory in Fine (1999). A version of Fine's theory of embodiments is used to account for groups in Uzquiano (2018) and social groups in Fine (2020). Finally, his theory of embodiments is employed to account for intentional collectives in Brouwer et al. (2021).

thing x existing at t, and the principle f fails to pick out anything at the times it is not defined.[7] Related to a principle f, there is a new thing, *the variable embodiment* /f/ of f that exists at those times at which the principle f is defined. Moreover, *the variable embodiment* /f/ of f at a certain time t is manifested by the thing f_t picked out by the principle f at t. Further, Fine stresses that *the variable embodiment* /f/ is different from its principle f.[8] Fine formulates several principles governing the notion of variable embodiment. We only provide some of them relevant for our investigation.[9] We keep Fine's nomenclature of these principles.

(V1) The variable embodiment /f/ exists at time t *iff* it has a manifestation at t – namely, *iff* the principle f is defined at that time.

(V2) If the variable embodiment /f/ exists at t, then its location is that of its manifestation f_t (assuming that f_t has a location).

(V3) The variable embodiments /f/ and /g/ are identical *iff* their principles f and g are identical.

(V4) Any manifestation of a variable embodiment at a given time is a temporary part of the variable embodiment at that time.

(V5a) If a is a timeless part of b that exists at t and if b is part of c at t, then a is a part of c at t.[10]

(V7) The pro tem properties of a variable embodiment /f/ at a given time t are the same as those of its manifestation f_t,

where the notion of pro tem property is defined by Fine (1999, p. 71) and Koslicki (2008, p. 80) as follows:

Definition 1 A property of a thing is a pro tem property if its holding at a time depends only upon how the thing is at that time.

We take the previous principles to be clear enough not to deserve clarification, except for (V7) and **Definition 1**. As Koslicki puts it, principle (V7) and **Definition 1** establish that 'a variable embodiment inherits those properties from its manifestations which depends only on "how the object is at that time" (whatever exactly that means)'.[11] For instance, suppose that an object o is conceived as the variable embodiment /f/. Then, /f/ will be red at a given time *iff* its manifestation is red at that time.

2.2 Objects as Variable Embodiments

Fine (1999, 2008) holds that those ordinary objects that may change over time, such as a car or a human body, are accounted for by combining the two theories of embodiments: the theory of rigid embodiments that accounts for their mereological structure

[7] Fine (1999, 2008, 2022).

[8] Fine, 1999, pp. 69-70.

[9] Fine, 1999, pp. 70-71.

[10] Principle (V5a) will be relevant to account for the mereological structure of those ordinary objects that, within Fine's framework, are conceived by combining the theories of rigid and variable embodiment. See Sect. 2.2.

[11] Koslicki, 2008, p. 81.

at a given time, and the theory of variable embodiments that accounts for their variation over time. A rigid embodiment is a *sui generis* kind of whole composed of some entities *a*, *b*, *c*, … that are modified or stuck together by a property or relation *R* they jointly possess. Neither is a rigid embodiment identical to the mereological sum of *a*, *b*, *c*, …, nor is it identical to the mereological sum of *a*, *b*, *c*, … and *R*. Indeed, such mereological sums may exist even though the entities *a*, *b*, *c*, … are not related by *R*. Instead, it is key for a rigid embodiment to exist that *a*, *b*, *c*, … are related by *R*. Moreover, a rigid embodiment cannot vary its constitution over time – namely, it has its parts timelessly. So, rigid embodiments with different parts are different. For the aim of this article, let us only focus on ordinary objects as things that may undergo change and let us clarify how the theory of variable embodiments accounts for this phenomenon.

According to the theory of variable embodiments, an ordinary object, such as a car *C*, is conceived as a variable embodiment /*f*/ whose principle *f* is a function from times to rigid embodiments that picks out, at any given time *t* the variable embodiment exists, its corresponding manifestation at the given time *t*, and it fails to pick out anything at any other time. Such a manifestation, call it "R_t", is a rigid embodiment existing at the given time *t*.[12] Moreover, given (V4), this rigid embodiment existing at *t* is a temporary part of the variable embodiment /*f*/ at *t*. The rigid embodiment R_t grounds the mereological structure of the car *C* at time *t* via (V5a). For instance, in the given situation, rigid embodiment R_t that manifests car *C* at *t* will be the various parts of the car (for instance, the chassis, the engine, the gearshift) arranged in a specific car-wise manner. These parts and the specific car-wise manner of composition are timeless parts of the rigid embodiment R_t. By (V5a), they are part of car *C* at *t*. This account explains the fact that car *C* changes its parts over time as follows. Car *C* is a variable embodiment /*f*/, its manifestation at *t* is rigid embodiment R_t that has an engine *E* as a timeless part. By (V5a), *E* is part of car *C* at *t*. Now, the manifestation of /*f*/ at a different time *t** is rigid embodiment R_{t*} that has a different engine *E** as a timeless part. Since the rigid embodiments R_t and R_{t*} have different parts, they are different. Further, by (V5a), *E** is part of car *C* at *t**. So, the theory of variable embodiments explains how an ordinary object changes its parts by interpreting it as a variable embodiment whose principle selects different rigid embodiments at different moments of time.

3 The Extensional Understanding of The Principle of A Variable Embodiment

According to Fine (1999, p. 68), the theory of variable embodiments will take the principle of a variable embodiment to be a function from times to things that picks out, at any given time the variable embodiment exists, its corresponding manifesta-

[12] Given the general theory of variable embodiments formulated in Sect. 2.1, the manifestation of a variable embodiment at a given time *t* may in principle be another variable embodiment. However, Fine (1999, pp. 69-70; 2008, pp. 115-116) holds that, given the specific theory of ordinary objects as variable embodiments, the manifestation of an ordinary object (or, at least of most ordinary objects), conceived as a variable embodiment, at a given time *t* is a rigid embodiment existing at that time *t*.

tion at the given time, and it fails to pick out anything at any other time. But how should this principle or function be understood? In other words, what is the nature of such a function? There are at least two conceptions of function. The first view, that is the most widespread, conceives functions in the mathematical-extensional sense as sets of tuples. According to the second conception, functions are rules that, given an argument, assign to it its value. For instance, such a view, derived from some pioneering ideas of Frege (1983), can be formally characterized in terms of λ-calculus.[13] In this article, we confine ourselves to the most widespread conception of function in the mathematical-extensional sense, and we assess the plausibility of several ways of understanding the idea that the nature of the principle of a variable embodiment is to be a function in such a sense. The first interpretation we take into consideration is based upon Fine's remark that the function in question "fails to pick out anything at any other time"[14]. Specifically, according to the first interpretation, the principle is a *partial function*, but not a total function, from times to things in the mathematical-extensional sense, where the definition of (partial and total) function we adopt is formulated in (Gallier, 2011, p. 106):

Definition 2 A *partial function* f is a triple $f = \langle A, G, B\rangle$, where A is a set called the *input domain of* f, B is a set called the *output domain of* f (sometimes *codomain of* f), and $G \subseteq A \times B$ is a functional relation called the *graph of* f; we let $graph(f) = G$. We write $f\colon A \to B$ to indicate that A is the input domain of f and that B is the codomain of f and we let $dom(f) = dom(G)$ and $range(f) = range(G)$. For every $a \in dom(f)$, the unique element $b \in B$, so that $\langle a, b\rangle \in graph(f)$ is denoted by $f(a)$ (so, $b = f(a)$). […] If $dom(f) = A$, we say that f is a *total function*, for short, a *function with domain* A.

Given **Definition 2**, we formulate three objections against the thesis that the principle of a variable embodiment is a partial (but not a total) function – a partial function, for short. However, when one says: "X is a partial function from a set A", one may mean something like: "X is a *total* function from a proper subset of A". In this sense, the principle of a variable embodiment is identified with what **Definition 2** calls "the graph G of function f" – or, more precisely, given **Definition 2**, with the *total* function $g = \langle dom(G), G, range(G)\rangle$. Crucially, if principle h of variable embodiment /h/ is identical to function g, then h is a *total* function whose input domain contains all and only the times at which variable embodiment /h/ exists. Further, on the ground of the Finean gloss, such an interpretation is in no way unprincipled. Thus, the second interpretation we assess is the one according to which the principle f of a variable embodiment /f/ is a *total* function whose input domain contains all and only the times at which variable embodiment /f/ exists. Given widely shared and plausible assumptions, we provide an objection against this understanding of the principle as a total function. The results of this section make it plausible that the principle of a variable embodiment cannot be understood as a partial or total function. Finally, since the theory of variable embodiments has been primarily developed to account for ordinary objects, the following arguments shall focus on ordinary objects construed as vari-

[13] Alama and Korbmacher, 2021, p. 1.

[14] Fine, 1999, p. 68.

able embodiments. However, these arguments are independent of such an assumption. Indeed, as the reader can easily check – given, for example, Fine's notion of process –, it is possible to formulate versions of such arguments that concern processes instead of ordinary objects.

3.1 Objection 1

A key premise of Objection 1 is that a metaphysical theory should adequately account for an entity or a phenomenon under investigation. When a metaphysical theory accounts for this entity or phenomenon, the account is a well-designed theory, otherwise it is a badly-designed theory. For instance, if the entity under investigation pre-theoretically concerns only certain properties, a theory is well-designed *only if* it provides an account of that entity that concerns all and only these pre-theoretically relevant properties. This premise is a key starting point both in conceptual modeling and metaphysics and ontology.[15]

Given the previous premise, focus on an ordinary object, such as Tom. Tom begins to exist at t_1, he keeps existing for some moments of time, and he comes to his end at t_n. It is natural to hold that, pre-theoretically, Tom only concerns moments from t_1 to t_n. Indeed, our world could finish right after t_n, and Tom would still have come to his end at t_n. Ditto for its starting moment. Thus, pre-theoretically, this object only concerns moments from t_1 to t_n – specifically, he doesn't concern moments later than t_n. Thus, a theory that accounts for what Tom is by referring to moments not included in the relevant interval $[t_1, t_n]$ – namely, by referring to moments at which he does not exist – is clearly a badly-designed theory.

Now, given the thesis that ordinary objects are variable embodiments, we test whether the principle of a variable embodiment is a *partial* function, where the definition of partial function is **Definition 2**. Let us focus on Tom as our case study, and suppose that Tom is variable embodiment /*f*/. Thus, the principle of a variable embodiment /*f*/ is defined as a partial function $f = \langle A, G, B \rangle$, where A is a set of times, B is a set of rigid embodiments, and *graph* G is a subset of $A \times B$.[16] Crucially, the definition of f makes indirect reference to the elements of A. Among these elements there are moments at which variable embodiment /*f*/ does not exist – given the fact that f is a partial function.[17] Now, since variable embodiment /*f*/ is characterized in terms of f – namely, since the Finean principles (V1)-(V7) fix what a variable embodiment /*f*/ is and since some of these principles (V1)-(V7) involve as an ineliminable constituent the principle f of variable embodiment /*f*/ –, then the full characterization of /*f*/ must also involve indirect reference to all moments of A – including those moments

[15] For conceptual modeling, see Guizzardi (2014). Fine (2022, §4) suggests a similar methodological premise.

[16] As stated in Sect. 2.2, according to the theory of ordinary objects as variable embodiments, the principle f of a variable embodiment /*f*/ is a function from times to rigid embodiments. This is the reason why B is a set of rigid embodiments.

[17] It is worth recalling that, given Fine's description of the principle of a variable embodiment ("it picks out, at any time t at which the river exists, the quantity of water in the river at that time (and fails to pick out anything at any other time" (Fine, 1999, p. 68), we are considering (and rejecting) the thesis that the principle is a partial function that is not total.

at which variable embodiment /*f*/ does not exist.[18] But, the pre-theoretical understanding of Tom, that variable embodiment /*f*/ aims to provide theoretical account of, does not involve any reference to moments at which he does not exist. So, given the understanding of the principle of a variable embodiment as a partial function as defined in **Definition 2**, the theory of ordinary objects as variable embodiments is an instance of a badly-designed theory.

3.2 Objection 2

Objection 1 relies on the notions of badly-designed and well-designed theory. However, these notions may be called into question. So, it is better to rephrase the objection by introducing the notion of *existential essential dependence* defined as follows:

Definition 3 *x* depends for its existence upon *y* =*df.* It is part of the essence of *x* that *x* exists only if *y* exists.[19]

Two facts concerning **Definition 3** are relevant. First, the notion of essence at stake is not to be understood in modal terms, but as a primitive notion – for instance, as the Finean notion of essence.[20] Second, this notion of ontological dependence is characterized as a transitive relation – namely, we are assuming a notion of essence akin to the Finean notion of mediate constitutive essence.[21]

Now, it is plausible to hold that it is pre-theoretically true that Tom – namely, object *o* –, that begins to exist at t_1 and comes to his end at t_n, does not depend for its existence upon the moment t_m (with *m* greater than *n*).[22] Consider how theoretical entity /*f*/ – that is called to account for what ordinary object *o* is – has been designed. Its principle is a *partial* function *f* whose definition, given **Definition 2**, makes reference to the set *A*, that in turn includes moments at which object *o* does not exist. So, given that a set existentially depends upon its members and given the transitivity of dependence, it follows that partial function *f* depends for its existence upon moments in *A*. In turn, since /*f*/ is characterized in terms of *f* – namely, since the Finean principles (V1)-(V7) fix what a variable embodiment /*f*/ is and since some of these principles (V1)-(V7) involve as an ineliminable constituent the principle *f* of variable embodiment /*f*/ –, then it is plausible that /*f*/ depends for its existence upon the existence of *f*. Specifically, since /*f*/ is characterized in terms of its principle *f*, it is plausible that it is part of the essence of /*f*/ that /*f*/ exists only if *f* exists. So, by **Definition 3**, /*f*/ depends for its existence upon *f*. Then, /*f*/ depends for its existence

[18] The fact that some of the principles (V1)-(V7) involve as an ineliminable constituent principle *f* means that none of the relevant principles can be formulated without involving as a constituent principle *f*. If there were such a formulation, then the full characterization of variable embodiment /*f*/ need not to involve indirect reference to all moments of *A*.

[19] See Tahko and Lowe (2020).

[20] See Fine (1994, 1995).

[21] See Fine (1995).

[22] It is key to notice that we are not holding that such an object pre-theoretically depends for its existence upon moments included in the interval $[t_1, t_n]$. For the aim of this article, we leave it as an open question whether objects pre-theoretically depend for their existence upon moments at which they exist.

upon moments in A at which object o does not exist. This result marks a difference concerning fundamental features possessed by the pre-theoretical entity o and the designed entity /*f*/. So, the designed entity /*f*/ does not provide a good account of o.[23] Thus, given Objections 1 and 2, the thesis that the principle of a variable embodiment is a *partial* function, as defined by **Definition 2**, is subject to considerable costs that justify its rejection.

Clearly, if one rejects the understanding of *partial* function provided by **Definition 2**, and instead holds that the expression "X is a partial function from a set A" means something like "X is a *total* function from a proper subset of A", Objections 1 and 2 fail. Given this interpretation, the principle *f* of a variable embodiment /*f*/ is plausibly identical to what **Definition 2** calls "the graph G of function *f*" – or, more precisely, given **Definition 2**, to the *total* function $g = \langle dom(G), G, range(G)\rangle$ – whose input domain contains all and only the times at which /*f*/ exists. In what follows, we also assess the plausibility of understanding the principle as a *total* function. Specifically, we formulate an objection that applies to the understanding of the principle as a *partial* function as defined in **Definition 2** as well as to its understanding as a *total* function as defined in **Definition 2**.

3.3 Objection 3

Objection 3 shows that the idea of considering the principle of a variable embodiment as a total or partial function as defined by **Definition 2** yields a metaphysics that is highly revisionary of our ordinary conception of what objects are.

Let us consider the Finean principles (V1)-(V7). It is natural to understand these principles as fixing, at least partially, what a variable embodiment /*f*/ is. In other words, these principles partially specify the nature or the identity of a variable embodiment. Crucially, the principle of variable embodiment is an ineliminable constituent of some of such principles – see, for instance, (V3). Now, suppose that such a principle is a total or partial function as defined by **Definition 2**. Call this thesis "**Assumption 1**". It follows that this principle is a set-theoretic entity. Then, given **Assumption 1**, principles (V1)-(V7), that fix what a variable embodiment /*f*/ is, involve as an ineliminable constituent a set-theoretic entity. Thus, if objects are variable embodiments, the principles (V1)-(V7) that specify what these entities are involve as an ineliminable constituent a set-theoretic entity. But according to our ordinary conception of what objects are, there is simply no set-theoretic entity that constitutes the nature of objects. We may let Fine (1994) himself to express such a concern:

> Intuitively, this is not so […]. There is nothing in the nature of a person, if I may put it this way, which demands that [Socrates] belongs to this or that set or which even demands that there be any sets.[24]

[23] The same result can be obtained by using the modal-existential analysis of ontological dependence – since also this relation is transitive.

[24] Fine, 1994, p. 4.

Thus, given **Assumption 1**, the thesis that objects are variable embodiments leads to a metaphysical position that is highly revisionary of our ordinary or intuitive picture of the world. This is a costly outcome, especially for those philosophers, such as Fine, whose goal is to provide metaphysical accounts that reflect our intuitive or ordinary picture of reality.[25]

Such result triggers a challenging dilemma:

(a) One accepts **Assumption 1** and its costly revisionism. In such a situation, one is compelled to justify why such a costly revisionism is worth being paid compared to other revisionary views, such as standard mereology and perdurantism, whose costly results have been judged reasons to reject them.[26] Pending any argument to that effect, it is better to examine the second horn of the dilemma.
(b) One rejects **Assumption 1**. Given that, one is free to hold, for instance, that Fine's claim that the principle of a variable embodiment is a function only has a representational import. The principle may be represented as an extensional function – along the lines of **Definition 2** – but it is not identical to such a function: their nature is different. Now, if one takes this route, one still has to clarify what the nature of the principle of a variable embodiment is – a clarification that cannot rely on any set-theoretic notion without triggering new versions of the previous objection. How to formulate such a view seems not to be a trivial matter.

Summing up, the thesis that the principle of a variable embodiment is a partial or total function understood in extensional terms (as defined in **Definition 2**) gives rise to a costly form of revisionary metaphysics concerning objects. Such costs undermine the plausibility of understanding the principle as a function in extensional terms and suggest investigating other interpretations of such a principle.

4 Towards an Intensional Understanding of the Principle of a Variable Embodiment

The principle of a variable embodiment cannot plausibly be understood as a partial or total function in extensional terms. Another natural option is to consider it as a function in intensional terms. Such an option is motivated by the following considerations: if the theory of variable embodiments is called to account for objects, then the same variable embodiment has to exist in different possible circumstances where its principle is defined for different sets of times. For instance, Tom as a variable embodiment may exist for interval $[t_1, t_n]$ in a possible circumstance, and for interval $[t_1, t_m]$ (with m greater than n) in a different possible circumstance. So, the same principle must be defined for different sets of times. Thus, it is plausible to understand

[25] Objections 1-3 depend upon **Definition2** that provides adequate definitions of partial and total function in the extensional sense. There may be other adequate definitions of partial and total function in the extensional sense. But the burden of the proof is on the opponent to argue that there are alternative adequate definitions that neutralizes Objections 1-3.

[26] For standard mereology, see Fine's (1999) Monster Objection. For a discussion of this objection, see Calosi et al., (2023). For the revisionary import of perdurantism, see Sattig (2003) and Varzi (2003).

the principle of a variable embodiment in intensional terms. The question is how to clarify what this option means.

Given the initial characterization of the principle as a function, one of the most natural interpretations is to analyze the principle in terms of a possible world semantics. According to such a view, the principle of a variable embodiment turns out to be a function *f* from possible worlds to sets of order pairs, where the first member is a time at which the variable embodiment exists, and the second member is the manifestation of the variable embodiment for the given time.[27] Now, there is the issue of providing an account of function *f*. The main options are those already listed: (i) interpreting it in the mathematical-extensional sense; (ii) interpreting it as a rule. In this article, we assumed to confine ourselves to the mathematical-extensional interpretation of a function. This means that function *f* is analyzed in terms of **Definition 2**, where the input domain of *f* is a set of possible worlds, and the output domain of *f* is a set of sets of order pairs characterized as above. The objections already discussed strike back. Suppose that function *f* is a partial function. Now, Tom does not exist in every possible world. So, Tom does not concern – and does not depend upon – worlds at which he does not exist. Thus, given the understanding of *partial* function provided by **Definition 2**, it is clear that such a strategy triggers possible world versions of Objections 1-2. Moreover, since this option still considers the principle of a variable embodiment as a set-theoretic entity, it also triggers Objection 3. Thus, also the understanding of function *f* as a total function (and so also the understanding of "*f* is a partial function from a set *A*" as meaning "*f* is a *total* function from a proper subset of *A*") is not plausible. As a consequence, the interpretation of the principle in terms of the previous version of a possible world semantics is not plausible.[28]

These results make clear why the principle of a variable embodiment cannot plausibly be understood as a function in the mathematical-extensional sense. Moreover, they also show that this principle cannot plausibly be understood as a function in intensional terms, where the notion of intension is analyzed in terms of a specific conception of a possible world semantics. Furthermore, the previous results suggest some constraints that an account of the principle should meet. First, given Objection 3, it is extremely costly to interpret the principle of a variable embodiment in set-theoretic terms. Thus, it is plausible that an account of this principle should not understand it in these terms. Second, the above considerations make it plausible that the principle has an intensional character. So, an account of this principle should clarify the notion of intension to be adopted. Third, these considerations make it plausible that the principle may be defined for different sets of times in different possible circumstances. Finally, this account should explain what makes it the case that a principle differs from another and what, in the nature of a principle, allows it to individuate a unique variable embodiment. Suppose, in particular, there to be in a Black (1952) possible world in which there are just two exactly similar spheres, and

[27] See Menzel (2016).

[28] Evnine (2016, §2.3.6) formulates a completely different argument against the extensional interpretation of the principle of a variable embodiment – where such an argument is based on his notion of metabolism of ordinary objects (§1.2.3). So, his argument stands or falls together with his thesis that ordinary objects have a metabolic nature (in his sense).

suppose also that these spheres are variable embodiments. Thus, by (V3), the principles of such spheres are different. Two questions arise. What makes the principle associated with one of those spheres different from that associated with the other sphere? What, in the nature of these principles, enables each one of them to individuate a specific sphere as opposed to the other?

In conclusion, given the results of this article, a further option one may consider is the idea that the principle of a variable embodiment is a function conceived as a rule. Whether such a conception avoids the previous objections and satisfies the former constraints as well as it is not subject to further problems is the proverbial story for another time.

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Competing interests The author reports there are no competing interests to declare.

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