The Epistemology of the Near Miss and Its Potential Contribution in the Prevention and Treatment of Problem-Gambling

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Abstract

The near-miss has been considered an important factor of reinforcement in gambling behavior, and previous research has focused more on its industry-related causes and effects and less on the gaming phenomenon itself. The near-miss has usually been associated with the games of slots and scratch cards, due to the special characteristics of these games, which include the possibility of pre-manipulation of award symbols in order to increase the frequency of these "engineered" near-misses. In this paper, we argue that starting from an elementary mathematical description of the classical (by pure chance) near-miss, generalizable to any game, and focusing equally on the epistemology of its constitutive concepts and their mathematical description, we can identify more precisely the fallacious elements of the near-miss cognitive effects and the inadequate perception and representation of the observational-intentional "I was that close." This approach further suggests a strategy of using non-standard mathematical knowledge of an epistemological type in problem-gambling prevention and cognitive therapies.

Keywords: near-miss, mathematical education, gambling mathematics, cognitive therapy, epistemology of mathematics, mathematical modeling

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1. Introduction

In problem-gambling research, the near-miss has received great attention as a complex game-related factor that influences problematic gambling behavior. Such complexity resides in the capability of the near-miss to produce psychologically rewarding experiences, even in losing situations. The immediate effects of these experiences are the excitement, arousal, thrill, and a decision to continue the gambling activity at the same or higher rate, due to the effect of the now paradigmatic "I was that close." In particular, the near-miss is a kind of intermediate reinforcer in the form of an encouraging sign, different from a gambler's continuous rewarding experiences in his/her reinforcement history (Parke & Griffiths, 2004, p. 407). Early studies have shown that gamblers become physiologically aroused when they either win or nearly win; other studies have shown a strong cognitive bias characterizing the psychological factors which cause persistent gambling, and near-miss experiences have their own such bias.[1] This double nature of the gambler's experience of a near-miss (that is, biological and psychological) imposed the new interdisciplinary line of research into the etiology of pathological gambling over the last two decades, which until then tended to emphasize psychosocial factors. This interdisciplinarity of the research, also part of the complexity of the near-miss, justifies M. Griffiths' (1991) calling it "the psychobiology of the near-miss."

Psychology of the near-miss came into focus with the work of Reid (1986), who argued that a near-miss may have the same conditioning effect on behavior as a success, being a strongly reinforcing factor (at no extra expense to the machine's owner). As such, a near-miss could produce some of the excitement of a win as a cognitive conditioning. Reid (1986) also pointed out that near-miss behavior can be explained in terms of frustration theory (Amstel, 1958) or cognitive regret (Kahneman & Tversky, 1982).[2] A recent study on scratch-card players showed that players interpreted near-misses as negatively valenced and frustrating losses, but even so, they moved on more quickly to the next game following this type of outcome than following winning outcomes.

[1] See (Griffiths, 1991) for a well-organized brief of these early results.
[2] According to frustration theory, failing to reach a goal (like winning) causes frustration, which fuels ongoing behavior toward that goal. According to cognitive-regret theory, the frustration caused by a near-miss induces a form of cognitive regret, which can be eliminated by playing again.

Additionally, near-misses were associated with the largest amount of change in SCLs[3] as the outcome was revealed (Stange et al., 2015). Such results support both the "psychobiological" approach of the near-miss and the frustration theory. Other similar studies on slots players found that near-misses triggered SCRs[4], further confirming the frustration hypothesis (Lobbestael et al., 2008; Dixon et al., 2011).

Reid (1986) also left us the definition of a near-miss as being "a failure that is close to being successful," which has been carried over literally, with no refinement, in the conceptual frameworks of contemporary research. This is the subject of the current paper. However, with the development of the games, especially slots, distinctions have been made regarding the types of near-misses. Thus, we distinguish between two types of near-misses: a *direct* near-miss (associated only with the outcoming sequence or combination of numbers, symbols, cards, etc. of the game – e.g., what appears on a payline in slots), and an *indirect* near-miss (associated with the area a game displays together with the outcome – e.g., what appears adjacently above or below the payline in slots). The latter type of near-miss is specific to the games of modern slots and some scratch cards and is seen as a structural feature of these games. In slots with virtual reels, manufacturers use a technique called "award symbol ratio" for the virtual reel mapping to create a high number of illusory near-misses above and below the payline. The direct effect of this technique is the increase of the frequency of so-called near-misses near the paylines. The effect of the indirect near-miss on the player is supported initially with the sensation that the player gained some insight into the functionality of the machine (since s/he can see on the display more than the outcome of the machine, including an illusory movement of the reels and implicitly a false chronology). The functionality of such techniques has been analyzed in detail in the work of Harrigan (2007, 2008, 2009), and also in (Turner & Horbay, 2004) as concerns their statistics, ethical and legal aspects, and their causal connection with player's distorted cognitions.

The pre-manipulation by manufacturers of the frequency of award symbols can also occur in direct near-misses through the same virtual reel mapping. Of course, since the reels do not "spin" independently, there is actually no chronology for such a near-

[3] skin conductance levels
[4] sin conductance responses

miss; however, the player perceives a false chronology with the illusory spin. We can talk about pre-manipulation even in slots with classical mechanical reels, given that the reels are not weighted equally and the pre-weighting can favor certain combinations holding award symbols on some but not all reels. Let us call this type of near-miss an engineering or *artificial* near-miss[5] (which may be direct or indirect, per the previous typology), and distinguish it from the near-miss by pure chance (which is also a direct near-miss; call it *chance* near-miss). Observe that the name "chance near-miss" is entirely justified only in relation to ethical aspects, namely, if certain statistical and probabilistic information about the game is available to the player, for if such information is hidden, we cannot distinguish clearly between *pure* chance and chance through manipulation (with respect to the near-miss). For more on the ethical aspects regarding the exposure of the mathematical parameters of the games of chance, see (Bărboianu, 2014). Such pre-manipulation yielding an artificial near-miss has its equivalent in scratch cards, in a flat form, where the award symbols are pre-printed in clusters to create the near-miss sensation when uncovering them. The technique has been analyzed recently in (Strange et al., 2017). Studies on scratch-card near-misses (Stange et al., 2016, 2017a, 2017b) found that such outcomes exert strong effects on a player's physiological arousal and frustration, and are different more in intensity than in nature from the effects reported in the case of slots.

The central role of the artificial near-miss effects in slots (especially) and scratch cards is perhaps one of the reasons that these games gained the focus of problem-gambling research in the last decade. However, classical chance near-misses specific to other games of chance generate their effects on the gambling behavior, too, and it may be hypothesized that such effects vary in degree or amount and not in nature from the effects of slots near-misses. The simplistic definition of the near-miss as "failure that is close to being successful" supports this hypothesis, because it is actually an equivalent rephrasing of the paradigmatic "I was that close," expressing player's immediate observation and emotions following the near-miss. Indeed, observe that our typology (direct-indirect, artificial-chance near-miss) is more an object of study for researchers, since for the players there is only one type of near-miss (per the simplistic definition) associated with a

[5] It is sometimes called "*built-in* near-miss".

certain moment of the game or with a certain part of an outcome. Whether players distinguish further between artificial and chance near-misses is a matter of personal information and education. If they have this information, it is possible that the near-miss effects change in intensity; however, there will always remain the chance near-misses, as with any game. If they don't have that information, they go ahead with the distorted cognition[6] that an artificial near-miss is a chance near-miss. But the most relevant question is this: Do players distinguish between the near-miss as a gaming phenomenon and its immediate effect? If the answer is negative, since the near-miss effects are the same in nature for all types of near-misses, research in cognitive-therapy methodology may concentrate on classical chance near-misses as being more accessible. If the answer is positive, a more precise descriptive definition for the near-miss is necessary, one not identifying the phenomenon with its effects, as is the case with the simplistic one used thus far.

Past research on the near-miss has had preponderantly as its target the industry-related causes of the near-miss phenomenon and the psychobiological effects on players, with less attention given to the prevention and counseling of the near-miss problem-gambling behavior. This observation is also in line with Blaszczynski and Silove's (1995) conclusions that research has focused on the etiology of cognitive distortions in gambling, but findings from these studies have not yet been systematically translated into treatment programs. In this paper we shall argue that a focus on the near-miss phenomenon itself may prove fruitful in that it is able to suggest strategies for cognitive interventions against excessive gambling due to near-miss, and these strategies can be generalized also to other game-related risk factors. Such a possibility and the suggestions themselves yield a framework in which hypotheses can be further tested through empirical research.

In section 2, we shall sketch in basic mathematical terms a definition of the chance near-miss, which applies to all games of chance. The definition will be designed to reflect both the near-miss as a gaming phenomenon (strictly related to the game itself) and as it is perceived by the player. The definition will incorporate, in mathematical

[6] This distortion is different in nature from the classical gambling fallacies regarding probability, randomness, etc. It is actually a lack of information (of an ethical type) which prevents a distinction in nature for a gaming phenomenon.

terms, the description of the concepts of 'closeness' and the measure of that closeness in the common usage of "I was that close." In section 2, we shall argue that cognitive distortions associated with both the observational and intentional character of "I was that close" are categorizable in what is usually called gambling fallacies and can easily be corrected or discarded through an elementary analysis of the mathematical relations between the probabilities associated with events expressed in the near-miss and their probability fields. In section 3, we identify the primary cause of both the erroneous perception of a near-miss and the gambling fallacies associated with it in a constitutive element of the definition of the near-miss, namely the dimension of the outcome (as vector or combination), or its split into matching and non-matching parts. Arguing that such a split has no relevance for any goal of a general or particular mathematical model representing that game, and using the epistemology of the constitutive concepts of our mathematical representation (in which the entire epistemology of the near-miss can be reduced to an atomic content-free non-interpretable set-theoretic description), we can extract a specific potential strategy, which we call the *no-split strategy*, to be tested and used against the effects of the near-miss. Further, we can consider a general *reduction strategy* for other risk game-related factors. In section 4, we provide a brief overview of the results of studies on whether mathematical knowledge can change gambling behavior and determine that these results are not conclusive. By generalizing the reduction strategy to involve other risk factors associated with the characteristics of the games, we sketch a primary framework for further research, within which we can test the general hypothesis that *standard* mathematical knowledge is not effective in prevention and cognitive therapies, and the possible missing complement might be the *epistemology* of the mathematics of gambling involved.

2. Definition of near-miss and its probabilistic model

Consider a game of chance and denote by O a final outcome, that is, the outcome not followed by another outcome in that game and on the basis of which the award is given if O matches a winning rule. O is a combination, arrangement, finite sequence, or vector[7] consisting of n partial outcomes $O=(e_1, e_2, \ldots, e_n)$, where e_i are one-dimensional outcomes (items) in the form of symbols, cards, or numbers, depending on the game. The partial outcomes e_i may physically occur at once or in sequential stages of the game. In the latter case (for instance, in lottery, bingo, poker games, blackjack, classical mechanical slots, or craps), the indexation (1 to n) represents the size and chronology of the outcome O. In the former case (for instance, in coin toss or one-die roll, or virtual-reel slots if we ignore the false spin of the reels, or instant lottery if we ignore the time of scratching), it represents only the size of O.

For a given n-size winning outcome W as defined in the game's rules, if W differs from O in only one element e_i ($O-W=\{e_i\}$ in set-theoretic denotation), then the occurrence of O is said to be a near-miss of W[8]. As order does not count, consider e_n as the missing element from W. In reality, in case of a real or illusory chronology of O, this situation is the most frustrating and emotional for the gambler.

The finite sequence $S_1=(e_1, e_2, ..., e_{n-1})$ is that part of the outcome O that matches the winning outcome W, while $S_2=(e_n)$ is that part that does not match. We can then represent the near-missed outcome as a bi-dimensional combination $O=(S_1, S_2)$, where S_1 is multi-dimensional and S_2 is one-dimensional. For example, in slots, considering a near-miss $O=(e_1, e_2, e_3)$ on a standard payline crossing over three reels which stop one after another, sequence S_1 consists of the partial outcomes on reels no.1 and no. 2, which are two-dimensional, while sequence S_2 consists only of the partial outcome on reel no. 3, which is one-dimensional. However, the representation can be further generalized so as to have any dimension of the two or even more, having more than two partial sequences S_i and allowing the non-matching part to be anywhere in the sequence, not necessarily the final one. Staying with such generalized representations of the near-miss, we can identify this phenomenon in all games of chance. Below are few examples:

[7] For our purposes, the order of the elements or the direction in which they appear is not important.
[8] The definition can be generalized further to have more than one partial outcome missed, or, better, to have a certain ratio between the number of missed items and the total size of W. In other words, we can gradualize the near-miss. For the purpose of simplicity and because the further arguments do not depend on such particularities of the near-miss, we shall retain the definition with one item missed.

In Texas Hold'em poker, a near-miss of a player can be represented as $O = (S_1, S_2, S_3, S_4)$, where S_1 consists of the player's two hole cards, S_2 of the three flop cards, S_3 of the turn card, and S_4 of the river card. The missed card or cards for a valuable formation can belong to any of the four sequences.

In five-draw poker, a near-miss can be represented for example as $O = (S_1, S_2)$, where S_1 consists of three cards hold and S_2 of two replacements (if two discarded).

In blackjack, assuming a bust after four hits, the near-miss can be represented as $O = (e_1, e_2, e_3, e_4, e_5)$, where e_5 is the busting card.

In sport betting, the near-miss has the same representation $O = (e_1, e_2, \ldots, e_n)$, where the first $n - 1$ elements of O are the outcomes of the matches/races correctly predicted and the nth element is the outcome of a match/race wrongly predicted.

We can call *combinatorial* near-misses those specific to games whose final outcomes are evaluated just as combinations of items and not through other amounted value except for the number of the particular items (as in slots, lottery, bingo, card games, etc.) and *cumulative* near-misses those specific to games whose final outcomes are evaluated through sums of values attached to the partial outcomes (as in blackjack, baccarat, and some gaming situations specific to dice games like craps and backgammon). There is also a trivial type of near-miss in roulette, which we can call *physical* near-miss, namely the ball landing on a number spatially adjacent on the roulette wheel to a number on which the player has placed a bet.[9] For both combinatorial and cumulative near-misses, the above definition applies, since a cumulative win can be represented though a combinatorial unfoldment of those combinations submitting to the winning rule (That is, for instance, instead of saying that the winning sum of the values of two cards should be 20, we can say that the winning combinations of cards are (10, 10), (10, J), (10, Q), (10, K), (J, 10), (J, J), and so on[10])

Overall, given the possibility of grouping or ungrouping the elements within a combination, we can generally represent a near-miss as a bi-dimensional combination of

[9] Physical near-misses would also be specific to other wheel games or physical-skill games like those based on throwing objects at a target. However, the results of the current section do not apply to physical near-misses.

[10] and unfolded down to show values and symbols

two sequences of one-dimensional partial outcomes $O=(S_1, S_2)$, that is, the matching and non-matching parts, where S_2 can be considered as one-dimensional (one missed item e_n) if any chronology is ignored. With this representation, let us retain that:

1. The near-miss as a gaming phenomenon refers to the entire vector or combination (S_1, S_2) and not to a part of it. Indeed, the "miss" refers to the whole winning outcome as being different from *O*, and the "near" refers to the same difference.

2. The near-miss effect follows the near-miss phenomenon; therefore, it appears *after* this phenomenon is complete, namely after S_2 occurs, regardless of whether the indexation (1, 2) represents a gaming chronology.

3. Within the near-miss effect, the gambler creates a mental representation of a *split* of the final outcome *O* in two parts, namely the matching part S_1 and the non-matching part S_2; this split can be a simple combinatorial split of *O*, a chronological split, or both; without this split, there will be no near-miss as defined.

The above observations help us to distinguish more clearly between the near-miss and the near-miss effect, as well as between the near-miss gaming phenomenon and the near-miss gambler's phenomenon (or gambling near-miss).

Let us see now what probabilities are involved in the near-miss as a probabilistic event. First, note that S_1 and S_2 may or may not be independent[11]. For instance, in dice games or classical mechanical slots, they are independent, since dice roll and reels spin independently of each other; in poker games, blackjack, and virtual-reel slots with RNG, they are not – in such card games, occurrences of certain cards depend on the cards already dealt; in such slots, the reels are illusory and the resulting symbols are combined in pre-determined combinations (which the RNG just randomly chooses at the "spin"). The probabilities of either *O*, *W*, S_1, or S_2 as gaming events, are defined and calculated in two probability fields with specific sample spaces characterized by the information available[12] before the measured event occurs.

[11] that is, the probability of one does not or does depend on the probability of the other, respectively

[12] The sample space is the set of all possible outcomes as elementary events (combinations of the same size in our context). The probability field consists of a sample space, the set of all subsets of this sample space (called the field of events, which is a Boolean algebra) and a probability-function defined on the field of

a) If S_1 and S_2 are independent, we have the probabilities:

$P_o(O) = P_o(S_1) \cdot P_o(S_2)$ as the probability of *O*, in the probability field of the information before S_1 occurring (this is denoted by the *o* (original) with P_o). (1a)

$P_o(W) = P_o(S_1) \cdot P_o(W - S_1)$, as the probability of *W*, in the same probability field as above.[13] (2a)

(1a) and (2a) just reflect the definition of two independent events, for which the probability of their conjunction is the product of their individual probabilities.

Now, right after S_1 occurs (at the moment of the split, before S_2 occurs)[14]:

$P_s(O) = P_s(S_2) = P_o(S_2)$, in the probability field of the information at the moment of the split [this is denoted by the *s* (split) with P_s]. (3a)

That information is the occurrence of S_1.

$P_s(W) = P_s(W - S_1) = P_o(W - S_1)$, in the same probability field as right above. (4a)

From 2a and 4a it follows through subtraction that:

$P_o(W) - P_s(W) = P_o(W - S_1) \cdot \left[P_o(S_1) - 1 \right] < 0$ (5a). Of course, it was expected that $P_o(W) < P_s(W)$ since a part of *W* occurs, as *a priori* and *a posteriori* probabilities.

It also follows through division that: $\dfrac{P_s(W)}{P_o(W)} = \dfrac{1}{P_o(S_1)}$ (6a). If we take here S_1 as a variable (*W* is given as per game's rules and thus $P_o(W)$ is constant, while $P_s(W)$ depends on S_1), we have that the product $P_s(W) \cdot P_o(S_1)$ is constant, as being equal to $P_o(W)$.

b) Similarly, if S_1 and S_2 are not independent, we have these probabilities:

$P_o(O) = P_o(S_1) \cdot P_s(S_2) = P_o(S_1) \cdot P_o(S_2 | S_1)$ (1b)

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events. The information available when measuring a gaming event in probability is actually the sample space at *that moment*, consisting of the *possible* outcomes. These are determined by taking into account the items out of play (cards already dealt, symbols already having occurred, and so on).

[13] The denotation of difference in the last factor is set-theoretic, that is, the elements that are in the first set and not in the second.

[14] Note that time reference (occurrence, moment, after, right after, before) in this part of the section has a probabilistic nature and is thus conventional. It amounts to the available probability information and not to placement on a real timeline, although this may be the case in particular games and situations.

$$P_o(W) = P_o(S_1) \cdot P_s(W - S_1) = P_o(S_1) \cdot P_o(W - S_1 | S_1) \quad (2b)$$

$$P_s(O) = P_s(S_2) = P_o(S_2 | S_1) \quad (3b)$$

$$P_s(W) = P_s(W - S_1) = P_o(W - S_1 | S_1) \quad (4b)$$

What you see in (1b) and (2b) with the first equality is the basic formula of the conditional probability for two non-independent events, or a trivial instance of Bayes's theorem. The second equality in (1b) and (2b) is just a symbolistic manipulation, for showing the conditionality and having the probabilities in the same probability field.

And similarly, from 2b and 4b through subtraction it follows that

$$P_o(W) - P_s(W) = P_o(W - S_1 | S_1) \cdot \left[P_o(S_1) - 1 \right] < 0 \quad (5b)$$

and through division that $\dfrac{P_s(W)}{P_o(W)} = \dfrac{1}{P_o(S_1)}$ (6b) , hence $P_s(W) \cdot P_o(S_1)$ is constant with S_1 as variable.

This is all we need for a complete description of a near-miss in basic mathematical terms. But why do we need the probability part? Why isn't the first combinatorial description enough? The answer is this: The near-miss is both a gaming phenomenon and a gambling phenomenon, and the latter is associated with the gambler. The near-miss gambling phenomenon is expressed through the paradigmatic "I was that close." This expresses both an observation and an intention. The observation is that of being very close to a win. But what is "close," and what is "that"? And where or when (in what time frame) is "was"? These can be answered in the combinatorial terms which define the near-miss: "Close" is – for the gambler – a measure of the size of a combination, that is, in *how many* elements the outcome differed from the winning outcome; it also can be considered as a metric distance in a discrete setup. "That" is the numeric amount of that difference; we took 1 as the standard size of a near-miss. "Was" is placed at the moment of the split which allows this measurement. The questions arise as to whether this measure is right or relevant and what would be the adequate measure for the near-miss. Outside this context, there is nothing wrong in defining a measure in this way, but the question indicates the implicit intentional aspect of "I was that close," saying in effect, "…therefore I will play again to be in that situation again, and perhaps next time I won't miss." This intentional aspect was confirmed by all the empirical studies on the near-miss effects. The intentional aspect involves prediction under uncertainty, and the only rigorous measure available for the quantification of uncertainty is probability; thus, we have to engage the probabilistic description in the definition of the near-miss for the clarification of "close" and "that." This argument also explains why the simplistic definition used in research must be reformulated and refined.

3. Cognitive distortions, fallacies and conceptual rearrangement

Assume now that we want to change the measure of the "closeness" from size-metric to probability. With this new measure, we want to evaluate the increase in probability of *W* from *a priori* to *a posteriori* (from $P_o(W)$ to $P_s(W)$), but how should we evaluate this increase, through difference ($P_s(W)-P_o(W)$) or ratio ($P_s(W)/P_o(W)$)? The difference takes values in interval (0,1) and the ratio in (0, ∞), so the former seems to be more appropriate since it can be a probability itself. However, observe in 2a, 6a, 2b, and 6b that both depend on probability $P_o(S_1)$, and this dependence is now problematic because the measurement of the "how close" should be defined only at a certain moment (that of "I was"), the moment of the split. S_1 is associated with that split (it is the split that determines the two sequences), but $P_o(S_1)$ is associated with the original sample space where the outcome *O* is an elementary event, and this means in reality *all possible* outcomes occurring in an infinite series of potential trials. This is the nature of probability. The definition of probability as a measure makes no sense for a finite number of trials, and even more so for a single instance of time. We can measure in probability the heads of the interval (*W* at the original and final moments), but we cannot measure how close to them is an intermediary[15] probability of *W*, because the measure we use should be defined in the same measure space.

Apparently paradoxically, "that close" in the gambler's metric is not adequate since it does not involve probability, while probability-based measurements are not

[15] in a chronological sense

consistent with the moment of “I was.” What options do we have? Let us measure relative to the probability of the sure event (1) instead of to the *a priori* probability of *W*. The “distance” (difference) looks like this: $1-P_s(W)$. This measure seems adequate at a first glance; it has the form of a probability and expresses how close (in probability) we come to the winning event *W*. However, $P_o(S_1)$ is again involved, this time in the information of the probability field in which P_s is defined, since P_s is conditional on $P_o(S_1)$. Nothing changed; instead of the constant $P_o(W)$ we have now constant 1, but the same problematic dependence.

Do we have an alternative? Yes, we can just change the whole foundation on the basis of which the near-miss was defined. Since the essential cause of the inability to define an adequate measure of “that close” was the representation of the split through the *size* of the outcome vector as (S_1, S_2), which forcibly induced an intermediary event to measure, we should drop it and leave *O* in its canonic form $O=(e_1, e_2, \ldots, e_n)$. In this form, we won’t have any mathematical near-miss, but this a good thing, as we argued previously. With this degeneration, a near-miss becomes a simple failure. There is nothing wrong with our representation $O=(S_1, S_2)$, and the entire math associated with it stands. But this mathematical model does not *represent* a real near-miss in all its relevant aspects since it cannot account for the adequacy of all “close,” “that,” and “I was.” This relation of the model to the reality is part of the epistemology of this mathematical representation.

We still have a gambling near-miss, so let us focus now on its intentional aspect, which is a double prediction with effects: (P1) “I will be in this (near-miss) situation again” *and* (P2) “At that time (or another ‘next time’), I will win (*W*)”.

In our probability terms, the realization of prediction P1 depends on $P_o(S_1)$, or in frequential terms, the higher this probability, the higher the frequency of S_1. The realization of prediction P2 depends on $P_s(W)$, and the two probabilities are related through their product’s being constant, according to 6a and 6b. Of course, the optimism is expressed through the confidence in the realization of *both* P1 and P2. However, since the product of probabilities of P1 and P2 is constant, the higher the one is, the lower the other. Any strong confidence in one prediction should thus weaken confidence in the other, and the overall confidence should not be influenced by either of the two alone. Moreover, as regards the actual numerical probabilities, it is well known that they are usually both very low in most of the games of chance.

Just to insert illustrative examples, let us take one from lottery and one from blackjack:

Consider a one-line ticket with six numbers in the 6/49 lottery and the winning outcome having four winning numbers. Let us assume you had three of them already hit in the first five numbers drawn and are awaiting the sixth to be drawn. The probability of hitting three winning numbers within the first five drawn is 0.004961, that is about 0.5% (Infarom, 2005), as the probability $P_o(S_1)$. The conditional probability of hitting the sixth drawn number is 0.06818 (about 6.8% – a big one!), that is $P_s(W)$. However, their product is 0.000338; that is almost 0%! It doesn't look like a *near*-miss any more, does it?

Consider a one-deck blackjack situation in which you are the only player, and you achieved 19 points from the first two cards, the dealer's card is not A or 2, and you expect a blackjack with the next card. The probability of achieving the 19 points with the first two cards is 6.03318% as $P_o(S_1)$, while that of getting A or 2 with the third card is then 8.16326% (Infarom, 2005) as $P_s(W)$. Their product is 0.4925%. Again, not that "near"! But anyway this is our "near" and not the gambler's, whose "near" is actually evaluated at the moment of the split.

Observe that any cognition, optimism, or behavior in opposition to the line of mathematical thought preceding these examples falls within what we usually call standard gambling fallacies. Our context reveals a special form of conjunction fallacy.
The conjunction fallacy is a general probability misconception based on a subjective, wrong estimation of the likelihood of both of two independent events occurring. It tends to overestimate the probability of the double occurrence (sometimes through the addition of the two likelihoods, and as such, thinking it to be even higher than each of the two likelihoods independently), by ignoring the mathematical facts: 1. the probability of such

conjunction is the product of the individual probabilities; 2. the product of two subunitary numbers is usually much lower than each factor (the lower the factors, the higher the difference between product and each factor).

The conjunction fallacy has been widely discussed in the field of cognitive sciences. Tversky and Kahneman (1983) have long argued that humans do not reason rationally and are subject to many "cognitive illusions," of which the conjunction fallacy is a special one. Hertwig and Gigerenzer (1999) claimed that this fallacy may result from the human capacity for inferring additional semantic information from social situations and not from "flaws" in human cognition. Recently, Costello and Watts (2017) provided a behavioral explanation for the high rate of the occurrence of this fallacy, claiming that people reason according to probability theory but are subject to random noise in the reasoning process. Gambling probability-related fallacies are believed to be an important cause of the development of problem gambling. However, this evidence is tenuous due to the lack of consensus on what exactly constitutes gambling fallacies and the adequacy of instruments that ostensibly measure them (Leonard & Williams, 2016). The most striking manifestation of the conjunction fallacy seems to be in sport betting, where many gamblers overload their tickets with several events considered as fairly predictable, ignoring (or not knowing) the fact that the likelihood of their conjunction is reduced almost exponentially with the number of events.

The conjunction fallacy of the near-miss is similar, but has its own specificity. First, the two linked events could also be non-independent, as we saw in the abstract part of the previous section (where denoted as S_1 and S_2). Second, the two events, although placed within one game with the gaming near-miss, become placed at different time (and game) moments with the gambling near-miss given the intentional aspect. This timing element adds a specific fallacy to the "product" fallacy: Even though the gambler may estimate correctly the probabilities of the various gaming events, and even though this person may multiply them instead of adding, once the gambler places himself/herself in the time of the prediction P2, s/he actually ignores the first factor of the product, namely $P_o(S_1)$. Ignoring this probability means also ignoring the *physical* possibility of the prediction P1. In gambling terms, how many games would be required for the gambler to be in the same position, so as to expect $W - S_1$ to occur as S_2?

$P_o(S_1)$ is usually lower than $P_s(W)$, and our previous gaming examples also reflect that. The gambler may perceive this difference in his/her gambling experience and this might be a factor influencing the near-miss effect and behavior. But $P_o(S_1)$ is always a low probability. Our lottery and blackjack examples in fact show unusually high values for this probability. Staying with the previous lottery example with 0.004961 as the value for $P_o(S_1)$, assuming one draw weekly, in frequency and mean terms, the lottery gambler would have to play for another 17 years to be in the same situation with three numbers hit from the first five drawn. Playing with more tickets is an option for reducing this time, but at the cost of increasing expenses accordingly. Finally, if somehow the gambler reaches that situation, P2 will happen with a 6.8% chance. If not, the gambler must be prepared to wait for another 17 years, and the "near" is actually "far." Of course, this example may not be the most relevant; it is just illustrative for the point made.

The special conjunction fallacy of the near-miss was clearly expressed mathematically in the previous section, and that description can stand for a potential cognitive intervention against it, since its main argument is the basic mathematics of a product that is constant and in which we cannot increase one factor without lowering the other. This is standard mathematical knowledge, as are the particular values of the two probabilities in various games and situations. Such knowledge is required for correcting the general conjunction fallacy. However, the discussion at the begining of this section in regard to the adequacy of measuring the closeness is not standard mathematical knowledge, but epistemic knowledge of the mathematical modeling, and it is not deliverable in standard mathematical education (either curricular/institutional or gambling-targeted current education). It is also the third element of specificity for the near-miss conjunction fallacy. Such knowledge is not about how mathematics works, but rather, what mathematics is about: its truths, the nature of its concepts down to the constitutive concepts, the adequacy of the definitions, about the intimate relation between mathematical concepts and reality as well as between mathematical models and the empirical contexts they represent and the justification for the inferences we made in the empirical contexts through these models. This is *active* knowledge, implying cognitive processes and behavior through critical thinking, organizing systems of beliefs, and even changing conceptual frameworks. It is different from the *plain* mathematical knowledge acquired through standard education.

In the particular case of the near-miss, the mathematical representation revealed the specific fallacy and provided the essential cognitive elements for fighting against it within a potential cognitive therapy, in the form of the (plain) mathematical explanation. However, the combinatorial definition of the near-miss failed to represent adequately the near-miss phenomenon in its complexity, and the conceptual rearrangement was that of dropping the *split* of the outcome combination in two sequences. With this change, the outcome is seen as nothing more than an elementary event in the primary sample space of the game; it has the same nature and status as other outcomes, winning or not. It is just an atomic *n*-dimensional element of a large set of combinations and has no other empirical content or interpretation given by any split of it. In this representation, there is no near-miss, nor near-miss effect. This epistemic action, however radical it may look, is consistent with the epistemology and practice of the probabilistic modeling of games of chance.

Indeed, in any mathematical model, the whole outcome O is considered, with no split, either merely combinatorial or chronological. There is no mathematical reason for treating S_1 and S_2 separately with respect to any of the functions and goals of a mathematical model (representation, prediction, measurements, optimization, etc.). O is an element of a mathematical structure having the same status as any other outcome; "parts" of O are irrelevant for the epistemology of the mathematical models. In a probabilistic model, O and sets of outcomes are events belonging to a Boolean structure, but such an event has nothing of the complexity of an event in real life; it is just denoted by the same word with a different semantics. Event O has a certain *a priori* probability of occurrence $P(O)$, which is the only probability that counts toward any theoretical or practical aim, since O is a *final* outcome. There is no reason to consider probabilities $P(S_1)$, $P(S_2)$, or $P(S_2|S_1)$ since they are not relevant for any objective measurement except perhaps for practical statistics or studies regarding the frequency of occurrence of the chance near-miss.

Not accidentally, the epistemology of the non-split outcome also indicates that the mathematical explanation of the special conjunction fallacy of the near-miss was adequate in the respect that probability cannot be limited to a single instance of time, not even to a limited interval,[16] but is constitutively dependent on the all possible instances. The split we made notwithstanding, the final probability of winning does not depend on that split and is the probability of a non-split outcome. It is an epistemic-mathematical form of explanation or revelation (for the gambler) that s/he is not constantly near-missing or near-winning (a well-known distorted belief as an effect of the near-miss), but constantly losing. Or, in other words, the near-miss is of no relevance and an ingenuine problem for mathematics due to the inadequacy of the possible measurements, and as such it should be for the gambler.

This is a case where epistemic knowledge is complementary to standard mathematical knowledge toward effectiveness. What this combined knowledge did in the analysis of the near-miss was to drop the split and to reduce the representation of the game to its primary mathematical model, in which the near-miss does not even exist. We may call this action for the particular cognition of the near-miss, the no-split strategy and observe that it has an immediate methodological generalization amounting to the *general* representation of a game through its mathematical models. Since the mathematical models reflect only what is *relevant* for the game (that is, its functionality, house edge, all probabilities of the winning outcomes, and objective statistical indicators such as expected value, all described with the adequate mathematical concepts), and as such the near-miss and other fallacious concepts from a gambler's perspective are not reflected, we can fairly assume that a cognition method based on this representation is worth exploring further. In the particular case of the near-miss, the no-split strategy aims to eliminate the concept of near-miss (and consequently, the near-miss effect), since with no split, there is no near-miss; also the near-miss is eliminated through non-representation based on criteria of relevance.

The no-split strategy can be applied to combat the effects of both the artificial near-miss and the chance near-miss (somehow abusively, in the sense that such

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[16] This is also specific to the classical gambling fallacy of perceiving randomness and independence of the events.

application is based on a possibly distorted cognition of the players about this distinction), as we argued in the introductory section. Both strategies (the particular and the general one) – yet to be tested for effectiveness through further research – use mathematical knowledge and epistemic knowledge, but their implementation in treatment sschemes should use the tools and methods of cognitive psychology.

4. Mathematical education in problem gambling. What is missing?

Past studies on the impact of a mathematical didactic intervention with gamblers, testing whether learning about mathematics of gambling does change gambling behavior, were mainly empirical (see Abbott & Volberg, 2000; Gerstein et al., 1999; Hertwig et al., 2004; Lambros & Delfabbro, 2007; Pelletier & Ladouceur, 2007; Peard, 2008; Steenbergh et al., 2004; Williams & Connolly, 2006). The experimental setup of those studies was of two types with respect to the teaching module, which was either assimilated with standard Probability Theory & Mathematical Statistics courses taught in secondary and post-secondary schools but including more applications from the games of chance, or was designed and taught outside the curricula. The content of most of the teaching modules fell within *Introduction to* and *Basics of Probability and Statistics*, covering definition and properties of probability, basics of descriptive and inferential statistics, discrete random variables, expected value, classical probability distributions, and central limit theorem. The modules were packed with examples and applications from games of chance and had lessons dedicated to demystifying mathematically the common gambling fallacies.

Both types of studies have yielded contradictory, non-conclusive results, and many of them unexpectedly tended to answer *no* to the hypothesis that gamblers receiving such specific mathematical education show a significant change in gambling behavior after the intervention. Keen et al. (2017), in their systematic review of empirically evaluated, school-based gambling education programs (20 papers and 19 studies), found that only nine of the studies attempted to measure intervention effects on behavioral outcomes, and only five of those reported significant changes in gambling behavior. Of these five, methodological inadequacies were commonly found including brief follow-up periods, lack of control comparison in *post hoc* analyses, and inconsistencies and misclassifications in the measurement of gambling behavior. Besides any discussion on the relevance and adequacy of the experimental setup of these studies concerning sampling, evaluation, and testing of hypotheses (which may provide a partial explanation for the contradictory results), two main questions arise:

1. What mathematical knowledge would an optimal teaching module contain with respect to the intended effect of limiting excessive gambling? In other words, what is missing in the current didactic interventions?

2. How important is the previous mathematical background of the gambler for reaching the intended effect? In other words, even if content and structure of the teaching module are optimal, is it enough for the student to understand and assimilate the mathematical facts presented, or is mathematical thinking necessary – not only computational but also conceptual inquiring, and highly critical thinking attainable only through long-term previous mathematical education and experience?

Both questions require further research, both theoretical and empirical as well as interdisciplinary. The interdisciplinary requirement is indicated even from our near-miss example, where cognitive sciences, psychology, education sciences, mathematics, and epistemology are involved.

Mathematics is strongly connected to gambling through the mathematical models underlying any game of chance. Games of chance are developed structurally and physically around abstract mathematical models, which are their mere essence, and the applications within these mathematical models represent the premises of their functionality and the business of their owners. No game is developed before its mathematical models are designed and checked mathematically, so the real game stems from its mathematical models and does function according to them. In problem-gambling research, treatment, and prevention, we cannot separate the gambler from the game he plays; therefore, mathematics should be given a central role in an optimal psychological intervention.

Thus far, the contribution of mathematics to psychological intervention in problem gambling was reduced to *facing the odds* and *correcting standard misconceptions* through *standard* mathematical knowledge. However, these components of the intervention are not enough, and some of the past empirical studies have confirmed this insufficiency (see Hertwig et al., 2004; Steenbergh et al., 2004; Williams & Connolly, 2006). Turner et al. (2008) found instead that such an intervention improved students' knowledge and critical thinking and offers promise with respect to future gambling behavior.

Our analysis of the near-miss phenomenon revealed that special kinds of misconceptions and fallacies can potentially be corrected through epistemic knowledge in conjunction with the standard mathematical knowledge – that is, what we called the no-split strategy in the previous section. It follows then to investigate theoretically whether other types of gambling misconceptions can potentially be eliminated through this conjunction of knowledge.

The mathematical description of the near-miss phenomenon also suggests a broad strategy that can be generalized to the entire spectrum of games of chance, the reduction strategy. This strategy of reducing the games, through mental representation, to their mathematical models, would theoretically place outside the mathematical models not only the near-misses and their effects, but also other factors of risk related to the characteristics of the games, such as the illusion of control, and of course, all sparkling cases, colors, lights, illusions, sounds and the like. All these suggestions draw upon a preliminary framework for further research, both theoretical and empirical, within which various hypotheses can be tested. Hypotheses related first to the potential of this strategy to inform new treatment approaches and second to the effectiveness of such approaches should be formulated in terms of problem-gambling psychology and addressed with the tools of this discipline.

In proposing further development and research, we must be aware of the size and complexity of that project. The project needs preliminary research and pilot studies at the professional, institutional, practice, and methodological levels especially concerning the management of the indisciplinary knowledge, language and collaboration. Theoretical research should follow regarding the best way to blend epistemic knowledge with mathematics of gambling on one hand, and this combined knowledge with cognitive psychology on the other hand. Next, theoretical research is needed for sketching the optimal principles, content, and organization of a didactic module based on this knowledge. Empirical studies are then needed to investigate how gamblers receive such interventions, how previous mathematical education influences their reception of the interventions, and to test a primary experimental behavior as effect of an intervention. On the other side, once the effectiveness of such interventions is confirmed, further practical-methodological projects would ensure the correct implementation of the results to the entities and into the resources dealing with problem-gambling prevention, reduction, and treatment, including special modules designed for counselors.

5. Conclusions

Starting with essential distinctions between the near-miss phenomenon and effect, the artificial (engineered) and the chance near-miss, the direct and the indirect near-miss, and the gaming and the gambling near-miss, we concluded that the simplistic definition of the near-miss (as being "failure that is close to a win") as used thus far needs reformulation and refinement for the needs of contemporary research into this phenomenon. We have offered a basic mathematical description of the near-miss as a gaming phenomenon consisting of a combinatorial description associated with the probabilistic model it involves. By this description, we argued that none of the concepts of closeness and measurement of that closeness is clear or adequate in the paradigmatic expression "I was that close," and the only available way to ensure adequacy is to drop the split (either combinatorial or chronological) that we made for defining the near-miss as reflecting gambler's perceptions of it. Looking at the implicit intentional aspect of the observational "I was that close," we have analyzed the special type of conjunction fallacy specific to near-miss, and we have offered a mathematical explanation for eliminating it. Both this explanation and the analysis of the adequacy of the mathematical definition of the near-miss suggested two strategies, one particular (for eliminating the near-miss effects), and one general (for eliminating the effects of other game-related risk factors), to be implemented potentially in further cognitive interventions. Both strategies used epistemic knowledge along with the standard mathematical knowledge, and it can be hypothesized that this is the missing element for the alleged non-effectiveness of didactical interventions based on mathematical knowledge, previously designed for prevention and to change gambling behavior. This study provides a primary framework for a further large project of research whose main lines of initial design were outlined in the previous section.

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