Universality, Topic-Neutrality, Monism, and Pluralism in Logic

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Abstract

According to several views, logic is universal and topic-neutral; i.e., any correct logic system should be applicable to any topic, context, domain, etc. in exactly the same way. Universality and topic-neutrality are central to common characterisations of logic. Moreover, some authors take monistic positions to naturally assume universality and topic-neutrality, whereas taking pluralistic positions to reject them. In this paper, I introduce a framework allowing for a more fine-grained characterisation of monistic and pluralistic positions in logic, and their possible relations with the universality and topic-neutrality theses.

Keywords: logical pluralism, logical monism, topic, topic-specificness, non-classical logics.

1 Introduction

According to most philosophers of logic, including logical pluralists, logic is *topic-neutral* and *universal*. This means that rules of inference can and must be applied to any topic, context, domain, etc., exactly in the same way. In other words, the rules of inference provided by any logic system which is deemed acceptable should be applicable to any topic whatsoever (universality) and indifferent to the specific topic under consideration (topic-neutrality). Both concepts are central to common definitions of logic [cf. 8, 16], and have also been invoked in characterising some differences between logical monistic and pluralistic positions. For example, it has been argued that monistic positions naturally assume the logic-neutrality and universality theses, whereas pluralistic positions reject them, and rather adopt the opposite views that logic is non-universal and *topic-specific* [see 6].

Despite the aforementioned centrality of the concept, there does not seem to be a standard formal definition of it. Such a definition is nevertheless important for philosophical logic on at least two grounds. First, it would provide a notion with respect to which we could be sure to agree or disagree whether logic is universal and topic-neutral or not (cf. [8] for a similar point about the related concept of *formality* in logic). Second, it would provide a framework to distinguish between several kinds of logical pluralism and monism, some of which assume universality and topic-neutrality, and others which reject at least one of them. As they currently stand, these concepts are not sufficiently clear as to understand the differences between non-universalistic and topic-specific variants of logical pluralism and logical monism, or if all of those variants would be possible.

In this paper, I will explore the relation between the theses of the topic-neutrality and universality in logic and logical pluralism and monism. Section 2 will provide initial definitions of these concepts with respect to the notion of logic system. We will see, however, that those concepts cannot be asserted from logic systems themselves, but from theories of logic. I will provide a definition of theory of logic in Section 3, which not only establishes a list of correct logic systems, but also the topics to which it can be correctly applied. Finally, Section 4 will provide formal definitions of our four concepts and examples of theories of logic featuring all possible combination of these theses.

2 Logic Systems and Theories of Logic

In a more or less standard view, a logic system is a formal system of inference or a relation of logical consequence defined for a some language. The following definition should capture the idea.

Definition 2.1 (Logic System)**.** A *logic system* is a pair $\mathsf{S} = \langle \mathcal{F}_\mathsf{S}, \mathsf{Cons}_\mathsf{S} \rangle$; where $\mathcal{F}_\mathsf{S}$ is a set of schematic formulae and $\mathsf{Cons}_\mathsf{S} \subseteq 2^{\mathcal{F}_\mathsf{S}} \times \mathcal{F}_\mathsf{S}$ is a relation of logical consequence or consequence relation for short.

The expressions $(\mathcal{A}, A) \in \mathsf{Cons}_\mathsf{S}$, $\mathsf{Cons}_\mathsf{S}(\mathcal{A}, A)$, and $A \in \mathsf{Cons}_\mathsf{S}(\mathcal{A})$ denote that $A \in \mathcal{F}_\mathsf{S}$ is a S-consequence of the set $\mathcal{A} \subseteq \mathcal{F}_\mathsf{S}$, i.e., it logically follows from the set $\mathcal{A} \subseteq \mathcal{F}_\mathsf{S}$ according to the logic system S.

Consequence relations can be defined in a much more complex way. First, we can extend Cons_S to be a relation between subsets of $\mathcal{F}_\mathsf{S}$, so that $\mathsf{Cons}_\mathsf{S} : 2^{\mathcal{F}_\mathsf{S}} \times 2^{\mathcal{F}_\mathsf{S}}$. In this way, we could represent all the conclusions that can follow from a set of premises. We can also take $\mathcal{A}$ to be a sequence of premises instead of a set, in which case the order of the premises would matter. Moreover, we can define $\mathsf{Con}_{\mathsf{S}}$ through meta-inferential rules of the form $(\mathcal{A}, A) \in \mathsf{Con}_{\mathsf{S}}$ & $(\mathcal{B}, B) \in \mathsf{Con}_{\mathsf{S}} \Rightarrow (\mathcal{C}, C) \in \mathsf{Con}_{\mathsf{S}}$. This would be important for defining logic systems where the specific sets (or sequences) of premises we are dealing with is relevant to the validity of an inference. Since this is a first approximation to the subject, I will only consider plain inferential rules defined for statements as conclusions and sets of them as premises.

I will not specify whether $\mathsf{Con}_{\mathsf{S}}$ is a syntactic or semantic consequence relation. This will depend on the *theory of logic* we are dealing with, which – as we will shortly see – is not the same as a *logic system.* A $\mathsf{Con}_{\mathsf{S}}$ will express the most complete set of possible inference which, according to the theory of logic at hand, it is possible to do according to the specific system S. This seems to make $\mathsf{Con}_{\mathsf{S}}$ closer to a semantic consequence relation. However, if we are dealing with a theory of logic which has a syntactic conception of the consequence relation, then its $\mathsf{Con}_{\mathsf{S}}$ relation will be better conceived as syntactic calculi.

In any case, logic systems and consequence relations are, thus, formal or mathematical structures which can be used for representing what follows from what. In this paper, I will take propositions as the kinds of entities for which consequence relations are defined. Propositions are statements which can be true or false. Hence, the formulas in the formal language of a logic system stand for arbitrary statements which can be true or false. I will also take truth-preservation from premises to conclusion as the relation which consequence relations are supposed to represent.

I must nevertheless note that logic systems could also be applied to other kinds of statements such as imperatives or questions. Similarly, there could be logic systems dealing with something other than truth-preservation, like, for instance, meaning-preservation [cf. 10].[1] Although I think that we can analyse logic systems in this way, this paper will nevertheless focus in this more or less standard understanding of it. Hence, we will discuss universality, topic-neutrality, monism, and pluralism in logic with regard to logic systems with alethic and truth-transmissive consequence relations, and leave for a future work a similar study of these notions in a broader domain of consequence relations.

This clarification is important because logical pluralism and monism are commonly defined as theses about the consequence relation [cf. 3]. I will translate those formulations in terms of logic systems as follows:

[1] I want to thank Tyko Schuff for making me aware of this possibility in personal conversations and in his talk 'Logic beyond truth preservations' in the event *Logical Pluralism* (LMU, Germany, 9–10 Oct. 2023).

Logical pluralism There is more than one correct logic system.

Logical monism There is only one correct logic system.

Logical nihilism There is no correct logic system.

(I include the definition of logical nihilism for completeness.)

The universality and topic-neutrality theses, as we will shortly see, are best formulated also as theses about logic systems. Instead, they are usually formulated as theses about logic itself, more or less in the following terms:

Topic-neutrality Logic must be applied independently of the topic, context, or purpose we are dealing with.

Universality Logic can be applied everywhere (and everywhen).

A they stand, both theses are a bit ambiguous and can be interpreted as being equivalent, as they often are. An alternative way to formulate them is as theses about logical consequence, as follows:

Topic-neutrality Correct logic systems can be correctly used for all topics, contexts, or purposes.

Universality For every topic, context, or purpose some logic system can be correctly applied.

In our definitions of universality, topic-neutrality, pluralism, and monism we notice that, although these are theses about logic systems, they are not theses of logic systems themselves. Logic systems just have inference rules ready to be applied here or there. There is however no information about where should the system itself be applied nor in what way nor for what purpose. A logic system, by itself, says nothing about where and how it should be applied, since it is just a formal structure without content. In fact, a logic system can be used in contexts unrelated to inference or reasoning. For example, classical sentential logic can be used for representing electrical circuits. Hence, no logic system states that there is only one correct logic system, or that logic is topic-neutral. We assert this from another kind of framework: a theory of logic.

Now, a theory of logic might only need to point to a logic system if we assume monistic and universalistic views. No further specification is needed regarding the contexts, domains, or topics to which our logic system can be correctly applied if we assume – as it is often done – that logic is universal and topic-neutral. Thus, at least for the standard logical monist, we can reduce the concept of theory of logic to the concept of logic system, since we can define a theory of logic as being identical to the one correct logic system.

Logical pluralism challenges this definition a bit, for there is more than one correct logic system in this view. However, if our logical pluralist assumes the universality and topic-neutrality of logic, a theory of logic would only need to indicate which are those logic systems that are correctly applied to all contexts. Thus, for logical pluralist of this kind, the concept of theory of logic can also be reduced to that of logic system, since we can define a theory of logic as being identical to the collection of correct logic systems.

The real challenge comes from questioning the theses of logic's universality and topic-neutrality. Although these are often regarded as a defining attributes of logic systems, some authors have questioned this idea stating that some logic systems might be appropriate to some contexts or topics but inappropriate to others. This has resulted in views such as the principle systematisation [7], logical contextualism [1] and logical modalism [4, 6], the latter explicitly claiming that logic is topic-specific.

Rejecting the universality and topic-neutrality of logic might seem beyond acceptable to some readers. However, in order to make sense of what a theory of logic is, we need to understand what would it mean for logic not to have these features. As already argued, a logic system cannot be characterised in such terms since it is just a mathematical structure with no specification on where it should be used. But a theory of logic is much more than a formalism applic*able* to this or that topic. It is, instead, a proposal about which logical systems are or are not correctly applied to this or to that or to no topic, and in what ways. This means that we have to incorporate topics or domains as part of our characterisation of a theory of logic. Consequently it also means that we can no longer reduce the concept of theory of logic to that of logic system. In the next section, I will provide a concept of theory of logic which satisfies this requirement.

3 What is a Theory of Logic?

As previously mentioned, a theory of logic is much more than a formalism applic*able* to this or that context. It is, instead, a proposal about which logical systems are or are not correctly applied to this or to that or to no topic, and in what ways. For instance, a way to formulate the classical theory of logic, or *standard monism*, is by stating that the classical logic system is the only one which is correctly applied to all topics. A non-classical monistic theory, instead, would formulate that the one correct logic is some non-classical system.

Other theories of logic might propose that more than one logic system can be correctly applied to all topics, which would be a pluralistic theory. This kind of pluralism, though, does not say that this logic system applies to these topics and not to these other ones, while that system applies or does not apply to some other combination of topics. It plainly states that more than one logic system is applicable to all topics, which means that it is a topic-neutral pluralism. If the theory, instead, assigns to each system a collection of topics to which it applies, while excluding others to which it does not, then we are facing topic-specific pluralism.

Now, how can we more formally define a theory of logic? In very simple terms, a theory of logic would provide a set of acceptable logic systems and a set of topics, and relate those systems and topics according to their correct applicability. The following definition is to formalise this idea.

Definition 3.1 (Theory of Logic)**.** A *theory of logic* is a three-tuple $\mathfrak{L} = \langle \mathscr{S}, \mathscr{T}, \mathscr{R} \rangle$ such that:

- $\mathscr{S} = \{\mathsf{S}_1, \mathsf{S}_2, ...\}$ is a non-empty set of logic systems;
- $\mathscr{T} = \{\mathbb{T}_1, \mathbb{T}_2, ...\}$ is a non-empty set of topics;
- $\mathscr{R} = \{\mathbf{R}_1, \mathbf{R}_2, ...\}$ is a non-empty set of also non-empty two-place relations $\mathbf{R}_i$ such that either $\mathbf{R}_i \subseteq \mathscr{S} \times \mathscr{T}$ or $\mathbf{R}_i \subseteq \mathscr{S}^2$ or $\mathbf{R}_i \subseteq \mathscr{T}^2$ (we will focus on the first case).

Each of these elements are independently defined by each theory of logic. Let us explain in detail what they stand for.

The notion of topic is intentionally left vague. They can be regarded as contexts of application, subject matters, etc. They are whatever a theory of logic establishes as areas which *might* have distinct logical needs. Providing a specific set of topics $\mathscr{T}$, however, does not commit it to provide each $\mathbb{T}_i \in \mathscr{T}$ with a distinct logical treatment. It just means that, according to this theory, there are reasons for entertaining that possibility. For instance, Beall and Restall [3] consider several topics of application for which some logic system would be better suited. In the end, however, they assert that all acceptable logic systems are correct accounts of inference for all topics.

$\mathscr{S}$ is the set of all logic systems which satisfy the minimal formal criteria which our theory establishes for logic systems and consequence relations. I myself am happy to be quite liberal in defining consequence relations (following [2, p. 150]), by requiring no inference rule as a necessary condition for them. Other authors, however, would establish some minimal requirements. For instance, Beall and Restall propose transitivity and reflexivity as minimal properties of consequence relations [3]. Tarski [15] also demands monotonicity. Others would consider a classical characterisation in broad terms, which includes some formulation of the laws of non-contradiction, *tertium non datur*, and identity [cf. 11].

In all these cases, there are infinitely many mathematical systems satisfying those properties, and not all of them will be considered to have interesting applications. As Schurz notices, not all mathematical logic systems are philosophical logic systems, in the sense that not all of them have applications which are philosophically relevant [cf 14, sec. 1.5.1]. Continuing with Beall and Restall, they consider that classical, intuitionistic, and relevance logics are (for the most part) the only ones which have interesting logical applications, despite there being many more systems satisfying those criteria. They consider a series of purposes or topics for which some of these systems are better positioned than the others. However, as they subscribe the topic-neutrality thesis (or at least the related thesis that logic is formal), they will in the end assign all topics to all of these three systems.

We can specify this with a relation $\mathbf{C} \in \mathscr{R}$ such that $\mathbf{C} = \mathscr{S}' \times \mathscr{T}' \subseteq \mathscr{S} \times \mathscr{T}$ representing which logic systems in $\mathscr{S}$ are correctly applied to which topics in $\mathscr{T}$.[2] In the case of Beall and Restall, $\mathscr{S}$ would be the set of all transitive and reflexive mathematical systems, and $\mathscr{S}'$ would be a proper subset of $\mathscr{S}$ containing only the classical (C), intuitionistic (I), and relevance (R) logic systems. In other words, the theory of logic by Beall and Restall states that, for all topics $\mathbb{T}_i$, it holds that $(\mathsf{C}, \mathbb{T}_i), (\mathsf{I}, \mathbb{T}_i), (\mathsf{R}, \mathbb{T}_i) \in \mathbf{C}$.

Once again, I must note that each theory of logic defines each of these items independently. There is no common concept of topic, logic system, nor common criteria of adequacy between logic systems and topics. Each theory represents a proposal for defining and relating all of these concepts. This does not mean that theories of logic cannot be compared. A main motivation of this framework is precisely to allow for such comparison. But we must do it by taking into account that what two different theories of logic consider as a topic might not be the same.

I will now analyse what monism, pluralism, topic-neutrality, and universality mean in this framework by providing definitions of these concepts and examples of theories featuring combinations of these theses.

4 Definitions and Cases

The definitions bellow provide formal accounts of the theses of universality, topic-neutrality, monism, and pluralism in logic. In what follows, $\mathscr{S}'$ and $\mathscr{T}'$

[2] We can also include a relation $\mathbf{P}$ corresponding to pragmatical uses (see Section 4.5).

will stand for arbitrary non-empty subsets of $\mathscr{S}$ and $\mathscr{T}$, respectively.

Definition 4.1 (Universality)**.** A *theory of logic* $\mathfrak{L} = \langle \mathscr{S}, \mathscr{T}, \mathscr{R} \rangle$ is *universalistic* (i.e., it states that logic is universal) iff, for all $\mathbb{T}_i \in \mathscr{T}$, there is a S_j such that $\mathbf{C}(\mathbb{T}_i, \mathsf{S}_j)$. Otherwise, it is *non-universalistic*.

Definition 4.2 (Topic-Neutrality)**.** A *theory of logic* $\mathfrak{L} = \langle \mathscr{S}, \mathscr{T}, \mathscr{R} \rangle$ is *topic-neutral* iff $\mathbf{C} = \mathscr{S}' \times \mathscr{T}'$. Otherwise, it is *topic-specific*.

Definition 4.3 (Monism)**.** A *theory of logic* $\mathfrak{L} = \langle \mathscr{S}, \mathscr{T}, \mathscr{R} \rangle$ is *monistic* iff there is exactly one logic system $\mathsf{S} \in \mathscr{S}$ such that $\mathbf{C} = \{\mathsf{S}\} \times \mathscr{T}'$.

Definition 4.4 (Pluralism)**.** A *theory of logic* $\mathfrak{L} = \langle \mathscr{S}, \mathscr{T}, \mathscr{R} \rangle$ is *pluralistic* iff $\mathbf{C} = \mathscr{S}' \times \mathscr{T}'$ where $\mathscr{S}' \subseteq \mathscr{S}$ has at least two elements.

In definition 4.2, $\mathscr{S}'$ and $\mathscr{T}'$ stand for the sets of all logic systems and topics, respectively, for which there is some use according to our topic-neutral theory of logic. Hence, requiring that $\mathbf{C} = \mathscr{S}' \times \mathscr{T}'$ means that all those logic systems are correctly applied to all those topics. In other words, all logic systems correctly applicable to some topic are also applicable correctly to all topics to which some logic can be correctly applied. This leads to the following corollary, expressing the concept of topic-specificness:

Corollary 4.5 (Topic-Specificness)**.** A *theory of logic* $\mathfrak{L} = \langle \mathscr{S}, \mathscr{T}, \mathscr{R} \rangle$ is *topic-specific* iff there are at least two $\mathsf{S}_i, \mathsf{S}_j \in \mathscr{S}$ and two $\mathbb{T}_k, \mathbb{T}_l \in \mathscr{T}$ such that $(\mathsf{S}_i, \mathbb{T}_k), (\mathsf{S}_j, \mathbb{T}_l) \in \mathbf{C}$ but $(\mathsf{S}_i, \mathbb{T}_l) \notin \mathbf{C}$.

I will now provide examples of theories of logic considering all our formal definitions so far. In presenting these examples, I will use accounts which have been published by other authors. However, I will describe them in their most general features and not always considering all their relevant details.

I will consider all possible combinations of logical theories in terms of universality vs non-universality, logical pluralism vs logical monism, and topic-neutrality vs topic-specificness. This division treats logical pluralism and logical monism as mutually exhaustive views, which they are not given logical nihilism. In any case, I consider that incorporating logical nihilism into the framework would require a more thorough philosophical analysis of this concept than I can provide here. Hence, theories of logic involving this view will have to wait for a later work to be analysed under into this framework.

In what follows, the symbols $\mathsf{C}, \mathsf{I}, \mathsf{R}, \mathsf{P}$ will stand for pertinent systems of classical, intuitionistic, relevance, and paraconsistent logics, respectively.

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