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2.1 Event \( E_1 \) – A specific symbol five times ......................

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1.2 A winning event on any of the vertical lines ..............

1.3 A winning event on any of the horizontal or vertical lines ...

1.4 A winning event on either or both of the diagonal lines ....

1.5 A winning event on any of the horizontal or diagonal lines.

1.6 A winning event on any of the vertical or diagonal lines ....

1.7 A winning event on any of the horizontal, vertical, or diagonal lines ..........................................................................

2. Case B – different numbers of stops and symbol distributions on the reel .................................................................

2.1 A winning event on any of the horizontal lines ............

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