UNDERSTANDING YOUR GAME: A MATHEMATICIAN'S ADVICE FOR RATIONAL AND SAFE GAMBLING

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Foreword

Games of chance have a long history, being traced back in antiquity to the Egyptian civilization. They have diversified and evolved over time and gained popularity that has reached its peak in present days. This particular category of games has become one of the most sought-after forms of entertainment and – for some people – a way of making money.

If we dig for the elements responsible for this popularity, the most relevant is related to the name of this category of games (chance), namely *uncertainty*. Such games run under conditions of uncertainty with respect to their outcomes, so that predictions on these outcomes are virtually impossible either in the short or the long term, either individually or cumulatively; as a result, any outcome triggers sensations of surprise, joy, or deception for the players. Those sensations are preceded by other emotions specific to the stage of waiting, when players has expectations and hopes for the outcome. These elements are of course of a psychological nature, related to our neural-biological constitution.

Another element counting for the popularity of games of chance is the *reward*. Games of chance generate monetary rewards for the lucky players, which is itself one of the main objectives of a gambler. We may qualify this element as having a social nature; however, psychologists have also identified other kinds of rewards in gambling – for instance, a near-miss may be perceived by the gambler as a reward (a kind of "I am getting closer"); even the interactions of all sorts of the gambler with the machine or the table (sounds, visuals, physical contact, and control) are considered as rewards at least for the neural system of the gambler. Feeling pleasure when scratching a lottery ticket or rolling a dice submits to this description. You have to admit that we do not have this kind of rewards in the games of chess or Go, for instance. These gambling rewards are still related to the uncertainty element. Even poker, considered by some experts as a game of skill more than one of chance, owes its popularity to the same uncertainty element.

The gambling reward is not only a cause of the popularity of games of chance, but also a psychological-biological reason for which many players become addicted to gambling. This addiction occurs because such rewards are related to what neurologists call *The Reward System*, which consists of several areas of the brain working together to regulate individuals' reactions towards or away from reward. Various factors, including biological, social, and cognitive-educational, may contribute in variable shares to the unwanted evolution from gambling to problem gambling, which includes forms of pathological gambling and addiction.

Even from this short analysis of the popularity of games of chance, one can figure about the *complexity* of such games. Games are so designed as to work for both the houses and the players; this functioning means the guarantee both that the houses won't go bankrupt and that the gamblers will continue to come and play even after they lose. The characteristics of the games (including the parameters of their constitutive elements, payout schedules, and playing rules) result from the use of rigorous mathematical models for their design; this is the gaming mathematician's job. Such models represent the first (and the most important) place where *mathematics* comes into play, and reflect the main premise that games of chance and gambling are analyzable scientifically.

But games are also analyzed relative to players' behavior, cognition, and also to their social effects. Therefore, other disciplines besides mathematics also have games of chance and gambling as objects of analysis and investigation. The most important is psychology (with its branches of addictive, cognitive, and behavioral psychology), then economics and education as social sciences. You may be surprised to find that philosophy is also involved in the study of gambling, more precisely in problem gambling, in the most applicative way possible.

But mathematics is not exclusive of these disciplines dealing with gambling and problem gambling. It is involved essentially together with philosophy (epistemology, philosophy of mathematics, and philosophy of science) in education and cognitive psychology. Mathematics collaborates with all of these disciplines in resolving issues related to the adequate education of gamblers in what concerns prevention of development of excessive or problematic gambling, as well as correcting the cognitive distortions responsible for such development. The mathematically-related principles of gambling and the associated recommendations for gamblers presented in this book are provided from that interdisciplinary perspective.

During a time in my youth when I was studying mathematics, I was fascinated by roulette. This game looked to me so simple and meanwhile so complex. Actually, I was perceiving its complexity when thinking of it as an application field for Probability Theory (a mathematical theory whose early creator – Pierre Laplace – was inspired by roulette in the 17th century). It was amazing for me to see probability theory "at work" and the "curious" way in which the concrete game reflected in reality the abstract probability laws, through the behavior of the roulette ball.

As a mathematician, I knew that you can't beat roulette, though you may still win from time to time. Besides the joy of analyzing mathematically the various systems of betting in roulette for saying what is right or wrong with them, one question was seeding in my mind: Is it possible to use mathematics not only to describe and analyze the games of chance, but also to temper other non-mathematically inclined people who have too high expectation in winning, and thereby to correct their play?

I have applied mathematics to describe the most popular games of chance and obtain the mathematical results needed for any kind of gambling strategy, covering all possible gaming situations. These mathematical facts and results were published in several books on gambling mathematics, to which I refer throughout the current book when needed. Although the information concerning mathematical facts of the games and gambling is important to acquire if aiming at a rational gambling behavior, these previous books were written and organized to serve more the player than the problem gambler.

After several years of dealing with applied mathematics in gambling, I have earned my PhD in philosophy of science with a research focus on philosophy of mathematics. Even in my early work as a researcher in problem gambling, I realized that gambling is a very complex phenomenon that cannot be approached exclusively mathematically; my new philosophical expertise reinforced this belief and moreover provided me with new scientific tools to deal with problem gambling in the gamblers' favor. The blending of mathematics with theoretical philosophy and psychology proved fruitful in this field, and the information offered in this guide book in the form of principles and recommendations is the outcome of the mathematician-philosopher vision and approach in a psychological framework of gambling as a human behavior around some special mathematically-conceived products – the games of chance.

The author

Introduction

Numerous titles pretending to have as their topic mathematics of gambling or mathematics of particular games of chance have been published over the last two decades. The profiles of their authors range from successful or experienced players to academics.

Every time a new book on gambling mathematics is published, the first question arising is whether its target audience is players or math students. Establishing one of these two options is not straightforward, and the criteria for putting a label on one or the other are dependent upon deep analysis since they have a cognitive, a pedagogical, and a gaming-behavioral dimension that are not independent of each other. In addition, generally a criterion that favors one audience disadvantages the other. This is why any possible claim of a publisher that a title in such a category has a double target audience (*both* players and math students) should be received with skepticism (such claims are actually present in the publishing industry).

On the one hand, gambling mathematics is supposed to provide useful and objective information for the players; on the other hand, gambling is an excellent field of application and exercises for probability theory and statistics. Whatever audience is concerned, the topic of gambling mathematics assumes unavoidable formal mathematics, mathematical models, explanations of the mathematical concepts involved and their reflection in the reality of gambling, and explanations about their application. Such content requires a certain type of language, a pedagogical approach, and organization for being delivered effectively to readers with a poor mathematical background. I myself have encountered these challenges and difficulties when writing my previous books on gambling mathematics.

But however skilled the author of such a book might be in that respect, and assuming that they succeeded in delivering the needed mathematical content (which the reader has understood even without going back to school or attending a course), how can the author be sure that the reader's acquired *mathematical* knowledge will be *used* further in a rational way in gambling? How can the author be sure that the reader's understanding of the mathematical concepts and facts implies the reader's understanding of the exact relationships of these concepts with the *real* world of gambling – and that that knowledge and understanding impacts positively their gambling behavior? The latter type of understanding requires going deeper into the nature of these concepts, of mathematics itself and its application in the real world, and this is no longer mathematical knowledge, but knowledge *about* mathematics. Assuming that the wide majority of gamblers do not even have the mathematical background required to understand the mathematical content, it is fair to hypothesize that they won't reach the other type of understanding either.

The present book is conceived so as to be pragmatic relative to the complexity of the knowledge required to understand the nature of games of chance and gambling, and to maintain an objective, healthy attitude toward gambling as a human activity. By exploiting the role of mathematics in a practical way – with respect to both the play itself and to preventing the development of problem gambling as based on scientific findings – the purpose of this work is to provide comprised knowledge in the form of mathematics-related principles that generate applicable recommendations for the gamblers.

It is a guide book addressed to both players and problem gamblers. The aim is for the players to understand the mathematical nature and functioning of the games of chance without being mathematicians and to use this knowledge – including information on strategy and optimal play – to their advantage as gamblers. As for problem gamblers (actual or potential), that same understanding of the mathematical nature of games, the relation between gambling mathematics and the real-life gambling, and the possible cognitive distortions related to these matters is supposed to steer them toward a healthy track of safe gambling. This double target is reflected in the labeling of the recommendations with letters O (for 'optimal play' or 'objective strategy') and S (for 'safe and rational gambling').

It is a conceptual approach of gambling mathematics rather than the formal-instrumental style that many readers of gambling-mathematics books have gotten used to, and it is this former type of approach that is required for the readers with poor or no mathematical background. The general premises and motivations for this kind of guide and the specific conceptual approach it advances are these:

● Mathematics governs games of chance and gambling in several ways

● The complexity of the gambling phenomenon includes its mathematics, but goes beyond mathematical formalism.

● Probability Theory and Mathematical Statistics is a tricky area for those unacquainted with that field; it has a specific philosophy and psychology, and (in relation to other non-mathematical factors) is responsible partially for some of the gambling cognitive distortions.

● Mathematics itself is usually hard to digest by non-mathematically inclined people.

● Humans are biologically predisposed to irrational beliefs and misconceptions, which many times result from misleading language that we use for communicating and acquiring knowledge.

Following the intended approach, the mathematical-philosophical-psychological knowledge is comprised and delivered in the form of seven general principles which are explained and further particularized for specific games and gambling contexts. The main chapter names enounce these principles. The principles have as subjects the play itself and the gamblers' perception of the play and its outcomes and effects (beliefs, expectations, illusions, etc.). At the end of each such chapter, the explained principle is "translated" into several enduser recommendations to be followed by gamblers in their gambling activity and reasoning about it.

One of the main purposes of this guide is for the reader to gain understanding about the mathematical facts of gambling in relation to his or her own gambling attitude, without learning in depth about the mathematical concepts and theories involved; the goal is not to teach the reader the mathematics of gambling. However, reference to these mathematical concepts is essential, and some mathematical content was unavoidable. I have tried to limit the presentation of the mathematical concepts and to show them in a descriptive and explicative language, using examples and sometimes metaphorical explanations. The explained definitions for the essential concepts as well as some philosophical-conceptual aspects of the mathematical concepts were separated by parentheses. This limitation and brevity of the mathematical-formal content should not affect the understanding of the general principles from which the recommendations are drawn. Besides, readers interested in a more intensive mathematical study of those concepts and their applications in gambling may consult the books dedicated to gambling mathematics in general, and to the mathematics of each game of chance respectively.

Even though some mathematical aspects of gambling explained in this guide will still remain unclear for some readers, the conceptual approach I have followed will at least trigger critical thinking in many directions that perhaps the reader has ignored or thought of as clear or unquestionable. This is itself a goal.

The current guide’s purpose is to inform, explain, make aware, stimulate, and correct. Its ultimate practical aim is for gamblers to understand properly the mathematics-related information in order to the have a *rational* play and a *safe* gambling activity.

Principle 1. **The premise of the existence and functioning of games of chance are the mathematical models behind them**.

1.1 Mathematical structures

Pure mathematics is the discipline of the mathematical structures. This of course does not exhaust the characterization of mathematics, nor clarify its nature. Mathematicians, scientists, and philosophers have failed in providing a definition and description of mathematics which reflects its complex and sometimes "mysterious" nature.

Mathematics as a discipline deals with concepts rigorously defined on the basis of logical methods and using a specific formal language whose aim is to be clear in reference and free of any interpretation (unlike natural languages), and to form propositions that can be operated with classical logic. The relations between the mathematical concepts are defined or constructed also on the basis of first- and second-order logic and these relations generate what we call *mathematical structures*.

A mathematical structure is defined as a set (the set of nodes) with a family of relations defined on it (the sets of connections between the nodes). As a trivial example, the set of natural numbers **N** with the order relation $<$ (less than) forms the mathematical structure (**N**, $<$), which has only one relation, in which ordered pairs of numbers such as (2, 3) or (15, 17) stand and others such as (3, 2) or (21, 22) do not stand. The addition as a relation between three numbers (a, b, and c stand in the relation of addition if $a + b = c$) generates a structure on $\mathbf{N}\times\mathbf{N}\times\mathbf{N}$ (the set of all triples of natural numbers); of course, not every triple stands in that relation.

Mathematical structures can be of any complexity (given by either the complexity of the set of nodes or of the family of

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probabilities assumes establishing several different probability fields for the events related to one's own hand and those events of type "at least one" related to the opponents' hands, which means a different probabilistic model for each application.

Every game of chance is represented by models from both the first and second categories, even though a model from the latter category may be trivial. This happens because outcomes and uncertainty are specific to games of chance by definition (thus explaining the existence of first-category models) and every probabilistic/statistical model needs a functional model in order to ensure the grounding mathematical structures necessary for the governing theories of the probabilistic/statistical model to be applied. For example, any probability computation within a probabilistic model needs *a priori* a grounding model representing the gaming events to be measured, which must belong to a Boolean structure, and this latter model is a functional one.

What is important to retain for the moment is that these mathematical models provide the mathematical facts associated with a game in the form of information to be *used*. There are two categories of users of this mathematical information: one consists of game developers, operators and gambling houses (casinos), actually the gambling industry; the other is the players.

1.4 **Mathematical conception of games of chance**

When developing a new game of chance, a company is aware that such a game will run under conditions of uncertainty regarding its outcomes, and the developers have to be sure that the company won't go bankrupt – or at least the chances of that happening are low enough to be worth the investment. The only scientific tool able to provide such a guarantee is mathematical modeling. There are the mathematical models describing that game and the behavior of its outcomes that will tell the developer – in mathematical terms and numbers – that a certain game design is or is not worth the risk. These models will also be the operational framework of the mathematician dealing with the creation of that game, by telling them (as result of applications and calculations) how to choose, adjust, or modify the parameters of the game so as to meet the expected or required conditions.

The game's rules, number and distribution of its variable elements, and payout schedule – what we usually call the characteristics of a game – are describable in mathematical terms and stand as the entry data for the applications using probability and statistical models. These applications yield mathematical information in regard to the outcomes of the game, in terms of Probability Theory and Statistics, namely probabilities and statistical indicators in the form of statistical means and averages. For example, in slots, the number of the reels, the distribution and weighting of the symbols on each reel, and the payout associated with each payline will provide the statistical indicators that the company will take as their "certificate of guarantee" that the game will run in their favor, as well as the desired behavior of the machine (how often it will return money to players, in what proportion, etc.). The main statistical indicator of a game is the *house edge* (HE, or house advantage), which should be positive for each possible bet in that game for the house to make a profit over the long run. Therefore, the characteristics of a new game are chosen to ensure a positive house edge. We shall define and explain this notion in a further section.

Old or classical games such as roulette, blackjack, baccarat, craps, bingo, and lottery have well known characteristics for which a positive house edge is ensured, as derived from their mathematical models.

The mathematical models provide not only the statistical indicators needed by the developers and the operators of the game, but all the mathematical facts and results related to *playing* that game. In game theory, some games allow what mathematicians call the *optimal play*, which is the best possible mathematical strategy to follow for winning that game, even if the uncertainty factor remains and the win is not sure. Hence,

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Necessary mathematical insights and explanations

Only a small part of the processes taking place in nature or society can be approached and analyzed in a deterministic mode. The wide majority of these processes are random and can be investigated only stochastically. Probability Theory and Mathematical Statistics provide the concepts and tools for such stochastic investigation, which is an important method for almost all sciences.

The core concept of these theories is that of *probability*. While it is easy to enounce its not-too-complex mathematical definition, it is pretty hard to explain it, just because the general concept of probability extends its reference beyond mathematical formalism and has connections with our intimate cognition.

■ Probability as a measure

The first thing to know is that probability is a *measure*; this is its main nature. Just as length measures distance, area measures surfaces, and volume measures the occupied space, probability measures something, but this measurable thing is not tangible like distances and such. It is about the *possibility* for an event to occur. Probability measures this possibility by actually measuring some information or evidences found in the context of that possible event. All these concepts are mathematically represented through specific structures allowing us to define a measure-function called probability.

In mathematics, measure has a rigorous definition and a whole theory attached – Measure Theory. Attempts at an adequate definition of probability were made by mathematicians and philosophers before the birth of Measure Theory (and some of them remained as valid mathematical notions), but Probability

Theory had to wait for the mathematician A. Kolmogorov to provide in 1933 the formalism that allows its integration in Measure Theory and which is accepted as canonical in the mathematical community.

The primitive (indefinable mathematically) notions that the definition of probability is based on are *experiment* and *event*. An experiment is an action that has associated events, which may or may not occur. For example, let us consider the experiment of rolling the die. The experiments actually performed generate *outcomes* (actual results). A performed experiment is usually called a *trial* (or test). Probability Theory deals with events generated by *random* experiments, that is, experiments in which the physical conditions or circumstances of performance are assumed to be so complex or unquantifiable that they do not influence the outcomes (although, deterministically speaking, they do influence them).

An event is seen as a *set* of possible outcomes of an experiment and can happen (occur) or not, as result of an actual experiment. If the outcome is in that set, the event is said to occur; otherwise, not. For example, in the experiment of rolling the die, here are some events: A – occurrence of number 2; B – occurrence of an even number; C – occurrence of a less than 3 number. In the experiment of spinning the roulette wheel, some events are: D – occurrence of a red number; E – occurrence of number 15; F – occurrence of number 2 or number 3.

These events can be described as sets if considering the possible outcomes of the experiment as independent non-decomposable elements: $A = \{2\}$; $B = \{2, 4, 6\}$; $C = \{1, 2\}$;
$D = \{1, 3, 5, 7, 9, 12, 14, 16, 18, 19, 21, 23, 25, 27, 30, 32, 34, 36\}$, $E = \{15\}$, $F = \{2, 3\}$.

The set of all possible outcomes of an experiment, denoted by Ω, is called the *sample space* of that experiment. Of course, for the die roll, $\Omega = \{1, 2, 3, 4, 5, 6\}$. Any event will be an element of the set $\mathcal{P}(\Omega)$, which is the set of parts of the set Ω (the set of all subsets of Ω, called the *power set* of Ω). Denoting by Σ the set of the events associated with an experiment having Ω

Statistical indicators in gambling are of importance for the houses, but they should be for the players as well. They provide mathematical information that might influence a player's behavior and decisions, as we shall see in the next chapters.

These are the basic mathematical definitions, results, and insights that are necessary to describe games of chance and their statistical models. Existent games of chance have finite sample spaces, and the notion of discrete probability on a finite field of events is sufficient for any probability or statistical computation. Still, the concept of probability as a measure and as a limit is important when dealing with the classical gambling cognitive distortions specific to problem gambling.

I have not included in this section the notions and results of Combinatorics, which are important, as many of the games of chance are combinatorial games, and probability computations for their events revert to combinatorial computations. You may find a detailed section of Combinatorics, with applications in games of chance, in the book *Understanding and Calculating the Odds: Probability Theory Basics and Calculus Guide for Beginners, with Applications in Games of Chance and Everyday Life*. Still, I will explain at the appropriate time how these combinatorial notions are involved in the functional models of the games, in the numerical behavior of probabilities, and in some cognitive distortions.

To be retained:

● Probability is a mathematical concept reflecting the likelihood of a random event occurring in conditions of uncertainty, as a measure of this possibility.

● Probability is conceived so as to be a mathematical measure, as a function obeying certain axioms, defined on a set (field) of events endowed with a certain structure (Boolean algebra). Mathematical probability makes no sense defined on any other kind of structure or on a set with no structure. Probability of an isolated event (not belonging to such a structure) makes no mathematical sense (although in ordinary language we can refer to it).

● A probability function is determined by its entire probability field, which reflects the information we use for assigning probabilities to events. If this information changes, we have a new probability field, and as such, a new probability function.

● Probability (in the Kolmogorovian axiomatics) is the most rigorous measure we have for physical possibility, although other kinds of probabilities may be defined. Yet, this does not mean that probability may tell us something precise about the occurrence of the events, in a deterministic sense. It counts only the evidences in the favor of an event or another.

● The Law of Large Numbers is the result that makes the (only) connection between the empirical (real) behavior of the random events and their mathematical probability. By LLN, probability is the limit of the sequence of relative frequencies and nothing more. Like any limit, it is approached to infinity and we have no sure information about any particular term or terms of this sequence, nor about their behavior on finite intervals.

● Even defined on a finite field of events, probability assumes potential infinity – first as a measure, through the countable-additivity axiom, and second, through the primary concept of random event, which is supposed to have the same probability whenever measured, that is, as a result of any imagined experiment performed under identical conditions; this is also reflected in LLN, where the sequence of the relative frequencies is infinite for producing a limit.

● The statistical indicators (expected value, variance, standard deviation) are defined through probability and thus carry its infinity feature, even though they may be computed for finite random variables. They are defined as averages or means, not in an arithmetical sense, but rather as probabilistic/statistical (in the same sense that probability is an average). They are mathematical descriptions or properties of the sequences of outcomes of trials as a whole and not of partial or finite portions.

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2.3 **Conclusions and the gambling case**

Randomness is conceptualized as a *disorder* (of the occurrences of the events for which causes are not known in their entirety), and the attempts to define it aimed at defining that special kind of disorder. This disorder reflects our lack of knowledge (or ignorance), and as such, it is first a feature of our reasoning and second of the phenomenological world, if this world is non-deterministic. However we think of randomness – as the opposite of law, rule, or purpose, of indeterminacy, irregularity, or a form of independence – concepts like prediction, causality, and dependence fall within the concept of randomness.

But randomness exists as a special type of disorder and is a sort of *total* disorder, characterizing all factual reality as seen through our reason. The 'total' attribute may be expressed through 'equally possible' or 'equally unknown' or just 'independent.' For science and mathematics randomness is just a convenient conceptual perquisite for probability theory and for making the probabilistic/stochastic method operational and effective in scientific reasoning, by allowing us to abstract and idealize non-deterministic conditions and to connect probability theory with other mathematical or scientific theories. We may call this *objective* or even *pragmatic* randomness.

However, all this latter characterization makes randomness in turn to be an *order*. The uniformity of randomness is in fact order rather than disorder. Moreover, the *infinite* feature of randomness also strengthens the qualification as order. Indeed, infinity is present in the concept of randomness. We cannot talk of something random without imagining it in an infinite context, in an infinite number of instances or infinite possibilities. Think of Borel's simple notion of random sequence. If it were finite instead of infinite, one would come up at any time with a personal rule of generating a term from the previous terms, even though it might not be the real rule that the creator of the sequence had in mind originally. Infinity has a homogenizing,

No. of trial:	1	2	3	4	5	6	7	8
Outcome:	T	H	H	H	H	H	H	H
Frequency:	0	1	2	3	4	5	6	7
Relative frequency: (fraction)	0	1/2	2/3	3/4	4/5	5/6	6/7	7/8
Relative frequency: (decimal fraction)	0	0.5	0.66	0.75	0.8	0.86	0.85	0.87

Over the interval of trials from 2 to 8 there were 7 heads recorded, instead of 3-4, what was expected as "normal," since the probability for a head is 1/2. A difference of about 3-4 units from the "normal" prior expectation is recorded. But if we look at how the relative frequency of heads changed over that interval, we see it ranging from 0.5 to 0.87, that is, with about 3 to 4 decimals. Moreover, imagine that the new pending outcome is T. At this ninth trial, the relative frequency of the heads is now 7/9 = 0.77. It decreased back 1 decimal for only one trial, coming closer to the probability of 0.5. With these numbers in hand, the "abnormal" behavior of the outcomes does not seem so hard to be restored to "normal" with the forthcoming trials.

The effect is visible also when the interval in question shows many Hs, but also Ts. Assume one T occurs in that interval: T H H T H H H H … The new table is as follows:

No. of trial:	1	2	3	4	5	6	7	8
Outcome:	T	H	H	T	H	H	H	H
Frequency:	0	1	2	2	3	4	5	6
Relative frequency: (fraction)	0	1/2	2/3	2/4	3/5	4/6	5/7	6/8
Relative frequency: (decimal fraction)	0	0.5	0.66	0.5	0.6	0.66	0.71	0.75

Again, there is a frequency difference of about 3 heads from the "normal" over that interval and only of 0.25 for the relative frequency. If a T occurs in the next trial, the relative frequency of heads becomes 0.66, so it decreases back to about 1 decimal for only one trial.

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This calculation does not take February 29 into account, or that birthdays have a tendency to concentrate higher in certain months rather than in others. The first circumstance diminishes the probability, while the second increases it. Of course, the more persons considered, the higher the probability. With over sixty persons, probability gets very close to certitude. For 100 persons, the chance of a bet on a coincidence is about 3,000,000:1. Obviously, absolute certitude can be achieved only with 366 persons or more.

Such erroneous estimations based on false intuition occur in gambling as well, and the plethora of examples is huge. For instance, here is a gaming situation requiring comparison of chances:

Assume you are playing a 5-draw poker game with a 52-card deck. The cards have been dealt and you hold four suited cards, but also a pair. For example, you hold 3♣ 5♣ 8♣ Q♣ Q♦. You must now discard and you ask yourself which combination of cards it is better to keep and which to replace.

To achieve a valuable formation, you will probably choose from the following two options:

– Keep the four suited cards and replace one card (expecting a flush); or

– Keep the pair and replace three cards (expecting 'three of a kind or better').

In this gaming situation, many players intuit that, by keeping the pair (which is a high pair in the current example), the chances for a Q (queen) to be drawn or even for all three replaced cards to have same value, are bigger than the chance for one single drawn card to be ♣ (clubs). And so, they choose to play for 'three of a kind or better'. Other players may choose to play for a flush, owing to the psychological impact of those four suited cards they hold. In fact, the probability of getting a flush is about 19% and the probability of getting three of a kind or better is about 6.3%, which is three times lower.

But if you are in a similar gaming situation (you hold four suited cards and a pair) in a 24-card deck 5-draw poker game, the

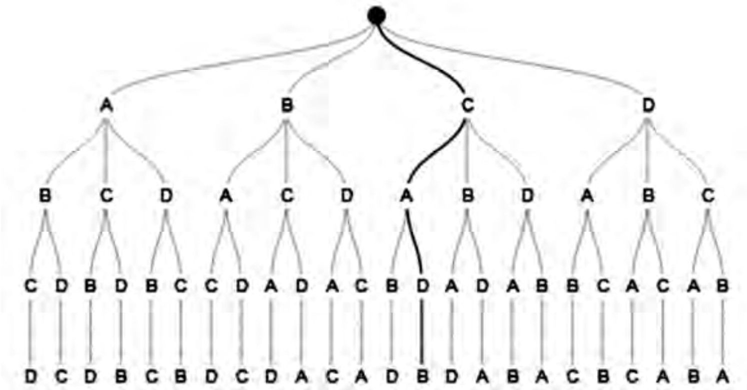
order of those probabilities is reversed: the probability of getting a flush is 10.5% and the probability of getting three of a kind or better is almost 50% percent and may help you to determine to keep the pair, assuming you know the correct figures.

Each game has huge number of gaming situations (especially card games), and each such situation generates plenty of probabilities for the events associated with it. If not known in advance from reliable sources or not calculated ad hoc, a player may estimate or compare these probabilities by wrong intuition at any time and in any game.

A good example for probabilities that defy intuition is lottery. Let us take, for instance, the *6 from 49* system. The probability of five numbers from your ticket being drawn is about 1/53,992, and the probability of all six numbers being drawn (the big hit!) is 1/13,983,816. For someone having no idea of combinations, these figures are quite unbelievable because that person initially faces the small numbers 5, 6 and 49, and does not see how those huge numbers are obtained. The explanation resides in the multiplication power of combinations.

Permutations, arrangements, and combinations are mathematical objects reflecting just the concepts that these words refer to in the real world. Combinatorics is concerned with their number or count. When it comes to a combinatorial count, that number is expresed as a function of the given parameters: the number of permutations of n objects, the number of arrangements of n objects taken each m ($n \geq m$), and the number of combinations of n objects taken each m ($n \geq m$). Each of these functions has a well known combinatorial formula having as variables those parameters.

The values that those combinatorial functions may take for given parameters are many times counterintuitive for those unaware, even when facing the formula. The combinatorial results are of a higher order of magnitude than multiplications and even power raising. For instance, as a low one, the number of permutations of 4 objects is $4! = 1\times2\times3\times4 = 24$; for 5 objects, it jumps to $5! = 1\times2\times3\times4\times5 = 120$. This power of multiplication reflects the power of unfoldment as a tree. In the diagram below is the permutation tree for four objects A, B, C, D:



The unfoldment generates 24 possible paths, each corresponding to a unique permutation of the four objects. Now imagine that the four objects represent the participants in a race where you bet on the final order. If you bet for instance on the final order C, A, D, B, represented by the bold path in the diagram, and assume that each order has the same chances to occur, you get a probability of 1 to 24 of winning your bet.

Combinations unfold in a more complex way, embedding several partial unfoldments of permutations such as that illustrated. Such unfoldment remains unseen for the person whose only concern is the number of the combinations and who may be surprised by the result provided by the formulas.

In the lottery example, the number of 13,983,816 combinations of 49 numbers taken each 6 is given by the computation 49!/(6!×43!). Looking at this numerical expression, one who is not math-inclined cannot see clearly that it may give such a huge order of magnitude for its result. That order is perceivable for such a person only with the unfoldments of the combinations, where the multiplication power becomes visible.

Therefore, a lottery player who does not know the real figures may play the lottery with the intuition-based belief that his or her chances are much higher that the real ones. This usually happens in combinatorial games (games whose outcomes are combinations of items such as cards, numbers, or symbols of any kind). This false belief about lottery chances can be also seen as the result of using a wrong measure for estimating the chance: Instead of a calculated probability, the subject makes a very rough estimation based of the *length* of the string of numbers expected to be drawn; even if the individual might estimate the likelihood as very low, such an estimation is very far from the real probability.

Such intuitive estimations or comparisons of probabilities occur in gambling and in daily life as well. Most of them are expected to be wrong, and some, luckily enough, are accurate. They involve inadequate measurements, incorrect calculations, or reasoning fallacies (individual or combined) and cannot be justified by any pressure of time against complex probability calculation, or by necessity of whatever estimation.

Erroneous intuitive probability estimations can be fairly qualified as cognitive distortions and have psychological causes as well besides the lack of probability knowledge. In either form, such intuitive estimation is again (partially or totally) an effect of relying on past experience – intuition worked for other estimations, that is, they reflected well the frequency or relative frequency of the measured event or the expectations – so the estimator is justified in bypassing again the probability calculation in the current situation. The behavior is also an emotional response to a questionable situation – a sort of personal non-expert opinion that is felt as necessary in the given situation.

3.2 **The conjunction fallacy**

The conjunction fallacy expresses a fallacious comparison of probabilities, namely the belief that the probability of a conjunction of two statements is higher than the probability assigned to at least one of the two constituent statements. Tversky

for that single bet are higher than those 24.1%. If the bettor does not have a clear answer for that, they might overestimate the probability of the conjunction, influenced by the good chances of the favorites in those matches.

Besides a possible conjunction fallacy, in terms of optimal play, it is safer to split a ticket with "sure" or "almost sure" events and its stake. In our previous example, playing two tickets – holding two matches each – instead of one and halving the stake in two is safer. Assume S is the stake of the original ticket with four matches and $S/2$ the stake of each reduced ticket:

Ticket A, stake $S/2$:
Match 1: victory of the favorite; ×1.2 odds
Match 2: victory of the favorite; ×1.2 odds
Ticket B, stake $S/2$:
Match 3: victory of the favorite; ×1.2 odds
Match 4: victory of the favorite; ×1.2 odds

Each ticket has ×1.44 payout odds. In case all four results are predicted well, the two winning tickets together give a profit of $0.44S$ (less the possible ticket fee charged by the agency; usually online agencies do not charge such fee), which, compared to the possible profit of the original unsplit ticket, namely $1.07S$, is indeed lower. However, if one result is predicted wrongly, one of the tickets A and B loses, and the overall loss is $(S/2) - 0.44\times(S/2) = 0.28S$, which is about three times lower than the possible loss with the initial ticket (S). If staying with the original ticket, it would be losing anyway in this situation. Therefore, it is safer to split the ticket in two, which supports the idea that there shouldn't be too many events on the same ticket, however "safe." Of course, if the possible profit of the split ticket is not satisfactory for the player, they can increase its stake, at the cost of a higher possible loss.

Splitting a combined bet is safer not only for multiple low-odds bets. It is possible for the split bets to have together zero profit (and loss) in case one of them is lost. Let's take a simple example, inspired by the arithmetical fact that $2 \times 2 = 2 + 2$.

Assume a combined bet for two events, each of them having ×2 payout odds (hence ×4 overall), with stake S. Possible profit is $4S - S = 3S$ and possible loss S. Now consider two separate tickets with the bets on those two events, each with the stake $S/2$. If one of the two separate bets is lost, there is an overall zero profit and loss in this case. If both are lost there is a loss in amount of S, while if both events are predicted well, the profit is S. If one of the events was not predicted well, the combined bet would be lost, but in the case of splitting, there is still the chance to have no loss. By splitting the combined bet, the loss is thus limited, but at the cost of a lower possible profit. The payout odds of the initial bet (×4) are actually maintained with the splitting if both events are predicted well; however, the possible loss is reduced if only one event is predicted well. Doubling the stake of the two smaller tickets doubles the possible profit ($2S$, still lower than the $3S$ of the initial ticket), and also the possible loss ($2S$), but that loss is in effect only when the player fails to predict well both events.

If we want to formally estimate the EV of the two bets (the combined bet on the one hand and the two separate bets taken together as one bet on the other hand), we have to assign a probability to the two events, say p and q. With stake S, the random variable having as value the profit of the bet is:

For the combined bet:

Profit:	$3S$	$-S$
Probability:	pq	$1 - pq$

EV is: $3Spq - S(1 - pq) = S(4pq - 1)$

For the two separate bets together:

Profit:	S	0	0	$-S$
Probability:	pq	$p(1 - q)$	$q(1 - p)$	$1 - pq$

EV is: $Spq - S(1 - pq) = S(2pq - 1)$

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its own probability, which should be subtracted from that sum; otherwise, it is counted twice.

Another example, still in Texas Hold'em, is related to the estimation of the chances that either opponent hits a certain draw. For instance, you play against five opponents, the flop board contains three suited cards, and you hold no card of that suit. Say you estimated the probability for one opponent to achieve a flush by river (which is 4.54%) and you want next to estimate the probability that either of your opponents (at least one) hits a flush. If you estimate it by adding 4.54% five times (giving 22.72%), it is not accurate, as two up to five opponents may hold two cards of that suit (there are 10 outs in play). The probabilities of the events 'two opponents achieve a flush' and 'four opponents achieve a flush' must be subtracted from the total, while those of the events 'three opponents achieve a flush' and 'five opponents achieve a flush' must be added to the total (as an application of the inclusion-exclusion principle in its general form, for more than two events). The last two probabilities are significantly lower than the first two, hence leaving the estimation of your initial total as generally an overestimation.

In conclusion, in form 2 the disjunction fallacy is just a mathematical error originating in the lack of knowledge about the inclusion-exclusion principle.

3.4 **The near-miss effect**

A frequent gambling situation encountered by every gambler playing regularly is the so-called near-miss, roughly defined as a failure that looks close to being successful. It is associated with an outcome that differs slightly from a winning one – for instance, by a symbol missing in a slots payline or a scratch-card payline, one or two numbers missing in the draw of a lottery, one card missing for a straight in a poker hand, and so on.

Principle 4. **The mathematical information about a game reflects the characteristics of that game and provides optimal play and objective strategies**.

I have explained in the first chapter how the mathematical models behind games of chance provide developers and operators with the guarantee that the games are profitable for them. This guarantee is expressed in statistical terms. Both the functional and statistical models of a game provide mathematical information about the game that stands at the base of the conception of that game and is actually used by all people involved in playing that game: players, developers, operators, and experts. In this chapter we shall focus on how players interact with and use the mathematical information provided by the mathematical models of a game.

A game of chance is known and evaluated not only through its rules of play and payout schedule, but also through the basic numerical information that is associated with the outcomes of the game. That information consists of probabilities of the winning events and statistical indicators such as expected value, house edge (or RTP), or variance. When talking about the *characteristics* of a game of chance, a non-mathematical description in terms of rules and appearance is not enough. The game is completely characterized only if mathematical information is included in its description.

Jumping from a high place and landing on your feet requires you to know the height of the place before you may bet on that; throwing an arrow at a target requires you to know the distance to the target and target dimensions; and playing Russian roulette requires you to know the number of bullet chambers in the cylinder. Even if such games did not involve betting, that numerical information would still be part of the characteristics of each game. If betting is involved, that information is required even more so, for ethical reasons as well.

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while looking at a roulette wheel and knowing that all those numbers arranged circularly have the same well-known likelihood of occurrence. But despite such equality and symmetry, roulette numbers do not all have the same status. This is just because on the table we cannot cover any group of numbers with unique placements of the same type, as we have seen in the previous section.

4.1.2.1 **High probability of winning against possible profit**

Surprisingly, roulette offers the highest probabilities of winning among all games of chance; we can find bets with over 90% winning probability. Indeed, such probability increases with the coverage of a bet. For instance, a complex bet consisting of 17 straight-up bets on black numbers with the stake of $1 each and a bet on *Red* with the stake of $18 gives a probability of winning of 92.09%. It is in the category of the so-called large-coverage disjointed bets.

But we shouldn’t be too excited about this: winning on either number or color will make you a profit of only $1. While you can still be satisfied with this and run this bet repeatedly, a possible failure would cost you $35, which would cancel $35 of your assumed previous profits or result in an overall loss. The expected value of that bet is –$1.84, as expected loss on average for the $35 bet, and in case of a profit, the profit rate (relative to stake as investment) is only about 2.85%. It’s all in the profit function of that bet.

To place a large-coverage disjointed bet, you may choose between categories of bets, and also between particular bets within the same category, by choosing the desired parameters of that bet. Let’s stay with the example of a large-coverage bet consisting of a color bet and several straight-up bets on numbers of the opposite color – the generalization of our initial example in this section. Of course, its coverage can be enlarged with the number of the straight-up bets, which is one of its parameters;

other parameters refer to the stakes of these simple bets. For simplicity, assume the straight-up bets have the same stake, and as such there is only one parameter left – the ratio between the stake of the color bet and the stake of a straight-up bet.

For generality and being precise, denote by n the number of straight-up bets and by c that ratio. Denote by S the stake of a straight-up bet (then, cS is the stake of the color bet). Let's stay in the case of American roulette. The possible events after the spin are: A – winning the bet on color, B – winning a bet on a number, and C – not winning any bet. Their probabilities sum 1, since the events are mutually exclusive and exhaustive.

Probability of A is $P(A) = 18/38 = 47.368\%$. In the case of winning the color bet, the player's profit is $cS - nS = (c - n)S$ (which can be also negative, that is, a loss). Probability of B is $P(B) = n/38$. In the case of winning a straight-up bet, the player's profit is $35S - (n - 1)S - cS = (36 - n - c)S$. The probability of C is $P(C) = 1 - P(A) - P(B) = 1 - 9/19 - n/38 = (20 - n)/38$. In the case of not winning any bet, the payer loses $cS + nS = (c + n)S$. The overall winning probability is $P(A) + P(B) = (18 + n)/38$.

These formulas tells us that the higher n, the higher the winning probability, which was expected. Second, if n increases, so does the possible loss in case event C happens. This means that the "safety" given by a high winning probability is decompensated by the possibility of losing a large amount if no number in your coverage is hit. Another factor may contribute to this decompensation: a possible low profit in the case of winning the color bet. For instance, choosing $n = 10$, $c = 11$ and $S = 1\$$ (10 straight-up bets with a stake of \$1 each and an \$11 stake on the color bet), we have a 72.67% winning probability, a possible loss of \$21 if not winning any bet and only \$1 profit in case of winning the color bet; this means a low profit rate relative to the investment (4.76%), along with the risk of a high loss.

How can we manage these parameters to suit our personal strategy? First of all, it is natural to put the condition of a positive profit in both cases A and B, which results in $n < c < 36 - n$. This is a relation between parameters n and c which is also sufficient

size of your bankroll, which should be large enough to sustain a loss as a result of a hypothetical long succession of failures. Limiting the cumulated loss has imposed constraints on the parameters of the martingale: the multiplier is usually chosen at a maximal value of 2; those simple bets having a probability of winning close to 1/2 are preferred – such as *Red/Black*, *Even/Odd*, or *Low/High* (this condition makes the martingale suitable for other games such as blackjack or baccarat); as for the initial stake, it is of course up to each player, but only relatively, based on the player's bankroll and level of afforded risk. Second, a solid bankroll might not be enough for playing the martingale effectively, because most of the casinos have an upper limit for the stake of a bet; if that limit is reached within the succession of failures, your martingale cannot go through.

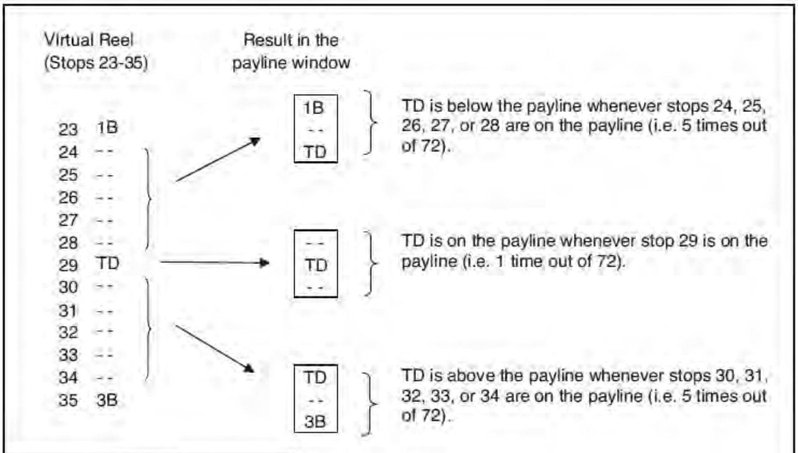
It appears then as a necessary condition for the player taking that risk to evaluate his or her bankroll against the possible failures, to check for the casino's upper limit for a stake, to calculate or simulate the possible losses relative to the various initial stakes, and then to choose the best option with respect to their strategy prior to playing the martingale.

Besides the practical constraints above, mathematics itself still has something to say against the idea that martingale is always a winning strategy. If algebra gives the impression that this is the case, for probability theory the martingale is a bet like any other and submits to the same universal probabilistic laws. Like any bet, a martingale has its own expected value. For the classical version with 2 as the multiplier, the expected value of the martingale is EV = $S\left[1-(2q)^n\right]$, where S is the initial stake, n the number of successive bets lost, and q the probability of winning the simple bet. Assuming q <1/2 (the case in our color bet example), the EV is positive, which supports the algebraic result that you always make an overall profit with the first winning bet. So if you have unlimited resources (and if the casino has no betting limit) you could in principle make money using

one is frequently visible in the payline window right above or below it.

For modern slots, the technique creating false near-misses adjacent to the payline is called *clustering* and relies on the flexibility of the symbol weighting and arrangement on the virtual reels. The basic idea is to place a stop with a high-award symbol on the virtual reel between several stops with low-award symbols or blanks, mapped into the corresponding positions on the physical reel or the virtual reel visible in the payline window. Here is a simple example of how clustering works in a real slots game:

In the next figure is a portion of an actual 72-stop virtual reel. The physical (visible) reel has 22 stops, half of which are blanks. TD is the highest paying symbol. The portion of the visible reel holding the TD symbol is: 1B - - TD - - 3B (the symbols are described in the figure's legend). TD is placed in virtual stop 29 and has five blanks above it (virtual stops 24–28) and five blanks below it (virtual stops 30–34) on the virtual reel.



Legend	
- -	Blank
1B	Single Bar
3B	Triple Bar
TD	Triple Diamond

Each stop on the virtual reel will occur on the payline on average 1 time out of 72. Thus, each of the five blanks in positions 24–28 will occur on average on the payline one time out of 72 for a total of five times. Because the five blanks 24–28 are adjacent to the TD, it follows that that the TD in position 29 will be just below the payline on the visible reel on average 5 times out of 72.

The TD in position 29 will occur on the payline in average 1 time out of 72 and as frequently on the visible reel. The five blanks in positions 30–34 will each occur on the payline on average 1 time out of 72 for a total of five times. Because the five blanks 30–34 are adjacent to the TD, the TD in position 29 will be just above the payline on average 5 times out of 72.

If the visible reel were to determine the outcome by random spin, the positions creating (by chance) near-misses for TD as in the figure would have occurred on average 2 times out of 22 (stops), that is, 1 time out of 11. Due to the mapping from the virtual reel, those positions will actually occur on average 10 times out of 72 (virtual stops), that is, 1 time out of 7.2. Therefore, the frequency of those positions has been increased artificially with the clustering technique.

There are only 11 blanks on the physical reel, but the chances the RNG will pick a blank are much higher than 1 in 11. In fact, the blanks immediately above and below the high-award symbol are favored on the physical reel. Hitting these blanks gives players the illusion that they almost landed the high-award symbol, because that symbol is physically close to the payline. But it is not mathematically close.

The techniques of creating false near-misses in slots are quite complex, and I have limited the presentation to a short description and simple example just for illustrating the essentials of these engineered near-misses.

The general advice against the near-miss effect following the formal presentation of the near-miss in its dedicated section was to not split the actual outcome into a matching and non-matching part and to treat it like any other non-winning outcome.

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4.3.2 **High-low-count optimal strategy**

Thorp began by replacing the assumption of the equal distribution of the card values with that of an arbitrary probability distribution. Then, he investigated how much the expectation changes if the card probabilities are slightly increased or decreased, and found that the changes are of the same order of magnitude as the expectation of loss when using the optimal fixed strategy. He also found that the play of cards 2 to 6 is positive for the player, while for the other cards, especially for 10 and ace, it is negative.

The basic idea was to assign a weight to each card value played, as follows: +1 for 2 through 6; –1 for 10, face cards, and ace; 0 for the remaining cards. This turned out to be a practical counting system based on an approximation, whose total at a certain moment (called the *count*) quantified sufficiently well the information given by the cards played for being transformed into a mathematical criterion for changing the strategy according to the count. It was called the *high-low system*. The high-low counting system, although approximal in reflecting the situation of the cards played, was proved by Thorp to reflect exactly the change in winning expectation. The High-Low technique was first introduced in 1963 by mathematician engineer Harvey Dubner.

Optimizing the strategy as based on the current count (C) proceeds with determining the supposed composition of the remaining cards. This goes by determining the probabilities of the individual card values in the rest of the deck/stack conditional on C and the number n of remaining cards in the deck/stack: Since the cards valued 7, 8, and 9 have no influence on the count, their conditional probabilities are still 1/13, whatever C. Probabilities P (for the low-valued cards 2 through 6) and Q (for the high-valued cards 10, picture, ace) depend on the count and are to be determined by the relation $P - Q = -(C/n)$ and the approximation $P + Q \approx 10/13$. The two equations yield the probabilities:

$\mathrm{P}(2) = \mathrm{P}(3) = \mathrm{P}(4) = \mathrm{P}(5) = \mathrm{P}(6) = P/5 \approx (1/13) - (C/10n)$

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I already mentioned that exact probability estimations are impossible in poker during the game, and even though mathematical sources exist listing all probabilities and formulas covering all gaming situations in Hold'em poker, it would be impossible to memorize them all for application during the game. The question is whether trained "by-eye" estimations and popular approximating rules such as the rule of 4 and 2 are sufficiently accurate relative to exact probability.

Let's take a few examples:

1) Assume the flop cards are (8 A 2), the hole cards are (8 7). The target draw that is the most advantageous is a full house of 8s, namely (88877). The probability of this full house to be hit by river is 0.555%. What the rule of 4 and 2 would give for this draw is 5 (outs) × 4 = 20, so 20% is the estimation, 36 times as high as the exact number. Not that close, right? Taking the target draw to be 'any full house of 8s', which has the probability 1.665%, maintains the inaccuracy: 11 (outs) × 4 = 44.

Let's continue this example also in the turn stage. Assume a 7 came as the turn card: (8 A 2 7). The probability for the full house (88877) to be hit by river is now 7.692%, while the rule of 4 and 2 gives: 2 (outs) × 2 = 4. Taking 'any full house' for the target draw (of 8s or of 7s), the numbers change as follows: the probability is 15.384% and the 4-and-2 estimation is 8%. The overestimation in the flop stage turned into an underestimation in the turn stage.

2) Assume the flop cards are (A 5 8), the hole cards are (8 7) and the target draw is the straight (56789), which has a 1.48% probability. The rule of 4 and 2 gives: 8 (outs) × 4 = 32, far enough. Changing the target to 'any straight' (56789 or 45678), with a probability of 2.98%, the error also gets doubled, since the 4-and-2 estimation would be 48%.

3) Assume the flop cards are (7 J 3), the hole cards are (5 7) and the target draw is 'trips of 7s or trips of 5s or better,' where "better" means 'full house of 7s or quads of 7s'. The probability for that draw is 10.359%, while the rule of 4 and 2 gives: 11 (outs) × 4 = 44, so 44%. Staying only with trips of 7s,

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The popular pot odds criterion conceived and used by the community of poker players still relies on an expectation that does not materialize in practice due to the same rules of poker. This criterion assumes an evaluation of the chances for the player's own hand that does not take into account seriously the opponents' chances. Besides, we saw that the odds estimation methods allegedly by approximation proposed by the same community are accurate only in a minute part of the entire spectrum of poker hands.

These popular strategic criteria, indicators, and methods of estimation had as motivation, and are entirely justified by, the overall high complexity and difficulty of the probability computations associated with the poker hands and the very short time allowed for a decision within a game. However, their objectivity and effectiveness are undermined by the mathematical models of the game, which are able to provide exact probability estimations but are unable to reflect all the rules of the game and the psychological aspects of players' behavior.

Obviously, poker is a game of skill and of chance. Skill assumes not only reading the opponents' behavior and strategies and adjusting the personal strategy to their actions, but also mastering the odds evaluations. This latter component renders self-study as a training requirement outside the game. Then, application of this acquired knowledge during the game is itself a skill, assuming quick observation, framing, computing, and recall.

The number of all possible Hold'em hands in flop, turn, and river stages (also taking into account the number of the opponents as a parameter along with the board and the player's own hand configuration) is 28,038,455,640, that is, over 28 billion. Each of these hands has a set of probabilities associated for the player's own hand and for the opponents' hands, for each target draw. Developing a skill of knowing and applying these odds requires far more effort than skilling to read your opponents in long practice. A perquisite of developing such skill includes these tasks: reducing the configurations to as large as possible

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information or justified belief that a wide majority have that behavior. Beyond that premise, it is all a matter of simple arithmetic.

But lottery playing exhibits curious behavior in its own right. From all games of chance, lottery offers by far the lowest odds of winning for the top prizes, on the order of one to millions or tens of millions for the first prize, and one to tens or hundreds of thousands for the second prize, for the common lottery designs. Most lottery players play regularly and are aware of the very low odds of winning. Even though most of them might not know the exact figures, all have a clue about the size order of these odds, knowing that they are very close to zero, because this information has spread widely enough in common communication between lottery players and through the media as well to become a proven well-known characteristic of lottery.

For instance, at 6/49 lottery, a certain 6-size combination of numbers is drawn with a probability of 1/13,983,816, sending the top prize to the players matching all six numbers. Some say that these chances are much lower than those of the player being struck by lightning, and I won't disagree with them. In frequential terms (so popular and proffered by regular gamblers, but so uncertain), a player playing one line in one lottery draw regularly (usually, once a week), will have to play 13,983,816 times to expect meeting once with the big hit. This would mean playing for about 291,392 years. If assuming the player plays 100 independent lines each time to enhance the probability of winning (which would be costly as a disadvantage), they would have to wait "only" about 2,914 years.

Even knowing that the winning odds are very low, lottery players still continue to buy tickets on a regular basis and the lottery has never lacked for business. So the question arises as to what actually makes lottery players persist despite the minute odds of winning?

Since lottery is not such an attractive game (much less attractive than slots or sport betting, for instance), it is hardly believable that fun it is the main reason. Then, it can be fairly

dollar bet in this game over the long run" may or may not eliminate the conflict, as we cannot be sure whether that addition induces to the reader the meaning of an infinite average, which is the mathematical meaning of EV as a mean weighted with probabilities; 'long run' and 'infinity' mean different things.

The best possible improvement of the stated text seems to be "*Expected* value as a percentage is minus 5%, so we *expect* to lose on average 5 cents for every dollar bet in this game over the long run," but uniform semantics for 'expectation' is attained only if the reader takes the mathematical meaning of "average" in this context, which is 'statistical average' based on infinity, which would justify no additional in-text clarification for 'the long run,' assumed infinite, even though in ordinary meaning it is finite. It seems that the elimination of the potential semantic conflict within the text itself is not achievable unless the reader acquires the required mathematical knowledge of the notion of EV and adjusts the meanings for the rest of the statement accordingly.

● "Probability of a pair of dice to roll a total of 11 is 1/18, so we *expect* that to happen about once in 18 rolls." This time expectation is not about profit or loss, but about the frequency of an event. The conflict in this example is not between two meanings of the term 'expectation,' but between 'expectation' and 'probability.' In the frequentist interpretation of probability, probability of an event is the limit of the relative frequencies of the occurrences of that event over an infinite sequence of trials and can be equated with a statistical average over that infinity of trials. In the given context, this mathematical description is explained in ordinary terms that cannot reproduce infinity. The "expectation" referred to there is over a finite number of rolls. One could take literally the prediction of having one occurrence in exactly 18 rolls, 18 certain rolls, or at every 18 rolls.

Even though "about" may induce a meaning of approximation (about once in 18 trials may mean about twice in 36, and the like upward), it still does not induce that of statistical average, which cannot defined for finite intervals of trials. If the statement contained only the second part "We *expect* that to happen about once in 18 rolls" without linking it to the mathematical probability, it would not be conflicting, but at most imprecise. It would be just a belief expressed on the basis on insufficient mathematical knowledge. However, in its entirety, the statement would also express that that expectation (in ordinary meaning) is entailed by the mathematical knowledge. Actually it is, but the conclusion (description/explanation) is inadequately formulated in the mixed language. As in the previous example, an enhancement such as "Probability of a pair of dice to roll a total of 11 is 1/18, so we *expect* that to happen on average about once in 18 rolls" would be effective only if the reader takes 'average' in its statistical meaning. A misinterpretation of the statement in our example is tightly related to the Monte Carlo fallacy.

Average. We have already seen in the previous sub-section and in other previous sections of this guide that the adequate interpretation of probability and expected value as averages should take 'average' in the sense of *statistical average* and not *arithmetical average* (mean). 'Average' is one of the mathematical terms widely imported in ordinary language and the most frequently assigned meaning is that of arithmetical average. In the ordinary language of gambling, 'average' is many times meant in this sense as well. Sometimes it is meant in the sense of 'approximately.'

However, in any gambling discourse using the term and relating it to probability, frequency, or expectation, 'average' should be meant as statistical average for being consistent with the properties of probability, which is a constitutive concept for 'statistical average.' If it is meant in the other way, the statement using it may lead to misconceptions and fallacious beliefs about the application of probability theory in gambling connected to some classical gambling cognitive distortions.

● Take as an example "This slot machine returns to the player on *average* 95% of the wagers," formulated by an expert.

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In the sections dedicated to these cognitive distortions, we saw that they are interrelated, and as such, even those that are not labeled as mathematics-related are still indirectly related to the mathematical dimension of gambling, in either their formation, manifestation, or correction. For instance, the Monte Carlo fallacy stems from a misconception of the randomness and statistical independence, which are primitive concepts for probability theory. The illusion of control, although forming and manifesting relative to the physical characteristics of a game, is indirectly related to the mathematics of that game, since its correction assumes a cognitive intervention based on a representation of the game by its mathematical model, which is free of that "control" feature.

These GCD are common among gamblers, regardless of their experience or level of education. Each gambler is affected to a certain degree by one of more of these distortions, which are recognized by problem-gambling researchers as risk factors in the developing of a problematic gambling behavior, including addictive behavior. But this does not mean that we should take them as a kind of disease. On the contrary, they are natural in the sense that games of chance themselves and the activity of gambling trigger and install them in our underlying cognitive system.

The palette of all cognitive distortions is very wide, per their classical taxonomy in psychology, stemming from and impacting our usual daily life; gambling-specific ones form only a very small part of them.

Research has shown that people develop cognitive distortions as a way of coping with adverse life events. The more prolonged and severe those adverse events are, the more likely it is that one or more cognitive distortions will form. Research has even suggested that human beings might have developed cognitive distortions as a kind of evolutionary survival method. In other words, stress could cause people to adapt their thinking in ways that are useful for their immediate survival, even though this thinking is not rational or healthy over the long term.

Therefore, we should not be too worried about the possible presence of cognitive distortions in our mind, but we should instead be worried about their possible harmful effects on our life and on the lives of others over the long term. In gambling, these possible effects are not only a fallacious way of reasoning extended perhaps beyond gambling, but also all the harmful effects (mental health and social) that problematic gambling might generate through its risk factors.

7.2 **Correction of gambling cognitive distortions**

I have already said, and it was seen from the presentation of each gambling cognitive distortion, that the GCD are correctable and the key cognitive-epistemic source of that correction are the concepts of gambling mathematics. The first step in correcting the GCD is knowing them – getting informed about what they are, and why and how they form. The current guide is sufficient in this respect. The next and decisive step is achieving the necessary cognitive assets for demounting the misconceptions and fallacious reasoning that constitute GCD, and reshaping them in the appropriate form and with the appropriate concepts. This process is an understanding and learning process whose functioning and effectiveness depend on the individual's universe of beliefs (including the level of education) and the degree to which an individual is predisposed to or affected by the GCD.

Essential for learning and understanding the concepts and knowledge required for correcting the GCD are the gambling-mathematics concepts that underlie them and the knowledge associated with the mathematical models of games of chance and gambling. It was seen in the sections dedicated to gambling mathematics and the GCD that it is not formal mathematics alone that can contribute to ensuring the required cognitive-epistemic assets in this respect, but rather the conceptual and epistemological approach to gambling mathematics. In other

…………….. missing part ………………

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