What is logical form?

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A few years ago, I wrote a book chapter called “Sobre la Naturaleza Múltiple de las Constantes Lógicas” [“On the mutiple nature of logical constants”] for a kind of *Festschrift* in honor of a former professor of mine, Raul Orayen, edited by my colleague Maite Ezcurdia and titled *Orayen: De la Forma Lógica al Significado [From Logical Form to Meaning]* (IIFs UNAM, 2007. ISBN 978-970-32-4534-5. Pp. 61-82, and I just noticed that a lot of the work I have been doing on the nature of logical form since then has deep roots in that text. The question I was trying to answer back then has been recently taken by Kit Fine (*forthcoming*), Andrea Iaccona (*forthcoming*) and others thus:

> “If a sentence *s* can rightfully be said to have a logical form *f*, then there must be some fact in virtue of which *s* has *f*, that is, some fact that constitutes grounds for the ascription of *f* to *s*.[1] So it may be asked what kind of fact it is. In other words, the question is what it means for *s* to have *f*, or equivalently, what it is for *f* to be the logical form of *s*.” (Iaccona *forthcoming*)

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1. A note on grounding. What does it mean for something to ground logical properties and relations. Once upon a time, philosophers –specially of a naturalist bent– were very weary of notions of fundamentality, so they hated this! So they opted for a more psychological notion of priority, so for example, Robert Stainton and Ray Elguardo (2001) formulated the condition thus:
"The logical form of *a* is that in virtue of which agents recognize *a*’s entailmente relations” (Stainton & Elguardo 2001, 393)
So the idea was that in the cognitive process through which we recognize the logical properties and relations of a proposition, we need to first detect the proposition’s logical form.

I consider three three sorts of possible answers to the question "what is a proposition's logical form?":

1. Structuralism: Having a certain form is being constructed in a certain way
2. Internalism: Logical form is a component of the proposition
3. Externalism:[2] Logical form is a relation (between the proposition and its constituentes)

This metaphysical question is centrally related to other fundamental questions in the philosophy of logic, questions like: what do formulas mean? what is the relation between logic and language? are formal languages genuine languages? what is the semantic value of a variable? is all logic formal? and if not, what role does its formal part play? is all content conceptual? and if not, how does it relate to conceptual content? is all thought computation? and, of course, what is logic itself? This is so because the notion of logical form has had a very lively and successful life throughout philosophy. The main applications of the notion have been, of course, in logic and (formal) semantics. In Iaccona's words, not only do we believe that a proposition's logical properties and relations are grounded on the proposition's logical form, it is also commonly assumed that "a systematic connection between logical form and semantic structure" exists so that logical form determines what each of the proposition's components contributes to the

2. I now prefer these terms over "analytic" and "synthetic" which I employ in (2003) and (2008b)

logical behaviour of the whole proposition (Iaccona *forthcoming*).[3] Thus, we would expect our metaphysical account to ground these two functions, i.e., metaphysically explain how logical form achieves to fulfill these two functions. In other words, a good metaphysical account of logical form, must make clear why logical form is logical.

One might think that the most straightforward answer (or at least one of them) would be to say that a proposition's logical form is just the *way* the propositions components compose the proposition. Unfortunately, this would only be close to a fruitful answer if we had a good account of what these so-called "**ways**" propositional components compose propositions are (or in general, what it means for some components to compose some thing in one way instead of another), unfortunately, we do not and thus, this would not. In other words, since we do not have a good account of *forms*, we cannot appeal to the notion of *form* to give a good account of *logical form*. I and others think we have a good metaphysical and epistemological account of forms, but mine like most of the accounts out there in the literature is controversial to say the least, so I will consider here only ways of trying to define logical form that do not appeal to some notion of form. In other words, I will exclude all attempts to define logical form as a form that is logical.

Our first hypothesis is closely linked it to the idea that there are so-called "logical" words (or constants) in natural language – words like "and", "or", etc. – whose

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3. This is weaker than what Iaccona calls the Davidonian or semantic notion of logical form, where, in the words of Petroski (2016), "following Davidson, one might say that the logical forms of expressions (of some natural language) just are the structures that determine the corresponding meanings given the relevant word meanings." The logical notion of logical form we are after must be related to the Davisonian one, but they are not the same; the Davisionian is stronger, so that propositions with the same logical logical form will have the same Davidsonian logical form, so that "the phenomenon of valid inference may be largely a by-product of semantic compositionality", but not necessarily the other way around.

contribution to the proposition expressed by the sentences they occur in is something like a logical operation. Unfortunately, trying to find a good criteria to identify when a word occurs as a logical constant in a sentence has proved elusive to say the least (Gómez-Torrente 2000). Against this view, in the aformentioned (2007) chapter, I argued that these words are better conceived as defeasible clues into the logical form of propositions. Thus, the reason why we think that logical forms are constituents is because, in natural language, we use syntactic and morphological elements to signal to our hearers how to grasp the propositions we want to communicate. We use, for example, a word like "and" to signal that the proposition we want to communicate must be constructed by conjunction from other propositions, etc.

However, there are well-known further and deeper metaphysical reasons to reject this first hypothesis, dating at least as back as Russell (and developed by Kit Fine, John MacFarlane, Cody Gilmore, among others). The major metaphysical issue is that, if logical form was an extra component in a proposition besides the usual propositional constituents there would be something extra necessary to bring them all together into a unified proposition. And this would be either some second sort of logical form or would make the logical form as component unnecessary.

The basic idea behind this first hypothesis is that a proposition *P*'s logical form *F* is a proper part of *P*. In order for logical forms not to be innocuous, *F* must be a proper part of *P*. Thus, let *C* be the remanent of *P* once we remove its logical form *F*. Presumably *C* would just be the unstructured sum or set of constituents of *P*, but this is not an essential part of the argument. What is essential is that *C* not be structured in such a way that C necessitates F or vice versa, this means that F must actually contribute something substantial and irreducible to the proposition. In other words, it must be possible for at least two different propositions P_1 and P_2 to be different only in their logical form, and vice versa, it must be possible for at least two different propositions P_1 and P_2 to be different yet have the same logical form. This means that there must be a C such that P_1 is the composite of C and logical form F_1 and P_2 is the composite of C and logical form F_2, and also there must be at least one logical form F such that there are at least two different propositions P_1 and P_2 such that P_1 is the composite of F and C_1 and P_2 is the composite of the same F but a different C_2.

Now, consider the following case: let *A* and *B* be two components of the same type, for example, two atomic propositions, also let 'if *A* then *B*' be the material implication with *A* as its antecedent proposition and *B* as its consequent proposition and let 'if *B* then *A*' be the material implication with *B* as its antecedent proposition and *A* as its consequent proposition.[4] Now, the key question here is whether 'if *A* then *B*' and 'if *B* then *A*' have the same logical form, but different constituents, or the same constituents but different logical form? Unfortunately, if logical form is a component of the proposition both horns of the dilemma are untenable. Consider first the horn according to which 'if *A* then *B*' and 'if *B* then *A*' have the same logical form –'if *X* then *Y*' –but different components. Thus, the remaining components cannot be just *A* and B (nor their sum, fusion or set), but need be something else, like the ordered sets <*A*, *B*> and <*B*, *A*> respectively. However, it has been argued that this solution is extremely ad-hoc (for

4. Any homogenous asymmetric relation would work just as well.

example, by Fine 2003), since there is no way of *ordering* the components *A* and *B* in such a way that, when combined when the conditional logical form, would make *A* become the antecedent (and *B* the consequent) –instead of the other way around–, that is neither compeltely *ad-hoc* (like talking of first and second places, since there is nothing in the material conditional that makes the antecedent be 'first' and the consequent 'second' in any substantial sense) nor circular (like talking of *A* and *B* ordered in corresponding antecedent and consequent places).

In other words, what we want is a way of understanding what are a proposition's constituents such that they combine with logical forms in a way that is both unique and natural (so that it is impossible for two different propositions to have the same constituents and logical form). Unfortunately, there does not seem to be such a way. There is no natural way to say that, for example, '*A* and not *B'* has the same components as 'if *A* then *B'* but not 'if *B* then *A'*. This makes the first horn of the dilemma untenable.

The second horn does not fare much better. If *C* and *D* are different only in their logical form, there must be a common element *E* and a pair of different logical forms F_1 and F_2 such that *C* is the composite of *E* and F_1 and *D* is the composite of *E* and F_2. However, if this were so, a dual version of the previous argument would reproduce here. The idea is that the logical forms F_1 and F_2 could not simply be the material conditional, but something more fine grained like the forms 'if *X* then *Y*' and 'if *Y* then *X*'. But again, there would be no ad-hoc way of making 'if *X* then *Y*' be the appropriate form for 'if *A* then *B*' instead of 'if *Y* then *X*'. Once again, there is nothing in variable *X* that makes it fit for being substituted by *A* instead of *B*. And once again, the only other option would be to make logical forms be too fine grained, i.e., fine grained enough that there would be practically one logical form for each proposition (or at least for non-symmetric propositions like these), so that the logical form of proposition 'if *A* then *B*' Were something like 'if *a* then *b*' where the place marked by *a* can only be substituted by proposition *A* and the place marked by *b* can only be substituted by proposition *B*. This would make components completely idle and thus is also an inadmissible solution. Both options thus lead to insatisfactory accounts, making the first hypothesis untenable.

In other words, the hypothesis that propositions are composed of two different and independent components, form and content, is metaphysically incoherent. It is torn apart by two nefarious forces, each one pulling us into absurdity from opposite directions: one pulling us into slicing logical forms in finer, and finer ways, until we reach the absurd conclusion that we have as much logical forms as propositions, that is, that every proposition has its own logical form; the other one pulling us into the opposite direction, into slicing the proposition's constituents in finer, and finer ways, until we reach the absurd conclusion that no two different propositions have the same constituents.

Option (2) also deserves detailed discussion. In (2002), I trace this hypothesis to a couple of passages in Quine's *Methods of Logic* and Orayen's *Lógica, Significado y Ontología,* even though the idea can also be found in John MacFarlane's dissertation[5] and now, I am sure I (just like MacFarlane) was overreaching in my interrpetation of those passages. According to this second way of conceiving of logical forms, having a given

5. MacFarlane's motivation is to account for the possibility of a language without logical words, but with other mechanisms to express logical form, like typographical conventions or diagrammatic means.

logical form is just having been constructed a certain way, so that for example, a proposition *P* has the form (*A* and *B*) iff it was built out of another two propositions *A* and *B* by the operation of conjunction, a proposition *P* has the form (*A* and (*B* or *C*)) iff *P* was built out of another two propositions *A* and *X* by the operation of conjunction (which explains why, at some level of abstraction, a proposition of the form (*A* and *B*) and a proposition of the form (*A* and (*B* or *C*)) are both conjunctions, despite having different logical forms), and *X* in turn was built out of another two propositions *B* and *C* by the operation of disjunction, etc. The main problem with this way of understanding what a logical form is is that it is not clear in what sense it is logical. This might give us the ellusive notion of *form* we were looking for earlier, but not the notion of *logical* form we were looking for.

In consequence, I consider a third way of understanding logical form. The basic idea there is that for a proposition *P* to have a particular form is to be related in a specific way to certain other propositions (C_1, C_2, … C_n) commonly, but misleadingly known as the proposition's constituents. I insist that this way of talking is misleading because there is no reason to think of them as "parts of" or "included in" the proposition (at least not without falling into the antinomies I mentioned regarding the first hypothesis. After all, I think this kind of talk is nothing but a leftover from hypothesis (1), which in turn is just the result of mistaking the syntactical components of a formula with the logical 'constituents 'of a proposition).

This third way of conceiving of logical forms seems to face two substantial problems: first, it makes logically equivalent propositions have the same logical form, and second, it is prone to something like a Benacerraff problem of underspecificity. Let me address each one of them.

First of all, this third approach is prone to a sort of Benacerraffian problem of underspecificity because there seem to be many ways of cashing out the 'particular way' propositions are related to their so-called constituents. For an inferentialist like Gentzen, for example, a proposition *P* has the form (*A* and *B*) iff *P* is related to another two propositions *A* and *B* in such a way that each one of them is a logical consequence of *P* and for any proposition *Q* that also has *A* and *B* among its logical consequences, *P* is a logical consequence of *Q*. From this inferentialist perspective, a proposition *P* has the form (*A* and (*B* or *C*)) iff *P* is related to propositions *A, B* and *C* in such a way that *A* is a logical consequence of *P*, and *P* is a logical consequence of *B* and *A* and of *C* and *A*, etc. For a representationalist, in contrast, a proposition *P* has the form (*A* and *B*) iff *P* is related to another two propositions *A* and *B* in such a way that *P* is true iff *A* and *B* are also true, a proposition *P* has the form (*A* and (*B* or *C*)) iff *P* is related to propositions *A*, *B* and *C* in such a way that *A* is true if and only if *A* is true and at least one of *B* and *C* is true as well, etc.

Finally, some people have argued that propositions that are logically equivalent might neverhteless have different logical form. For example, the following pairs of propositions are logically equivalent, yet do not seem to have the same logical form:

1a. '*A* and *B*'

1b. '*B* and *A*'

2a. 'A and (B or A)'

2b.’A’

3a. ‘A or not A’

3b. ‘if *C* then (if *D* then *C*)’

However, if one takes a closer look as to the grounds one might have to hold that each of these pairs constitutes propositions with different logical forms, these seem weak. For example, there seems to be nothing but strong intuitions behind the claim that *1a* and *1b* have different logical forms. however, I would argue that these are just different ways of expressing the same logical form in our formal language. Our linguistic conventions cosntrain us to sequences of symbols, as such it is necessary for a symbol to be first and another later, but we need not interpret this as saying anything about what the symbols stand for.

Regarding 2a and 2b, it could be argued that they cannot have the same logical form because their constituents are different (Iaccona *forthcoming*). This might be backed up by the idea that logical forms tell you how to combine constituents into propositions and thus, if the constituents are different, the way they are combined cannot be but different. However, I have already dismissed the idea that forms are ways components are combined. I think a better way of thinking about the relation between a proposition’s logical form and its so-called constituents would be that logical forms determine the contribution the proposition’s constituents make to the proposition. But once we think of logical forms this way, since *B* makes the same contribution to the propositions *2a* and *2b* –that is, *nothing–,* then there is no difference at this level either between *2a* and *2b*.

Finally, one could appeal to something like what Frege called the informativity of *3a* and *3b* to claim that even though they are logically equivalent they cannot have the same logical form, for they are radically different propositions (with substantially different epistemic profiles). However, once again, proper groundign for the inferential step from being different propositions to having different logical forms is still missing. After all, logical forms usually are postulated to play two roles: to determine the logical properties and relations and to determine the semantic contribution of the propositions constituents (Iaccona, manuscript), and logically equivalent propositions certainly have the same logical properties and relations –after all, that is just what it means to say that two propositions are logically equivalent. Similarly, as I just started arguing shortly above, the components of logically equivalent propositions make the same semantic contributions in logically equivalent propositions ('no logical difference without a semantic difference' might be a good slogan for the relation between logic and semantics): *A* makes the same semantic contribution to the content of *1a* as to *1b* (that of being one of the necessary conditions for the proposition's truth, for example), as aformentioned, *B* makes the same contribution to *2a* as to *2b,* that is, no contribution at all; and similarly for *A*, *B* and *C* in *3a* and *3b*: they make no contribution to the proposition's truth conditions, for *3a* and *3b* are true regardless of what *A*, *B* and *C* mean and whether they are true or not. Thus to postulate different logical forms for logically equivalent propositions would be to introduce a distinction without a difference.

These intuitions and argument for logical forms being finer than logical equivalence stem from a confusion on the role of formal languages in logic: they serve to make salient the logical form of propositions, but they can do this very well without there being a one to one correspondence between formulas and forms. In other words, different formulas may express the same logical form and this might be a substantial discovery, which would account for the epistemic opacity of the equivalences between 1a and 1b, 2a and 2b, 3a and 3b, etc.

I favour this third way of understanding logical form, because it has the main advantage of being eminently *logical* (specially from an inferentialist perspective). In other words, it explains why a proposition's logical form is logical. Furthermore, at least for classical logic, there is a way out of the Benacerrafian problem, since one can argue that the inferentialist and referentialist accounts are just different ways of identifying the same relation (I develop this argument in further detail in a posterior paper "Inferential Patterns" from 2008a on *Crítica* 40 (120): 3-35). Thus, it seems like the most promising option.

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