

## **Intensional entities**

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Intensional entities are such things as concepts, propositions and properties. What makes them ‘intensional’ is that they violate the principle of extensionality; the principle that equivalence implies identity. For example, the concept of being a (well-formed) creature with a kidney and the concept of being a (well-formed) creature with a heart are equivalent in so far as they apply to the same things, but they are different concepts. Likewise, although the proposition that creatures with kidneys have kidneys and the proposition that creatures with hearts have kidneys are equivalent (both are true), they are not identical. Intensional entities are contrasted with extensional entities such as sets, which do satisfy the principle of extensionality. For example, the set of creatures with kidneys and the set of creatures with hearts are equivalent in so far as they have the same members and, accordingly, are identical. By this standard criterion, each of the following philosophically important types of entity is intensional: qualities, attributes, properties, relations, conditions, states, concepts, ideas, notions, propositions and thoughts.

All (or most) of these intensional entities have been classified at one time or another as kinds of universals. Accordingly, standard traditional views about the ontological status of universals carry over to intensional entities. Nominalists hold that they do not really exist. Conceptualists accept their existence but deem it to be mind-dependent. Realists hold that they are mind-independent. *Ante rem* realists hold that they exist independently of being true of anything; *in re* realists require that they be true of something.

### **1 History**

### **2 Extensional reductions**

### **3 Non-reductionist approaches**

#### **1 History**

Contemporary use of the term ‘intension’ derives from the traditional logical doctrine that an idea has both an extension and an intension (or comprehension). This doctrine is explicit in most modern logicians of the nineteenth century, implicit in many medieval logicians and, arguably, present in Porphyry and Aristotle. Although there is divergence in formulation, most of these thinkers accept that the extension of an idea (or concept) consists of the subjects to which the idea applies, and the intension consists of the attributes implied by the idea. The Port Royalists (see [Arnauld 1662](#)) tell us, ‘the comprehension of the idea of triangle includes extension, figure, three lines, three angles, and the equality of these three angles to two right angles, and so forth’. This yields the ‘law of inverse ratio’: the larger the intension, the smaller the extension. (Note that these traditional doctrines were expressed with the use of plurals and without explicit commitment to classes or sets.) If the extension consists of the individuals (as opposed to species) to which an

idea applies, it is evident that two ideas could have the same extension but different intensions. This is the tie to Russell's subsequent notion of an intensional propositional function (1910–13) and leads to our contemporary usage of 'intensional entity' and 'extensional entity'.

In contemporary philosophy, it is linguistic expressions, rather than concepts, that are said to have intensions and extensions. The intension is the concept expressed by the expression, and the extension is the set of items to which the expression applies. On the standard picture, that set is the same as the set of items to which the concept applies, hence the standard view that intension determines extension. This usage resembles Mill's use of 'denotation' and 'connotation' and Frege's use of '*Bedeutung*' and '*Sinn*' (see [Mill, J.S. §§2–3](#); [Frege, G. §3](#)).

The systematic study of intensional entities has been pursued largely in the context of intensional logic; that part of logic in which the principle of substitutivity of equivalent expressions fails. For example, 'Necessarily, creatures with kidneys have kidneys' is true whereas 'Necessarily, creatures with hearts have kidneys' is false. Following Frege (1892), Whitehead and Russell (1910–13), Carnap (1947) and Church (1951), the now standard explanation of this failure of substitutivity is this: the truth-value of these sentences is determined, not (just) by the extension of 'creature with kidney' and 'creature with heart' (that is, the set of creatures with kidneys) but (also) by the intensions of these expressions, that is, by the intensional entities which they express (the concept of being a creature with a kidney and the concept of being a creature with a heart). Because these intensions are different, the sentences can have different truth-values.

This account yields a method for determining precise intensional identity conditions: intensional entities should be as finely discriminated as is necessary for explaining associated substitutivity failures in intensional logic. For example, this method has led most philosophers to hold that logically equivalent concepts can be distinct, and even that in a correct definition the definiens and the definiendum can express different concepts. This is not to say that the properties corresponding to those concepts would be distinct; on the contrary, the definition would be correct only if the properties were identical. Such considerations lead to the view that there are also more coarsely grained intensional entities (properties, relations, conditions in the world). Independent arguments in metaphysics, epistemology, philosophy of science and aesthetics seem to support the same conclusion (see, for example, [Armstrong 1978](#), [Bealer 1982](#), [Lewis 1986](#)). On this view, properties and relations (as opposed to concepts) play a primary role in the non-arbitrary categorization and identification of objects, in the constitution of experience, in description and explanation of change, in the theory of inductive inference, in the analysis of objective similarities and in the statement of supervenience principles. In the resulting picture there are both fine-grained intensions, which play the role of cognitive and linguistic contents, and coarse-grained intensions, which play a constitutive role in the structure of the world.

Systematic theories of intensional entities have often incorporated some form of extensional reductionism. The leading reductions (possible worlds, propositional complexes, propositional functions) are critically reviewed below; non-reductionist approaches to intensional entities are then discussed. At issue is the question of what intensional entities *are*. Are they identical to extensional functions, ordered sets, sequences and so on? Or are they *sui generis* entities, belonging to an altogether new category?

## 2 Extensional reductions

On the possible worlds reduction (see [Lewis 1986](#); [Stalnaker 1984](#)), a proposition is either a set of possible worlds or a function from possible worlds to truth-values, and properties are functions from possible worlds to sets of possible (usually non-actual) objects (see [Semantics, possible worlds §9](#)). Many people find the possible worlds theory intuitively implausible. Are familiar sensible properties (for example, colours, shapes, aromas) really functions? When I am aware that I am in pain, is a set (function) of possible worlds really the object of my awareness? Besides this intuitive objection, there are familiar epistemological and metaphysical objections to a theory that is truly committed to the existence of things that do not exist actually. Finally, the possible worlds reduction implies that necessarily equivalent intensions are identical, but as seen above this is implausible in the case of concepts and propositions. Certain possible worlds theorists (for example, [Cresswell 1985](#)) have responded to this problem by holding that concepts and propositions are ordered sets (sequences, abstract trees) whose elements are possible worlds constructs. This revisionary view is a variation of the propositional-complex reduction.

According to this reduction (see [Perry and Barwise 1983](#)), concepts and propositions are nothing but ordered sets (sequences, abstract trees, partial functions) whose ultimate elements are properties, relations and perhaps individuals. For example, the proposition that you are running is the ordered set  $\langle \text{running}, \text{you} \rangle$ ; the proposition that you are running and I am walking is

$\langle \text{conjunction}, \langle \text{running}, \text{you} \rangle, \langle \text{walking}, \text{me} \rangle \rangle$ ;

and so forth. As with the possible worlds theory, this theory clashes with intuition. When I am aware that I am in pain, is an ordered set the object of my awareness? When I see that you are running, do I see an ordered set? Furthermore, there is in principle no way to determine which ordered set I allegedly see. Is it  $\langle \text{running}, \text{you} \rangle$ ? Or is it  $\langle \text{you}, \text{running} \rangle$ ? The choice seems utterly arbitrary, but if the reduction were correct, there would have to be a fact of the matter.

A rather different kind of difficulty arises in connection with 'transmodal quantification'. Consider the following intuitively true sentence:

(1) Every  $x$  is such that, necessarily, for every  $y$ , it is either possible or impossible that  $x = y$ .

In symbols,

(2)  $(\forall x)\Box(\forall y)(\text{Possible } [x = y] \vee \text{Impossible } [x = y])$ .

By the propositional-complex theory, this is equivalent to

(3)  $(\forall x)\Box(\forall y)(\text{Possible } \langle x, \text{identity}, y \rangle \vee \text{Impossible } \langle x, \text{identity}, y \rangle)$ .

The singular term ' $\langle x, \text{identity}, y \rangle$ ' may be thought of as a definite description, 'the ordered set whose elements are  $x$ , identity and  $y$ '. Accordingly, there are two ways to understand (3). On the first construal (corresponding to the narrow scope reading of the definite description), (3) implies

$(\forall x)\Box(\forall y)(\exists v)v = \langle x, \text{identity}, y \rangle$ .

However, by the principle that, necessarily, a set exists only if its elements exist, this implies

$(\forall x)\Box(\exists v)v=x$ .

That is, everything necessarily exists. A manifest falsehood. On the alternative construal of (3), corresponding to the wide scope reading of the definite description, ‘the ordered set  $\langle x, \text{identity}, y \rangle$ ’, (3) implies that every  $x$  is such that, necessarily, for all  $y$ , there exists an actual set  $\langle x, \text{identity}, y \rangle$ . That is

$$(\forall x)\Box(\forall y)(\exists_{\text{actual}}v)v = \langle x, \text{identity}, y \rangle.$$

But, by the principle that, necessarily, a set is actual only if its elements are actual, this implies

$$\Box(\forall y)y \text{ is actual.}$$

That is, necessarily, everything (including everything that could exist) is already actual. Another manifest falsehood. So either way, (3) implies something false. But (3) is the propositional-complex theorists’ way of representing the true sentence (2). So the propositional-complex theory appears unable to handle intuitively true sentences such as (2). (This style of argument can be exploited in a defence of *ante rem* realism.)

According to the propositional-function theory (see [Whitehead and Russell 1910–13](#)), a property is a function from objects to propositions, where propositions are taken to be primitive entities. For example, the property ‘being red’ is the function  $(\lambda x)(\text{the proposition that } x \text{ is red})$ . For any given object  $x$ , the proposition that  $x$  is red is the result of applying this function to the argument  $x$ . That is, the proposition that  $x$  is red is  $(\lambda x)(\text{the proposition that } x \text{ is red})(x)$ . But are the familiar sensible properties really functions? Once again, this is highly implausible and should be rejected on this ground alone if an acceptable alternative exists.

Besides this sort of intuitive difficulty, there are several technical difficulties. Consider an illustration involving properties of integers. Being even = being an  $x$  such that  $x$  is divisible by two; and being self-divisible = being an  $x$  such that  $x$  is divisible by  $x$ . Given the propositional-function theory, the following identities hold:

$$\begin{aligned} \text{that two is even} &= (\lambda x)(\text{that } x \text{ is even})(\text{two}) \\ &= (\lambda x)(\text{that } x \text{ is divisible by two})(\text{two}) \\ &= \text{that two is divisible by two} \\ &= (\lambda x)(\text{that } x \text{ is divisible by } x)(\text{two}) \\ &= (\lambda x)(\text{that } x \text{ is self-divisible})(\text{two}) \\ &= \text{that two is self-divisible} \end{aligned}$$

However, that two is even and that two is self-divisible are plainly different propositions: certainly someone could be consciously and explicitly thinking the former while not consciously and explicitly thinking the latter. Indeed, someone who is thinking that two is even might never have considered the concept of self-divisibility. (Analogous difficulties arise for Church’s somewhat similar propositional-function theory of concepts ([1951](#)). Incidentally, since properties are not propositional functions, using function abstracts to denote them invites confusion. A growing practice is to use  $\ulcorner [v: A] \urcorner$  to denote the property of being a  $v$  such that  $A$ , just as in set theory  $\{v: A\}$  denotes the set of things  $v$  such that  $A$ .)

### 3 Non-reductionist approaches

Each of the preceding problems arises from the attempt to reduce intensional entities of one kind or another to extensional entities; either extensional functions or sets. These difficulties have led some theorists to adopt non-reductionist approaches (see [Bealer 1979, 1982](#)). Consider the following truisms. The proposition that  $A \& B$  is the conjunction of the proposition that  $A$  and the proposition that  $B$ . The

proposition that not  $A$  is the negation of the proposition that  $A$ . The proposition that  $Fx$  is the predication of the property  $F$  of  $x$ . The proposition that there exists an  $F$  is the existential generalization of the property  $F$ , and so on. These truisms tell us what these propositions essentially are: they are by nature conjunctions, negations, singular predications, existential generalizations and so on. These are rudimentary facts which require no further explanation and for which no further explanation is possible. Until the advent of extensionalism, this was the standard view.

By adapting techniques developed in the algebraic tradition in extensional logic, one is able to develop this non-reductionistic approach. Examples such as those just given isolate fundamental logical operations: conjunction, negation, singular predication, existential generalization and so on. Intensional entities are then taken as *sui generis* entities; the aim is to analyse their behaviour with respect to the fundamental logical operations.

An intensional algebra is a structure consisting of a domain  $D$ , a set of logical operations and a set of possible extensionalization functions. The domain divides into subdomains: particulars, propositions, properties, binary relations, ternary relations and so on. The set of logical operations includes those listed above plus certain auxiliary operations. The possible extensionalization functions assign a possible extension to relevant items in the domain: each proposition is assigned a truth-value; each property is assigned a set of items in  $D$ ; each binary relation is assigned a set of ordered pairs of items in  $D$ , and so on. One extensionalization function is singled out as the actual extensionalization function: the propositions which are true relative to it are the propositions which are actually true, and so on. (More formally, an intensional algebra is a structure  $\langle D, \tau, K, G \rangle$ .  $D$  divides into subdomains:  $D_{-1}, D_0, D_1, D_2, \dots$ .  $D_{-1}$  consists of particulars;  $D_0$ , propositions;  $D_1$ , properties;  $D_2$ , binary relations, and so on.  $\tau$  is a set of logical operations on  $D$ .  $K$  is a set of extensionalization functions.  $G$  is a distinguished function in  $K$  which is the actual extensionalization function.)

To illustrate how this approach works, consider the operation of conjunction, conj. Let  $H$  be an extensionalization function. Then, conj satisfies the following: for all propositions  $p$  and  $q$  in  $D$ ,  $H(\text{conj}(p, q)) = \text{true}$  iff  $H(p) = \text{true}$  and  $H(q) = \text{true}$ . Similarly, if neg is the operation of negation, then for all propositions  $p$  in  $D$ ,  $H(\text{neg}(p)) = \text{true}$  iff  $H(p) = \text{false}$ . Likewise, for singular predication preds, which takes properties  $F$  in  $D$  and items  $y$  in  $D$  to propositions in  $D$ :  $H(\text{pred}_s(F, y)) = \text{true}$  iff  $y$  is in the extension  $H(F)$ .

This non-reductionistic approach can be extended to more complex settings in which, for example, both fine-grained intensional entities (concepts, propositions) and coarse-grained intensional entities (properties, relations, conditions) are treated concurrently and in which a relation of correspondence between the two types of intensional entities can be characterized in terms of the fundamental logical operations. The non-reductionist approach can thus accommodate the fine-grained intensional entities which serve as cognitive and linguistic contents and also the more coarsely grained intensional entities which play a constitutive role in the structure of the world.

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