that the classical results of mathematical analysis can be formally obtained in conservative extensions of first-order arithmetic. For the metamathematical work Gentzen's introduction of sequent calculi and the use of transfinite induction along constructive ordinals turned out to be very important, as well as Gödel's primitive recursive functionals of finite type. The methods and results of proof theory are playing, not surprisingly, a significant role in computer science.

Work in proof theory has been motivated by issues in the foundations of mathematics, with the explicit goal of achieving epistemological reductions of strong theories for mathematical practice (like set theory or second-order arithmetic) to weak, philosophically distinguished theories (like primitive recursive arithmetic). As the formalization of mathematics in strong theories is crucial for the metamathematical approach, and as the programmatic goal can be seen as a way of circumventing the philosophical issues surrounding strong theories, e.g., the nature of infinite sets in the case of set theory, Hilbert's philosophical position is often equated with formalism – in the sense of Frege in his *Über die Grundlagen der Geometrie* (1903–06) and also of Brouwer's inaugural address *Intuitionism and Formalism* (1912). Though such a view is not completely unsupported by some of Hilbert's polemical remarks during the 1920s, on balance, his philosophical views developed into a sophisticated instrumentalism, if that label is taken in Ernest Nagel's judicious sense (*The Structure of Science,* 1961). Hilbert's is an instrumentalism emphasizing the contentual motivation of mathematical theories; that is clearly expressed in the first chapter of Hilbert and Bernays's *Grundlagen der Mathematik I* (1934). A sustained philosophical analysis of proof-theoretic research in the context of broader issues in the philosophy of mathematics was provided by Bernays; his penetrating essays stretch over five decades and have been collected in *Abhandlungen zur Philosophie der Mathematik* (1976).

See also CONSISTENCY, FORMALIZATION, GÖDEL'S INCOMPLETENESS THEOREMS, HILBERT'S PROGRAM, METAMATHEMATICS. W.S.

propensity, an irregular or non-necessitating causal disposition of an object or system to produce some result or effect. Propensities are usually conceived as essentially probabilistic in nature. A die may be said to have a propensity of "strength" or magnitude $1/6$ to turn up a 3 if thrown from a dice box, of strength $1/3$ to turn up, say, a 3 or 4, etc. But propensity talk is arguably appropriate only when determinism fails. Strength is often taken to vary from 0 to 1.

Popper regarded the propensity notion as a new physical or metaphysical hypothesis, akin to that of forces. Like Peirce, he deployed it to interpret probability claims about single cases: e.g., the probability of *this* radium atom's decaying in 1,600 years is $1/2$. On relative frequency interpretations, probability claims are about properties of large classes such as relative frequencies of outcomes in them, rather than about single cases. But single-case claims appear to be common in quantum theory. Popper advocated a propensity interpretation of quantum theory. Propensities also feature in theories of indeterministic or probabilistic causation.

Competing theories about propensities attribute them variously to complex systems such as chance or experimental set-ups or arrangements (a coin and tossing device), to entities within such set-ups (the coin itself), and to particular trials of such set-ups. *Long-run* theories construe propensities as dispositions to give rise to certain relative frequencies of, or probability distributions over, outcomes in long runs of trials, which are sometimes said to "manifest" or "display" the propensities. Here a propensity's strength is identical to some such frequency. By contrast, *single-case* theories construe propensities as dispositions of singular trials to bring about particular outcomes. Their existence, not their strength, is displayed by such an outcome. Here frequencies provide evidence about propensity strength. But the two can always differ; they converge with a limiting probability of 1 in an appropriate long run.

See also CAUSATION, DETERMINISM, DISPOSITION, PEIRCE, PROBABILITY, QUANTUM MECHANICS. D.S.

proper class. See CLASS.

properly basic relief. See EVIDENTIALISM, PLANTINGA.

proper names, causal theory of. See CAUSAL THEORY OF PROPER NAMES.

proper sensibles. See ARISTOTLE.

proper symbol. See SYNCATEGOREMATA.

properties of terms, doctrine of. See SHERWOOD.

property, roughly, an attribute, characteristic, feature, trait, or aspect.

Intensionality. There are two salient ways of talking about properties. First, as *predicables* or *instantiables.* For example, the property red is predicable of red objects; they are instances of it. Properties are said to be *intensional* entities in the sense that distinct properties can be truly predicated of (i.e., have as instances) exactly the same things: the property of being a creature with a kidney ≠ the property of being a creature with a heart, though these two sets have the same members. Properties thus differ from sets (collections, classes); for the latter satisfy a principle of *extensionality:* they are identical if they have the same elements. The second salient way of talking about properties is by means of *property abstracts* such as 'the property of being *F*'. Such linguistic expressions are said to be intensional in the following semantical (vs. ontological) sense: 'the property of being *F*' and 'the property of being *G*' can denote different properties even though the predicates '*F*' and '*G*' are true of exactly the same things. The standard explanation (Frege, Russell, Carnap, et al.) is that 'the property of being *F*' denotes the property that the predicate '*F*' expresses. Since predicates '*F*' and '*G*' can be true of the same things without being synonyms, the property abstracts 'being *F*' and 'being *G*' can denote different properties.

Identity criteria. Some philosophers believe that properties are identical if they necessarily have the same instances. Other philosophers hold that this criterion of identity holds only for a special subclass of properties – those that are purely qualitative – and that the properties for which this criterion does not hold are all "complex" (e.g., relational, disjunctive, conditional, or negative properties). On this theory, complex properties are identical if they have the same form and their purely qualitative constituents are identical.

Ontological status. Because properties are a kind of universal, each of the standard views on the ontological status of universals has been applied to properties as a special case. *Nominalism:* only particulars (and perhaps collections of particulars) exist; therefore, either properties do not exist or they are reducible (following Carnap et al.) to collections of particulars (including perhaps particulars that are not actual but only possible). *Conceptualism:* properties exist but are dependent on the mind. *Realism:* properties exist independently of the mind. Realism has two main versions. *In rebus* realism: a property exists only if it has instances. *Ante rem* realism: a property can exist even if it has no instances. For example, the property of being a man weighing over ton has no instances; however, it is plausible to hold that this property does exist. After all, this property seems to be what is expressed by the predicate 'is a man weighing over a ton'.

Essence and accident. The properties that a given entity has divide into two disjoint classes: those that are essential to the entity and those that are accidental to it. A property is essential to an entity if, necessarily, the entity cannot exist without being an instance of the property. A property is accidental to an individual if it is possible for the individual to exist without being an instance of the property. Being a number is an essential property of nine; being the number of the planets is an accidental property of nine. Some philosophers believe that all properties are either essential by nature or accidental by nature. A property is *essential by nature* if it can be an essential property of some entity and, necessarily, it is an essential property of each entity that is an instance of it. The property of being self-identical is thus essential by nature. However, it is controversial whether every property that is essential to something must be essential by nature. The following is a candidate counterexample. If this automobile backfires loudly on a given occasion, loudness would seem to be an essential property of the associated bang. That particular bang could not exist without being loud. If the automobile had backfired softly, that particular bang would not have existed; an altogether distinct bang – a soft bang – would have existed. By contrast, if a man is loud, loudness is only an accidental property of him; he could exist without being loud. Loudness thus appears to be a counterexample: although it is an essential property of certain particulars, it is not essential by nature. It might be replied (echoing Aristotle) that a loud bang and a loud man instantiate loudness in different ways and, more generally, that properties can be predicated (instantiated) in different ways. If so, then one should be specific about which kind of predication (instantiation) is intended in the definition of 'essential by nature' and 'accidental by nature'. When this is done, the counterexamples might well disappear. If there are indeed different ways of being predicated (instantiated), most of the foregoing remarks about intensionality, identity criteria, and the ontological status of properties should be refined accordingly.

See also ESSENTIALISM, INTENSIONALITY, RELATION. G.B.