

On The Rabbinical Exegesis of an Enhanced Biblical Value of π

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January 13, 2011

AMS 1980 Mathematics Subject Classification: 01 A 15, 01 A 17

Key words: history of **Pi**, Rabbinical exegesis.

Abstract

We present here a biblical exegesis of the value of π , $\pi_{\text{Hebrew}} = 3.141509\dots$, from the well known verse **1 Kings 7:23**. This verse is then compared to **2 Chronicles 4:2**; the comparison provides independent supporting evidence for the exegesis.¹

1.The **Hebrew Bible** often speaks the language of numbers and measurements [Feldman 1965]; the Western tradition rarely², if at all [Hoyrup

¹An earlier version has appeared in the Proceedings of the *XVIIth Canadian Congress of History and Philosophy of Mathematics*, Queen's University, Kingston, Ontario, May 27-29, 1991, pp. 93-101.

²One of those rare cases is Isaac Newton's "*obsession with the [King Solomon's] temple's plan and dimensions... Being the man he was, he plunged into an extensive program of reading in Josephus, Philo, Maimonides, and the Talmud scholars*" [Westfall 1987, pp. 346-348]. Newton's inspirations were conjectured by Frank Manuel [Manuel 1974] in the following form: "*The temple of Solomon was the most important embodiment of a future extramundane reality, a blueprint of heaven; to ascertain every last fact about it was one of the highest forms of knowledge, for here was the ultimate truth of God's kingdom expressed in physical terms*" (quoted in [Brooke 1988], p. 177.)

1989], understands this language, and the case of Biblical value of π could be seen as both a remarkable exception of this rule and its striking confirmation.

As a recent publication in *The American Mathematical Monthly* puts it, “*the ancient Hebrews regarded π as being equal to 3*” [Almkvist, Berndt 1988, p. 599]. This claim (as several identical claims made by both working mathematicians [Borwein, *et al.*1989] and historians of science [Bell 1945], [Beckmann 1971]) is based on the plain meaning of the following verse of the Hebrew Bible, **1 kings 7:23**, giving the dimensions of a tank in the First Temple³:

“*And he made a molten sea [tank], ten cubits from the one brim to the other: it was round all about, and its height was five cubits: and a line of thirty cubits did circle it round about*”. [Holy Scriptures, p. 412]

As a matter of fact, after mentioning this verse, people either can not⁴, or do not want^{5 6}, to hide (or are even happy for some ideological reasons, to emphasize⁷) their surprise by such a low accuracy of the Biblical approximation, $\pi_0 = 3$, especially in the light of well-documented evidence that the ancient Babylonians and Egyptians used for π much better approximations [Neugebauer 1969], [Gillings 1972] many hundred years before this part of

³Built by the King Solomon, the ninth century BCE; the water of the tank was used by priests for ritual ablutions. “*The molten sea was a large, bronze water reservoir set on backs of twelve bronze oxen and placed in the court of Solomon’s temple. . . The diameter was about 5 m (16 feet), the height about 2.5 m (8 feet), and the volume amounted to roughly 45,000 litres (12,000 U.S. gallons). There can be little doubt that it was one of the greatest engineering works ever undertaken in the Hebrew nation. Its size is comparable to some of the largest church bells cast in modern times*” [Zuidhof 1982, p. 179].

⁴“*but several difficulties complicate the analysis of the design of the vessel, its dimensions and the volumetric capacity . . . The sea apparently was not the typical straight-walled mathematical cylinder. . . a brim and a lily has outward curving petals. . . The biblical account mentions first the brim to brim diameter of ten cubits. A line stretched across the top would easily have measured this. . . It is then reasonable to conclude that the 30-cubit circumference was measured below the brim*” [loc.cit., pp. 179-181].

⁵“*It has been suggested, perhaps by someone who believes that ‘God makes no mistakes’, that ‘round’ and ‘depth’ are to be interpreted loosely, and that the tank was elliptical in shape*” [Almkvist, Berndt 1988, p. 599].

⁶“*Not all ancient societies were as accurate, however - nearly 1500 years later the Hebrews were perhaps still content to use the value 3*” [Borwein, *et al.*1989, p. 204].

⁷“*The inaccuracy of the biblical value of π is, of course, no more than amusing curiosity. Nevertheless, with the hindsight of what happened afterwards, it is interesting to note this little pebble on the road to the confrontation between science and religion*” [Beckmann 1971, p. 13-14].

the Hebrew Bible was written:

$$\pi_{\text{Babylon}} = 3\frac{1}{8}, \quad 0.017 > \pi - \pi_{\text{Babylon}} > 0.016;$$

$$\pi_{\text{Egypt}} = 3\frac{13}{81}, \quad 0.019 > \pi_{\text{Egypt}} - \pi > 0.018$$

Thus, it seems both appropriate and interesting at this point to give a Rabbinical interpretation of the above verse and the way the number π , implicitly⁸ defined in this verse, has to be computed [Max Munk 1962, 1968] (see also two popular and slightly different accounts in [Posamentier, Gordan 1984] and [Roiter 1993]). We do not claim, however, that the Rabbinical folklore has preserved either the *mathematical method* which was used in this approximation of π , or its *historical origins*: all that was left to us is an extremely natural and concise *mnemonic rule of the reconstruction of π_{Hebrew}* (see more about it in [Max Munk 1962, 1968]).

Such an absence of mathematical justification is, of course, well known to historians; as, e.g., a researcher writes about the value of π_{Egypt} : “*Just how this remarkably close approximation was found, we do not know, but we can offer a suggestion on examining the diagram of RMP 48*” (cited in [Gillings 1972, p. 142]). In our case, no diagrams were preserved; one could even doubt that such diagrams ever existed: “ancient Hebrews” have never regarded mathematical or, for that matter, any other scientific knowledge *per se* as deserving to be developed, preserved, and disseminated *in the written form*, as they were not interested (with the Jewish Temple being a notable exception) in creating numerous and splendid monuments of their religion and culture.

2. The key to an alternative reading of the verse **1 Kings 7:23** is to be found in the very ancient Hebrew tradition (see, e.g., [Britannica 1985], [Banon 1987, pp. 52, 53]) to *differently write* (spell) and *read* some words of the Bible; the *reading* version is usually regarded as a correct one (in particular, it is always correct from the point of view of the Hebrew *grammar*, and this is why it could be easily either remembered or reconstructed from the written version), whereas the *written* version slightly deviates from the correct spelling. (Another approach, involving the comparison between *written*

⁸ “Also, the ratio between circumference and diameter (π) of the circular vessel is not mentioned in the Bible. . .” [Zuidhof 1982, p. 180]

forms of the same words in **1 Kings 7:23** and **Chronicles 4:2** is cited in [Posamentiern, Gordan 1984]⁹; see more about this version of the exegesis in 4).

Such a disparity is a common feature for all Books of the Hebrew Bible; *and in any such case* there exists (or existed: some of this knowledge was definitely lost) a Rabbinical folklore (in fact, *strict Rabbinical hermeneutical rules* [Steinsaltz 1976, part three: *Method*], [Britannica 1985], [Banon 1987]) of interpretation of the difference in question.

In our case there is such a disparity for the word “line”: in Hebrew, it is written as “*QVH (Qof, Vav, Hea)*”, but it has to be read as “*QV (Qof, Vav)*” (the reader is advised to look at any edition of the Hebrew Bible with the Hebrew text and its translation; all disparities are either marked by an *atersik*, or the reading version is written on the margins).

Tradition asserts that not only does this disparity testify to an *approximate* character of the given length of the *line* circling around the “sea” (tank), — a much more accurate approximation to π , π_{Hebrew} , is hidden in the *choice* of the written version!

The letters of the Hebrew alphabets were traditionally used (well before the building of the First Temple [Guitel 1975]) for numerical purposes and, thus, have had numerical values¹⁰. Using these values, one can calculate values of words (as sums of values of letters, but also in several other, less obvious and/or more involved ways); these methods became later known as **gematria** [Michael Munk 1983, p. 163], [Britannica 1985]. Here are the standard numerical equivalents of the letters of the Hebrew alphabet:

*Aleph=1, Beth=2, Gimel=3, Daled=4, Hea=5, Vav=6, Zain=7, CHet=8, Tet=9,
Yod=10, Caf=20, Lammed=30, Mem=40, Noon=50, Samech=60, Aiin=70, Pea=80, TSadik=90,
Qof=100, Reish=200, Shin=300, Tav=400.*

In particular, the numerical equivalent of the *written* version, “*QVH*”, is $Qof+Vav+Hea=100+6+5=111$, whereas the numerical equivalent of the *reading* version, “*QV*”, is $Qof+Vav=106$.

⁹Who attribute their exegesis to Rabbi Eliyahu of *Vilna*, alias *Gaon-mi-Vilna*, the famous Talmudic scholar of the late eighteenth century; unfortunately, the author was unsuccessful in locating the related reference to works of *Gaon-mi-Vilna*

¹⁰Analogous numeric systems were used later, and, without doubt, following the Hebrew tradition, in the Arabic, Greek, and Cyrillic texts [Guitel 1975]

Using these numerical equivalents, one defines π_{Hebrew} as follows:

$$\begin{aligned} \pi_{\text{Hebrew}} &= \pi_0 \times \frac{\text{the numerical equivalent of the written version}}{\text{the numerical equivalent of the reading version}} = \\ &= 3 \times \frac{111}{106} = \frac{333}{106} = 3 \frac{15}{106} \end{aligned}$$

Thus,

$$\pi = 3.1415926\dots, \quad \pi_{\text{Hebrew}} = 3.1415094\dots, \quad |\pi_{\text{Hebrew}} - \pi| < 0.000084 .$$

3. Quantitatively, this is quite a remarkable approximation! However, it is even more remarkable qualitatively. Here is a finite section of the (infinite) continued fraction of the number π :

$$\pi = 3 + \frac{1}{7 + \frac{1}{15 + \frac{1}{1 + \frac{1}{292 + \frac{1}{1 + \dots}}}}}$$

and here are the *convergents* (see, e.g., [Khinchine 1963]) corresponding to the first five sections of π :

$$[3;] = 3; \quad [3; 7] = 3 \frac{1}{7}; \quad [3; 7, 15] = 3 \frac{15}{106};$$

$$[3; 7, 15, 1] = 3 \frac{16}{113}; \quad [3; 7, 15, 1, 292] = 3 \frac{4687}{33102}$$

One immediately observes that, firstly, $\pi_{\text{Hebrew}} = [3; 7, 15]$, and, secondly, π_{Hebrew} is the *second* (after $\pi_1 = [3; 7, 15, 1]$) *best convergent* with a denominator under 30,000 ! Notice also that the preceding convergent, $[3; 7]=22/7$, was known to ancient Greeks.

4. It is worthwhile to mention here a remarkable fact, namely, that in the case of the verse **1 Kings 7:23** we have an *independent confirmation* of the above mentioned *written vs. reading* disparity.

Namely, it could be easily seen that the verse **2 Chronicles 4:2** of the Hebrew Bible repeats **1 Kings 7:23** almost verbatim [Holy Scriptures, p. 988]. Looking at the Hebrew text, one immediately observes that the Hebrew word translated as line is traditionally *spelled* (written) here **identically** to its *reading* version. Thus, even if somebody would rebuff as irrelevant the

problem of interpretation of the disparity *written* vs. *reading* version of the word line in **1 Kings 7:23** (because he does not trust the *oral* tradition of transmission of Biblical texts), he would still have to explain the disparity between two different *written* versions of the same word (with only one version being grammatically correct) in two almost identical verses of the Bible! This last disparity is chosen as the point of departure for the Rabbinical exegesis in [Posamentier, Gordan 1984].

One could ask, why would be this important hint to the enhanced value of π omitted from the Books of Chronicles? An answer might be that the Books of Chronicles were written more than four hundred years after the Books of Kings, and the author of the Chronicles (traditionally identified with the Scribe Ezra) was much more preoccupied with rebuilding the Temple and preserving the spirit of the Torah, than with the “correct” value of π hidden in the descriptions of dimensions of the sacred objects in the First Temple; still, Ezra has faithfully reproduced these dimensions in his book.

A methodological remark: whereas the exegesis based on comparison of *written-vs.-reading* versions of a verse is a very general method in the Rabbinical tradition [Munk 1962, 1968], [Banon 1987], the above exegesis exploits a more rare event: the existence of two almost identical verses.

5. The following question arising from the above analysis has to be, at least briefly, touched upon: if the author of the first Book of Kings (traditionally identified with Prophet Jeremiaia) *actually knew* the value π_{Hebrew} and *intentionally exploited* the aforementioned *written-vs.-reading* disparity to encode it, why couldn't he simply write this value down in his text?

The answer might be that the value $\pi_0 = 3$, implicitly given in the text, plays an important rôle as an approximation which was regarded (and *is still regarded*) as best suited for all ritual purposes in the everyday life of a common practitioner (possibly, mathematically illiterate) of the Jewish law. Thus, our verse serves, in fact, (and so, we conjecture, *was it conceived* by its author) as the [only] *textual basis* for the following *legal* definition of π : “Any [circle] which has a circumference of three fists has a diameter of one fist” [Mishnah 1983, p. 23] (this important dictum is encountered in at least four different places of the Babylonian Talmud [Max Munk 1962, 1968]).

Still, all legal texts thoroughly investigate the problem [Max Munk 1962, 1968], [Scherman 1980], [Mishnah 1983, p. 22] and confirm that the *real* value of π is “slightly bigger” than 3, with some commentators advancing an almost modern point of view on irrational nature of π (the irrationality of

π was strictly proved only in the late eighteenth century); thus, *Rambam*¹¹ comments: “. . . the [exact] ratio of the diameter of a circle to its circumference cannot be known [is irrational]. . . but it is possible to approximate it. . . and the approximation used by scientists [Greeks and Arabs] is the ratio of one to three and one seventh. . . Since it is impossible to arrive at a perfectly accurate ratio, . . . they [the Jewish Sages] assumed a round number and said: ‘Any [circle] which has a circumference of three fists has a diameter of one fist’. And they relied on this for all the measurements they needed” [Mishnah 1983, p. 22].

It should be stressed that the purposed interpretation of the two-level semantical structure of a Biblical verse (in our case, **1 Kings 7:23**), one level for legal purposes, and another one for “connoisseurs”, is not only a typical phenomenon in the Rabbinical tradition, - in a sense, such a multi-level approach to texts is the main methodological legacy of this tradition [Steinsaltz 1976, Part Three: *Method*], [Banon 87]. As Rabbi Moshe ben Nachman¹² writes: “Everything that was transmitted to Moses our teacher through the forty-nine gates of understanding was written in the Torah explicitly or by implication in words, in the numerical value of the letters or in the form of the letters, that is, whether written normally or with some change in form, such as bent or crooked letters, and other deviations. . .” [Ramban 1971, Vol.1, p. 10].

Of course such an approach makes sense only if applied to texts which are faithfully transmitted from generation to generation; in fact, Judaism possesses elaborated institutions for such a transmission¹³. In this sense, it is (and always was) similar to modern science, with its elaborated institutions of training and supporting professionals, whose duty is to discover, accumulate, and transmit knowledge.

6. With all this understanding, gained thus far, we are, as yet, unable to elucidate the way the exegesis of the verse **1 Kings 7:23** has come to us: was it rediscovered by Rabbi Matityahu Hakohen Munk on his own [Max

¹¹A Rabbinical authority, codifier, philosopher, and royal physician, Rabbi Moshe ben Maimon (1135-1204), known by his acronym, *RAMBAM*, and as *Maimonides*, was one of the most illustrious figures in Judaism of all time.

¹²A Rabbinical authority, codifier, philosopher, physician, and poet; born in 1195, died circa 1270; known by his acronym, *RAMBAN*, and as *Nachmanides*

¹³A historian comments: *Josephus, writing not long after 70 CE boasts of the existence of a longstanding fixed text of the Jewish Scriptures*” [Britannica 1985, vol.14, p. 760].

Munk 1962, 1968], or was it transmitted to him? Is there another source in the Rabbinical literature for the exegesis?

A formidable *a priori* difficulty in answering these and similar questions is related to unpleasant two-thousands years old legacy of Judaism: as a religion, it invariably remained during this period an *underdog*, prone to persecutions and derision. This external pressure, together with related to its scarcity of social resources, explain why Rabbis have strictly separated legal matters (as, e.g., the legal definition of π_0) from “esoteric” knowledge available to them (our exegesis possibly included). In fact, it would be a nightmare scenario for *Rambam*, or any other Jewish scholar who lived two hundred years ago, or more, to advance a better approximation of π , *without being able* (as we now are) *to confirm this value scientifically*.

This fundamental difficulty still remains the main obstacle to scientific “customization” of the vast body of esoteric knowledge accumulated, commented upon, and faithfully transmitted by Jewish scholars. The author hopes to be able to contribute more to our better understanding of this precious intellectual and spiritual heritage.

Acknowledgements. Any acknowledgements would be both incomplete and difficult to appreciate without some rather personal remarks about the history of the writing of the present paper.

The author has acquired the knowledge of the Rabbinical exegesis of the verse **1 Kings 7:23** from Rabbi Haim Roth, of *Mevasseret Yerushalaim*, eleven years ago (the winter of 1979-1980); since then, several scholars in Talmudic studies have confirmed the existence of the exegesis, however, no sources for it were ever mentioned.

The author decided to publicize the exegesis, in the fall of 1990, after he stumbled upon two recent papers in *The American Mathematical Monthly* (written for a wide mathematical audience and devoted to new methods of computation of π), which claimed, in a matter-of-fact manner, that “*the ancient Hebrew regarded π as being equal to \mathcal{P}* ”, - citing, of course, the verse **1 Kings 7:23 !**

The first draft of the paper appeared in October 1990, with a very gratifying response from both the Talmudic and scientific communities. The comments of Rabbi Naftali Gut, of *Zürich*, were most inspiring. Rabbi Dr. Henri Biberfeld, Rabbis Daniel Mund and Arye Posen, of *Montréal*, suggested several important Talmudic and Halachic sources. Rabbi Dr. Nachum L. Ra-

binovich, of *Maaleh Adumim*, read the paper and suggested an important correction. Discussions with Prof. Louis Charbonneau, of *Montréal*, and his colleagues were helpful in adjusting the presentation to tastes of practitioners of history of mathematics; the references [Feldman 1965], [Hoyrup 1989] belong to Prof. Charbonneau. Later, he introduced the author to Prof. Roger Herz-Fischler, of *Carleton*, to whom belongs the reference [Zuidhof 1982]. Monsieur Luc Gagnon, the student of Prof. Jacques Lefebvres, *Montréal*, supplied the reference [Posamentier, Gordan 1984]. Several manifestations of utmost disbelief (in few cases, bordering on ridiculous¹⁴), on the part of colleagues with, apparently, no previous exposure to Jewish studies, helped the author to contain excitement and avoid self-congratulations.

Finally, and miraculously, Prof. Edward Reingold, of *Urbana*, whose enthusiasm for the subject was most encouraging, introduced the author to Rabbi Dr. Zeharia Dor-Shav, of *Bar-Ilan*, who, by sheer coincidence, has just become aware about the existence of an exegesis and started to look for its source. In a week or so, the crucial references [Max Munk 1962, 1968] were found and transmitted to the author, - and all this has happened in the last week of April 1991, after eleven years of unsuccessful search for such a source! After hearing about the author's difficulties to locate the (Hebrew) references in *Montréal*, Prof. Reingold has found the articles in *Urbana* and sent the copies to the author.

Still, with all the aforementioned interest and encouragement, the risky endeavor to bridge the gap between the Rabbinical tradition and modern history of science would be impossible without the steadfastness and support of the author's family.

¹⁴As an anonymous reviewer has written on the third draft of the present paper (which went in all through a dozen of drafts), "*Il n'aurait pas à adhérer à un acte de foi, comme celui décrit en p.2 ni comme en p.3-4: '(...) Ezra has faithfully reproduced these dimensions in his book'*". The present author does not remember now what exactly has the reviewer referred to on the page 2 (nor was it clear to the author immediately after he has received the reviewer's text), but the author's statement about the "*faithfulness of Ezra*" has survived all changes (see the end of 4), to testify that no "*act of faith*" is needed to compare two verses and to conclude that the second one is a faithful copy of the first one.

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Note: The Rabbinical literature on the subject which are dealt with (or only briefly mentioned) in this paper is enormous. However, the present author has intentionally restricted his choice to such English (and, in three cases, French) references which are widely available in modern libraries. The only (and, unfortunately, unavoidable) exceptions are the original papers of Rabbi Max Munk, written in Hebrew and never translated in any of Western languages.

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